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A Study of Bianchi Type cosmological models with bulk viscous fluid in modified theory of gravity

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Abstract:
This paper introduces non-singular Bianchi Type I and V cosmological models utilizing bulk viscous fluid within the context of f (R, T) gravity theory. Exact solutions to the field equations are obtained by choosing a specific form of the function f (R, T).

Keywords: Bianchi Type-I, Cosmological Models, Modified Gravity, Scale Factor Evolution, Field Equations, Cosmological Solutions.

1. Introduction

The most basic depiction of the expanding cosmos is provided by Friedmann-Robertson-Walker models, characterized by spatial homogeneity and isotropy. These models serve as a reasonable global approximation of the contemporary universe. It is, however, illogical to presume that the initial phases of the universe's evolution can be adequately characterized by the uniform expansion forecasted by these models. The objective of contemporary cosmology is to examine the historical development, current condition, and future progression of the cosmos. Recent observational data suggest that our universe is undergoing acceleration (Riess et al. [1], Perlmutter et al. [2]).

Observations, including cosmic microwave background radiation (Spergel et al. [3]) and large-scale structure (Tegmark et al. [4]), furnish indirect evidence for the late-time fast expansion of the universe.This acceleration is elucidated through the concept of dark energy. In light of the late-time acceleration of the cosmos and the presence of dark energy and dark matter, numerous modified theories of gravitation have been offered as alternatives to Einstein's general theory of relativity. Among them, the f(R) gravity hypothesis is of significant cosmic importance. f(R) gravity theory has been demonstrated to be a viable alternative to general relativity, remaining consistent during the dark epoch. It has been proposed that cosmic acceleration may be attained by substituting Einstein's Hilbert action of general relativity with a generic function of the Ricci scalar. R. Nojiri and Odintsov [5] created a comprehensive framework for the unification of the matter-dominated era with the accelerated phase for scalar fields.

Tensor theory or black fluid. Nojiri and Odintsov [6] provided a comprehensive assessment of modified gravity theories, seen as a gravitational alternative to dark energy. Bertolami et al. [7] presented a generalization of f(R) gravity theory by incorporating an explicit coupling of an arbitrary function of the Ricci scalar R to the matter Lagrangian density. Shamir [8] has proposed a feasible f(R) gravity model that unifies early-time inflation with late-time acceleration. Shamir and Jhangeer [9] examined static plane symmetric vacuum solutions in f(R) gravity within (n + 1) dimensional spacetime. Adhav [10] has investigated a Bianchi type-III cosmological model within the framework of f(R) gravity, incorporating the influence of cosmic strings. Several authors, including Misner, Murphy, and Belinsky and Khalatnikov, have examined the influence of viscosity and the resulting dissipative mechanisms in cosmology. The elevated entropy per baryon in the microwave background offers valuable insight into the characteristics of the early cosmos. A potential explanation for this substantial entropy per baryon is that it was produced by physical dissipative processes occurring at the onset of evolution. Dissipative processes may be responsible for the attenuation of early anisotropies (Weinberg [15]). Misner [11, 12] proposed that neutrino viscosity in the early universe may have significantly diminished the current anisotropy of the black-body during the evolutionary process. Belinsky and Khalatnikov [14] delineated some general properties of anisotropic cosmological models in the context of viscosity. Bulk viscosity is the sole dissipative phenomenon present in FRW models and plays a crucial role in inducing the accelerated expansion of the universe, referred to as the inflationary phase, as examined by Setare and Sheyki [16]. Murphy [13] has derived a zero curvature Friedmann-Robertson-Walker model solely incorporating bulk viscosity, which possesses the intriguing characteristic that the big-bang singularity emerges in the infinite past. Roy and Tiwari [17] introduced plane symmetric solutions to Einstein's field equations that depict inhomogeneous cosmological models characterized by a viscous fluid and constant bulk viscosity. Szyddowski and Heller [18] developed models of the universe containing interacting matter and radiation, incorporating dissipation attributed to bulk viscosity. Mohanty and Pradhan [19] derived a category of precise non-static solutions in closed elliptic form. Robertson-Walker spacetime populated by a viscous fluid in the presence of an attractive scalar field. Banerjee et al. [20] derived several Bianchi type-I solutions for stiff matter, assuming that the shear viscosity coefficient is a power-law function of energy density. Goener and Kowalski [21] devised a technique for acquiring irrotational anisotropic properties.

Viscous fluid solutions of a Bianchi type I model featuring a barotropic equation of state. Banerjee and Sanyal [22] introduced an irrational Bianchi type V model influenced by shear and bulk viscosity, along with heat flow. Coley [23], along with Coley and Hoogan [24], expanded upon the research of Coley and Tupper [25] by examining diagonal Bianchi type-V imperfect fluid models that incorporate both viscosity and thermal conditions, with and without the inclusion of a cosmological factor. Bali and Meena [26] have examined tilted cosmological models populated with disordered radiation for ideal fluid and thermal conduction. Bali and Sharma [27] investigate a tilted Bianchi type I cosmological model for a perfect fluid distribution in the presence of a magnetic field. Bali and Anjali [28] introduced Bianchi type-I bulk viscous fluid string dust magnetized cosmological models within the framework of general relativity. Adhav et al. [29] investigated Bianchi type-III anisotropic cosmological models with a variable Λ. Baghel and Singh [30] examined a spatially homogeneous and anisotropic Bianchi type-V spacetime characterized by a bulk viscous fluid source, together with a time-varying gravitational constant G and cosmological term Λ. Numerous authors have examined the significance of bulk viscosity in the primordial development of the cosmos across various physical frameworks. Harko et al. [31] formulated a variant of Einstein's gravitational theory, termed f (R; T) gravity theory, in which the gravitational Lagrangian is an arbitrary function of the Ricci scalar R and the trace T of the energy-momentum tensor Tij. The dependence on T may be attributed to exotic imperfect fluids or quantum phenomena. The authors have formulated the field equations of f (R, T) gravity by altering the action of the gravitational field equations concerning the metric tensor, presenting a physically plausible model based on a specific selection of the function f (R; T). Subsequently, other authors, including Myrzabulov [32], Adhav [33], Reddy et al. [34], Chaubey and Shukla [35], Ram et al. [36], and Chandel and Ram [37], introduced spatially homogeneous Bianchi type cosmological models incorporating a perfect fluid inside the framework of f (R, T) gravity theory. Samanta [38] examined the Kantowski-Sachs spacetime cosmological model populated with perfect fluid matter inside the framework of f (R, T) gravity. Additionally, Reddy et al. [39] and Ram and Priyanka [40] have examined five-dimensional Kaluza-Klein cosmological models populated with perfect fluid within the framework of f (R, T) gravity theory. Naidu et al. [41] examined a Bianchi type-V bulk viscous string cosmological model within the framework of f (R, T) gravity theory. Reddy et al. [42] examined LRS Bianchi type II spacetime and derived solutions to the field equations incorporating a cosmic string and bulk viscous fluid inside the f (R, T) theory of gravity. Recently, Ahmed and Pradhan [43] examined a

10.48047/jocaaa.2023.31.04.29

cosmological model with f (R T) gravity of Bianchi type V by positing f(R.T) = $f_1(R) + f_2(T)$. Chakraborty et al.

