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# Application of Grey Wolf Optimization for Controller Design using Indirect Approach

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## Abstract

For the controller design of the higher-order models, a novel system minimization technique is put forward in this study. This study uses the Grey Wolf Optimization approach to obtain a reduced order model. The PID controller is made for the large-scale system utilizing this reduced-order concept. The step responses of the complete and lower-order models are compared in order to verify the accuracy and efficacy of the suggested approach. Step response and performance error indices are used to illustrate how well the suggested method performs. The performance of the suggested model-order reduction technique and the efficacy and accuracy of the constructed controller are validated using a single standard numerical system.

***Keywords***: Model reduction technique, Step response, Reduced-order system, Grey Wolf optimization algorithm, controller design, ISE, ITAE, IAE, ITSE.

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## Introduction

Large-scale model research and synthesis are challenging tasks that result in an ongoing attempt to reduce the complexity of the higher-order model. Aeronautic systems [1], jump systems, control systems [2], multilayer systems, electromagnetic systems, power systems [3], thermodynamics, and regulator problems [4] are only a few of the technical and scientific domains where higher dimensional systems are found. A system reduction technique aims to create a plant that is less complex than the original system while preserving its key characteristics. A common subject in the fields of biological systems, control systems [5,6], electromagnetic fields, mechanical engineering, power systems [7], chemical engineering, etc., is the model diminution of the complex system. The literature contains a number of methods for order diminution of the transfer function of the higher-order linear time invariant (LTI) dynamic models in the frequency domain [8–15]. The most popular model diminution strategies among these are the temporal moment matching technique [15] and the Padé approximation approach [12], which are appropriate procedures for matching the static responses of the reduced system with the original system [16,17]. Because these methods produce unstable reduced models even

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when the original higher order systems are stable, they can occasionally fail to reduce the complexity of large-scale systems [18,19]. Another widely utilized technique for the order decrease of higher-order plants is the routh stability scheme [9], which is also a well-liked scheme for matching the transient reactions of the lower-order system and the higher-order plant [16,17]. Additionally, this method has certain drawbacks, including the inability to preserve the dominant poles of non-minimum phase complex systems in their reduced system and non-uniqueness (occasionally providing the same lower-order plant for the various large-scale models) [20,21].

Another well-liked model reduction technique for reducing higher-order linear systems in the frequency domain is the Routh approximation; however, it is only applicable to complex systems with strictly appropriate transfer functions [10]. Although [8] describes a stability equation technique for the model reduction in minimum phase higher-order plants, it is inconvenient for large-scale non-minimum phase systems. The factor division algorithm for approximating large-scale systems into lower-order approximants is provided in [11]. For the reduction of minimum and non-minimum large-scale models, Sinha and Pal suggested the pole clustering approach [13]. Some disadvantages of this method include the need for a gain adjustment factor to match the steady-state responses of the lower-order model with the large-scale plant and a tuning factor to match transient responses. One of the best techniques for figuring out the denominator of the simplified plant in the frequency domain is the Mihailov stability methodology [14]. As long as the higher-order model is stable, the reduced model is always stable according to the Mihailov stability criterion approach. The reduced model will be stable if the original system is stable because the reduced denominator polynomial is obtained so that the reduced system's Mihailov frequency characteristic matches the original system's characteristic.

"Evolutionary Techniques," a subject that draws comparisons to social or natural systems, has emerged as one of the most promising research areas from past many years. Because of their adaptability and capacity to optimize in intricate multimodal search spaces when applied to non-differentiable objective functions, evolutionary approaches are becoming more and more popular in the research community as design tools and issue solvers. Techniques like Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) emerged as promising algorithms for solving optimization challenges. GA is based on Darwinian concepts of biological evolution, reproduction, and "the survival of the fittest," and can be thought of as a general-purpose search method, optimization technique, or learning process. [22]. PSO draws inspiration from how herds of animals, schools of fish, and flocks of birds use information exchange to adapt to their surroundings, locate abundant food supplies, and evade predators. As part of a socio-cognitive study exploring the idea of collective intelligence in biological populations, the PSO approach was developed in the mid-1990s in an effort to mimic the coordinated, elegant motion of swarms of birds [23].

