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A Study on Homogenous cosmological model in f (R, T) Theory of gravity

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Abstract:

This study introduces non-singular Bianchi type I and V cosmological models together with a specific value for the average scale factor, related to a time-dependent deceleration parameter. The cosmological models first demonstrate acceleration for a limited period before shifting to deceleration. The physical and kinematic properties of the universe models are analyzed.

Keywords:

Bianchi Type-V cosmological models, f (R, T) Theory of gravity, Deceleration parameter, Bulk viscous fluid, Scale Factor Evolution, Field Equations, Cosmological Solutions and properties.

1. Introduction

The diagonal version of the metric for the Bianchi type V cosmological model is represented by

$$ds^2 = dt^2 - A^2dx^2 - e^{2mx}[B^2dy^2 + C^2dz^2] \tag{1}$$

Here, A, B, and C are also cosmic scale factors and an arbitrary constant.
Utilizing Eqs. (1), (4), (5), and (45), we derive the subsequent set of equations.

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - 3\frac{m^2}{A^2} = (8\pi + 3\lambda)\rho - \lambda\bar{p} \tag{2}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = \lambda\rho - (8\pi + 3\lambda)\bar{p} \tag{3}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} - \frac{m^2}{A^2} = \lambda\rho - (8\pi + 3\lambda)\bar{p} \tag{4}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = \lambda\rho - (8\pi + 3\lambda)\bar{p} \tag{5}$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \tag{6}$$

Upon integrating Equation (50), we obtain $A^2 = kBC$, where k represents an integration constant. Assuming k=1 without loss of generality. The identical methodology employed for the Bianchi type-I solution is utilized here to resolve the specified equations. Utilizing Eq. (50), we derive the constraint equations as follows

$$p_1 = 1 \ , p_2 = p_3^{-1} = P \quad , q_1 = 0 \ , q_2 = q_3 = Q \tag{7}$$

From equations (46) to (51), we can quickly obtain

$$A = a\,, \quad B = aPe^{[Q\int\frac{dt}{a^3}]}$$

$$C = aP^{-1}e^{[Q\int\frac{dt}{a^3}]} \tag{8}$$

By substituting the value, a(t) provided in Eq. (31) into Eqs. (46) and (49), we get the metric functions A, B, and C as follows:

$$A = \left(t^2 + \frac{2a}{3}\right)^{\frac{1}{3}} \tag{9}$$

$$B = \left(t^2 + \frac{2a}{3}\right)^{\frac{1}{3}} Pe^{[Qtan^{-1}\left(\frac{3}{2\alpha}\right)^{\frac{1}{2}}t]} \tag{10}$$

$$C = \left(t^2 + \frac{2a}{3}\right)^{\frac{1}{3}} P^{-1}e^{[-Qtan^{-1}\left(\frac{3}{2\alpha}\right)^{\frac{1}{2}}t]} \tag{11}$$

The energy density and bulk viscous pressure for the Bianchi type-V space-time model are specified as

$$\rho = \frac{1}{9(8\pi + 4\lambda)(8\pi + 2\lambda)\left(t^2 + \frac{2\alpha}{3}\right)^2}[(8\pi + 3\lambda)(12 - 36Q^2)t^2 - \lambda\left(12 - t^2\left(12 - t^2(6 + 34Q^2)\right)\right) - \frac{3m^2(8\pi+2\lambda)}{\left(t^2+\frac{2a}{3}\right)^{\frac{1}{3}}}] \tag{12}$$

$$\bar{p} = \frac{1}{9\left(t^2 + \frac{2a}{3}\right)^{\frac{1}{3}}}[\frac{(8\pi + 3\lambda)}{\lambda(8\pi + 2\lambda)(8\pi + 4\lambda)}[(8\pi + 3\lambda)(12 - 36Q^2)t^2 - \lambda] - (12 - 36Q^2)t^2] - [\frac{(8\pi+3\lambda)}{\lambda(8\pi+2\lambda)(8\pi+4\lambda)} - 1]\frac{m^2}{3\left(t^2+\frac{2a}{3}\right)^{\frac{2}{3}}} \tag{13}$$