Al. [44] formulated an alternative gravity theory f (R, T) addressing the dark energy problem. Recently, Sharif and Zubair examined Bianchi type-I anisotropic models in f (R, T) gravity theory. Sahoo et al. [46] examined an axially symmetric space-time in the presence of a perfect fluid source within the context of f (R, T) gravity theory. Mishra and Sahoo [47] examined Bianchi type VI cosmological models populated by perfect fluid within the context of f (R, T) gravity theory. The spatially homogenous and entirely anisotropic Bianchi type-II cosmological solutions of massive strings in the presence of a magnetic field within the f (R, T) theory of gravity have been examined by Sharma and Singh [48]. Singh and Singh [49] demonstrated the cosmological plausibility of reconstructing an alternative gravitational theory, specifically the modified f (R, T) gravity theory. Inspired by the aforementioned works, we examine novel classes of spatially homogenous Bianchi type I and V bulk viscous fluid cosmological models inside the f (R, T) theory of gravity. We will also explore certain physical and kinematical characteristics of the cosmological models.

1.1. Equation of Field

We postulate that cosmic matter may be described by the energy-momentum tensor of an imperfect bulk viscous fluid.

$$T_{ij} = (\rho + \bar{p})u_i u_j - \bar{p} g_{ij} \tag{1}$$

Where as $\bar{p}$, the effective pressure ids given by

$$\bar{p} = p - \gamma u_i^i \tag{2}$$

Satisfies equation of states in linear form

$$p = \varepsilon\rho, \qquad 0 \leq \varepsilon \leq 1 \tag{3}$$

Here, p is the equilibrium pressure, ρ represents the energy density of matter, γ is the coefficient of bulk viscosity, and u^i is the flow vector of the fluid, meeting the condition $u^i u_i = 1$. The semicolon represents covariant differentiation. In thermodynamic contexts, the bulk viscosity coefficient C is positive, indicating that viscosity drives the dissipative pressure $\bar{p}$ towards negative values. Nonetheless, the adjustment made to the thermodynamic pressure p due to bulk viscous pressure is minimal. Consequently, the dynamics of cosmic evolution are not substantially affected by the incorporation of the viscous element in the energy-momentum tensor.

The field equations in f (R, T) gravity theory with the specific selection of the function f (R,T), expressed as

$$f(R,T) = R + 2f(T) \tag{4}$$

When the matter source is a bulk viscous fluid, these are provided by (Reddy et al. [30]):

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [2\bar{p}f'(T) + f(T)]g_{ij} \tag{5}$$

We additionally select that f(T) = λT, where λ is a constant. This f (R, T) gravity model is identical to a cosmological model with an effective cosmological constant Λ equal to H^2, where H represents the Hubble function. It is also noteworthy that, generally for this selection of f (R, T), the gravitational coupling transforms into an effective and time-dependent coupling, represented as $G_{eff} = G \pm 2f'(T)$. Consequently, the term 2f(T) in the gravitational action alters the gravitational interaction between matter and curvature, substituting G with a dynamic gravitational coupling parameter.

1.2. Bianchi type I model

We examine a spatially homogeneous Bianchi type-I metric defined as:

$$ds^2 = dt^2 - A^2dx^2 - B^2dy^2 - C^2dz^2 \tag{6}$$

where A, B, and C are cosmic scale functions.

10.48047/jocaaa.2023.31.04.29

To examine the kinematic properties of the models, we introduce the expansion scalar (θ), shear scalar (σ), average Hubble parameter (H), and anisotropy parameter (A_m) for the metric (6) as follows:

$$V = a^3 = ABC \tag{7}$$

$$H = \frac{1}{3}\frac{\dot V}{V} = \frac{1}{3}\left(\frac{\dot A}{A} + \frac{\dot B}{B} + \frac{\dot C}{C}\right) \tag{8}$$

$$\theta = 3H = \frac{\dot V}{V} = \left(\frac{\dot A}{A} + \frac{\dot B}{B} + \frac{\dot C}{C}\right) \tag{9}$$

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot A}{A}\right)^2 + \left(\frac{\dot B}{B}\right)^2 + \left(\frac{\dot C}{C}\right)^2\right] - \frac{1}{6}\theta^2 \tag{10}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{11}$$

Where as $\Delta H_i = H_i - H, (i = 1,2,3) and\ H_1 = \frac{\dot A}{A}, H_2 = \frac{\dot B}{B}, H_3 = \frac{\dot C}{C}$ as the parameter of directional Hubble.

An essential observational metric is the deceleration parameter (DP), defined as

$$q = -\frac{a\ddot a}{\dot a^2} \tag{12}$$

Here, the dot signifies derivatives with respect to time, t. The sign of q determines whether the model inflates or not. The positive sign of q corresponds to a conventional decelerating model, whereas the negative sign indicates inflation. For the metric (6), the field equations (1), (4), and (5) in comoving coordinates yield the following set of equations.

$$\frac{\ddot B}{B} + \frac{\ddot C}{C} + \frac{\dot B\dot C}{BC} = \lambda\rho - (8\pi + 3\lambda)\bar p \tag{13}$$

$$\frac{\ddot C}{C} + \frac{\ddot A}{A} + \frac{\dot C\dot A}{CA} = \lambda\rho - (8\pi + 3\lambda)\bar p \tag{14}$$

$$\frac{\ddot A}{A} + \frac{\ddot B}{B} + \frac{\dot A\dot B}{AB} = \lambda\rho - (8\pi + 3\lambda)\bar p \tag{15}$$

$$\frac{\dot A\dot B}{AB} + \frac{\dot B\dot C}{BC} + \frac{\dot C\dot A}{CA} = (8\pi + 3\lambda)\rho - \lambda\bar p \tag{16}$$

These are four highly non-linear equations involving six unknowns: A, B, C, ρ, $\bar p$, and C. Consequently, to identify a consistent solution to these equations, acceptable assumptions must be established to simplify the physics or mathematics involved. By subtracting Equation (14) from Equation (13), Equation (15) from Equation (14), and Equation (15) from Equation (13), and thereafter integrating the resulting equations, we derive

$$\frac{B}{A} = d_1 e^{\left(c_1 \int \frac{dt}{a^3}\right)} \tag{17}$$

$$\frac{C}{B} = d_2 e^{\left(c_2 \int \frac{dt}{a^3}\right)} \tag{18}$$

$$\frac{A}{B} = d_3 e^{\left(c_3 \int \frac{dt}{a^3}\right)} \tag{19}$$

Where $c_1, c_2, c_3\ and d_1, d_2, d_3$ are integration constants which satisfies the relations

$$c_1 + c_2 + c_3 = 0\ and\quad d_1 d_2 d_3 = 1 \tag{20}$$

From equations (17) to (19), we may clearly derive the scale factors A, B, and C metric functions as

$$A = a p_1 e^{\left(q_1 \int \frac{dt}{a^3}\right)} \tag{21}$$

$$B = a p_2 e^{\left(q_2 \int \frac{dt}{a^3}\right)} \tag{22}$$

$$C = ap_3 e^{\left(q_3 \int \frac{dt}{a^3}\right)} \tag{23}$$

Where $p_1 = (d_1^{-2} d_2^{-1})^{\frac{1}{3}}$, $p_2 = (d_1 d_2^{-1})^{\frac{1}{3}}$, $p_1 = (d_1 d_2^{-1})^{\frac{1}{3}}$ (24)

And $q_1 = -\frac{2c_1+c_2}{3}$, $q_2 = \frac{c_1-c_2}{3}$, $q_3 = \frac{c_1+2c_3}{3}$ (25)

