Two level production inventory model for a deteriorating item with preservation technology using time varying demand under permissible delay in payment

Dr. Debasis Das

Department of Mathematics, Ramananda Centenary College, Laulara, Purulia, W.B, India, Pin-723151, Email: debasis_opt@yahoo.co.in

Received: 05.03.2024 ***Revised: 03.04.2024*** ***Accepted: 07.04.2024***

Abstract

This paper presents a two level production inventory model for a deteriorating item with preservation strategy under permissible delay in payment in an infinite time horizon. Here two different rates of productions are considered, initially production started at one rate and after some time it may be switched over another rate such a situation is desirable in the sense that by starting at a low rate of production, a large quantum stock of manufacturing items at the initial stage is avoided, leading to reduction in the holding cost. The demand rate is assumed as time varying demand and the stock itself is depleted due to demand and deterioration. The supplier offers the retailer fully permissible delay in payment. Here M be the period of permissible delay in settling account without extra charges. But if the retailer settles the account after M, he/she will have to pay with interest for the inventory not sold after the due date M. The model has been formulated as profit maximization problem with respect to the retailer and is solved numerically through modified genetic algorithm (MGA). Finally, the model is illustrated through a numerical ex- ample, and sensitivity analyses have been done with the variation of demand and deterioration parameters on optimal profit.

Key words: *Two level production; Inventory; Time-varying demand; Permissible delay in payment; Deteriorating items; Preservation strategy; Modified genetic algorithm.*

1 Introduction

The production base economic quantity model deals with different parameters such as production rate, demand rate, shortages, deterioration etc. In economic production quantity model, the rate of production is so vital. In the present competitive market, the production house, owner/retailer /supplier influence the customers in many different ways to capture the market. Goyal and Gunasekaran(1995) have developed an integrated production-inventory-marketing model involving deteriorating items for a multi- stage economic production lot-size and economic order quantity system. Sivashankari and Panayappan (2015) developed a production inventory model for two level production with deteriorative items and shortages. Also Kumar et al. (2018) developed a two level production inventory model with exponential demand and time dependent deterioration rate. Das et al. (2010) presented an EPQ model with stock-dependent demand rate under inventory control. Patra and Maity (2017) developed a production inventory model in which variable production rate, deterioration and two type of demand rates for defective and non-defective items are considered.

In production base economic quantity inventory management, the system deals with demand and supply chain, i.e; the business is totally depending on demand and supply of goods. So demand is one of the important parameter. Donaldson (1977) develops an inventory policy for the case of a linear trend in demand over a finite time horizon. After that a lot of research work has been done by Mitra et al. (1984), Chung and Ting (1993), Gupta and Agarwal (2000) etc. incorporating a time-varying demand under a variety of circumstances. But none of them considered inventory model with time-varying demand under permissible delay in payment. Das et al. (2011) develops an inventory policy for a retailer under permitted delay in payment by the wholesaler with time dependent demand over a finite time horizon.

The inventory model under permissible delay in payments is among the extensions found in the literature. Goyal (1985) develops an inventory model under the condition of permissible delay in payments. Later, Aggarwal and Jaggi (1995) extend the Goyal (1985) model to consider an inventory model of deteriorating items with permissible delay in payments. A group of researchers have considered inventory control systems with permissible delay in payments in their research such as Chung and Huang (2003), Ouyand et al. (2005), Huang (2007), Das (2019) etc. Das et al. (2015) has developed multi- warehouse inventory system for deteriorating item under inflation when delay in payment is permissible.

Deterioration is one of the factors of decrease in inventory level and profit. In general, deterioration is defined as spoilage, decay, damage, loss of utility, obsolescence etc. like fruits, fertilizer, food items, medicine. Some items have low rate of deterioration such as hardware, toys, glassware etc. Therefore, rate of deterioration is also different for different items. Ghare and Schrader (1963) first focus on the effect of decay in the inventory analysis. Singh and Pattnayak (2013) discuss on the deterioration of inventory with variable deterioration and partial backlogging. Das et al. (2012) presented EPQ models with deteriorating item with finite and random product life cycle. To reduce the deterioration rate, the retailer use the preservation technology. Hsu et al. (2010) invented a deteriorating model using the preservation technology. Again Dye and Hsieh (2012) extended the model of Hsu et al. by assuming the preservation technology cost, which is a function of the length of the replenishment cycle. Dye (2013), Das and Jana (2019), Das (2019), Das et al. (2020), Das (2023) focus on the effect of preservation technology on a non-instantaneous deteriorating inventory.

In the present study, I consider two level production system with time varying demand and permissible delay in payment using preservation technology having constant deterioration rate under infinite time horizon. Here both the production rates are constant, demand is the function of time which is increasing exponentially. The objective of this model is to find the optimal cycle length to maximize the total average profit. Here modified genetic algorithm (MGA) is used to solve the model. The theoretical results are illustrated using a numerical example and sensitivity analyses on some parameters.

Table-1: Summary of related literature for inventory models with infinite time horizon

Authors	One/Two level, EPQ/EOQ	Time-dependent demand	Preservation technology	Partial/full backlogging	Delay in payment
Das et al. (2010)	One, EPQ	No	No	No	Yes
Das et al. (2011)	One, EOQ	Yes	No	No	Yes
Dye and Hsieh(2012)	One, EOQ	No	Yes	Yes	No
Das et al. (2015)	One, EOQ	No	No	Yes	Yes
Sivashankari and Panayappan, (2015)	Two, EPQ	No	No	Yes	No
Kumar et al. (2018)	Two, EPQ	Yes	No	No	No
Das (2019)	One, EPQ	No	Yes	No	Yes
Das et al. (2020)	One, EOQ	No	Yes	Yes	No
Das (2023)	One, EPQ	No	Yes	Yes	No
Present paper	Two, EPQ	Yes	Yes	No	Yes

2 Assumptions and Notations

Two level production inventory model under permissible delay in payment is developed on the basis of following assumptions and notations:

2.1 Assumptions

The following assumptions are made:

(i) Inventory system involves two level production system.

(ii) The time horizon is infinite.

(iii) Shortages are not allowed.

(iv) Lead time is zero.

(v) Demand is time dependent.

(vi) The constant fraction of on-hand inventory gets deteriorated per unit time.

(vii) There is no quantity discount.

(viii) Set-up time is negligible.

(ix) Inspection cost is negligible.

(x) Two rates of production are considered.

(xi) The production rate is always greater than or equal to sum of the demand rate and deterioration rate.

2.2 Notations

The following notations are introduced:

(i) C_3 =Ordering cost per order

(ii) C_1 =The unit holding cost per unit time excluding interest charges.

(iii) t_1 =The production time period of first level.

(iv) t_2 =The production time period of second level.

(v) P = The production rate per unit time for the time period $[0, t_1]$.

(vi) λP = The production rate per unit time for the time period $[t_1, t_2]$, where λ is a constant and > 0.

(vii) W =On-hand inventory level at time t_1.

(viii) C_p = Production cost per unit time.

(ix) $D(t)$= Demand rate function varying with time t and is of the form:

$$D(t) = \alpha.e^{\beta.t}$$

where α and β are both constants such that $\alpha > 0$ and $\beta > 0$.

(x) $q(t)$ = On-hand inventory of the item at time $t \geq 0$.

(xi) s = Selling price per unit, $s > C_p$.

(xii) M = The period of permissible delay in settling account without extra charges.

(xiii) I_p = The interest paid per dollar in stocks per unit time by the supplier when the retailer pays after M.

(xiv) I_e = The interest earned per dollar per unit time.

(xv) θ = The constant deterioration rate.