The equation of state parameter yields the bulk viscosity coefficient.

$$C= \frac{1}{36t(8\pi+4\lambda)(8\pi+2\lambda)\left(t^2+\frac{2\alpha}{3}\right)} * [(8\pi + 3\lambda)(12 - 36Q^2)\varepsilon + \lambda(8\pi + 2\lambda)(8\pi + 4\lambda)(12 - 36Q^2) - (8\pi + 3\lambda)(12 - 36Q^2)] + \frac{1}{36t(8\pi+4\lambda)(8\pi+2\lambda)\left(t^2+\frac{2\alpha}{3}\right)} * \left[\lambda\left(12 - t^2(6 + 34Q^2)\right) - \lambda^2\varepsilon * \left(12 - t^2(6 + 34Q^2)\right)\right] - \frac{m^2}{12(8\pi+4\lambda)(8\pi+2\lambda)\left(t^2+\frac{2\alpha}{3}\right)^{\frac{4}{3}}} * [\varepsilon(8\pi + 2\lambda) + (8\pi + 3\lambda)] + \frac{m^2}{\left(t^2+\frac{2a}{3}\right)^{\frac{4}{3}}} \tag{14}$$

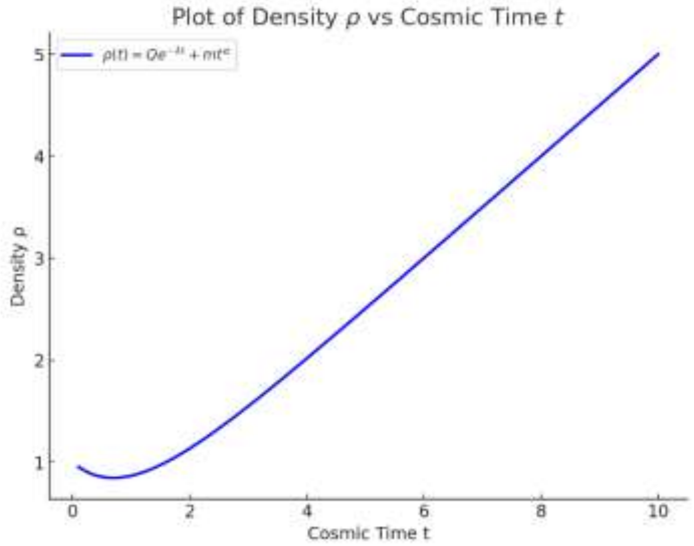

Figure 1. The plot of density ρ verses cosmic time t, Q=1, λ=1, m=0.5, α=1.

The directional Hubble parameters H_1, H_2, and H_3 are presented in the form

$$H_1 = \frac{2t}{3\left(t^2+\frac{2a}{3}\right)} \tag{15}$$

$$H_2 = [3Q+1]\frac{2t}{3\left(t^2+\frac{2a}{3}\right)} \tag{16}$$

$$H_2 = [-3Q+1]\frac{2t}{3\left(t^2+\frac{2a}{3}\right)} \tag{17}$$

The anisotropic parameter A_m has a specific value.

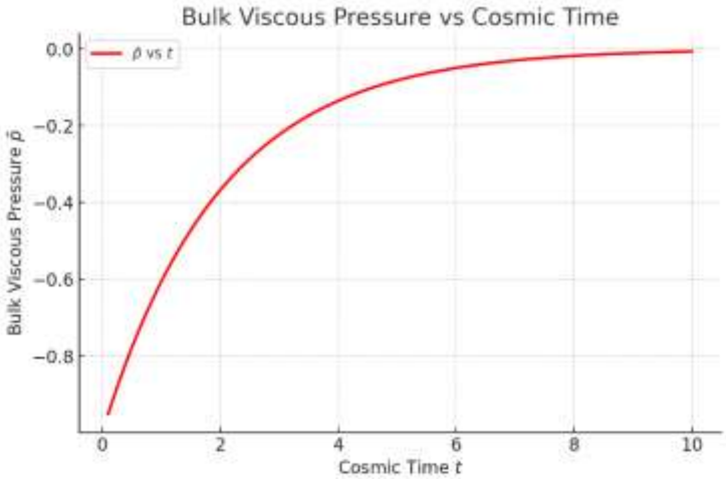

Figure 2. The plot of bulk viscous pressure verses cosmic time t, Q=1, λ=1, m=0.5, α=1.