Where the constants q_1, q_2, q_3 and p_1, p_2, p_3 are in the relations as

$$q_1 + q_2 + q_3 = 0 \; and \quad p_1 p_2 p_3 = 1 \tag{26}$$

It is evident that we ascertain the scale factors A, B, and C from Eqs. (21)-(23) given that the average scale factor a(t) is known. In the construction of physically realistic cosmological models, the Hubble parameter and the deceleration parameter (DP) are of significant importance. It has been a regular practice to utilize a continuous DP. Berman and Gomide proposed a law of variation for the Hubble parameter inside the FRW model that results in a constant value of DP, which subsequently leads to power-law and exponential forms of the average scale factor. The new observations of Type Ia supernovae (Riess et al. [1], Perlmutter et al. [2]) indicate that the universe is currently experiencing rapid expansion, having previously had decelerated expansion. The universe is transitioning between these two phases of expansion at present. Consequently, DP is anticipated to be variable rather than constant, functioning as a function of time.Several writers have suggested time-dependent variants of DP and generated a differential expression for the average scale factor of the model. Conversely, certain authors have selected the average scale factor and subsequently derived the time-dependent DP.Equation (12) can also be expressed as

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) \tag{27}$$

Abdussattar and Prajapati [53] offered a solution for the time-dependent form of q as.

$$q = (\beta - 1) - \left(\frac{\alpha}{t^2}\right) \tag{28}$$

Figure 1 illustrates the behavior of the deceleration parameter over time.
Equation (12) can be integrated to provide the scale factor a(t) as

$$a(t) = e^{\delta} e^{\int \frac{dt}{\int (1+q)dt + \gamma}} \tag{29}$$

where γ and δ are arbitrary constants of integration. By substituting Equation (28) into Equation (29) and integrating the result, Abdussattar and Prajapati [53] generated three distinct forms of a(t), the simplest of which is expressed as

$$a(t) = e^{\delta} \left(t^2 + \frac{\alpha}{\beta}\right)^{\frac{1}{2\beta}} \tag{30}$$

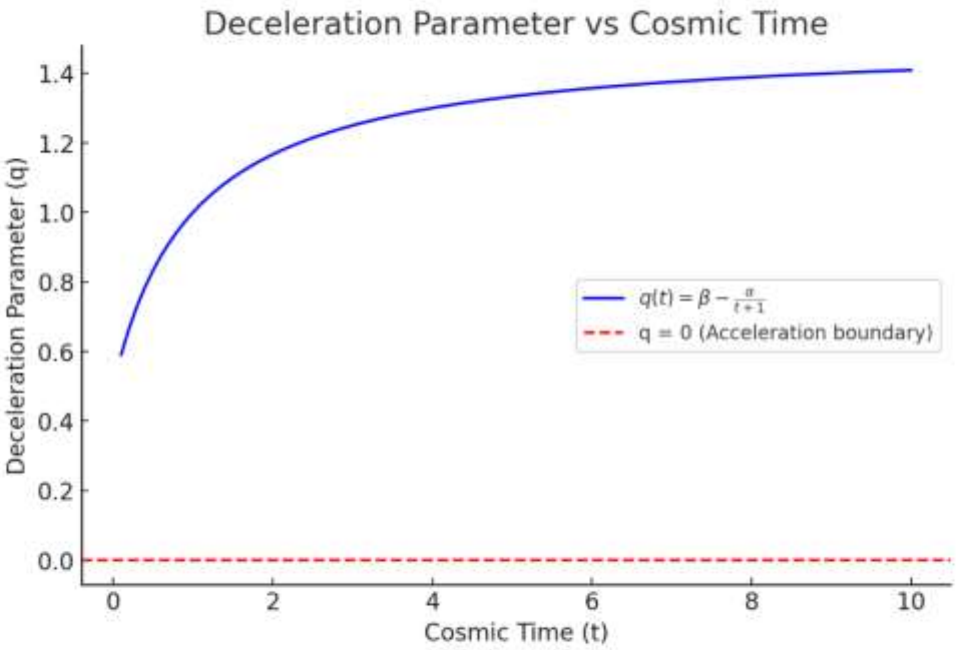

Figure 1 illustrates the behavior of the deceleration parameter over time.

They have also examined the non-singular bouncing FRW cosmological models with a(t) defined by Equation (30).

We use this form of (t) to ascertain the scale factors A, B, and C from Equations (21)-(23). Utilizing the value of a(t) in equations (21)-(23) renders the integration somewhat challenging. Consequently, we set δ=0 and β=3/2 in Equation (30) such that

$$a(t) = e^{\delta}\left(t^2 + \frac{2\alpha}{\beta}\right)^{\frac{1}{\beta}} \tag{31}$$

By substituting Eq. (31) into Eqs. (21)–(23) and integrating, we derive expressions for the metric functions as follows:

$$A = p_1\left(t^2 + \frac{2\alpha}{\beta}\right)^{\frac{1}{\beta}}\ e^{\left[q_1 tan^{-1}\left(\frac{3}{2\alpha}\right)^{\frac{1}{2}} t\right]} \tag{32}$$

$$B = p_2\left(t^2 + \frac{2\alpha}{\beta}\right)^{\frac{1}{\beta}}\ e^{\left[q_2 tan^{-1}\left(\frac{3}{2\alpha}\right)^{\frac{1}{2}} t\right]} \tag{33}$$

$$C = p_3\left(t^2 + \frac{2\alpha}{\beta}\right)^{\frac{1}{\beta}}\ e^{\left[q_3 tan^{-1}\left(\frac{3}{2\alpha}\right)^{\frac{1}{2}} t\right]} \tag{34}$$

For the model described by metric functions in (32)-(34), the energy density ρ and the bulk viscous pressure $\bar{p}$ are provided by

$$\rho = \frac{1}{9(8\pi + 2\lambda)(8\pi + 4\lambda)\left(t^2 + \frac{2\alpha}{3}\right)^2\ \left[t^2 \begin{Bmatrix} (8\pi + 3\lambda)(12 + 18q_1^2) - (q_1^2 + q_3^2)(144\pi + 64\lambda) - \\ 18\lambda(q_2 + q_3) - \lambda(8t + 12\alpha(q_2 + q_3) \end{Bmatrix}\right]}$$

….. (35)

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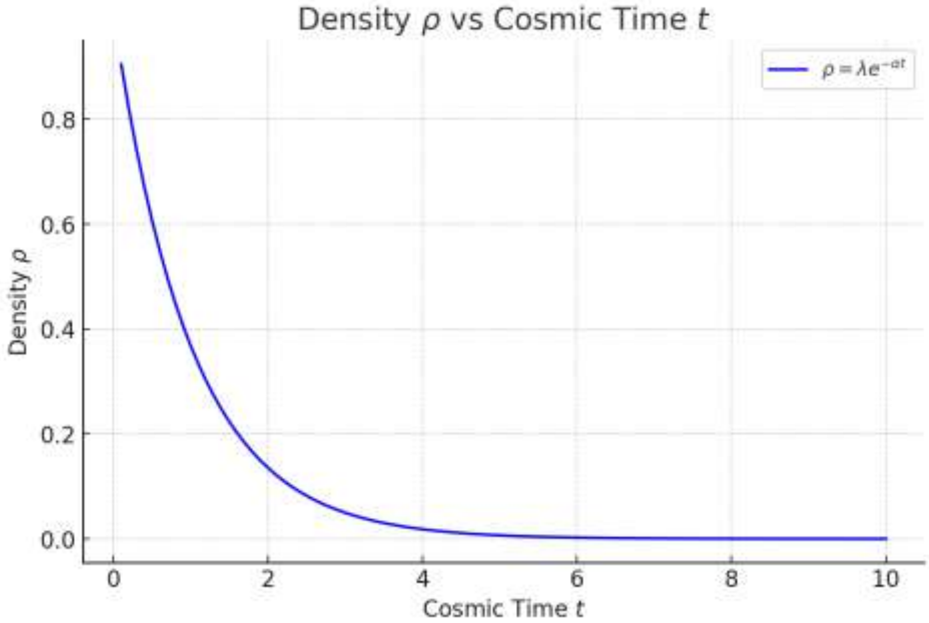