(xvi) ξ = The cost of preservation technology investment per unit time for reducing deterioration rate in order to preserve the products. According to our assumption the corresponding reduced deterioration rate is $\theta.m(\xi)$ per unit time when there is an investment on preservation technology, where $m(\xi) = e^{-a.\xi}, a > 0$.

(xvii) T = Duration of complete cycle where inventory level vanishes.

(xviii) TAP = Total average profit.

3 Mathematical Formulation

The behavior of inventory level in the two level production system time varying demand under permissible delay in payment is depicted in Fig.-1. Here the initial production starts at time $t = 0$ and continue up to $t = t_1$ and during the time interval $[0, t_1]$ the production rate is P and the demand rate is $\alpha.e^{\beta t}$. At time $t = t_1$, let W is the maximum inventory for first level. During the time interval $[t_1, t_2]$ the production rate is $\lambda.P$ and demand rate is also $\alpha.e^{\beta t}$. where $\lambda > 0$ is constant. At time $t = t_2$, the production stops and the inventory reaches maximum level for second level of production. After $t = t_2$, the inventory gradually depletes to zero at the end of the production cycle $t = T$ due to demand and deterioration.

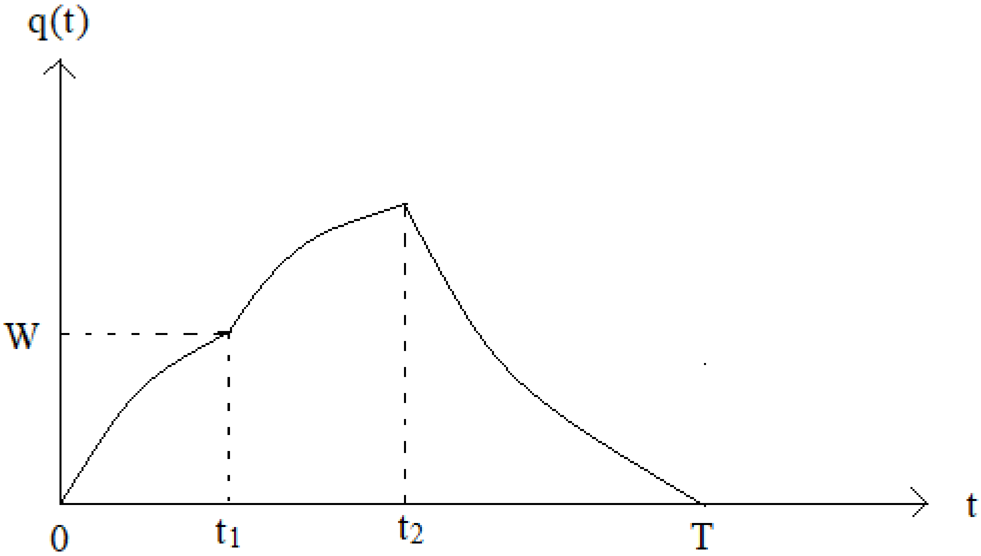

Fig.1. Graphical representation of a two level production system with time-dependent demand

The change of inventory level with respect to time can be described by the following differential equations:

$$\frac{dq(t)}{dt} + \theta.m(\xi).q(t) = \begin{cases} P - \alpha e^{\beta t}, & 0 \le t \le t_1 \\ \lambda P - \alpha e^{\beta t}, & t_1 \le t \le t_2 \\ -\alpha e^{\beta t}, & t_2 \le t \le T \end{cases} \quad (1)$$

with the boundary conditions $q(0) = 0$, $q(t_1) = W$, $q(T) = 0$.

Then the solutions of the differential equations in (1) are represented by

$$q(t) = \begin{cases} \frac{P}{\theta m(\xi)}\left\{1 - e^{\theta m(\xi)t}\right\} + \frac{\alpha}{\beta + \theta m(\xi)}\left\{e^{-\theta m(\xi)t} - e^{\beta t}\right\}, & 0 \le t \le t_1 \\ We^{-\theta m(\xi)(t-t_1)} + \frac{\lambda P}{\theta m(\xi)}\left\{1 - e^{-\theta m(\xi)(t-t_1)}\right\} + \frac{\alpha}{\beta + \theta m(\xi)}\left\{e^{\beta t_1 - \theta m(\xi)(t-t_1)} - e^{\beta t}\right\}, & t_1 \le t \le t_2 \\ \frac{\alpha}{\beta + \theta m(\xi)}\left\{e^{(\beta+\theta m(\xi))T - \theta m(\xi)t} - e^{\beta t}\right\}, & t_2 \le t \le t_2 \end{cases} \quad (2)$$

In addition to the continuity conditions at $t = t_1$ and $t = t_2$, I can derive the following:

$$W = \frac{P}{\theta m(\xi)}\left\{1 - e^{\theta m(\xi)t_1}\right\} + \frac{\alpha}{\beta+\theta m(\xi)}\left\{e^{-\theta m(\xi)t_1} - e^{\beta t_1}\right\} \quad (3)$$

and

$$T = \frac{1}{\beta+\theta m(\xi)} . ln\left[\frac{W(\beta+\theta m(\xi))}{\alpha}e^{\theta m(\xi)t_1} + \frac{\lambda P(\beta+\theta m(\xi))}{\alpha\theta m(\xi)}\left\{e^{\theta m(\xi)t_2} - e^{\theta m(\xi)t_1}\right\} + e^{(\beta+\theta m(\xi))t_1}\right] \quad (4)$$

Now the total holding cost (C_H) during the period (0, T)is given by,

$$\begin{aligned} C_H &= C_1\left[\int_0^{t_1} q(t)\,dt + \int_{t_1}^{t_2} q(t)\,dt + \int_{t_2}^{T} q(t)\,dt\right] \\ &= C_1\Big[\frac{P}{\theta^2 m^2(\xi)}\left(\theta m(\xi)t_1 + e^{-\theta m(\xi)t_1} - 1\right) - \frac{\alpha}{\beta+\theta m(\xi)}\Big\{\frac{1}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_1} - 1\right) \\ &+ \frac{1}{\beta}\left(e^{\beta t_1} - 1\right)\Big\} - \frac{We^{\theta m(\xi)t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2} - e^{-\theta m(\xi)t_1}\right) + \frac{\lambda P}{\theta m(\xi)}\Big\{(t_2 - t_1) \\ &+ \frac{e^{\theta m(\xi)t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2} - e^{-\theta m(\xi)t_1}\right)\Big\} - \frac{\alpha}{\beta+\theta m(\xi)}\Big\{\frac{1}{\beta}\left(e^{\beta t_2} - e^{\beta t_1}\right) \\ &+ \frac{e^{(\beta+\theta m(\xi))t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2} - e^{-\theta m(\xi)t_1}\right)\Big\} - \frac{\alpha}{\beta+\theta m(\xi)}\Big\{\frac{1}{\beta}\left(e^{\beta T} - e^{\beta t_2}\right) \\ &+ \frac{e^{(\beta+\theta m(\xi))T}}{\theta m(\xi)}\left(e^{-\theta m(\xi)T} - e^{-\theta m(\xi)t_2}\right)\Big\}\Big] \end{aligned} \quad (5)$$

Total amount of produced item (PI) is given by

$$\begin{aligned} PI &= \int_0^{t_1} P\,dt + \int_{t_1}^{t_2} \lambda P\,dt \\ &= P[t_1 + \lambda(t_2 - t_1)] \end{aligned} \quad (6)$$