In order to simplify and develop a controller for the higher-order system, a new system reduction technique is presented in this article. Grey Wolf optimization is used to lower the error between the original system and the suggested reduced system. The Padé approximation technique's characteristics are preserved in the suggested approach [24, 25]. The GWO technique is used in this method to determine the denominator and numerator polynomials. To demonstrate the efficacy of the suggested model, one case is taken into consideration [8–12] and has been compared. The lower-order plant's stability for the stable original system is guaranteed by the straightforward model reduction that is suggested. The remainder of the paper is organized as in Section 2, where the system reduction problem statement is provided. Section 3 outlines the fundamental steps of the suggested model reduction technique. The new approach for designing a controller for the large-scale system is demonstrated in Section 4. One well-known numerical example from the literature is used in Section 5 to validate the suggested techniques. Section 6 provides the paper's conclusion.

### 2 Problem of Statement

Equation (1) provides the transfer function for an $n^{th}$ order linear dynamic single input single output continuous system:

$$H(s) = \frac{N(s)}{D(s)} = \frac{u_{n-1}s^{n-1} + \cdots + \mathrm{us} + u_0}{v_n s^n + \cdots + v_1 s + v_0} \quad (1)$$

where $u_i; 0 \leq i \leq n-1$ $and$ $v_i; 0 \leq i \leq n$ are known scalar constants.

Using equation (2), the goal is to create a $r^{th}(r < n)$ reduced order model such that $H_r(s)$ closely approximates $H(s)$ which is provided by [15]:

$$H_r(s) = \frac{N_r(s)}{M_r(s)} = \frac{k_{r-1}s^{r-1} + \cdots + k_1 s + k_0}{m_r s^r + \cdots + m_1 s + m_0} \quad (2)$$

Where $m_i; 0 \leq i \leq r-1$ $and$ $k_i; 0 \leq i \leq r$ are unknown scalar constant

## 3. Grey Wolf Optimization Algorithm

Grey Wolf Optimization (GWO) is a nature-inspired algorithm that can be effectively utilized for model order reduction [33-38]. The process begins with the initialization of a population of "wolves," each representing a potential model of varying order. These wolves are randomly placed in the solution space, and an objective function is defined to evaluate their fitness, typically by measuring the error between the original model and its reduced counterpart. The wolves are then sorted based on their fitness, identifying the Alpha (α), Beta (β), and Delta (δ) wolves as the top three solutions, with all others classified as Omega (ω).

The algorithm updates the positions of the wolves by using the positions of α, β, and δ, reflecting the social hierarchy and hunting behavior of Grey wolves. This involves mathematical updates to the model orders based on the best-found solutions, ensuring that the positions remain within feasible bounds. Convergence is checked by assessing whether the improvements in the objective function have firmed up or if the maximum number of iterations has been reached. If convergence criteria are met, the best-found model order is selected as the reduced model. Otherwise, the process iterates from the evaluation step until a satisfactory solution is achieved.

In summary, GWO for model order reduction starts by initializing a population of model candidates, evaluates and ranks them based on fitness, and then updates their positions according to the best solutions found. The algorithm iterates until convergence, efficiently balancing accuracy and complexity by mimicking the social behavior of Grey wolves [33-38]. The basic equations are illustrated in [35]

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Key actions in a search for grey wolves are:

- Following, tracking, and getting close to the target.
- When the target stops moving, follow it; after that, encircle and harass it.
- Take direct aim at the goal.