$$A_m = 6(1+72Q^2) \tag{18}$$

The shear scalar for this model is provided by

$$\sigma^2 = \left(\frac{2Qt}{t^2+\frac{2a}{3}}\right)^2 \tag{19}$$

Figures illustrate the temporal change of ρ, $\bar{p}$, C, ρ+p, H, θ, and σ. From the aforementioned results, it can be observed

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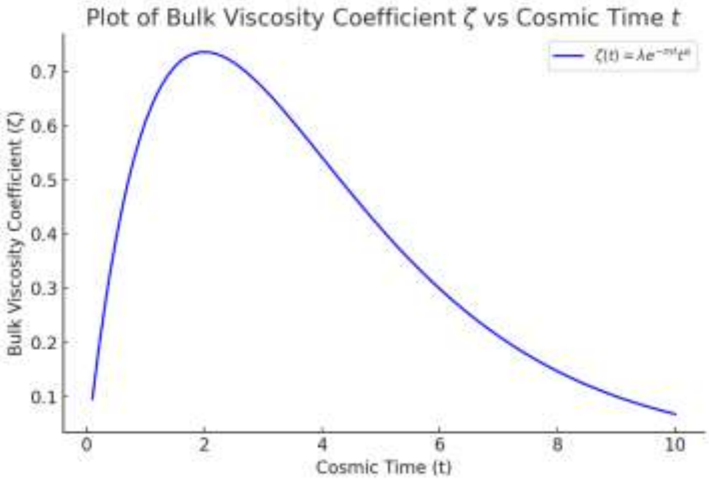

Figure 3. The plot of Bulk viscosity coefficient ζ verses cosmic time t, Q=1, λ=1, m=0.5, α=1.

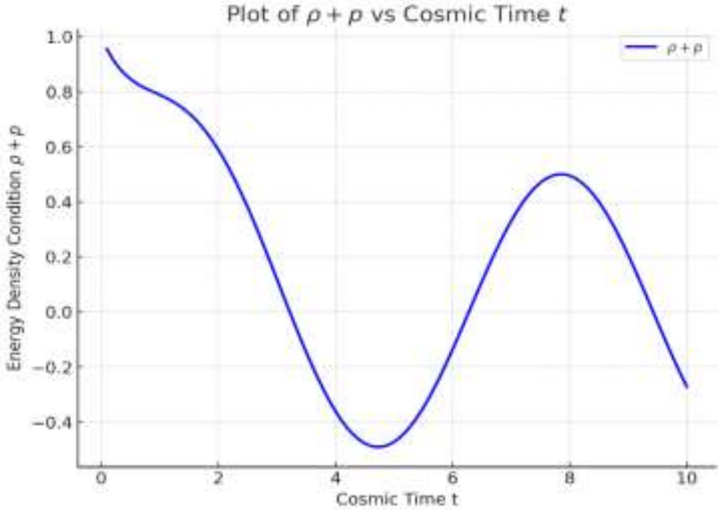

Figure 4. The plot of Energy density condition ρ+p verses cosmic time t,Q=1 λ=1,m=0.5,α=1

