

On \mathcal{N}_b -bc Irresolute Functions in Nano Topological Spaces

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Abstract: The purpose of this paper is to introduce and study the concept of \mathcal{N}_b -bc irresolute function in in nano topological spaces ($\mathcal{N}_b\mathcal{T}$ space). Also we discussed some of its characterizations. Additionally we introduce contra \mathcal{N}_b -bc irresolute functions. A thorough verification has been carried out using relevant cases for the irresolute function defined above.

Keywords: \mathcal{N}_b -bc-open set, \mathcal{N}_b -bc closed set, \mathcal{N}_b -bc continuous function, \mathcal{N}_b -bc irresolute function, contra \mathcal{N}_b -bc irresolute function.

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1. INTRODUCTION

In 1972, the concept of irresolute functions were introduced and studied in Crossley and Hildebrand .He proved that irresolute functions are stronger than semi continuous but are independent of continuous functions. The study of *nano topology* was started by M. Lellis Thivagar et al [8] with regard to a subset X of a universe that is described in terms of lower, upper and boundary approximations of X. He additionally described *nano interior* and *nano closure* in *nano topological spaces*. (or briefly $\mathcal{N}_b\mathcal{T}$ Spaces). Andrijevic [1] presented and studied a category of generalized open sets in a topological space referred to as b-open sets. Further C. Indirani et al [4] created and studied *nano b-open sets* (\mathcal{N}_b bo sets) in *nano topological spaces* ($\mathcal{N}_b\mathcal{T}$ Spaces). Bc open sets were first introduced in topological spaces by Hariwan Z. Ibrahim [6].

In this paper, we introduce and study the new classes of irresolute mappings namely *nano bc irresolute mappings and contra nano bc irresolute mappings in $\mathcal{N}_b\mathcal{T}$ spaces* and we examine its characteristics.

2. PRELIMINARIES

Definition 2.1: [8] Let \tilde{U} denote a non-empty finite set of elements called the universe and \square represents an equivalence relation on \tilde{U} named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (\tilde{U}, \square) is called the **approximation space**. Let $X \subseteq \tilde{U}$.

(i) **The lower approximation** of X with respect to \square is the set of all elements, which can be for certain classified as X with respect to \square and it is denoted by $L_R(X)$. That is, $L_R(X) = \cup_{x \in \tilde{U}} \{ \square_x : \square_x \subseteq X \}$ where \square_x denotes the equivalence class determined by $x \in \tilde{U}$.

(ii) **The upper approximation** of X with respect to \square is the set of all elements, which can be possibly classified as X with respect to \square and it is denoted by $U_R(X)$. That is $U_R(X) = \cup_{x \in \tilde{U}} \{ \square_x : \square_x \cap X \neq \emptyset \}$.

(iii) **The boundary region** of X with respect to \square is the set of all elements, that can be classified neither as X nor as not- X with respect to \square and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2: [8] Let \tilde{U} represent the universe and \square represent an equivalence relation on \tilde{U} . Then $N_R(x) = \mathcal{N}_R^T = \{ \tilde{U}, \square, L_R(X), U_R(X), B_R(X) \}$ where $X \subseteq \tilde{U}$. Then, $N_R(x)$ satisfies the axioms listed below.

(i) \tilde{U} and $\square \in N_R(x)$.

(ii) The union of the elements of any sub collection of $N_R(x)$ is in $N_R(x)$.

(iii) The intersection of the elements of any finite sub- collection of $N_R(x)$ is in $N_R(x)$. Therefore, $N_R(x)$ is a topology on \tilde{U} called the **nano topology** (\mathcal{N}_R^T) on \tilde{U} with respect to X . We call $(\tilde{U}, N_R(x))$ (or) $(\tilde{U}, \mathcal{N}_R^T)$ as the **nano topological space** (\mathcal{N}_R^T TS-in short). The elements of \mathcal{N}_R^T are called **nano open sets** (briefly, \mathcal{N}_R -open sets). The complement of nano open sets are **nano closed sets** (briefly, \mathcal{N}_R -closed sets).