Total production cost (PC) is given by

$$PC = C_P[t_1 + \lambda(t_2 - t_1)]P \quad (7)$$

The amount of inventory items deteriorated during the time period (0, T) is given by

$$D_T = \theta m(\xi)\left[\int_0^{t_1} q(t)dt + \int_{t_1}^{t_2} q(t)dt + \int_{t_2}^{T} q(t)dt\right]$$
$$= \theta m(\xi)\Big[\frac{P}{\theta^2 m^2(\xi)}\left(\theta m(\xi)t_1 + e^{-\theta m(\xi)t_1} - 1\right) - \frac{\alpha}{\beta+\theta m(\xi)}\Big\{\frac{1}{\theta m(\xi)}(e^{-\theta m(\xi)t_1} - 1)$$
$$+\frac{1}{\beta}\left(e^{\beta t_1} - 1\right)\Big\} - \frac{We^{\theta m(\xi)t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2} - e^{-\theta m(\xi)t_1}\right) + \frac{\lambda P}{\theta m(\xi)}\{(t_2 - t_1)$$
$$+\frac{e^{\theta m(\xi)t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2} - e^{-\theta m(\xi)t_1}\right)\} - \frac{\alpha}{\beta+\theta m(\xi)}\Big\{\frac{1}{\beta}\left(e^{\beta t_2} - e^{\beta t_1}\right)$$
$$+\frac{e^{(\beta+\theta m(\xi))t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2} - e^{-\theta m(\xi)t_1}\right)\Big\} - \frac{\alpha}{\beta+\theta m(\xi)}\Big\{\frac{1}{\beta}\left(e^{\beta T} - e^{\beta t_2}\right)$$
$$+\frac{e^{(\beta+\theta m(\xi))T}}{\theta m(\xi)}\left(e^{-\theta m(\xi)T} - e^{-\theta m(\xi)t_2}\right)\Big\}\Big] \quad (8)$$

Total cost for preservation technology C_{pr} is given by

$$C_{pr} = T.\xi \quad (9)$$

Total selling price S_T is given by

$$S_T = s(PI - D_T) \quad (10)$$

Let M be the period of permissible delay in settling account without extra charges. Hence the interest payable per cycle and the interest earned per cycle for four cases be given as follows:

Case − I. $0 < M \le t_1$, **Case − II.** $t_1 \le M \le t_2$, **Case − III.** $t_2 \le M \le T$, **Case − IV.** $T \le M$.

We discuss each case in detail as follows:

Case −I. $0 < M \le t_1$

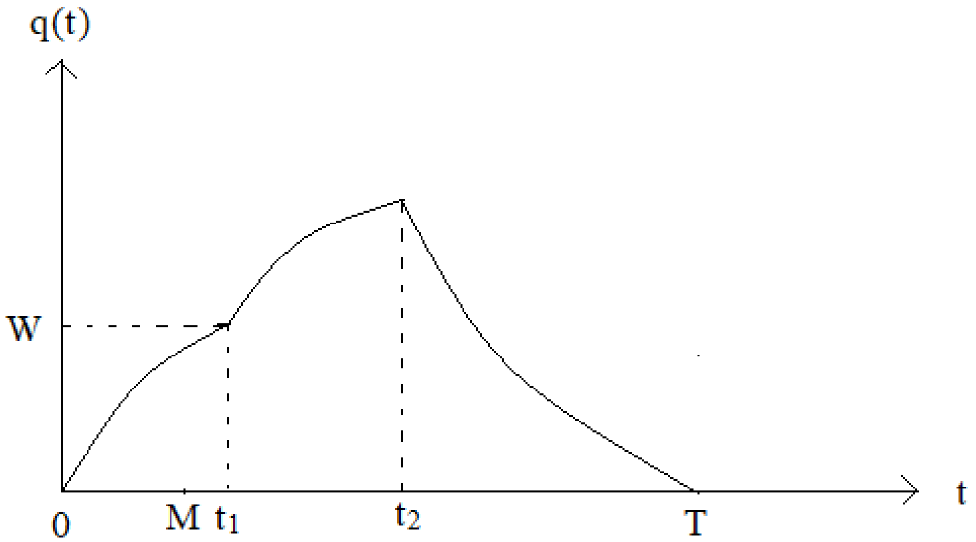

Fig.-2. Graphical representation of a two level production system with time-dependent demand for Case- I

Therefore the interest payable per cycle for the inventory not sold after the due date M is given by (Fig. 2)

$$P_T = C_p I_p \int_M^T q(t)dt$$

$$= C_p I_p \left[\int_M^{t_1} q(t)dt + \int_{t_1}^{t_2} q(t)dt + \int_{t_2}^{T} q(t)dt\right]$$

$$= C_p I_p \left[\frac{P}{\theta^2 m^2(\xi)}\left\{\theta m(\xi)(t_1 - M) + \left(e^{-\theta m(\xi)t_1} - e^{-\theta m(\xi)M}\right)\right\}\right.$$

$$-\frac{\alpha}{\beta + \theta m(\xi)}\left\{\frac{1}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_1} - e^{-\theta m(\xi)M}\right) + \frac{1}{\beta}\left(e^{\beta t_1} - e^{\beta M}\right)\right\}$$

$$-\frac{W e^{\theta m(\xi)t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2} - e^{-\theta m(\xi)t_1}\right) + \frac{\lambda P}{\theta m(\xi)}\{(t_2 - t_1)$$

$$+\frac{e^{\theta m(\xi)t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2} - e^{-\theta m(\xi)t_1}\right)\} - \frac{\alpha}{\beta + \theta m(\xi)}\left\{\frac{1}{\beta}\left(e^{\beta t_2} - e^{\beta t_1}\right)\right.$$

$$+\frac{e^{(\beta+\theta m(\xi))t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2} - e^{-\theta m(\xi)t_1}\right)\} - \frac{\alpha}{\beta + \theta m(\xi)}\left\{\frac{1}{\beta}\left(e^{\beta T} - e^{\beta t_2}\right)\right.$$

$$\left.+\frac{e^{(\beta+\theta m(\xi))T}}{\theta m(\xi)}\left(e^{-\theta m(\xi)T} - e^{-\theta m(\xi)t_2}\right)\}\right] \quad (11)$$

Interest earned per cycle I_T is given by

$$I_T = s.I_e \int_0^T D(t).t\,dt$$

$$= \frac{s\alpha I_e}{\beta^2}\left[e^{\beta T}(\beta T - 1) + 1\right] \quad (12)$$

Case − II. $t_1 \le M \le t_2$

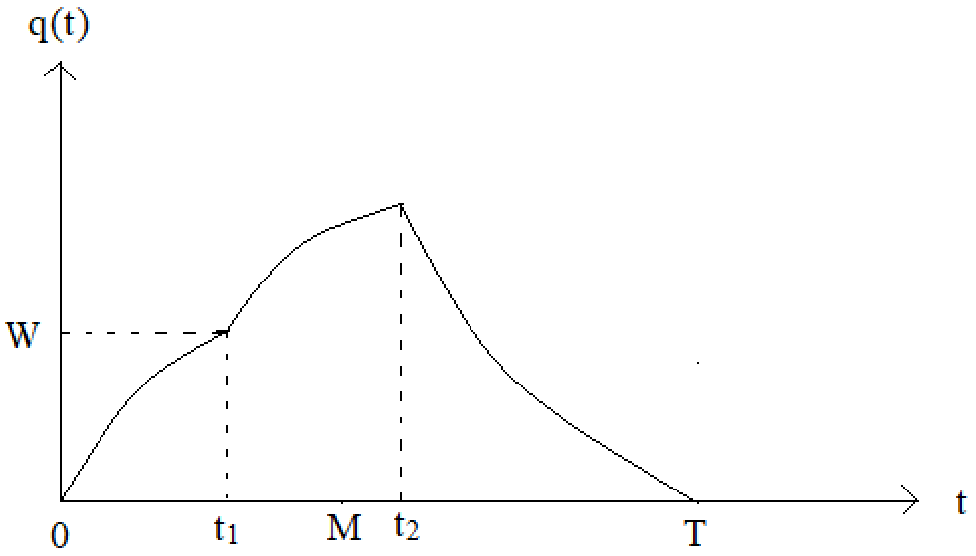

Fig.-3. Graphical representation of a two level production system with time-dependent demand for Case- II