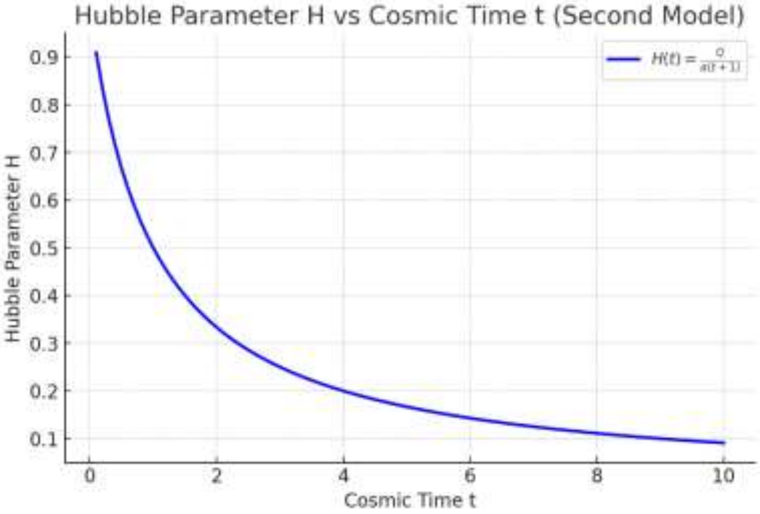

Figure 5. The plot of Hubble parameter H (for second model) verses cosmic time t, Q=1, α=1.

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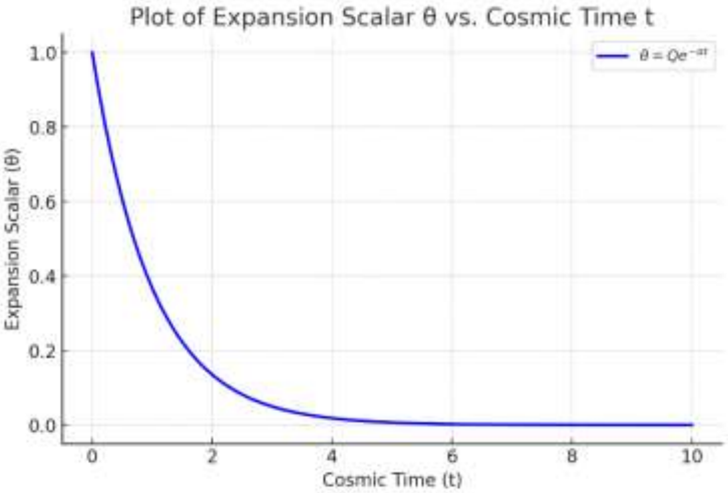

Figure 6. The plot of expansion scalar θ (for model second) verses cosmic time t, Q=1, α=1.

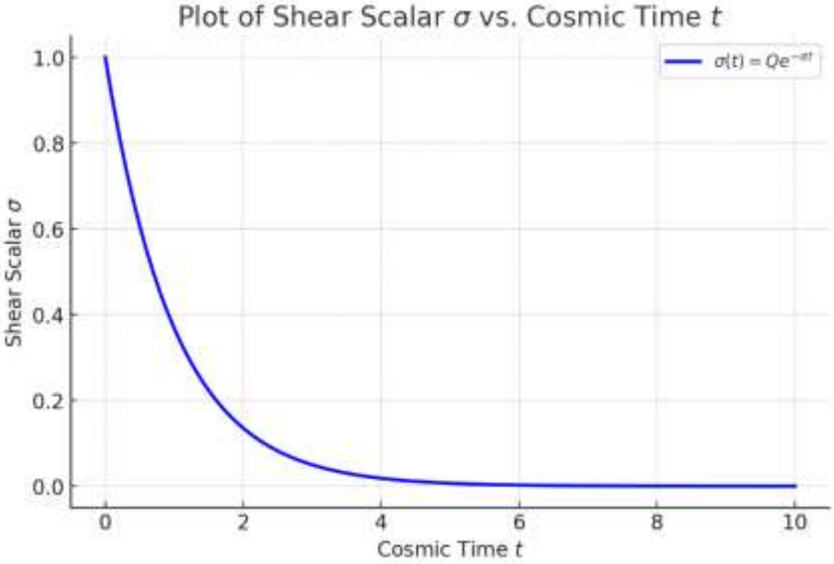

Figure 7. The plot of shear scalar σ cosmic time t, Q=1, α=1.