Equations (3) & (4) provide a mathematical description of the encircling process:

$$\overrightarrow{U} = \left|\vec{V}\overrightarrow{N_\iota}(t) - N(t)\right| \tag{3}$$

$$\vec{\mathrm{N}}(t+1) = \overrightarrow{N_\iota}(t) - \overrightarrow{W}.\overrightarrow{X} \tag{4}$$

where $\overrightarrow{W}$ and $\vec{V}$ are the coefficient vectors, the position vectors of the grey wolf and its prey are $\vec{N}$ and $\overrightarrow{N_\iota}$ , $t$ is the current iteration in equation (3) and (4). Equation (5) and (6) are used to find the coefficient vectors $\overrightarrow{W}$ and $\vec{V}$:

$$\overrightarrow{W} = 2\vec{b}.r_1 - \vec{b} \tag{5}$$

$$\vec{V} = 2.r_2 \tag{6}$$

where $r_1$ $and$ $r_2$ are random vectors and *b* is a variable changing from 2 to 0

When grey wolves identify potential prey, they encircle it to close in. During this process, the Betas and Deltas assist the Alphas in steering the hunt. The optimal solution is directed by the three leading positions—those of the Alphas, Betas, and Deltas—while the remaining solutions, including the Omegas, are adjusted. Equations (7) to (12) detail the updates for the positions of wolves α, β, and δ as they surround the prey

$$\overrightarrow{X_\alpha} = \left|\overrightarrow{V_1}\overrightarrow{N_\alpha} - \vec{N}\right| \tag{7}$$

$$\overrightarrow{X_\beta} = \left|\overrightarrow{V_2}\overrightarrow{N_\beta} - \vec{N}\right| \tag{8}$$

$$\overrightarrow{X_\delta} = \left|\overrightarrow{V_3}\overrightarrow{N_\delta} - \vec{N}\right| \tag{9}$$

Thus using equation (4)

$$\overrightarrow{N_1} = \overrightarrow{N_\alpha} - \overrightarrow{W_1}.\left(\overrightarrow{X_\alpha}\right) \tag{10}$$

$$\overrightarrow{N_2} = \overrightarrow{N_\beta} - \overrightarrow{W_2}.\left(\overrightarrow{X_\beta}\right) \tag{11}$$

$$\overrightarrow{N_3} = \overrightarrow{N_\delta} - \overrightarrow{W_3}.\overrightarrow{(X_\delta)} \tag{12}$$

and finally, update the position of the current search agent using equation (13)

$$\vec{N}(t+1) = \frac{(\overrightarrow{N_1}+\overrightarrow{N_2}+\overrightarrow{N_3})}{3} \tag{13}$$

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One of the advantages of GWO is that there are very less parameters to adjust. In model order reduction, GWO provides fast convergence, robust performance, minimal parameter tuning, and effective handling of complex systems, making it a powerful tool for simplifying large-scale models efficiently. The flowchart of Grey Wolf Optimization Technique is given in Figure 1

To evaluate the suggested technique for MOR a continuous time SISO system has been taken from literature. By lowering the performance index namely ISE, which is offered by the recommended technique, the present methodology has been put side by side. ISE is reduced using Grey Wolf Optimization.