Figure2. The plot of density ρ verses cosmic time t, λ=1, α=1.

$$\begin{aligned}\bar{p} = \frac{1}{9\lambda\left(t^2+\frac{2\alpha}{3}\right)^2}&[(18+q_1^2+q_2^2+q_3^2)-12\\&+\tfrac{(8\pi+3\lambda)}{(8\pi+2\lambda)(8\pi+4\lambda)}(12+18q_1^2(8\pi+3\lambda))\\&-(q_2^2+q_3^2)(144\pi+64\lambda)-18(q_2+q_3))\,t^2\\&-\tfrac{(8\pi+3\lambda)}{(8\pi+2\lambda)(8\pi+4\lambda)}\lambda(8t+12\alpha(q_2+q_3))]. \qquad (36)\end{aligned}$$

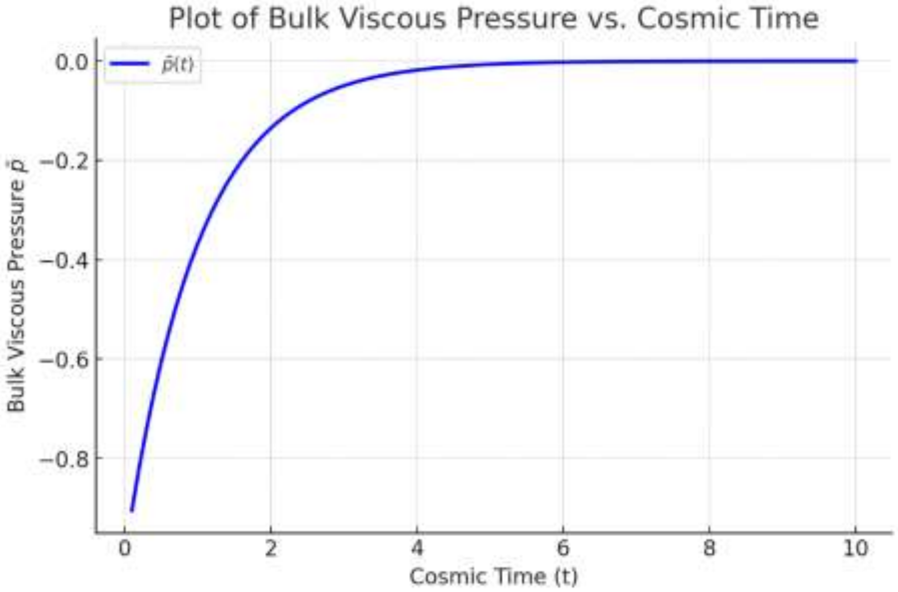

Figure3. The plot of bulk viscous pressure $\bar{p}$ verses cosmic time t, λ=1, α=1.

Figures 2 and 3 illustrate the dynamics of energy density and bulk viscous pressure in relation to cosmic time, respectively. The barotropic equation of state parameter can be utilized to derive the coefficient of bulk viscosity, which is determined from Equations (4) and (37) as follows:

$$C= \frac{t}{54\lambda(8\pi+4\lambda)(8\pi+2\lambda)\left(t^2+\frac{2a}{3}\right)} * \big[\varepsilon\,\lambda(8\lambda+3\lambda)(12+18q_1^2)-(q_1^2+q_2^2)(144+64\lambda)- 18\lambda(q_2+q_3)-(8\pi+3\lambda)(12+18q_1^2)(8\pi+3\lambda)\big)-(q_1^2+q_2^2)(144+64\lambda)-18\lambda(q_2+q_3)$$

$$-\frac{1}{54\lambda(8\pi+4\lambda)(8\pi+2\lambda)t\left(t^2+\frac{2a}{3}\right)} * [\varepsilon t^2\big(8t+12\alpha(q_2+q_3)\big)+18(8\pi+2\lambda)(8\pi+4\lambda)(q_1^2+q_2^2+q_3^2) -12(8\pi+2\lambda)(8\pi+4\lambda)+(8\pi+3\lambda)\lambda(8t+12\alpha(q_2+q_3))] \quad (37)$$

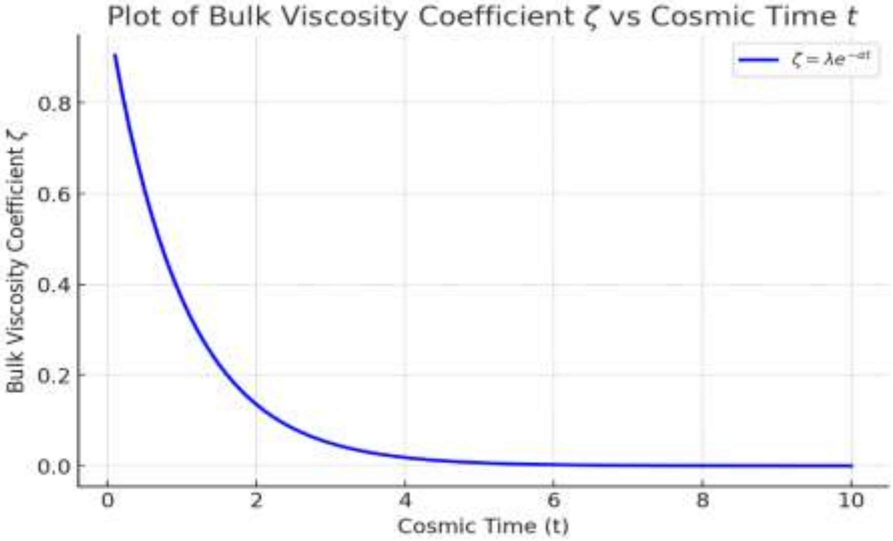

Figure 4 illustrates the behavior of the bulk viscosity coefficient throughout cosmic time.

For model 1, the energy density criteria ρ + p ≥ 0 and ρ + 3p ≥ 0 are equally satisfied, as illustrated in Fig. 5.

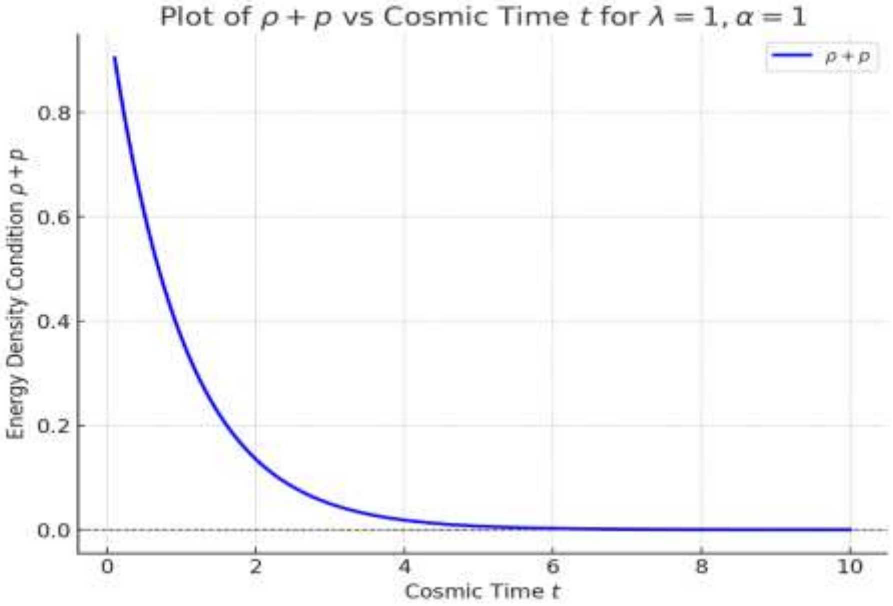

Figure 5. The plot of Energy density condition ρ+p verses cosmic time t, λ=1, α=1.

