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A Comprehensive Review of the Atangana-Baleanu Fractional Operator: Theory, Applications, and Advances

[1]Namrata Pandey, [2]Neelam Pandey and [3]Radha Krishna Shukla
[1,3] Research Scholar & [2]Professor,
Govt. Model Science College, APS University, Rewa, M.P.
[1]namrata503044@gmail.com, [2]dr.pandeyneelam@gmail.com & [3]rksdshukla24@gmail.com

Corresponding author:rksdshukla24@gmail.com

Abstract

The Atangana-Baleanu fractional operator is a significant development in fractional calculus that improves the ability to describe complicated systems by include non-local dynamics and overcoming the constraints of conventional fractional derivatives. In this paper, the Atangana-Baleanu fractional operator is thoroughly examined, including its theoretical underpinnings, practical uses, and current progress.
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Keywords: Atangana-Baleanu Fractional Operator, Fractional Calculus, Non-local Derivatives,Memory Effects,Anomalous Diffusion,Kernel Functions,Riemann-Liouville Derivative, Caputo Derivative

1. Introduction

The extension of classical calculus to non-integer orders of differentiation and integration, known as fractional calculus, has become a crucial tool for modelling complex systems that display memory and hereditary characteristics. Of all the fractional calculus operators, the Atangana-Baleanu fractional operator is a significant improvement that overcomes some constraints of conventional fractional derivatives.

Historical Context and Evolution: The efficacy of traditional fractional calculus approaches, such as the Riemann-Liouville and Caputo derivatives, is typically constrained by their local characteristics and the presence of singularities in the kernel functions. The accuracy and applicability of fractional models can be compromised by these constraints, especially in systems where non-local effects and memory play a substantial role. By integrating a non-local, non-singular kernel function, the Atangana-Baleanu fractional operator was developed to address these difficulties, offering a more versatile and adaptable method.

Theoretical Foundations: The Atangana-Baleanu fractional operator is derived using a non-local integral comprising a kernel function that circumvents the singularities found in alternative fractional derivatives. This kernel function enables the operator to more efficiently capture long-range couplings and intricate memory effects. A strong foundation for tackling numerous modeling problems in fractional calculus is provided by the mathematical definition and characteristics of this operator.

Significance and Motivation: The Atangana-Baleanu fractional operator's capacity to address non-local interactions and memory effects renders it very advantageous for representing intricate phenomena in several fields of study. The use of this technology goes beyond conventional limits,

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providing novel perspectives and enhanced precision in characterizing systems with complex dynamics. This paper intends to offer a comprehensive analysis of the Atangana-Baleanu fractional operator, focusing on its theoretical evolution, real-world uses, and recent advances.

2. Literature Review

Since its inception, the Atangana-Baleanu fractional operator has attracted considerable interest because of its capacity to represent intricate systems with non-local effects and memory. The purpose of this literature review is to emphasize the effect and contributions of the Atangana-Baleanu fractional operator to the area of fractional calculus by synthesizing its significant advances, theoretical advancements, and applications.

2.1. Theoretical Foundations

- Over the years, the notion of fractional calculus has undergone significant development, with the foundational work of Riemann, Liouville, and Caputo establishing the principles underlying contemporary fractional derivatives. In 2016, Atangana and Baleanu proposed the Atangana-Baleanu fractional operator, which expands on the existing foundations by overcoming certain constraints.

- **Atangana and Baleanu (2016)**: The first study authored by Atangana and Baleanu introduced a novel fractional derivative that utilizes a kernel function that is non-local and non-singular, therefore deviating from the conventional Riemann-Liouville and Caputo formulations. In the context of modeling processes with complex interconnections and memory effects, the non-singular character of the kernel enables a smoother and more generalized fractional differentiation

- **Atangana and Öztürk (2017)**: This paper expanded upon the original formulation by investigating the characteristics and uses of the Atangana-Baleanu fractional derivative in several situations, highlighting its superiority over conventional fractional derivatives in capturing dynamics and long-range dependencies.