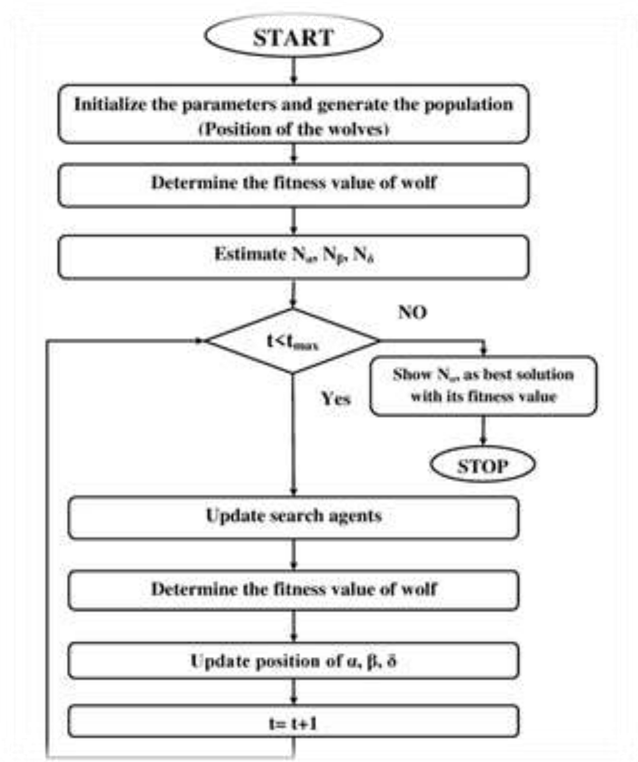


**Figure.1.** Flow chart of GWO

## 4 Design of PID Controller

Large-scale system simulation and controller design are challenging and time-consuming processes. The simulation time and controller design cost both rise in direct proportion to the dynamic system's complexity. It is possible to identify a "good" approximation model for the complex model and use it to create the controller in order to get around these kinds of restrictions. For the construction of feedback controllers, a higher dimensional model requires a large number of sensors to sense the state variables. Series controllers are therefore preferable to feedback controllers.

A reference model $(M(s))$ is developed based on the specified specifications to achieve the desired performance of the real-time dynamic system. This ensures that the closed-loop characteristic of the controlled plant with unity feedback is fully matched with the characteristic of the computed reference model. More information on how to calculate the reference system from the desired specification may be found in [26,27]. Examine a controller that uses proportional-integral derivatives (PIDs) to achieve the intended closed-loop behavior
.

$$G_c(s) = k_p + \frac{k_i}{s} + k_d s \tag{14}$$

From the closed-loop reference system $(M(s))$, the open-loop reference system $\left(\widetilde{M}(s)\right)$ is computed for the purpose of building the PID controller utilizing the estimated system.

$$\widetilde{M}(s) = \frac{M(s)}{1-M(s)} \tag{15}$$

The PID controller is made to ensure that an open-loop controlled system's performance is identical to that of the open-loop reference system.

$$G_c(s)G(s) = \widetilde{M}(s) \tag{16}$$

$$G_c(s) = \frac{\widetilde{M}(s)}{G(s)} = \frac{\sum_{i=0}^{2} e_i s^i}{s} \tag{17}$$

where $e_i (i = 0,1,2)$ are the Taylor series coefficients about *s=0,* and determined by using moment generating algorithm [12, 13]. The original system's transfer function, G(s), can be substituted with a comparable approximated model to reduce the amount of time needed for simulation and mathematical calculations. By comparing equation (14) and equation (17) as follows, the unknown scalar constants of the PID controller are determined:

$$k_p + \frac{k_i}{s} + k_d s = \frac{e_0 + e_1 s + e_2 s^2}{s} \tag{18}$$

$$\frac{k_i + k_p s + k_d s^2}{s} = \frac{e_0 + e_1 s + e_2 s^2}{s} \tag{19}$$

Therefore, $k_p = e_1, k_i = e_0, k_d = e_2$. After finding the scalar constants of the controller, the transfer function of the closed-loop plant can be written as

$$G_{cl}(s) = \frac{G_c(s)G(s)}{1+G_c(s)G(s)} \tag{20}$$

Now the closed loop transfer function $G_{cl}(s)$is reduced to lower order to obtain reduced closed loop transfer function $R_{cl}(s)$

## 5. Simulation Result and Comparison

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GWO is used to calculate the performance error index (ISE) below, which compares the suggested approach to a few other conventional and recently suggested system reduction techniques.