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We now explore the physical and kinematical behaviors of the Bianchi type-I cosmological model with metric functions specified by the equations. (32)-(34). The directional Hubble parameters and the average Hubble parameter are provided by

$$H_1 = \frac{2t}{3\left(t^2+\frac{2\alpha}{3}\right)}(3q_1+1) \tag{38}$$

$$H_1 = \frac{2t}{3\left(t^2+\frac{2\alpha}{3}\right)}(3q_2+1) \tag{39}$$

$$H_1 = \frac{2t}{3\left(t^2+\frac{2\alpha}{3}\right)}(3q_3+1) \tag{40}$$

$$H = \frac{2t}{3\left(t^2+\frac{2\alpha}{3}\right)} \tag{41}$$

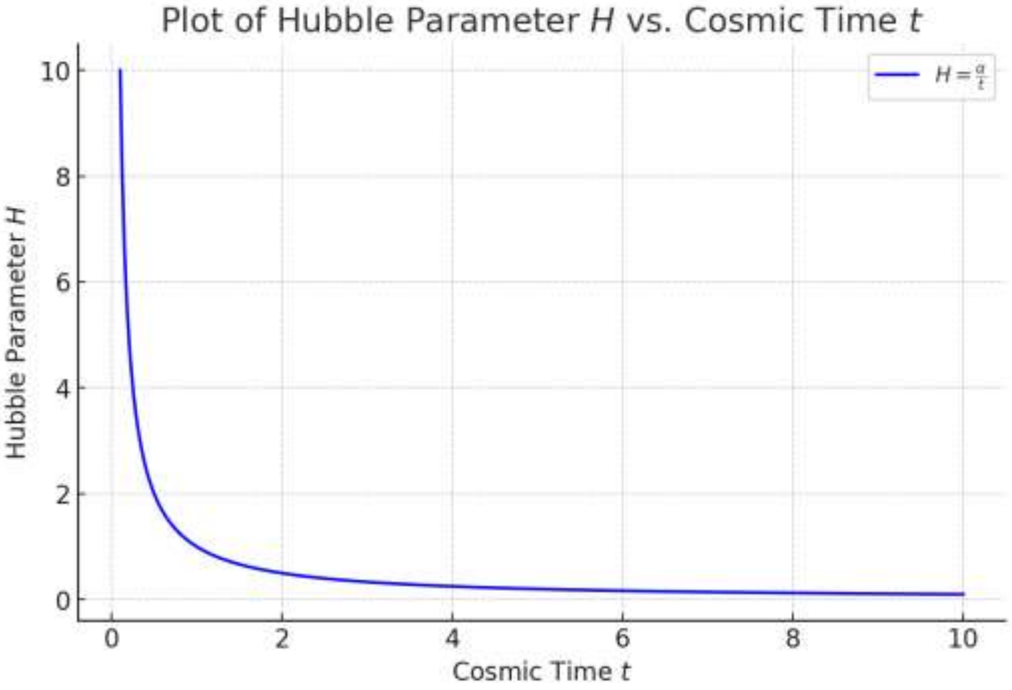

Figure 6. The plot of Hubble parameter H verses cosmic time t, α=1.

The expansion scalar, shear scalar, and mean anisotropic parameters are determined as follows:

$$\theta = 3H = \left(\frac{6t}{t^2+\frac{2\alpha}{3}}\right) \tag{42}$$

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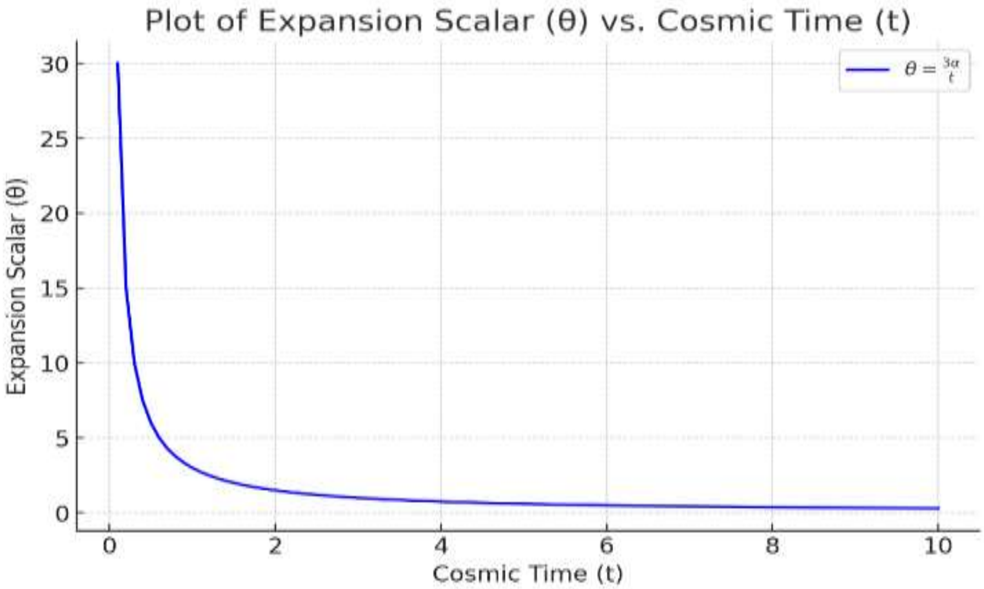

Figure 7. The plot of expansions scalar θ verses cosmic time t, α=1.

$$\sigma^2 = \left(\frac{2t^2}{(t^2+\frac{2\alpha}{3})^2}\right)(q_1^2+q_2^2+q_3^2) \tag{43}$$

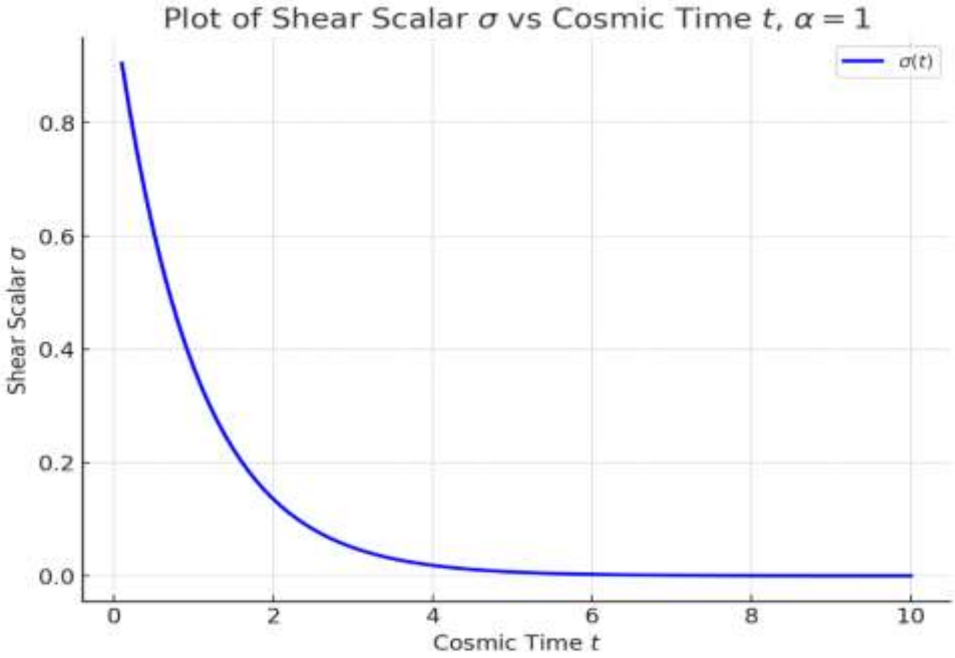

Figure 8. The plot of shear scalar σ verses cosmic time t, α=1.

$$A_m = \frac{1}{3}((q_1^2+q_2^2+q_3^2) \tag{44}$$

Figures 6, 7, and 8 illustrate the fluctuation of H, θ, and σ, respectively. We see that the model exhibits no initial singularity at t=0. Additionally, we observe that H, θ, σ, $\bar{p}$, ρ, and ξ are finite at t=0. These parameters are decreasing functions of time that approach zero for high values of time. Since $\sigma^2/\theta^2 \neq 0$, the model is anisotropic throughout the evolution of the universe.