The model has no singularity at t=0, and the spatial volume expands as time progresses, resulting in the accelerated expansion of the universe. In this model, we also remark that σ^2, $\bar{p}$, p, ρ, and C are finite at t=0 whereas they diminish for infinitely long t. However, $\sigma^2/\theta^2 \neq 0$, which indicates that the model does not converge towards isotropy for large time intervals. From Eq. [$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-3\frac{\beta^2}{A^2}=-\left(\frac{8\pi+\lambda}{\lambda}\right)p-\Lambda$

we observe that $q < 0$ for $t < \sqrt{(2\alpha)}$ and $q > 0$ for $t > \sqrt{(2\alpha)}$. It is noteworthy that Shamire et al. [54] Additionally, accurate solutions of Bianchi type I and V models in f (R, T) gravity theory have been shown by employing the variation law of Hubble's parameter proposed by Berman and further developed by Berman and Gomide. Various models were employed in that instance.

10.48047/jocaaa.2024.33.06.54

2. Properties of the Model for m≠3

For m≠ 3, by equation [$A(t) = a, B(t) = c_1 a e^{\left(c_2 \int a^{-3} dt\right)}$ and $C(t) = \frac{a}{c_1} a e^{\left(-c_2 \int a^{-3} dt\right)}$] we get

$$A(t) = \left[\frac{(3-m)k}{3k_1} t + k_2\right]^{\frac{1}{3-m}} \tag{20}$$

$$B(t) = c_1 \left[\frac{(3-m)k}{3k_1} t + k_2\right]^{\frac{1}{3-m}} e^{\frac{-3k_1 c_2}{mk}\left(\frac{(3-m)k}{3k_1} t + k_2\right)^{\frac{m}{3-m}}} \tag{21}$$

$$C(t) = \frac{1}{c_1} \left[\frac{(3-m)k}{3k_1} t + k_2\right]^{\frac{1}{3-m}} e^{\frac{-3k_1 c_2}{mk}\left(\frac{(3-m)k}{3k_1} t + k_2\right)^{\frac{m}{3-m}}} \tag{22}$$

The metric (18) takes the following form after an appropriate coordinate transformation.

$$ds^2 = -dt^2 + \tau^{\frac{2}{3-m}} dx^2 + \tau^{\frac{2}{3-m}} \left[e^{\left(2\beta x - \frac{6k_1 c_2}{mk} \tau^{\frac{m}{m-3}}\right)} dy^2 + e^{\left(2\beta x + \frac{6k_1 c_2}{mk} \tau^{\frac{m}{m-3}}\right)} dz^2\right]$$

where $\tau = \frac{(3-m)k}{3k_1} t + k_2$ (23)

In the model ($\tau = \frac{(3-m)k}{3k_1} t + k_2$), the spatial volume (V), Hubble parameter (H), expansion scalar (θ), deceleration parameter (q), and shear scalar (σ) are defined as follows:

$$V = a^3 = \tau^{\frac{3}{3-m}} \tag{24}$$

$$H = \frac{k}{3k_1 \tau} \tag{25}$$

$$\theta = \frac{k}{k_1 \tau} \tag{26}$$

$$q = 2 - m \tag{27}$$

$$\sigma = \frac{c_2}{\tau^{\frac{3}{3-m}}} \tag{28}$$

The anisotropy parameter (A_m) for the model can be articulated as

$$A_m = \frac{6c_2^2 k_2^2}{k^2} \frac{1}{\tau^{\frac{2m}{3-m}}} \tag{29}$$

By employing equations ($A(t) = \left[\frac{(3-m)k}{3k_1} t + k_2\right]^{\frac{1}{3-m}}$,

$$B(t) = c_1 \left[\frac{(3-m)k}{3k_1} t + k_2\right]^{\frac{1}{3-m}} e^{\frac{-3k_1 c_2}{mk}\left(\frac{(3-m)k}{3k_1} t + k_2\right)^{\frac{m}{3-m}}},$$