Therefore the interest payable per cycle for the inventory not sold after the due date M is given by (Fig. 3)

$$P_T = C_p I_p \int_M^T q(t)dt$$

$$= C_p I_p \left[\int_M^{t_2} q(t)dt + \int_{t_2}^{T} q(t)dt\right]$$

$$= C_p I_p \left[\frac{We^{\theta m(\xi)t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)M} - e^{-\theta m(\xi)t_2}\right) + \frac{\lambda P}{\theta m(\xi)}\{(t_2 - M)\right.$$

$$+ \frac{e^{\theta m(\xi)t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2} - e^{-\theta m(\xi)M}\right)\} - \frac{\alpha}{\beta + \theta m(\xi)}\left\{\frac{1}{\beta}\left(e^{\beta t_2} - e^{\beta M}\right)\right.$$

$$+ \frac{e^{(\beta+\theta m(\xi))t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2} - e^{-\theta m(\xi)M}\right)\} - \frac{\alpha}{\beta + \theta m(\xi)}\left\{\frac{1}{\beta}\left(e^{\beta T} - e^{\beta t_2}\right)\right.$$

$$\left. + \frac{e^{(\beta+\theta m(\xi))T}}{\theta m(\xi)}\left(e^{-\theta m(\xi)T} - e^{-\theta m(\xi)t_2}\right)\}\right] \tag{13}$$

Interest earned per cycle I_T is given by

$$I_T = s.I_e \int_0^T D(t).t\,dt$$

$$= \frac{s\alpha I_e}{\beta^2}\left[e^{\beta T}(\beta T - 1) + 1\right] \tag{14}$$

Case − III. $t_2 \leq M \leq T$

Therefore the interest payable per cycle for the inventory not sold after the due date *M* is given by (Fig. 4)

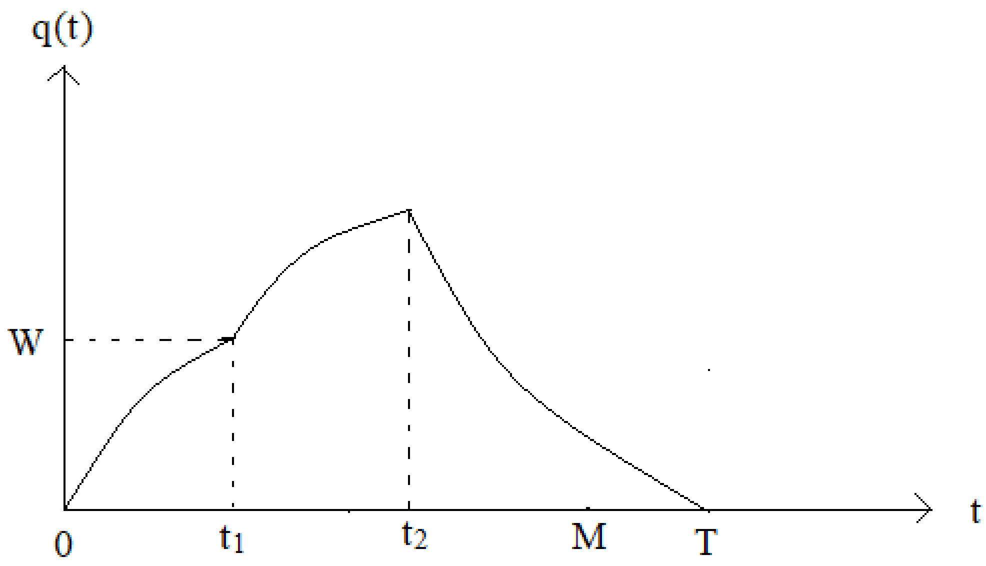

Fig.-4. Graphical representation of a two level production system with time-dependent demand for Case- III

Therefore the interest payable per cycle for the inventory not sold after the due date *M* is given by (Fig. 4)

$$P_T = C_p I_p \int_M^T q(t)dt$$

$$= C_p I_p \left[\frac{\alpha}{\beta+\theta m(\xi)}\left\{\frac{1}{\beta}\left(e^{\beta M} - e^{\beta T}\right) + \frac{e^{(\beta+\theta m(\xi))T}}{\theta m(\xi)}\left(e^{-\theta m(\xi)M} - e^{-\theta m(\xi)T}\right)\right\}\right] \quad (15)$$

Interest earned per cycle I_T is given by

$$I_T = s.I_e \int_0^T D(t).t\,dt$$

$$= \frac{s\alpha I_e}{\beta^2}\left[e^{\beta T}(\beta T - 1) + 1\right] \quad (16)$$

Case − IV. $T \leq M$

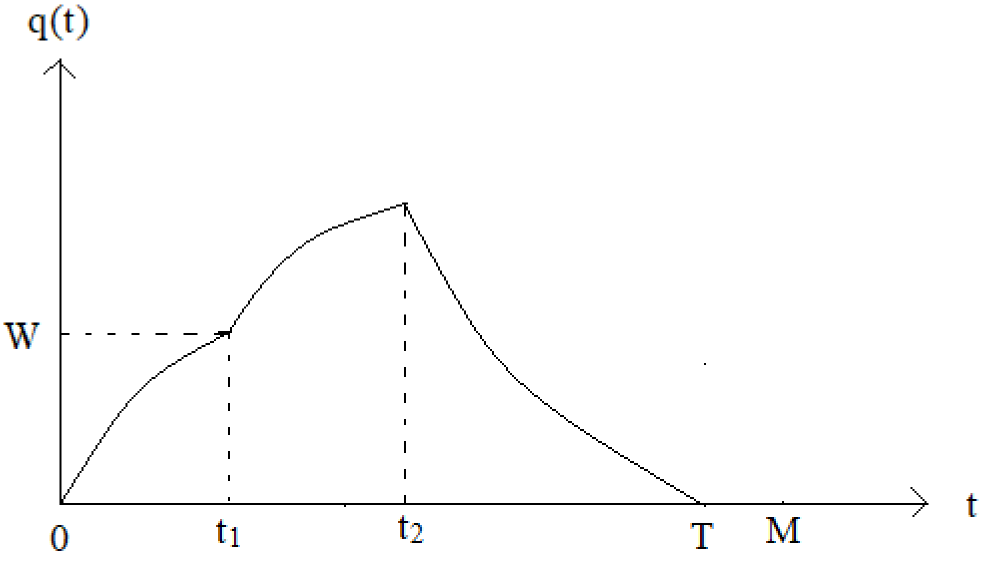

Fig.-5. Graphical representation of a two level production system with time-dependent demand for Case- IV

In this case there is no interest payable. Interest earned per cycle I_T is given by (Fig. 5)

$$I_T = s.I_e\left[\int_0^T D(t).t\,dt + (M-T)\int_0^T D(t)\,dt\right]$$

$$= \frac{s\alpha I_e}{\beta^2}\left[\left\{e^{\beta T}(\beta T - 1) + 1\right\} + (M-T)\beta\left(e^{\beta T} - 1\right)\right] \qquad (17)$$

Therefore, the total relevant cost for each case is obtained as follows:

$$[TC]_k \quad = C_3 + C_H + PC + C_{pr} + P_T - I_T \qquad (18)$$

where $k = I,\ II,\ III,\ IV$.