ISE is reduced using Grey Wolf Optimization.

$$ISE = \int_0^\infty [\emptyset(t) - \emptyset_r(t)]^2 dt$$

where $\emptyset(t) \rightarrow$ Step response of original Higher Order System

$\emptyset_r(t) \rightarrow$ Step response of Lower Order System

**Example 1:** Consider a plant transfer function taken from [39]

$$G_p(s) = \frac{2s^5 + 3s^4 + 16s^3 + 20s^2 + 8s + 1}{2s^6 + 33.6s^5 + 155.94s^4 + 209.5s^3 + 102.42s^2 + 18.3s + 1}$$

$$M(s) = \frac{25}{s^2 + 20s + 25}$$

The equivalent open loop transfer function is obtained by using equation (15)

$$\widetilde{M}(s) = \frac{M(s)}{1 - M(s)} = \frac{25}{s(s+20)}$$

In order to match the response of closed loop system $G_{cl}(s)$ exactly with that of the reference model M(s), the required controller is given by $G_c(s)G_p(s) = \widetilde{M}(s)$

$$G_c(s) = \frac{\widetilde{M}(s)}{G_p(s)} = \frac{e_0 + e_1 s + e_2 s^2 + \cdots}{s} = \frac{k_i + k_p s + k_d s^2}{s}$$

$$G_c(s) = \frac{\widetilde{M}(s)}{G_p(s)}$$

$$= \frac{4.242s^7 + 131.3s^6 + 1684s^5 + 11810s^4 + 4.9e04s^3 + 1.17e05s^2 + 140780s + 62500}{s^7 + 21.9s^6 + 217.8s^5 + 1217s^4 + 3991s^3 + 7057s^2 + 5090s}$$

$$= \frac{1}{s}(1.25 + 12.8125s - 0.6156s^2 + \cdots)$$

Hence, $k_p = 12.8125, k_i = 1.25, k_d = -0.6156$. By using this controller, the closed loop system is computed as

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$$G_{cl}(s) = \frac{G_p(s)G_c(s)}{1 + G(s)G_c(s)}$$

$$G_{cl}(s) = \frac{-1.2312s^7 + 23.7782s^6 + 31.0879s^5 + 196.4380s^4 + 271.3252s^3 + 126.8844s^2 + 22.8125s + 1.2500}{0.7688s^7 + 57.3782s^6 + 187.0279s^5 + 405.9380s^4 + 373.7452s^3 + 145.1844s^2 + 23.8125s + 1.2500}$$

The high order closed loop transfer function $G_{cl}(s)$ is reduced to second order model $R_{cl}(s)$ using proposed method

$$R_{cl}(s) = \frac{0.0207s + 0.04223}{0.02298s^2 + 0.05382s + 0.04225}$$

The lower order plant obtained by the proposed technique is given as follows:

$$R_p(s) = \frac{0.161s + 0.2883}{1.383s^2 + 3.144s + 0.2881}$$

The time responses of the full-order and reduced models produced by the suggested approach as well as other conventional techniques are displayed in Figure 2. This figure shows that the system's reaction achieved using the suggested method is substantially closer to the full-order model's response. Table 1 shows a quantitative comparison of the lower-order models calculated using the suggested method and a few other well-known approaches in terms of performance indicies viz. ISE, IAE, ITAE and ITSE values. It is evident that out of all the performance indexes, the suggested approach provides the lowest values. As a result, the suggested approach is both better and on par with certain other common system reduction techniques.