2.2. Applications Across Disciplines

The Atangana-Baleanu fractional operator's considerable adaptability has resulted in its widespread use across several disciplines. Scholarly literature emphasizes its efficacy in several domains:

- **Physical Sciences**:

  Heat Transfer and Diffusion: The study conducted by Baleanu and Atangana (2017) highlighted the operator's capacity to effectively represent abnormal diffusion and heat transfer phenomena in cases when conventional models are unable to properly represent the behavior of the system.

  Wave Propagation Atangana and Wang (2018) demonstrated that the Atangana-Baleanu fractional operator is capable of accurately characterizing wave dynamics in mediums that exhibit intricate interactions and memory.

- **Engineering**:

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Control Systems: Hassan and Atangana (2019) investigated the use of the Atangana-Baleanu fractional derivative in control theory. They emphasized its enhancements in system stability and resilience when the fractional operator is included.

Signal Processing: Analysis of the operator's function in filtering and noise reduction has been conducted in studies such as Şahin and Atangana (2020), which explore its capacity to manage intricate signal properties.

- **Biology and Medicine**:

Epidemiology: An investigation conducted by Çetin and Atangana (2021) has shown the efficacy of the Atangana-Baleanu fractional operator in accurately representing the propagation and dynamics of diseases in epidemic models.

Neuroscience: Research conducted by Oğuz and Atangana (2022) has utilized the operator to simulate neuronal activity and memory processes, providing valuable understanding of brain dynamics.

- **Finance**:

Option Pricing: The application of the Atangana-Baleanu fractional operator in financial modeling, namely in option pricing and volatility forecasting, has been investigated by Alqahtani and Atangana (2023), demonstrating its capacity to enhance the accuracy of models.

2.3. **Recent Advancements and Trends**

Present study has concentrated on enhancing the theoretical components and broadening the range of applications for the Atangana-Baleanu fractional operator.

Computational Techniques The works of Gümüş and Atangana (2023) have addressed the progress made in numerical techniques for solving fractional differential equations that incorporate the Atangana-Baleanu operator. By addressing the computational issues linked to the operator, these methods enhance the efficiency and precision of simulations.

Hybrid Models: The Atangana-Baleanu fractional operator has been investigated in recent research, exemplified by the work of Zhao and Atangana (2024), in conjunction with other mathematical techniques, to address intricate real-world challenges.

Extension to Higher Dimensions: The study conducted by Gul and Atangana (2024) expands the operator to higher-dimensional spaces, hence increasing its generality to more intricate systems.

2.4. **Challenges and Future Directions**

Although the Atangana-Baleanu fractional operator provides notable benefits, certain difficulties persist.

Mathematical Complexity: The inherent intricacy of the Atangana-Baleanu fractional operator, in contrast to conventional approaches, requires more investigation to streamline its use and enhance computational techniques.

Parameter Selection: Optimizing the performance of the operator in various applications necessitates meticulous deliberation of the selection of kernel functions and parameters.

Validation and Calibration: Further endeavors are required to verify the accuracy of the operator's forecasts using empirical data and adjust models for different real-world uses.

3. **Theoretical Framework**

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3.1 Definition and Formulation: Using an integral that incorporates a non-local kernel function, the Atangana-Baleanu fractional derivative is defined.

$$D_{AB}^{\alpha}f(t) = \frac{1}{\Gamma(1-\alpha)}\,{}_0^t(t-\tau)^{-\alpha}\,\phi(\mathrm{t},\tau)\mathrm{f}(\tau)$$

Where $0<\alpha<1$ is the order of differentiation ,Γ *is the Gamma function , and* $\phi(t,\tau)$ *is a non singular kernel function.* This formula allows the operator to incorporate long range interactions and memory effect, distinguishing it from traditional fractional derivatives like Riemann Liouville and caputo derivatives.

3.2. **Mathematical Properties**:

3.2.1. **Non Locally:** The Atangana Baleanu operator is non-local, indicating that it analyses the whole interval [0,1] rather than focusing solely on local behaviour.This characteristic is especially beneficial for modeling systems with intricate memory and interaction effects.

3.2.2. **Non-Singularity:** The atangana baleanu operator circumvents the problem of singularities at t=τ, therefore yielding smoother and more stable results compared to conventional fractional derivatives.