2. **Bianchi Type - V model**

10.48047/jocaaa.2023.31.04.29

2.1. Introduction

The investigation of Bianchi Type-V cosmological models is more compelling than that of isotropic special examples, allowing for arbitrarily small anisotropic models at specific moments in cosmic time. The natural extensions of open FRW models include various Bianchi type-V universes that ultimately achieve isotropy and are crucial for comprehending processes such as galaxy formation in the early cosmos. The quadrature representation of metric functions for Bianchi Type-V cosmological models, incorporating both perfect and viscous fluids, has been derived based on the studies of Saha [1] and Singh and Chaubey [2,3]. Numerous writers [4-11] have derived solutions to the Einstein Field Equations for homogeneous yet anisotropic models employing various generation strategies. Singh and Baghel [12] have proposed Bianchi Type-V cosmological models including a constant deceleration parameter within the framework of general relativity. Kumar and Yadav [13] have examined several Bianchi-V models of an accelerating cosmos characterized by dark energy. Tiwari and Singh [14] examined Bianchi type-V cosmological models featuring time-varying cosmological and gravitational constants alongside a perfect fluid distribution.

Numerous theoretical models have been proposed to elucidate the characteristics of dark energy and accelerated expansion, including quintessence, phantom energy, k-essence, tachyon, f-essence, and Chaplygin gas, among others. Dark energy plays a crucial role in elucidating recent cosmic findings. Considering the recent acceleration of the cosmos and the presence of dark energy and dark matter, numerous valuable modified theories of gravity have been formulated and examined. The principal objective of contemporary cosmology is to ascertain the large-scale architecture of the Universe. Astronomical observations of type-I a supernovae experiments [15-17] indicate that the observable The Universe is expanding; measurements like cosmic microwave background radiation and large-scale structure offer indirect evidence for the accelerated expansion of the Universe in recent times.

Dark Energy can be investigated by altering the geometric component of the Einstein-Hilbert action [20]. This method is regarded as the most efficient among other approaches to investigate dark energy. As a result of the alterations, numerous other theories of gravity emerged. f(R), f(T), f(G), and f(R,T) gravity are various modified theories of gravity. These simulations are designed to investigate dark energy and other cosmological issues. Sharif and Azeem [21] examined the cosmic development of Dark Energy Models within the framework of f (T) Gravity.

Among the diverse models of Dark Energy, the modified gravity theories that elucidate the late-time accelerated expansion of the Universe are f(R) gravity and Gauss-Bonnet gravity. Shamir [32] examined the f (R) gravity model that elucidates the union of early inflation and late acceleration. Harko et al. [33] formulated the f(R,T) theory of gravity utilizing an arbitrary function of the Ricci scalar R and the trace T of an energy-momentum tensor. Ahmed and Pradhan [34] have examined cosmological models using a cosmological constant within f (R, T) gravity across several Bianchi kinds of spacetime.

In recent years, the Bianchi universe has significantly contributed to observational cosmology, necessitating an enhancement in the WMAP data [35-37], the standard cosmological model with a positive cosmic constant. The universe is expected to have a somewhat anisotropic special geometry, despite the inflationary dynamics contrary to typical inflationary models [38-44], to account for the flatness and homogeneity of the

10.48047/jocaaa.2023.31.04.29

expanding universe. This increase is typically seen as an exponential growth [40-42]. Typically, an accelerating cosmos is characterized within the context of Friedman-Robertson-Walker (FRW) cosmology. Diverse writers have examined specific instances of anisotropic models and have determined that the FRW model, along with the anisotropic Bianchi models, holds unique scholarly significance.

Motivated by the above analysis, we propose to investigate cosmological models under f (R, T) gravity in Bianchi type-V spacetime, utilizing the formulation f (R, T) = $f_1(R) + f_2(T)$. This paper's framework adheres to the gravitational field equation inside the f(R,T) modified gravity theory outlined in delineates explicit field equations in f(R,T) gravity for a broad category of Bianchi cosmological models. , we examine a relation defined as $\sigma / \theta = \frac{k_1}{a^m}$ .The answers to the field equations and the physical characteristics of the models are examined in detail. The condensed conclusion of the proposed work.

2.2.f (R, T) Modified Gravity Theory

According to modified gravity, the following actions can be delineated:

$$S = \int \sqrt{-g}\left(\frac{-1}{16\pi G}\right) f(R,T)d^4x \; + \; L_m\sqrt{-g}\; d^4x \quad , \tag{45}$$

where f(R, T) is a general function dependent on the Ricci scalar R and the trace T of the stress-energy tensor $T_{\mu\nu}$. L_m represents the matter Lagrangian density. The stress-energy tensor of matter can also be defined as:

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}} . \delta \frac{\sqrt{-gL_m}}{\delta g^{\mu\nu}} \tag{46}$$

where the trace T is defined as $g^{\mu\nu} T_{\mu\nu}$. Assuming that the Lagrangian density L_m of matter relies solely on the components of the metric tensor $g_{\mu\nu}$ and not on its derivatives results in

$$T_{\mu\nu} = g^{\mu\nu} L_m - 2\frac{\delta \mathrm{Lm}}{\delta g^{\mu\nu}} \tag{47}$$

where the units of G and c are both equal to 1. Prior theories indicate that Harko and Lobo [45] have investigated a framework in which the Lagrangian density is characterized by an arbitrary function of R, with the Lagrangian density of matter represented as f(R, Lm). In a similar vein, Poplawski [46] has proposed a theory in which the cosmological constant is expressed as a function of the trace of the stress-energy tensor, Λ(T).

By varying the action S concerning the metric tensor components $g_{\mu\nu}$, the gravitational field equations of f (R, T) gravity are derived as

$$f_R(R,T)R_{\mu\nu} - \frac{1}{2}f(R,T)g_{\mu\nu} + \left(g_{\mu\nu}W\nabla_\mu\nabla_\nu\right)f_R(R,T)$$

$$= 8\pi T_{\mu\nu} - f_T(R,T)T_{\mu\nu} - f_T(R,T)\Theta_{\mu\nu} \tag{48}$$

10.48047/jocaaa.2023.31.04.29

Where $\Theta_{\mu\nu} \equiv g^{ij}\left(\frac{\delta T_{ij}}{\delta g^{\mu\nu}}\right)$,that derives from the relationship$\delta\left(\frac{g^{ij}T_{ij}}{\delta g^{\mu\nu}}\right) = T_{\mu\nu} + \Theta_{\mu\nu}\ and\ \blacksquare = \nabla^i\nabla_j$,