Table 1:Comparison of various MOR methods in terms of Performance Indicies

| Reduced Techniques | Reduced Model | ISE | ITSE | IAE | ITAE |
|---|---|---|---|---|---|
| Proposed Method | $\frac{0.161s + 0.2883}{1.383s^2 + 3.144s + 0.2881}$ | 0.00056318 | 0.0099 | 0.1734 | 13.3671 |
| Improved Pade Approximation and Mihailov Stability Criterion[40] | $\frac{5.934s + 1}{101s^2 + 16.23s + 1}$ | 0.1322 | 2.0962 | 2.4442 | 60.5823 |
| Routh approximation[10] | $\frac{0.0879s + 0.011}{s^2 + 0.2012s + 0.011}$ | 0.0063 | 0.0727 | 0.4903 | 11.6848 |
| Stability Equation[8] | $\frac{8s + 1}{101s^2 + 18.3s + 1}$ | 0.0250 | 0.3555 | 1.0488 | 26.4291 |
| Pade Approximation[12] | $\frac{0.09711s + 0.0001}{s^2 + 0.0987s + 0.0001}$ | 0.0362 | 0.5689 | 2.088 | 15.365 |
| Routh Stability[9] | $\frac{7.106s + 1}{87.38s^2 + 15.94s + 1}$ | 0.2594 | 0.3548 | 1.543 | 17.365 |
| Balanced truncation[29], Schur method[30] | $\frac{0.0961s + 0.0042}{s^2 + 0.1342s + 0.0046}$ | 0.0487 | 0.8941 | 1.854 | 12.548 |
| Balanced truncation and factor division[31] | $\frac{0.188s + 0.04}{2s^2 + 0.6s + 0.04}$ | 0.0012 | 0.0069 | 0.1564 | 2.1661 |

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| | | | | | |
|---|---|---|---|---|---|
| Hankel-norm approximation[32] | $\frac{0.0492s + 0.0896}{s^2 + 0.9811s + 0.0953}$ | 0.6133 | 69.4583 | 10.8243 | 1.174e+03 |

The full-order and reduced models' time responses, as determined by the suggested approach and other conventional techniques, are displayed in Figure 2. The reaction of the system produced using the suggested method is far closer to the response of the full-order model, as this figure shows. In terms of ISE, IAE, ITAE and ITSE values, Table 1 presents a quantitative comparison of the lower-order models calculated using the suggested method and a few other well-known approaches. It is evident that the suggested approach provides the lowest values across all performance indexes. As a result, the suggested approach is better than several other common system reduction techniques while remaining similar.