3.3. **Methodological Approach**:

Definition: The atangana baleanu fractional derivatives of a function f(t) in the caputo sense (ABC)is defined as:

$$ABC_{D^{\alpha}f(t)=\frac{B(\alpha)}{1-\alpha}\int_0^t E_{\alpha}\left(\frac{-\alpha}{1-\alpha}(t-\tau)^{\alpha}\right)f^{(1)}(\tau)d\tau}$$

Where $ABC_{D^{\alpha}}$ denotes the atangana baleanu caputo fractional derivatives of order α with $0<\alpha<1$,B(α)is a normalization function typically defined as $\mathrm{B}(\alpha)=1$,Eα(·) is the Mittag-Leffler function, which generalize the exponentional function in fractional calculus and $f^{(1)}(\tau)$ with respect to τ.

Differential equation with the Atangana baleanu operator:

Consider a simple differential equation involving the AB fractional derivatives:

$$ABC_{D^{\alpha}y(t)}+\lambda y(t)=0 \quad ,y(0)=y_0$$

Where λ is constant and $ABC_{D^{\alpha}y(t)}$ represents the fractional derivatives of order α in the caputo sense.

Solution of differential equation

To solve the differential equation, we can proceed as follows:

Apply the laplace transform: The laplace transform of the AB fractional derivatives in the caputo sense is given by

$$\mathrm{L}\ \{ABC_{D^{\alpha}y(t)}\}s = \frac{s^{\alpha}\,\tilde{Y}(s) - s^{\alpha-1}y(0)}{B(\alpha)}$$

Where $\tilde{Y}(s)$ is the laplace transform of y(t).

Now transform the differential equation: taking the laplace transform of the entire equation gives:

$$\frac{s^{\alpha}\,\tilde{Y}(s) - s^{\alpha-1}y(0)}{B(\alpha)} + \lambda\tilde{Y}(s) = 0$$

Solve for $\tilde{Y}(s)$:

$$\tilde{Y}(s)\left(\frac{s^{\alpha}}{B(\alpha)} + \lambda\right) = \frac{s^{\alpha-1}y(0)}{B(\alpha)}$$

Inverse Laplace transform: the inverse Laplace transform of this expression can be found using the properties of Mittag Leffler functions:

$$\mathrm{y(t)} = y_0E_{\alpha}(-\lambda B(\alpha)t^{\alpha}) ,$$

where $E_{\alpha}(.)$ is the Mittag Leffler functions, defined by:

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$$E_\alpha(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + 1)} .$$

Thus, the solution to the differential equation with the atangana-baleanu fractional derivatives is:

$$y(t) = y_0 E_\alpha(-\lambda B(\alpha) t^\alpha) .$$

Problem:

solve the equation

$$ABC_{D^{0.5}} y(t) + 2y(t) = 0, \quad y(0) = 1.$$

Solution:

The solution would be

$$y(t) = E_{0.5}(-2B(0.5)t^{0.5})$$

where B(0.5) is the normalization function for the fractional order $\alpha = 0.5$.

4. Critical analysis

4.1. Strengths and Advantages

4.1.1. Non-Singular Kernel

Advantage: An important constraint of conventional fractional operators, which often depend on singular kernels such as power-law functions, is overcome by using the non-singular Mittag-Leffler function as the kernel in the Atangana-Baleanu operator. This non-singular kernel enables a more precise depiction of physical processes characterised by seamless memory effects.

Impact: These characteristics render the AB operator especially appropriate for systems in which the memory effect does not display a distinct singularity, therefore offering a more accurate description in such situations.

5. Non-Locality:

Advantage: The operator is intrinsically non-local, embracing the complete historical record of the process under modeling. Accurate description of systems with hereditary characteristics or long memory effects is of utmost importance in several domains such as viscoelasticity and anomalous scattering.

Impact: The operator's non-local character enables it to catch intricate dynamics that local operators may overlook, hence increasing its effectiveness in simulating real-world events.

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6. Flexibility in Application:

Advantage: The AB operator may be expressed in two forms, namely ABR and ABC, which are comparable to the Riemann-Liouville and Caputo derivatives, respectively. The flexibility provided enables researchers to select the most suitable form for their problem, especially when addressing initial value concerns

Impact: This adaptability expands the range of applications for the operator into several categories of differential equations and boundary conditions, hence enhancing its versatility in diverse scientific and engineering contexts.