Then, the total relevant cost $[TC]_I$ for case-I is given by,

$$[TC]_I = C_3 + C_H + PC + C_{pr} + P_T - I_T$$

$$= C_3 + C_1\Big[\frac{P}{\theta^2 m^2(\xi)}\left(\theta m(\xi)t_1 + e^{-\theta m(\xi)t_1} - 1\right) - \frac{\alpha}{\beta+\theta m(\xi)}\Big\{\frac{1}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_1} - 1\right)$$
$$+\frac{1}{\beta}\left(e^{\beta t_1} - 1\right)\Big\} - \frac{We^{\theta m(\xi)t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2} - e^{-\theta m(\xi)t_1}\right) + \frac{\lambda P}{\theta m(\xi)}\{(t_2 - t_1)$$
$$+\frac{e^{\theta m(\xi)t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2} - e^{-\theta m(\xi)t_1}\right)\} - \frac{\alpha}{\beta+\theta m(\xi)}\Big\{\frac{1}{\beta}\left(e^{\beta t_2} - e^{\beta t_1}\right)$$
$$+\frac{e^{(\beta+\theta m(\xi))t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2} - e^{-\theta m(\xi)t_1}\right)\} - \frac{\alpha}{\beta+\theta m(\xi)}\Big\{\frac{1}{\beta}\left(e^{\beta T} - e^{\beta t_2}\right)$$

$$+\frac{e^{(\beta+\theta m(\xi))T}}{\theta m(\xi)}\left(e^{-\theta m(\xi)T}-e^{-\theta m(\xi)t_2}\right)\}\Big]+C_P[t_1+\lambda(t_2-t_1)]P+T.\xi$$

$$+C_pI_p\Bigg[\frac{P}{\theta^2m^2(\xi)}\left\{\theta m(\xi)(t_1-M)+\left(e^{-\theta m(\xi)t_1}-e^{-\theta m(\xi)M}\right)\right\}$$

$$-\frac{\alpha}{\beta+\theta m(\xi)}\left\{\frac{1}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_1}-e^{-\theta m(\xi)M}\right)+\frac{1}{\beta}\left(e^{\beta t_1}-e^{\beta M}\right)\right\}$$

$$-\frac{We^{\theta m(\xi)t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2}-e^{-\theta m(\xi)t_1}\right)+\frac{\lambda P}{\theta m(\xi)}\{(t_2-t_1)$$

$$+\frac{e^{\theta m(\xi)t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2}-e^{-\theta m(\xi)t_1}\right)\}-\frac{\alpha}{\beta+\theta m(\xi)}\left\{\frac{1}{\beta}\left(e^{\beta t_2}-e^{\beta t_1}\right)\right.$$

$$+\frac{e^{(\beta+\theta m(\xi))t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2}-e^{-\theta m(\xi)t_1}\right)\Big\}-\frac{\alpha}{\beta+\theta m(\xi)}\left\{\frac{1}{\beta}\left(e^{\beta T}-e^{\beta t_2}\right)\right.$$

$$+\frac{e^{(\beta+\theta m(\xi))T}}{\theta m(\xi)}\left(e^{-\theta m(\xi)T}-e^{-\theta m(\xi)t_2}\right)\}\Big]-\frac{s\alpha I_e}{\beta^2}\left[e^{\beta T}(\beta T-1)+1\right] \quad (19)$$

Then, the total relevant cost $[TC]_{II}$ for case-II is given by,

$$[TC]_{II}=C_3+C_H+PC+C_{pr}+P_T-I_T$$

$$=C_3+C_1\Bigg[\frac{P}{\theta^2m^2(\xi)}\left(\theta m(\xi)t_1+e^{-\theta m(\xi)t_1}-1\right)-\frac{\alpha}{\beta+\theta m(\xi)}\left\{\frac{1}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_1}-1\right)\right.$$

$$\left.+\frac{1}{\beta}\left(e^{\beta t_1}-1\right)\right\}-\frac{We^{\theta m(\xi)t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2}-e^{-\theta m(\xi)t_1}\right)+\frac{\lambda P}{\theta m(\xi)}\{(t_2-t_1)$$

$$+\frac{e^{\theta m(\xi)t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2}-e^{-\theta m(\xi)t_1}\right)\}-\frac{\alpha}{\beta+\theta m(\xi)}\left\{\frac{1}{\beta}\left(e^{\beta t_2}-e^{\beta t_1}\right)\right.$$

$$+\frac{e^{(\beta+\theta m(\xi))t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2}-e^{-\theta m(\xi)t_1}\right)\Big\}-\frac{\alpha}{\beta+\theta m(\xi)}\left\{\frac{1}{\beta}\left(e^{\beta T}-e^{\beta t_2}\right)\right.$$

$$+\frac{e^{(\beta+\theta m(\xi))T}}{\theta m(\xi)}\left(e^{-\theta m(\xi)T}-e^{-\theta m(\xi)t_2}\right)\}\Big]+C_P[t_1+\lambda(t_2-t_1)]P+T.\xi$$

$$+C_pI_p\Bigg[\frac{We^{\theta m(\xi)t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)M}-e^{-\theta m(\xi)t_2}\right)+\frac{\lambda P}{\theta m(\xi)}\{(t_2-M)$$

$$+\frac{e^{\theta m(\xi)t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2}-e^{-\theta m(\xi)M}\right)\}-\frac{\alpha}{\beta+\theta m(\xi)}\left\{\frac{1}{\beta}\left(e^{\beta t_2}-e^{\beta M}\right)\right.$$

$$+\frac{e^{(\beta+\theta m(\xi))t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2}-e^{-\theta m(\xi)M}\right)\Big\}-\frac{\alpha}{\beta+\theta m(\xi)}\left\{\frac{1}{\beta}\left(e^{\beta T}-e^{\beta t_2}\right)\right.$$

$$+\frac{e^{(\beta+\theta m(\xi))T}}{\theta m(\xi)}\left(e^{-\theta m(\xi)T}-e^{-\theta m(\xi)t_2}\right)\}\Big]-\frac{s\alpha I_e}{\beta^2}\left[e^{\beta T}(\beta T-1)+1\right] \quad (20)$$

Then, the total relevant cost $[TC]_{III}$ for case-III is given by,

$$[TC]_{III}=C_3+C_H+PC+C_{pr}+P_T-I_T$$

$$=C_3+C_1\Bigg[\frac{P}{\theta^2m^2(\xi)}\left(\theta m(\xi)t_1+e^{-\theta m(\xi)t_1}-1\right)-\frac{\alpha}{\beta+\theta m(\xi)}\left\{\frac{1}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_1}-1\right)\right.$$

$$\left.+\frac{1}{\beta}\left(e^{\beta t_1}-1\right)\right\}-\frac{We^{\theta m(\xi)t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2}-e^{-\theta m(\xi)t_1}\right)+\frac{\lambda P}{\theta m(\xi)}\{(t_2-t_1)$$

$$+\frac{e^{\theta m(\xi)t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2}-e^{-\theta m(\xi)t_1}\right)\}-\frac{\alpha}{\beta+\theta m(\xi)}\left\{\frac{1}{\beta}\left(e^{\beta t_2}-e^{\beta t_1}\right)\right.$$

$$+\frac{e^{(\beta+\theta m(\xi))t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2}-e^{-\theta m(\xi)t_1}\right)\Big\}-\frac{\alpha}{\beta+\theta m(\xi)}\Big\{\frac{1}{\beta}\left(e^{\beta T}-e^{\beta t_2}\right)$$

$$+\frac{e^{(\beta+\theta m(\xi))T}}{\theta m(\xi)}\left(e^{-\theta m(\xi)T}-e^{-\theta m(\xi)t_2}\right)\Big\}\Big]+C_P[t_1+\lambda(t_2-t_1)]P+T.\xi$$

$$+C_pI_p\left[\frac{\alpha}{\beta+\theta m(\xi)}\left\{\frac{1}{\beta}\left(e^{\beta M}-e^{\beta T}\right)+\frac{e^{(\beta+\theta m(\xi))T}}{\theta m(\xi)}\left(e^{-\theta m(\xi)M}-e^{-\theta m(\xi)T}\right)\right\}\right]-\frac{s\alpha I_e}{\beta^2}\left[e^{\beta T}(\beta T-1)+1\right] \quad (21)$$