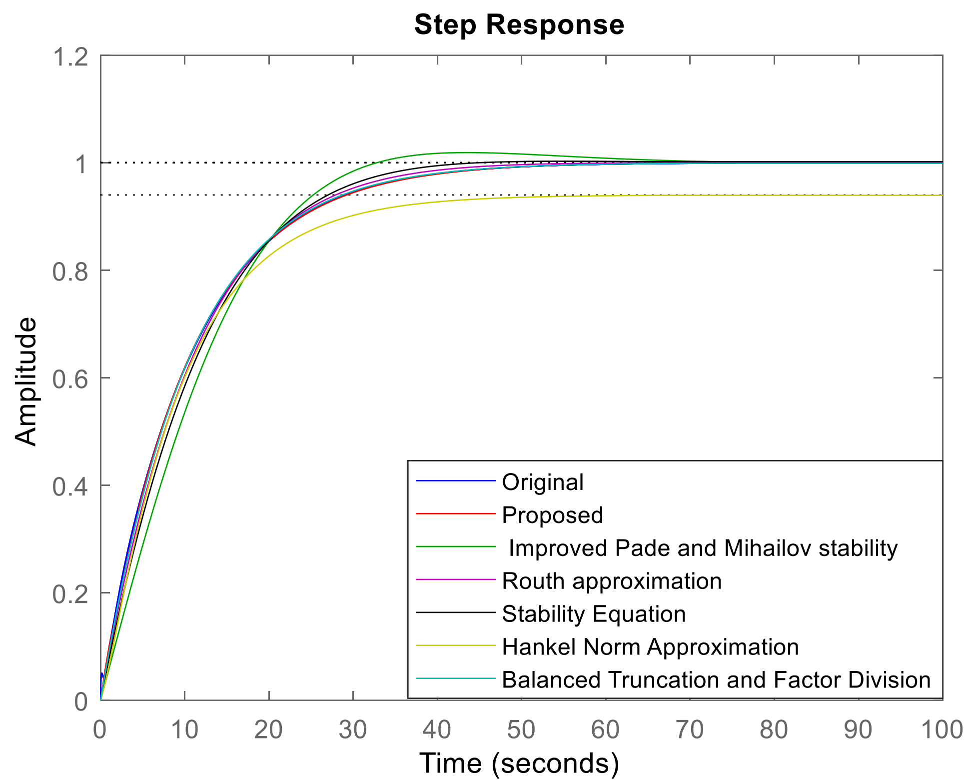


Figure 2 Qualitative comparisons of system reduction methods in terms of the step response

The step responses of the reference model and the closed-loop model with PID controllers, which were produced by employing the optimization technique, are contrasted in Figure 3. It is evident that in both the steady state and the transient zone, every response of the closed-loop plant with PID controllers roughly matches the reference model. Table 2 lists the closed-loop system with controllers' time-domain specifications. It is clear from this table that the time-domain specifications of the closed-loop system with the controller created using optimization technique are almost identical to those of the closed-loop plant with the controller computed using the original system. Compared to designing the controller using the original full-order

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system, designing the controller using the optimization technique is rather simple. Additionally, this table shows that the closed-loop plant with the controllers has the same time-domain parameters as the reference system.