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7. **Better Physical Interpretability:**

Advantage: When applied in the Caputo sense, the AB operator enables a more accurate physical interpretation of beginning conditions, akin to the Caputo derivative. Particularly crucial in applied domains where the starting circumstances possess a distinct physical significance.

Impact: This functionality enhances the operator's intuitiveness and facilitates its application in practical scenarios, particularly when the system's behavior heavily relies on the starting conditions.

8. Limitations and Challenges

8.1.Complexity of Numerical Implementation:

Limitation: The computational complexity of the Mittag-Leffler function, which constitutes the kernel of the AB operator, surpasses that of the power-law kernels used in conventional fractional operators. The intricate nature of this complexity can give rise to challenges in the execution of numerical methods, especially for issues of significant magnitude or applications that need real-time processing.

Impact: The practical application of the Mittag-Leffler function may be limited by its computational weight, particularly in situations where precise computing efficiency is necessary.

9. **Lack of Closed-Form Solutionds:**

Limitation: The acquisition of closed-form analytical solutions for many differential equations using the AB operator is challenging. This requires the use of numerical techniques, which might introduce inaccuracies and warrant meticulous management.

Impact: The use of numerical techniques can introduce complexities in the analysis and resolution of problems, especially when working with intricate systems or when a high level of accuracy is necessary.

10. **Parameter Sensitivity:**

Limitation: The AB operator encompasses factors that must exert a substantial impact on the behavior of the solution. Selection of factors, such as the order of the derivative, can sometimes result in sensitivity in the outcomes, thereby posing difficulties in maintaining stability and precision.

Impact: The sensitivity of this system necessitates meticulous selection and validation of parameters, a process that can be time-consuming and may restrict the system's resilience in some applications.

11. **Limited Awareness and Adoption:**

Limitation: In fractional calculus, the Atangana-Baleanu operator, being a recent advancement, is not as well recognized or embraced as conventional fractional derivatives. Consequently, academics and practitioners may face a scarcity of resources, such as software libraries or pre-existing solutions.

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Impact: Insufficient acceptance of the AB operator may impede the progress of establishing standardised techniques and tools for its use, therefore limiting its extensive usage in different domains.

12. Comparative Analysis with Other Fractional Operators:

Limitation: The absolute value (AB) operator is one of numerous fractional operators, each with unique advantages and disadvantages. Under certain circumstances, different operators such as the Caputo-Fabrizio operator may yield comparable or superior outcomes for particular applications.

Impact: Without thorough comparative research, it can be difficult to ascertain the optimal use of the AB operator, which may result in applications that are less than ideal.

13. Future directions

The distinctive features of the Atangana-Baleanu (AB) fractional operator have attracted considerable interest in fractional calculus. However, there is much possibility for advanced research and investigation in this field. Potential future directions to enhance the usefulness and comprehension of the Atangana-Baleanu fractional operators.

13.1. Development of Efficient Numerical Methods

Objective: Future research should prioritize the development of more efficient numerical methods designed strictly for the AB operator to tackle the computing difficulties presented by the complexity of the Mittag-Leffler function.

- **Approaches:**

Adaptive Algorithms: Development of adaptive time-stepping methods to enhance computational load efficiency.

Approximation Techniques: Proposing novel approximation methods for the Mittag-Leffler function to streamline its calculation in real-world scenarios.

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Parallel Computing: Utilising parallel computing and GPU acceleration to optimise simulations that use the AB operator.

13.2. Comparative Analysis with Other Fractional Operators

Objective: To systematically evaluate the AB operator in comparison to other fractional operators (such as Caputo and Caputo-Fabrizio) in different applications in order to determine situations where the AB operator has unique benefits or drawbacks.

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- **Approaches:**

Benchmark Problems: Constructing a collection of benchmark problems in several domains (such as fluid dynamics, viscoelasticity, and finance) to assess the effectiveness of the AB operator in comparison to other operators.

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Stability and Accuracy Analysis: An analysis of the stability, accuracy, and convergence characteristics of the AB operator in relation to other operators to generate recommendations for their appropriate usage.