Then, the total relevant cost $[TC]_{IV}$ for case-IV is given by,

$$[TC]_{IV} = C_3 + C_H + PC + C_{pr} + P_T - I_T$$

$$= C_3+C_1\Big[\frac{P}{\theta^2m^2(\xi)}\left(\theta m(\xi)t_1+e^{-\theta m(\xi)t_1}-1\right)-\frac{\alpha}{\beta+\theta m(\xi)}\Big\{\frac{1}{\theta m(\xi)}(e^{-\theta m(\xi)t_1}-1)$$

$$+\frac{1}{\beta}\left(e^{\beta t_1}-1\right)\Big\}-\frac{We^{\theta m(\xi)t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2}-e^{-\theta m(\xi)t_1}\right)+\frac{\lambda P}{\theta m(\xi)}\{(t_2-t_1)$$

$$+\frac{e^{\theta m(\xi)t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2}-e^{-\theta m(\xi)t_1}\right)\}-\frac{\alpha}{\beta+\theta m(\xi)}\Big\{\frac{1}{\beta}\left(e^{\beta t_2}-e^{\beta t_1}\right)$$

$$+\frac{e^{(\beta+\theta m(\xi))t_1}}{\theta m(\xi)}\left(e^{-\theta m(\xi)t_2}-e^{-\theta m(\xi)t_1}\right)\Big\}-\frac{\alpha}{\beta+\theta m(\xi)}\Big\{\frac{1}{\beta}\left(e^{\beta T}-e^{\beta t_2}\right)$$

$$+\frac{e^{(\beta+\theta m(\xi))T}}{\theta m(\xi)}\left(e^{-\theta m(\xi)T}-e^{-\theta m(\xi)t_2}\right)\Big\}\Big]+C_P[t_1+\lambda(t_2-t_1)]P+T.\xi$$

$$-\frac{s\alpha I_e}{\beta^2}\left[\left\{e^{\beta T}(\beta T-1)+1\right\}+(M-T)\beta\left(e^{\beta T}-1\right)\right] \quad (22)$$

Therefore, the total average profit per unit time for each case is obtained as follows:

$$[TAP]_k=\frac{1}{T}[S_T-[TC]_k] \quad (23)$$

where $k = I,\ II,\ III,\ IV$.

So, the above problem can be formulated as,

$$Maximize \quad [TAP]_k, \quad k = I,\ II,\ III,\ IV.$$

4 Solution Procedure

Here we propose a nobility in crossover operator of the Modified Genetic Algorithm (MGA) using probabilistic selection (Boltzmann Probability), IVF comparison crossover and sigmoid random mutation among a set of potential solutions to obtain a new set of solutions. The proposed algorithm is continued until terminating conditions are encountered. The proposed modified GA and its procedures are presented further.

4.1 Probabilistic Selection

In the present study, we proposed a predefined value, for example probability of selection parameter (p_s). Each solution randomly generates a number $r \in [0, 1]$. If $r < p_s$, the corresponding chromosome is stored to form the matting pool. To maximize the profit, selecting a chromosome in the neighborhood of the maximum solution of the entire solution space, so it propagates a higher convergences rate. From the initial population to the best fitted chromosome for EPQ model is chosen as the most maximum fitness (because EPQ model is a maximizing model) value. To form the matting pool, we use the Boltzmann-Probability of each chromosome from the initial population.

4.2 IVF Comparison Crossover

In IVF cases, in additional to the original parents (father and mother), a surrogate mother actively participates in the process of production a child. A new approach with three parents (first two are original parents and the third one say dummy parent) has been used to produce offspring. In the proposed crossover strategy, three parents (i.e., chromosomes) are randomly selected from the matting pool. Further, comparing the profits from each chromosome. On the basis of the above idea, we construct the crossover algorithm as follows:

Step1 : Start the algorithm.
Step2 : Initialize the three parents ($P_r1, P_r2, and\, P_r3$) depending on the probability of crossover p_c.
Step3 : Generate a random number in between zero and the node (e.g., a_i).
Step4 : Update parents by placing a_i in the position of each parent.
Step5 : In the first child place a_i in the first position.
Step6 : Find the minimum cost between a_i and each next node of given parents.
Step7 : Place s_1 (for example) of the first child at the second place, and update each parent with s_1 in the second place.
Step8 : Repeat steps 6 and 7 up to the end of the nodes.
Step9 : End the algorithm.

Sigmoid random mutation:
In the present study, in the place of constant p_m, we dynamically update the mutation probability by making it decreasing with generations. In our current implementation mechanism, sigmoid function returns a value in [0, 1] depending on generation number and adjustment parameter λ. We understand the diminishing requirement of perturbation as the quality of solution increases with generation. Following the expression of sigmoid function, it returns a value between 0 and 1, used as the probability of mutation. In- creasing generation compels the value of p_m to decrease with increasing return. In the initial few iterations, high value of p_m maintain the exploration in the solution space and gradually it stabilizes for convergence. The mutation process is as follows:

(a) Generation dependent p_m : To acquire the probability of mutation (p_m) by $p_m = \lambda(1 + e^{-g})$, $\lambda \in [0, 1]$, where g is the current generation number.

(b) Selection for mutation: To select the chromosome for mutation, produce a random number $r \in [0, 1]$. When $r < p_m$, the corresponding chromosome is selected for mutation. Here, p_m decreases smoothly as the generation increases. In a single point random mutation, two solutions are randomly chosen from each chromosome and interchanged to create the new offspring set.

4.3 Modified GA Procedure

Procedure name: Modified GA.
Input: Max gen (S_0), population size (pop_-size), probability of selection (p_s), probability of crossover (p_c), probability of mutation (p_m).
Output: Optimum and near-optimum solutions.
Step 1. Start.
Step 2. Set the initial generation $t \leftarrow 0$.
Step 3. (Initialization) Randomly generate initial population $p(t)$ where $f(x_i), i = 1, 2, ..., (pop_-size)$ are the chromosomes, and a_k number of nodes in each chromosome represent a solution of the problem.
Step 4. Evaluate the fitness of each solution of the initial population $p(t)$.
Step 5. Check the condition while ($t \leq S_0$) do up to step 14.
Step 6. Update the generation $t \leftarrow t + 1$.
Step 7. Selection Procedure.
Step 8. Determine the Boltzmann Probability (p_s).
Step 9. Create the mating pool based on (p_s) and (p_B).
Step 10. Invoke the crossover procedure based on (p_c).
Step 11. Invoke mutation based on (p_m).
Step 12. Store new offspring into the offspring set.
Step 13. Compare the fitness and store the local and near-optimum solutions.
Step 14. Repeat Steps 5 to 14.
Step 15. (Optimum Solution) Store global optimum and near-optimum results.
Step 16. Stop.