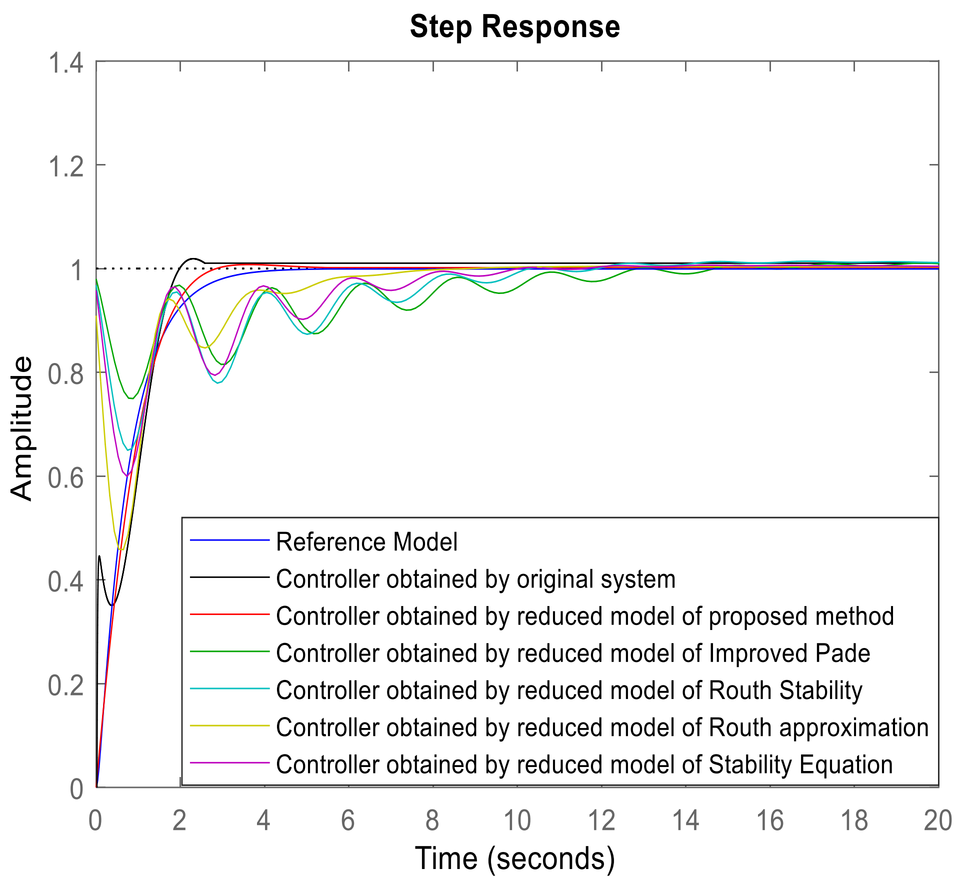


Figure 3: Comparison of Step response of close loop performance

Table 2 lists the closed-loop system with controllers' time-domain specifications. It is clear from this table that the time-domain specifications of the closed-loop system with the controller created using optimization technique are almost identical to those of the closed-loop plant with the controller computed using the original system. Compared to designing the controller using the original full-order system, designing the controller using the optimization technique is rather simple. Additionally, this table shows that the closed-loop plant with the controllers has the same time-domain parameters as the reference system.

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Table 2:Comparison of closed loop response in terms of transient parameters for various MOR techniques

| Reduced Techniques | Reduced Model | Rise Time | Settling Time | Peak Overshoot | Peak Time |
|---|---|---|---|---|---|
| - | Reference Model | 1.6462 | 2.9758 | 0.9993 | 5.4654 |
| - | Original System | 1.2234 | 1.7183 | 1.6016 | 0 |
| Proposed Method | $\frac{0.161s+0.2883}{1.383s^2+3.144s+0.2881}$ | 1.6080 | 2.3948 | 1.0078 | 3.6180 |
| Improved Pade Approximation and Mihailov Stability Criterion[40] | $\frac{5.934s+1}{101s^2+16.23s+1}$ | 4.2307 | 32.2273 | 1.0108 | 21.7507 |
| Routh approximation[10] | $\frac{0.0879s+0.011}{s^2+0.2012s+0.011}$ | 5.5007 | 6.8630 | 1.0042 | 12.5141 |
| Stability Equation[8] | $\frac{8s+1}{101s^2+18.3s+1}$ | 8.0030 | 9.5900 | 1.0065 | 14.525 |
| Pade Approximation[12] | $\frac{0.09711s+0.0001}{s^2+0.0987s+0.0001}$ | 8.0117 | 9.7275 | 1.0068 | 14.6093 |
| Routh Stability[9] | $\frac{7.106s+1}{87.38s^2+15.94s+1}$ | 3.9812 | 25.3871 | 1.0141 | 16.8365 |
| Balanced truncation[29], Schur method[30] | $\frac{0.0961s+0.0042}{s^2+0.1342s+0.0046}$ | 2.109 | 71.7394 | 1.0255 | 29.2094 |
| Balanced truncation and factor division[31] | $\frac{0.188s+0.04}{2s^2+0.6s+0.04}$ | 0.9415 | 5.2054 | 1.0025 | 10.6508 |
| Hankel-norm approximation[32] | $\frac{0.0492s+0.0896}{s^2+0.9811s+0.0953}$ | 4.0188 | 5.6033 | 1.0053 | 10.6508 |

## 6. Conclusion and Future Scope

This article suggests a novel method for lowering the complex SISO systems' transfer function's order. The Grey Wolf Optimization approach is used in this method to calculate the denominator and numerator coefficients of the simplified model. One typical numerical example has been used to validate this technique, and Fig. 2 shows a graphic comparison of the time responses of the reduced-order plants and the full-order model. Table 1 presents a quantitative comparison of different performance indices, including ISE and ITSE. If the higher-order plant is stable and precisely matches the steady-state value of the real system, this procedure validates the stability of the approximation plant. Additionally, a novel controller design methodology is suggested. Both the large-scale system and the smaller models are used in the controller design process. Compared to the original large-scale system, the controller's design is simpler and easier when employing the simplified model. Fig. 3 and Table 1 validate and verify this design process. The suggested works in this contribution are implemented on single input single output (SISO) LTI continuous systems, but they can also be expanded to large-scale discrete and multi-input multi-output (MIMO) systems.

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