$C(t) = \frac{1}{c_1} \left[\frac{(3-m)k}{3k_1} t + k_2\right]^{\frac{1}{3-m}} e^{\frac{-3k_1 c_2}{mk}\left(\frac{(3-m)k}{3k_1} t + k_2\right)^{\frac{m}{3-m}}}$)

Within

[$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\beta^2}{A^2} = \left(\frac{8\pi+\lambda}{\lambda}\right) p - \Lambda$, $\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\beta^2}{A^2} = \left(\frac{8\pi+\lambda}{\lambda}\right) p - \Lambda$,

$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\beta^2}{A^2}=\left(\frac{8\pi+\lambda}{\lambda}\right)p-\Lambda$ and $\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-3\frac{\beta^2}{A^2}=-\left(\frac{8\pi+\lambda}{\lambda}\right)p-\Lambda$] and resolving with ($\Lambda=\frac{1}{2}(\rho-p)$), we derive the expressions for pressure (p), energy density (ρ), and cosmological constant (Λ) for model ($\tau=\frac{(3-m)k}{3k_1}t+k_2$) as follows:

$$p(t)=\frac{c_3}{\tau^2}+\frac{c_4}{\tau^{\frac{2}{3}-m}}+\frac{c_5}{\tau^{\frac{6}{3}-m}}-\frac{c_6}{\tau^{\frac{6-m}{3-m}}} \tag{30}$$

$$\rho(t)=\frac{c_7}{\tau^{\frac{2}{3}-m}}+\frac{c_8}{\tau^2}+\frac{c_5}{\tau^{\frac{6}{3}-m}}-\frac{c_6}{\tau^{\frac{6-m}{3-m}}} \tag{31}$$

$$\Lambda(t)=\left(\frac{\lambda}{8\pi+\lambda}\right)\left\{2\beta^2\frac{1}{\tau^{\frac{2}{3}-m}}-\frac{1}{\tau^2}\frac{mk^2}{9k_1^2}\right\} \tag{32}$$

The model indicates that the spatial volume (V) is zero at τ=0, while the expansion scalar θ is infinite, demonstrating that the cosmos begins its evolution with zero volume at τ=0, consistent with the big bang hypothesis. The Hubble parameter H and the shear scalar σ diverge at τ=0.

From equations [($A(t)=\left[\frac{(3-m)k}{3k_1}t+k_2\right]^{\frac{1}{3-m}}$,

$$B(t)=c_1\left[\frac{(3-m)k}{3k_1}t+k_2\right]^{\frac{1}{3-m}}e^{\frac{-3k_1c_2}{mk}\left(\frac{(3-m)k}{3k_1}t+k_2\right)^{\frac{m}{3-m}}},$$

$C(t)=\frac{1}{c_1}\left[\frac{(3-m)k}{3k_1}t+k_2\right]^{\frac{1}{3-m}}e^{\frac{-3k_1c_2}{mk}\left(\frac{(3-m)k}{3k_1}t+k_2\right)^{\frac{m}{3-m}}}$], we note that the scale factors A, B, and C are zero at the first epoch τ=0. The model possesses Singularity at τ=0 is referred to as a point-type singularity. The scientific quantities isotropic pressure (p), proper energy density (ρ), and cosmological constant (Λ) become infinite at τ=0. At τ=0, the anisotropy parameter (Am) is indefinitely big, indicating that the universe in the model was anisotropic at the beginning moment. As time approaches infinity (τ → ∞), the Hubble parameter H, expansion scalar θ, shear scalar σ, pressure p, density ρ, and cosmological constant Λ become unimportant, while the scale factors A, B, and C tend toward infinity. As τ approaches infinity, spatial volume becomes infinite and the anisotropy parameter (A_m) approaches zero. This indicates that the cosmos in the model becomes isotropic at a later time.