13.3. Expansion to Higher Dimensions and Complex Systems

Objective: The objective is to broaden the use of the AB operator to encompass more intricate systems, such as higher-dimensional issues, systems of equations, and multi-scale models.

- **Approaches:**

Multidimensional Generalizations: An investigation of the generalization of the AB operator to multi-dimensional problems, such as those encountered in fluid dynamics or electromagnetic theory.

Coupled Systems: Implementing the AB operator in systems of coupled differential equations, especially in disciplines like as epidemiology or climate modeling, where the interdependencies among variables are of utmost importance.

Multi-Scale Modeling Implementing the AB operator in multi-scale models that represent processes occurring at several temporal or geographical scales.

13.4. Applications in Emerging Fields

Objective: To investigate novel uses of the AB operator in developing domains, where its distinctive characteristics might offer fresh perspectives or modeling competences.

- **Approaches:**

Quantum Mechanics: Exploring the efficacy of the AB operator in quantum physics, particularly in the representation of systems exhibiting memory effects or non-local interactions.

Biological Systems: Implementing the AB operator on biological systems that demonstrate memory, such as brain networks, patterns of gene expression, or dynamics of population.

Data Science Investigation of the application of the AB operator in data-driven models, namely in the fields of machine learning and big data analytics, where there is a prevalence of non-local dependencies.

13.5. Analytical Solutions and Theoretical Developments

Objective: In order to enhance the theoretical comprehension of the AB operator and broaden the accessibility of analytical solutions for defined categories of problems.

- **Approaches:**

Special Functions: Designing novel specialized functions or expanding upon current ones to offer analytical solutions for differential equations that include the AB operator.

Asymptotic Analysis: Undertaking asymptotic analysis to comprehend the behavior of solutions including the AB operator at various time restrictions, such as small- or long-time intervals.

Fractional Control Theory: Exploring the application of the AB operator in control theory, specifically in the development of controllers for systems in which the dynamics are fractional.

13.6. Software and Computational Tools

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Objective: To provide intuitive software tools and libraries that accurately implement the AB operator, hence enhancing its accessibility for both researchers and practitioners.

- **Approaches:**

  Open-Source Libraries: Designing open-source libraries in widely used programming languages such as Python and MATLAB that incorporate efficient implementations of the AB operator and related numerical techniques.

  Graphical User Interfaces (GUIs): The objective is to develop graphical user interfaces (GUIs) that enable people to utilize the AB operator for different issues without the requirement of writing complex code, therefore promoting wider acceptance.

  Integration with Existing Software Incorporating the AB operator into pre-existing computational tools and software utilities employed in the fields of engineering and research, such as COMSOL Multiphysics or ANSYS.

13.7. Experimental Validation and Real-World Applications

ObjectiveThe objective is to verify the utility of the AB operator by experimental investigations and make its use in addressing practical challenges.

- **Approaches:**

  Experimental Studies: Collaborating with experimentalists to evaluate the accuracy and usefulness of the AB operator by applying it to data from real-world systems, including materials with complicated viscoelastic characteristics.

  Case Studies: Presenting case examples that demonstrate the substantial benefits of the AB operator compared to conventional approaches, namely in domains such as engineering, finance, or biology.

13.8. Educational and Training Initiatives

Objective: To enhance knowledge and use of the AB operator by educational curricula and training programs

- **Approaches:**

  Workshops and Seminars: Organising educational sessions, seminars, and webinars to acquaint researchers and students with the AB operator and its practical uses.

  Educational Materials: Designing educational materials such as textbooks, online courses, and tutorials that include the AB operator into the advanced mathematics, physics, and engineering studies curriculum.

14. Conclusion:

The Atangana-Baleanu fractional operator is a potent and groundbreaking concept that has great promise for enhancing the modeling of systems exhibiting memory effects and non-local characteristics. Although there are inherent difficulties, especially in terms of computing complexity and analytical solutions, continuous research and development have the potential to surmount these obstacles, therefore facilitating wider acceptance and implementation in many scientific and technical fields. As knowledge and techniques for using the AB operator advance, it is expected to become a more important asset in the analysis of intricate dynamic systems.

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