The above model is solved by using Modified GA approach, discussed in article-4. Our MGA consists of parameters, population size (N) = 50, probability of crossover(p_c) = 0.2, probability of mutation (p_m) = 0.2, and maximum generation = 50. A real number presentation is used here. In this representation, each chromosome X is a string of n numbers of MGA, which denote the decision variable. For each chromosome X, every gene, which represents the independent variables, is randomly generated between their boundaries until it is feasible. In this MGA, arithmetic crossover and random mutation are applied to generate new off springs.

5 Empirical Tests

The optimal profit for time-dependent demand with preservation strategy under permissible delay in payment for time period M in settling account without extra charges has been treated with numerical data. An example is presented to illustrate the effect of the inventory model developed here with the following numerical data:

$C_3 = 45$, $C_1 = 0.25$, $C_p = 6.5$, $s = 8.5$, $P = 100$, $\lambda = 1.5$, $C_3 = 45$, $\theta = 0.15$, $\alpha = 30$, $\beta = 0.3$, $a = 0.5$, $I_p = 0.2$, $I_e = 0.15$ in appropriate units.

According to the proposed solution procedure (MGA) the results listed in Table-2 are obtained for M = 2.5, 2.6, 2.7, 2.8, 2.9 of Cases I, II, III, IV using preservation technology and the results listed in Table-3 are obtained for M = 2.5, 2.6, 2.7, 2.8, 2.9 of Cases I, II, III, IV without using preservation technology.

Table-2: Optimal solutions for illustrated example of Cases I, II, III and IV with preservation technology.

M	Case-I				Case-II				Case-III				Case-IV			
	t_1	t_2	ξ	TAP	t_1	t_2	ξ	TAP	t_1	t_2	ξ	TAP	t_1	t_2	ξ	TAP
2.5	2.54	4.94	4.94	600.531	0.02	4.97	4.98	725.470	0.01	2.45	0.27	1241.854	0.02	0.85	0.14	1763.332
2.6	2.69	4.94	4.96	602.597	0.02	4.97	4.98	728.324	0.03	2.58	0.21	1273.710	0.02	0.92	0.14	1785.984
2.7	2.71	4.94	4.95	605.369	0.01	4.98	4.98	733.455	0.03	2.68	0.81	1295.107	0.02	0.95	0.14	1800.799
2.8	2.86	4.97	4.96	606.930	0.01	4.97	4.98	734.989	0.04	2.79	1.92	1325.258	0.02	0.95	0.14	1810.653
2.9	3.02	4.94	4.95	608.012	0.02	4.97	4.94	737.015	0.03	2.85	2.03	1351.715	0.03	1.05	0.14	1829.688

Table-3: Optimal solutions for illustrated example of Cases I, II, III and IV without preservation technology.

M	Case-I			Case-II			Case-III			Case-IV		
	t_1	t_2	TAP	t_1	t_2	TAP	t_1	t_2	TAP	t_1	t_2	TAP
2.5	2.77	4.99	473.485	1.20	4.98	477.194	0.02	2.46	1232.729	0.01	0.86	1761.669
2.6	2.77	4.99	476.586	1.31	4.98	481.659	0.01	2.59	1270.483	0.01	0.88	1784.031
2.7	2.74	4.99	479.133	1.53	4.99	485.958	0.02	2.69	1293.965	0.01	0.95	1798.541
2.8	2.86	4.99	482.287	1.60	4.99	490.040	0.03	2.86	1320.715	0.03	1.02	1808.037
2.9	2.96	4.99	485.633	1.70	4.99	492.835	0.03	2.86	1344.711	0.03	1.10	1828.554

From the Table-2 and Table-3, it is observed that, for fixed α, β and θ as M increases total average profit also increases and these observations are realistic. And also from Table-2 and Table-3, it is observed that, in Case-I and Case-II preservation technology is very more beneficial and in Case-III and Case-IV preservation technology is very little beneficial.

5.1 Sensitivity Analysis

For the given numerical example mentioned in section 5, sensitivity analyses are performed to study the effect of changes of different values of the demand parameters α, β and deterioration parameter θ on maximum total average profit of the system. It is observed that for different values of α as β increases when θ is fixed, total average profit increases and also for different values of β as θ increases when α is fixed, total average profit decreases. All these observations agree with the reality.

Table-4: The sensitivity analysis of the demand parameters α and β when $\theta = 0.15$ and $M = 2.7$ for Cases I and II using preservation technology.

		Case-I				Case-II			
α	β	t_1	t_2	ξ	TAP	t_1	t_2	ξ	TAP
25	0.25	2.71	4.94	4.98	464.648	0.01	4.97	4.98	578.632
	0.30	2.71	4.94	4.98	526.704	0.01	4.98	4.98	657.667
	0.35	2.71	4.94	4.94	588.206	0.01	4.98	4.98	733.260
30	0.25	2.71	4.94	4.98	536.957	0.01	4.98	4.98	654.837
	0.30	2.71	4.94	4.95	605.369	0.01	4.98	4.98	733.455
	0.35	2.71	4.94	4.95	666.194	0.01	4.98	4.98	810.246
35	0.25	2.71	4.94	4.95	608.424	0.01	4.98	4.98	726.947
	0.30	2.71	4.94	4.95	675.407	0.01	4.98	4.98	806.061
	0.35	2.71	4.94	4.95	742.998	0.01	4.98	4.98	883.549

Table-5: The sensitivity analysis of the demand parameters α and β when $\theta = 0.15$ and $M = 2.7$ for Cases III and IV using preservation technology.

		Case-III				Case-IV			
α	β	t_1	t_2	ξ	TAP	t_1	t_2	ξ	TAP
25	0.25	0.03	2.69	4.98	1073.345	0.02	0.73	0.02	1606.742
	0.30	0.03	2.69	4.98	1182.318	0.02	0.81	0.14	1659.122
	0.35	0.03	2.68	4.98	1284.905	0.02	0.87	0.08	1716.481
30	0.25	0.03	2.68	0.22	1191.142	0.02	0.89	0.10	1747.888
	0.30	0.03	2.68	0.81	1295.107	0.02	0.95	0.14	1800.799
	0.35	0.03	2.68	1.34	1401.853	0.02	0.95	0.14	1843.951
35	0.25	0.03	2.69	0.22	1296.282	0.02	1.03	0.14	1879.314
	0.30	0.03	2.68	0.22	1417.754	0.02	1.09	0.19	1942.737
	0.35	0.03	2.69	0.22	1526.318	0.04	1.15	0.22	1990.617

Table-6: The sensitivity analysis of the deterioration parameter θ when $\alpha = 30$ and $M = 2.7$ for Cases I and II using preservation technology.

		Case-I				Case-II			
β	θ	t_1	t_2	ξ	TAP	t_1	t_2	ξ	TAP
0.25	0.125	2.71	4.94	4.94	539.807	0.01	4.98	4.98	660.194
	0.150	2.71	4.94	4.98	536.957	0.01	4.98	4.98	654.837
	0.175	2.71	4.94	4.98	533.899	0.01	4.98	4.98	647.923
0.30	0.125	2.71	4.94	4.94	608.994	0.01	4.98	4.98	738.487
	0.150	2.71	4.94	4.95	605.369	0.01	4.98	4.98	733.455
	0.175	2.71	4.94	4.94	598.768	0.01	4.98	4.98	728.469
0.35	0.125	2.71	4.94	4.94	668.313	0.01	4.98	4.98	814.956
	0.150	2.71	4.94	4.95	666.194	0.01	4.98	4.98	810.246
	0.175	2.71	4.94	4.95	664.091	0.01	4.98	4.98	805.574

Table-7: The sensitivity analysis of the deterioration parameter θ when $\alpha = 30$ and $M = 2.7$ for Cases III and IV using preservation technology.