It is evident that $\frac{\sigma^2}{\theta^2}$ approaches 0 as τ → ∞ approaches infinity, indicating that the model converges to isotropy, consistent with the findings of Collins and Hawking [59]. Equation [q = 2-m] indicates that q remains constant. For m < 3, q is positive, indicating that the universe is in a decelerating phase of expansion; conversely, for m > 3, q is negative, signifying that the cosmos in the model is in an accelerating phase.

3. Properties of the Model for m = 3

Now for m = 3, equation [$A(t)=a$, $B(t)=c_1ae^{(c_2\int a^{-3}dt)}$ and $C(t)=\frac{a}{c_1}ae^{(-c_2\int a^{-3}dt)}$] gives

$$A(t)=k_3e^{\frac{k}{3k_1}t} \tag{33}$$

$$B(t)=c_1k_3e^{\frac{k}{3k_1}t}.e^{\frac{-k_1c_2}{kk_3^3}e^{\frac{-k}{k_1}t}} \tag{34}$$

$$\mathrm{C}(t)=\frac{k_3}{c_1}e^{\frac{k}{3k_1}t}.e^{\frac{-k_1c_2}{kk_3^3}e^{\frac{-k}{k_1}t}} \tag{35}$$

In this solution, the metric ($ds^2=-dt^2+a^2(t)e^{2m_1x}dx^2+a^2(t)e^{2m_2}dy^2+a^2(t)e^{2m_3}dz^2$) takes the following form after an appropriate coordinate transformation.

10.48047/jocaaa.2024.33.06.54

$$ds^2=-dt^2+k_3^2e^{2Nt}dx^2+e^{2\beta x}[c_1^2k_3^2e^{\left(2Nt-\frac{4c_2}{3k_3^3}.\frac{1}{N}e^{\frac{-3Nt}{2}}\right)}dy^2+\frac{1}{c_1^2}k_3^2\ e^{\left(2Nt+\frac{4c_2}{3k_3^3}\frac{1}{N}.e^{\frac{-3Nt}{2}}\right)}dz^2]$$

where $N=\frac{k}{3k_1}$ (36)

For m = 3, the spatial volume (V), Hubble parameter (H), expansion scalar (θ), deceleration parameter (q), and shear scalar (σ) assume the following forms

$$V=a^3=k_3^3e^{3Nt} \tag{37}$$

$$H=N \tag{38}$$

$$\theta=3N \tag{39}$$

$$q=-1 \tag{40}$$

$$\sigma=\frac{c_2}{k_3^3}e^{-3Nt} \tag{41}$$

The anisotropy parameter (A_m) is expressed as

$$A_m=\frac{2c_2^2}{3k_3^6N^2}e^{-6Nt} \tag{42}$$

Utilizing equations ($A(t)=k_3e^{\frac{k}{3k_1}t}, B(t)=c_1k_3e^{\frac{k}{3k_1}t}.e^{\frac{-k_1c_2}{kk_3^3}e^{\frac{-k}{k_1}t}}$ *and* $C(t)=\frac{k_3}{c_1}e^{\frac{k}{3k_1}t}.e^{\frac{-k_1c_2}{kk_3^3}e^{\frac{-k}{k_1}t}}$) within [$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}-\frac{\beta^2}{A^2}=\left(\frac{8\pi+\lambda}{\lambda}\right)p-\Lambda$, $\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\frac{\beta^2}{A^2}=\left(\frac{8\pi+\lambda}{\lambda}\right)p-\Lambda$,
$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\beta^2}{A^2}=\left(\frac{8\pi+\lambda}{\lambda}\right)p-\Lambda$ and $\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-3\frac{\beta^2}{A^2}=-\left(\frac{8\pi+\lambda}{\lambda}\right)p-\Lambda$] and solving with ($\Lambda=\frac{1}{2}(\rho-p)$, we derive the formulas for pressure (p), energy density (ρ), and cosmological constant (Λ) for model