$f_R(R,T) \equiv \frac{\partial f(R,T)}{\partial R}, f_T(R,T) \equiv \frac{\partial f(R,T)}{\partial T}\ and\ \nabla_i$indicates the covariant derivative. The contraction of Equation (4) produces gives $f_R(R,T)R + 3\blacksquare f_R(R,T) - 2f(R,T) = \left(8\pi - f_T(R,T)\right)T - f_T(R,T)\Theta$ with $\Theta \equiv g^{\mu\nu}\Theta_{\mu\nu}$.By integrating Equation (4) with the condensed equation and removing the term $\blacksquare\ f_R(R,T)$ we derive

$$f_R(R,T)\left(R_{\mu\nu} - \frac{1}{3}Rg_{\mu\nu}\right) + \frac{1}{6}f(R,T)g_{\mu\nu}$$
$$= \left(8\pi - f_T(R,T)\right)\left(T_{\mu\nu} - \frac{1}{3}Tg_{\mu\nu}\right) - f_T(R,T)(\Theta_{\mu\nu} - \frac{1}{3}\Theta g_{\mu\nu}) + \nabla_\mu\nabla_\nu f_R(R,T) \tag{49}$$

conversely, by the covariant divergence of Eq. (1) and the energy-momentum conservation rule

$$\nabla^\mu\left[f_R(R,T) - \frac{1}{2}f(R,T)g_{\mu\nu} + \left(g_{\mu\nu}W\nabla_\mu\nabla_\nu\right)f_R(R,T)\right] = 0,$$

corresponding to the divergence of the left-hand side of Eq. (1), we obtain the divergence of $T_{\mu\nu}$ as

$$\nabla^\mu T_{\mu\nu} = \frac{f_T(R,T)}{8\pi - \frac{1}{2}f_T(R,T)}[\left(T_{\mu\nu} + \Theta_{\mu\nu}\right)\nabla^\mu ln\ f_T(R,T) + \nabla^\mu\Theta_{\mu\nu}] \tag{50}$$

With add on, from $T_{\mu\nu} = g_{\mu\nu}L_m - 2(\frac{\partial L_m}{\partial g^{\mu\nu}})$ we got

$$\frac{\partial T_{ij}}{\partial g^{\mu\nu}} = \left(\frac{\partial g_{ij}}{\partial g^{\mu\nu}} + \frac{1}{2}g_{ij}g_{\mu\nu}\right)L_m - \frac{1}{2}g_{ij}T_{\mu\nu} - 2\frac{\partial^2 L_m}{\partial g^{\mu\nu}\partial g^{ij}} \tag{51}$$

By the relation,

$\frac{\partial g_{ij}}{\partial g^{\mu\nu}} = -g_{i\gamma}g_{j\sigma}\delta_{\mu\nu}^{\gamma\sigma}$ with $\delta_{\mu\nu}^{\gamma\sigma} = \frac{\delta g^{\gamma\sigma}}{\delta g^{\mu\nu}}$which follows by $g_{i\gamma}g^{\gamma j} = \delta_i^j$. Then we get $\Theta_{\mu\nu}$ is given by

$$\Theta_{\mu\nu} = -2T_{\mu\nu} + g_{\mu\nu}L_m - 2g^{ij}\frac{\partial^2 L_m}{\partial g^{\mu\nu}\partial g^{ij}} \tag{52}$$

Assuming matter is considered a perfect fluid, the stress-energy tensor of the matter Lagrangian is expressed as

$$T_{\mu\nu} = (\rho + p)u_\mu v_\nu - pg_{\mu\nu} \tag{53}$$

Where $u^\mu = (0,0,0,1)$The four-velocity in the moving coordinates meets the requirements $u^\mu u_\nu = 1\ and\ u^\mu\nabla_\nu u_\mu = 0$. ρ and p denote the energy density and pressure of the fluid, respectively. Utilizing equation (8), we derive

$$\Theta_{\mu\nu} = -2T_{\mu\nu} - pg_{\mu\nu} \tag{54}$$

The field equations of $f(R,T)$ gravity is influenced by the physical characteristics of the matter field (via the tensor Θμν), allowing for the derivation of multiple theoretical models corresponding to each selection of f. Harko et al. [33] have examined three distinct formulations of the functional form of f.

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases}$$

The cosmological implications of the class $f(R,T) = R + 2f(T)$ have recently been examined comprehensively by numerous writers [47-52,53]. Shamir et al. [54] and Chaubey & Shukla

[47] have examined Bianchi type-I and V, as well as a broad class of Bianchi models, in the context of $f(R,T)$ gravity, specifically with $f(R,T) = R + 2f(T)$. This study examines the cosmological implications of the class defined by $f(R,T) = f_1(R) + f_2(T)$. Our developed cosmological model is entirely distinct and novel compared to that of the other writers referenced herein. The cosmological word Λ, a potential candidate for dark energy, has thus far received insufficient attention. Consequently, our generated model may enhance comprehension of the properties of Bianchi type-V models.

The equation for the gravitational field (4) is expressed as

$$f_1(R)R_{\mu\nu} - \frac{1}{2}f_1(R)g_{\mu\nu} + \left(g_{\mu\nu}W\nabla_\mu\nabla_\nu\right)f_1(R)$$

$$= 8\pi T_{\mu\nu} - f_2(T)T_{\mu\nu} + \left(f_2(T)p + \tfrac{1}{2}f_2(T)\right)g_{\mu\nu} \tag{55}$$

where the prime symbol indicates differentiation concerning the variable. The field equations of standard f(R) gravity can be derived for p = 0 (the dust scenario) and $f_2(T) = 0$. We examine a specific form of the functions $f_1(R) = \lambda_1 R$ and $f_2(T) = \lambda_2 T$, where λ_1 and λ_2 are arbitrary parameters. In this article, we set $\lambda_1 = \lambda_2 = \lambda$, resulting in $f(R,T) = \lambda\,(R + T)$.

Equation (11) can now be reformulated as

$$\lambda R_{\mu\nu} - \frac{1}{2}\lambda(R+T)g_{\mu\nu} + \left(g_{\mu\nu}W\nabla_\mu\nabla_\nu\right)\lambda$$

$$= 8\pi T_{\mu\nu} - \lambda T_{\mu\nu} + \lambda\left(2T_{\mu\nu} + pg_{\mu\nu}\right) \tag{56}$$

Now, we have to set $\left(g_{\mu\nu}W\nabla_\mu\nabla_\nu\right)\lambda = 0$, we obtain

$$\lambda G_{\mu\nu} = 8\pi T_{\mu\nu} + \lambda T_{\mu\nu} + \left(\lambda p + \tfrac{1}{2}\lambda T\right)g_{\mu\nu} \tag{57}$$

Where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is Einstein tensor. This will be rearranged as

$$G_{\mu\nu} - \left(p + \tfrac{1}{2}T\right)g_{\mu\nu} = \frac{8\pi+\lambda}{\lambda}T_{\mu\nu} \tag{58}$$

Revisiting Einstein's equations with the cosmic constant

$$G_{\mu\nu} - \Lambda g_{\mu\nu} = -8\pi T_{\mu\nu} \tag{59}$$

We select a tiny negative value for the arbitrary λ to ensure that the right-hand sides of (13) and (14) share the same sign; we maintain this choice of λ consistently. The expression $\left(p + \frac{1}{2}T\right)$ can now be considered a cosmological constant. Therefore, we compose

$$\Lambda = \Lambda(T) = \left(p + \tfrac{1}{2}T\right) \tag{60}$$

Poplawski [46] previously emphasized the reliance of the cosmological constant Λ on the trace of the energy-momentum tensor T, indicating that the gravitational Lagrangian was a consequence of this trace. According to Poplawski [46], the model was referred to as "Λ(T) gravity". Moreover, it has been proposed (Magnano [55], Poplawski [56,57]) that cosmic data supports a changing cosmological constant that aligns with Λ(T) gravity. In the context of the perfect fluid scenario, the trace T = -3p + ρ for our model simplifies Equation (16) to

$$\Lambda = \tfrac{1}{2}(\rho - p) \tag{61}$$

2.3. Metric and Field Equations

According to references [35-37], the universe cannot be deemed entirely symmetric based on contemporary observations. Therefore, to characterize the universe that possesses greater symmetry than the conventional FRW models. Bianchi models, which depict spatially homogeneous and anisotropic spaces, would be more suitable. Consequently, the overarching category of Bianchi cosmological models is delineated as follows:

10.48047/jocaaa.2023.31.04.29

$$ds^2=-dt^2+a^2(t)e^{2m_1}xdx^2+a^2(t)e^{2m_2}dy^2+a^2(t)e^{2m_3}dz^2 \tag{62}$$

where β is a constant, and the functions A(t), B(t), and C(t) represent the three anisotropic directions of expansion in standard three-dimensional space. The average scale factor a, the spatial volume V, and the average Hubble parameter H are defined as follows:

$$V=a^3=ABC \tag{63}$$

$$H=\frac{1}{3}(H_1+H_2+H_3) \tag{64}$$

Let $H_1=\frac{\dot{A}}{A}, H_2=\frac{\dot{B}}{B}\ and\ H_3=\frac{\dot{C}}{C}$ represent the directional Hubble parameters in the x, y, and z directions, respectively. In the subsequent equation and in all instances henceforth, the dot signifies differentiation with regard to cosmic time t. The key physical quantities of interest in cosmology are the expansion scalar θ, the deceleration parameter q, and the shear scalar σ, which are defined as follows:

$$\theta=3H \tag{65}$$

$$q=-1+\frac{d}{dt}\left(\frac{1}{H}\right) \tag{66}$$

$$\sigma^2=\frac{1}{2}\sigma_{ij}\sigma^{ij} \tag{67}$$

The anisotropy parameter (A_m) is defined as

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2 \tag{68}$$

The current cosmological equations for the energy-momentum tensor [$T_{\mu\nu}=(\rho+p)u_\mu v_\nu-pg_{\mu\nu}$] and the metric (62) are

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}-\frac{\beta^2}{A^2}=\left(\frac{8\pi+\lambda}{\lambda}\right)p-\Lambda \tag{69}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\frac{\beta^2}{A^2}=\left(\frac{8\pi+\lambda}{\lambda}\right)p-\Lambda \tag{70}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\beta^2}{A^2}=\left(\frac{8\pi+\lambda}{\lambda}\right)p-\Lambda \tag{71}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-3\frac{\beta^2}{A^2}=-\left(\frac{8\pi+\lambda}{\lambda}\right)p-\Lambda \tag{72}$$

$$2\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0 \tag{73}$$

In the subsequent section of the study, we assume the constant (β) to be unity without any loss of generality. By integrating Eq. (73) and incorporating the integration constant into B or C, we derive

$$A^2=BC \tag{74}$$

From equations (69) to (72), we derive the subsequent three relations:

$$\frac{A}{B}=\alpha_1e^{\left(\beta_1\int\frac{dt}{a^3}\right)} \tag{75}$$

$$\frac{A}{C}=\alpha_2e^{\left(\beta_2\int\frac{dt}{a^3}\right)} \tag{76}$$

$$\frac{B}{C}=\alpha_3e^{\left(\beta_3\int\frac{dt}{a^3}\right)} \tag{77}$$

Where $\alpha_1,\alpha_2 and\ \alpha_3,\beta_1,\beta_2 and\ \beta_3$are constants. Final we use $a=(ABC)^{\frac{1}{3}}$. We express the metric functions from (75–77) in explicit form as follows

$$A(t)=m_1ae^{\left(n_1\int a^{-3}dt\right)} \tag{78}$$

$$B(t)=m_2ae^{\left(n_2\int a^{-3}dt\right)} \tag{79}$$

$$C(t)=m_3ae^{\left(n_3\int a^{-3}dt\right)} \tag{80}$$

10.48047/jocaaa.2023.31.04.29

Where $m_1 = \sqrt[3]{\alpha_1}\alpha_2$, $m_2 = \sqrt[3]{\alpha_1^{-1}}\alpha_3$, $m_3 = \sqrt[3]{(\alpha_2\alpha_3)^{-1}}$ (81)

And $n_1 = \frac{\beta_1+\beta_1}{3}$, $n_2 = \frac{\beta_3-\beta_1}{3}$ and $n_3 = -(\frac{\beta_2+\beta_3}{3})$ (82)

Where the constants n_1, n_2, n_3 *and* m_1, m_2, m_3 satisfies the relation

$$n_1 + n_2 + n_3 = 0 \; and \; m_1 m_2 m_3 = 1 \tag{83}$$

Substitute the equation (74) in equation [78-80], we get

$$m_1 = 1, m_2 = m_3^{-1} = c_1, n_1 = 0, n_2 = -n_3 = c_2 \tag{84}$$

where c_1 and c_2 represent constants. By putting Equation (84) into Equations (78−80), the quadrature representation of the metric functions in relation to the average scale factor a can be expressed as

$$A(t) = a \tag{85}$$

$$B(t) = c_1 a e^{\left(c_2 \int a^{-3} dt\right)} \tag{86}$$

$$C(t) = \frac{a}{c_1} a e^{\left(-c_2 \int a^{-3} dt\right)} \tag{87}$$

2.4. Solution of the field equations

Recent investigations of CMBR (Cosmic Microwave Background Radiation) indicate that the cosmos has been expanding isotropically since the period it became indisputably transparent to radiation. We have examined the anisotropy parameter as a function of the scale factor. Consequently, the examination of cosmological theories in which the initial anisotropy of space is mitigated throughout time, resulting in an isotropic phase, is essential. The anisotropy of space is represented as $A_m = 6\frac{\sigma^2}{\theta^2}$.

The equations of modified gravity field can be resolved under the assumption that anisotropy (σ/θ) is inversely proportional to the mth power of the scale factor a [58]. This provides

$$\frac{\sigma}{\theta} = \frac{k_1}{a^m} \tag{88}$$

Where k_1 and m are constants.

From equation (88) we get average scale factor a as,

$$a = \left[\frac{(3-m)k}{3k_1} t + k_2\right]^{\frac{1}{3-m}} \quad \text{for } m \neq 3 \tag{89}$$

$$= k_3 e^{\frac{k}{3k_1} t} \quad \text{for } m = 3 \tag{90}$$

3.Conclusion.

This study examines spatially homogeneous and anisotropic cosmological models of Bianchi types I and V, populated with bulk viscous fluid, within the context of f(R,T) gravity theory. The lack of an early temporal singularity in both models is a significant feature of the results. The scale factors maintain constant values throughout the universe's early epochs (t→ 0), gradually growing with cosmic time without exhibiting any initial singularity, ultimately nearing infinity as t → ∞. Thus, the universe illustrated by both models begins with zero volume in the primordial past and experiences exponential expansion, approaching infinite volume. The expansion scalar θ and shear scalar σ are decreasing functions of time, ultimately converging to 0 as time advances. The ratio σ/θ converges to a constant as t approaches infinity, therefore the anisotropy in both models is maintained over time. The deceleration parameter q is negative for $t < \sqrt{(2\alpha)}$ and positive for $t > \sqrt{(2\alpha)}$. Thus, the cosmological models initially undergo acceleration for a defined period before shifting to deceleration. The attributes of bulk viscosity are illustrated visually in Figure 4.

10.48047/jocaaa.2023.31.04.29

The progression of the Bianchi type-V cosmological model is analyzed within the framework of a perfect fluid and a variable cosmological constant in the f (R, T) theory of gravity [Harko et al.33]. Every pick of the function f (R, T) produces unique theoretical models. This study formulates the gravitational field equation as outlined in (ii). We have analyzed a particular variation law wherein the anisotropy (σ/θ) is contingent upon the scale factor a.

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