		Case-III				Case-IV			
β	θ	t_1	t_2	ξ	TAP	t_1	t_2	ξ	TAP
0.25	0.125	0.03	2.68	0.22	1193.622	0.02	0.89	0.08	1749.217
	0.150	0.03	2.68	0.22	1191.142	0.02	0.89	0.10	1747.888
	0.175	0.03	2.68	0.22	1189.916	0.02	0.91	0.14	1744.579
0.30	0.125	0.03	2.68	0.64	1298.758	0.02	0.94	0.14	1805.686
	0.150	0.03	2.68	0.81	1295.107	0.02	0.95	0.14	1800.799
	0.175	0.03	2.66	0.22	1293.462	0.02	0.96	0.14	1796.605
0.35	0.125	0.03	2.66	1.34	1403.019	0.02	1.03	0.08	1864.254
	0.150	0.03	2.68	1.34	1401.853	0.02	0.95	0.14	1843.951
	0.175	0.03	2.68	1.41	1400.031	0.02	0.98	0.17	1827.217

6 Discussions

From the results, I observe that permissible delay in payment plays a vital role in two level production based economic quantity model. From Tables 2 and 3, it is observed that in Case-III and Case-IV the effect of permissible delay in payment is more beneficial as time span of permissible delay in payment is longer than the Case-I and Case-II. Also in Tables 4, 5 when $M = 2.7$ and $\theta = 0.15$ for different values of α as β increases and in Tables 6, 7 when $M = 2.7$ and $\alpha = 30$ for different values of β as θ increases, it is observed that in Case-III and Case-IV the effect of permissible delay in payment is more beneficial as time span of permissible delay in payment is longer than the Case-I and Case-II. All these observations agree with the reality.

7 Conclusion and Future Scope

In this paper, a two level production-inventory model for non-instantaneous deteriorating items and time-dependent demand has been considered using preservation technology under permissible delay in payment over an infinite time horizon. The model is solved numerically by Modified Genetic Algorithm (MGA) and then compared. Sensitivity analyses are also performed for different parameters to study the effect of the decision variables. Here I consider, M be the period of permissible delay in settling account without extra charges. Results in this study provide a valuable reference for decision makers in planning and controlling the inventory. Finally, for future research, one can incorporate more realistic assumptions in the proposed model, such as variable deterioration rate, stochastic nature of demand and production rate with two or more warehouses.

References

[1] Aggarwal, S.P. and Jaggi, C.K., (1995), *Ordering policies of deteriorating items under permissible delay in payments.* J. Oper. Res. Soc. **46**, 658-662.

[2] Chung, K.J. and Huang, Y.F., (2003), *The optimal cycle time for EPQ inventory model under permissible delay in payments.* International Journal of Production Economics, **84**, 307-318.

[3] Chung, K.J. and Ting, P.S., (1993), *A heuristic for replenishment of deteriorating items with a linear trend in demand.* J. Oper. Res. Soc. **44(12)**, 1235-1241.

[4] Das, D., Roy, A. and Kar, S., (2010), *Improving Production Policy for a Deteriorat- ing Item under Permissible Delay in Payments with Stock-Dependent Demand Rate.* Computers and Mathematics with Applications, **60**, 1973-1985.

[5] Das, S.C., Zidan, A.M., Manna, A. K., Shaikh, A. A. and Bhunia, A. K., (2020), *An application of preservation technology in inventory control system with price dependent demand and partial backlogging.* Alexandria Engineering Journal, **59(3)**, 1359-1369.

[6] Das, D., (2023), *Two warehouses production inventory model for deteriorating items with stock-dependent demand under preservation technology using modified genetic algorithm.* ICTACT Journal on Soft Computing, **13(4)**, 3075-3087.

[7] Das, D., (2019), *Production Inventory Model for a Deteriorating Item using Preservation Technology under Permissible Delay in Payment.* International Journal of Re- search and Analytical Reviews, **6(2)**, 369-382.

[8] Das, D., Roy, A. and Kar, S., (2011), *Optimal payment time for a retailer under permitted delay of payment by the wholesaler with dynamic demand and hybrid number cost parameters.* OPSEARCH, **48(3)**, 171-196.

[9] Das, D., Roy, A. and Kar, S., (2012), *A multi-item inventory model for two-stage production system with imperfect processes using differential evolution and credibility measure.* International Journal of Operations Research, **9(2)**, 87-99.

[10] Das, D., Roy, A. and Kar, S., (2015),*A multi-warehouse partial backlogging inventory model for deteriorating items under inflation when a delay in payment is permissible.* Annals of Operations Research, Springer, **226(1)**, 133-162.

[11] Das, D. and Jana, T., (2019), *An EPQ Model for a Deteriorating Item using Preservation Technology under Stock-Dependent Demand Rate.* International Journal of Re- search and Analytical Reviews, **6(2)**, 281-292.

[12] Dye, C.Y. and Hsieh, T.P., (2012), *An optimal replenishment policy for deteriorating items with effective investment in preservation technology.* European journal of operational research, **218(1)**, 106-112.

[13] Dye, C.Y., (2013), *The effect of preservation technology investment on a non-instantaneous deteriorating inventory model.* Omega, **41**, 873-880.

[14] Donaldson, W.A., (1977), *Inventory replenishment policy for a linear trend in demand: an analytical solution.* Oper. Res. Q. **28**, 663-670.

[15] Ghare, P.M. and Schrader, G.H., (1963), *A Model for an Exponentially Decaying Inventory.* Journal of Industrial Engineering, **14(5)**, 238-243.

[16] Goyal, S. K., (1985), *Economic order quantity under conditions of permissible delay in payments.* Journal of the Operational Research Society, **36**, 335-338.

[17] Goyal, S.K. and Gunasekaran, A., (1995), *An integrated production-inventory-marketing model for deteriorating items.* Computers and Industrial Engineering, **28**, 755-762.

[18] Gupta, P.N. and Agarwal, R.N., (2000), *An order level inventory model with time dependent deterioration.* OPSEARCH, **37(4)**, 351-359.

[19] Huang, Y.F., (2007), *Economic order quantity under conditionally permissible delay in payments.* European Journal of Operational Research, **176**, 911924. *An overtime production inventory model for deteriorating items with nonlinear price and stock de-pendent demand.* Alexandria Engineering Journal, **60(3)**, 2779-2786.

[20] Hsu, P., Wee, H. and Teng, H., (2010), *Preservation technology investment for deteriorating inventory.* International Journal of production Economics, **124 (2)**, 388-394.

[21] Kumar, N., Yadav, D. and Kumari, R., (2018), *Two level production inventory model with exponential demand and time dependent deterioration rate.* Malaya Journal of Matematik,**1**, 30-34.

[22] Mitra, A., Cox, J.F.and Jesse, R.R., (1984), *A note on deteriorating order quantities with a linear trend in demand.* J. Oper. Res. Soc. **35**, 141-144.

[23] Ouyand, L.Y., Teng, J.T., Chuang, K.W. and Chuang, B.R., (2005), *Optimal inventory policy with non-instantaneous receipt under trade credit.* International Journal of Production Economics, **98**, 290-300.

[24] Patra, K. and Maity, R., (2017), *A single item inventory model with variable production rate and defective items.* International Journal of Applied and Computational Mathematics, **03**, 1-11.

[25] Sivashankari, C.K. and Panayappan, S., (2015), *Production inventory model for two-level production with deteriorative items and shortages.* Int. Adv. Manuf. Techonl., **76**, 2003-2014.

[26] Singh, T. and Pattnayak, H.P., (2013), *An EOQ Model for Deteriorating Items with Linear Demand, Variable Deterioration and Partial Backlogging.* Journal of Service Science and Management, **6(2)**, 186-190.