$$[ds^2=-dt^2+k_3^2e^{2Nt}dx^2+e^{2\beta x}[c_1^2k_3^2e^{\left(2Nt-\frac{4c_2}{3k_3^3}.\frac{1}{N}e^{\frac{-3Nt}{2}}\right)}dy^2+\frac{1}{c_1^2}k_3^2\ e^{\left(2Nt+\frac{4c_2}{3k_3^3}\frac{1}{N}.e^{\frac{-3Nt}{2}}\right)}dz^2]$$

where $N=\frac{k}{3k_1}$ as follows:

$$p(t)=k_4+k_5e^{\left(\frac{-2k}{k_1}\right)t}+k_6e^{\left(\frac{-2k}{3k_1}\right)t} \tag{43}$$

$$\rho(t)=k_7+k_5e^{\left(\frac{-2k}{k_1}\right)t}+k_8e^{\left(\frac{-2k}{3k_1}\right)t} \tag{44}$$

$$\Lambda(t)=\left(\frac{\lambda}{8\pi+\lambda}\right)\left\{2\beta^2\frac{1}{k_3^2}e^{\left(\frac{-2k}{3k_1}\right)t}-\frac{k^2}{3k_1^2}\right\} \tag{45}$$

Subsequently, by employing the exponential representation of the average scale factor, we derived the solutions for the scale factors. The scale factor assumes constant values during the early epochs of the cosmos (t→0). The spatial volume V expands exponentially with cosmic time, ultimately reaching infinity in advanced epochs. The expansion scalar for these scale factors demonstrates a constant value of θ = 3N. This indicates consistent exponential expansion across time, meaning the cosmos expands uniformly. Given that N =H, the mean Hubble parameter remains constant, but the directional Hubble parameters exhibit dynamism. The deceleration parameter q = -1 indicates an accelerating expansion of the cosmos, consistent with exponential volumetric expansion. We also note that σ, A_m, p, ρ, and Λ diminish exponentially with cosmic time t.

Since σ/θ ≠ 0 at t = 0 and σ/θ = 0 at t = ∞, this indicates that the universe in the model reaches Isotropy in the late temporal phase.

4. Conclusion:

The bulk viscosity decreases over time, leading to models that ultimately become inflationary (Padmanabhan and Chitre [55]). The matter pressure and energy density are consistently decreasing functions of time, ultimately tending towards 0 as time advances. Thus, the models would accurately depict an empty planet over prolonged durations. The conditions (a) $\rho + p = 0$ and (b) $\rho + p = 0$ are equivalently satisfied. This study's models may clarify the importance of bulk viscosity in explaining both decelerating and accelerating processes, as well as in understanding cosmic structure development.

The model delineates two types of cosmologies, specifically for $m \neq 3$ and $m = 3$, respectively. In cosmology, a number of $m\neq3$ indicates a power law expansion of the cosmos, whereas a value of $m=3$ represents an exponential expansion of the universe:

(i)

The power law expansion demonstrates the distinctive concept wherein the spatial scale factors and volume scalar converge to zero at $\tau=0$.The pressure, energy density, and Hubble parameter are infinite at the initial epoch. As τ approaches infinity, the pressure (p), density (ρ), and enthalpy (H) tend toward zero. A and σ are initially significant but decrease over cosmic time, ultimately tending towards zero as τ approaches infinity. The model demonstrates isotropy in the later phases of its development. Under the power law expansion, the deceleration parameter q is expressed as $2 - m$. The exponential solution represents a model of the cosmos devoid of singularities.

(ii)

The exponential solution represents a model of the cosmos devoid of singularities. This scenario depicts uniform expansion, leading to exponential volume growth over time. All other physical constants, specifically p, ρ, σ, Λ, and A_m, remain constant at $t=0$ but decrease exponentially with cosmic time t, becoming negligible at later times, although H and θ remain constant throughout the evolution. The deceleration parameter, specified as $q = -1$, signifies that the model produces the highest value of the Hubble parameter and the swiftest pace of expansion of the observable universe.

(iii)

For $m < 3$, the universe undergoes a decelerating phase, whereas $m > 3$ leads to an accelerating phase.

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