Example 2.3: [8] Let $\tilde{U} = \{z_1, z_2, z_3, z_4\}$ with $\tilde{U} / \square = \{ \{z_1\}, \{z_2\}, \{z_3, z_4\} \}$ and $X = \{z_1, z_3\} \subset \tilde{U}$. Then $N_R(x) = \mathcal{N}_R^T = \{ \tilde{U}, \square, \{z_1\}, \{z_3, z_4\}, \{z_1, z_3, z_4\} \}$.

Remark 2.4 :[8] If $N_R(x) = \mathcal{N}_R^T$ is the nano topology on \tilde{U} with respect to X and \mathcal{B}_N is a nano subset of \mathcal{N}_R^T Space $(\tilde{U}, \mathcal{N}_R^T)$, then $\mathcal{B}_N = \{ \tilde{U}, L_R(X), B_R(X) \}$ is referred to as the basis for $N_R(X)$.

Definition 2.5: [8] If $(\tilde{U}, \mathcal{N}_R^T)$ is a nano T. space with respect to X where $X \subseteq \tilde{U}$ and if A_N is a nano subset in \mathcal{N}_R^T TS and if $A_N \subseteq \tilde{U}$, then

- (1) The **Nano interior** of A_N is defined as the union of all nano-open subsets of A and it is denoted by $\mathcal{N}_R \text{ int}(A_N)$. That is, $\mathcal{N}_R \text{ int}(A_N)$ is the largest nano-open subset of A_N .
- (2) The **Nano closure** of A_N is defined as the intersection of all nano closed sets containing A_N and it is denoted by $\mathcal{N}_R \text{ cl}(A_N)$. That is, $\mathcal{N}_R \text{ cl}(A_N)$ is the smallest nano closed set containing A_N .

Definition 2.6 : Let $(\tilde{U}, \mathcal{N}_R^T)$ be a \mathcal{N}_R^T TS and $A_N \subseteq \tilde{U}$. Then A_N is said to be

- (1) **Nano-semiopen set** (\mathcal{N}_R -SO set) [8] if $A_N \subseteq \mathcal{N}_R \text{ cl}[\mathcal{N}_R \text{ int}(A_N)]$ and **Nano semi- closed** (\mathcal{N}_R -SC set) [7] if $\mathcal{N}_R \text{ int}[\mathcal{N}_R \text{ cl}(A_N)] \subseteq A_N$.
- (2) **Nano-preopen set** (\mathcal{N}_R -PO set) [8] if $A_N \subseteq \mathcal{N}_R \text{ Int}[\mathcal{N}_R \text{ cl}(A_N)]$ and **Nano pre-closed** (\mathcal{N}_R -PC set) [7] if $\mathcal{N}_R \text{ cl}[\mathcal{N}_R \text{ int}(A_N)] \subseteq A_N$.

- (3) **Nano- θ open** set (N_0 - θ O set) [3] if for each $x \in A_N$, there exists a nano open set (N_0 -OS) G such that $x \in G \subset N_0 \text{ cl}(G) \subset A_N$.
- (4) **Nano- θ semiopen** (N_0 - θ SO) [3] if for each $x \in A_N$, there exists a nano semi open set (N_0 -SO set) G such that $x \in G \subset N_0 \text{ cl}(G) \subset A_N$.
- (5) **Nano-regular open** set (N_0 -RO set) [8] if $A_N = N_0 \text{ Int}[N_0 \text{ cl}(A_N)]$ and **Nano regular-closed** (N_0 -RC set) [7] if $N_0 \text{ cl}[N_0 \text{ int}(A_N)] = A_N$.

N_0 -SO(\tilde{U}, X), N_0 -PO(\tilde{U}, X), N_0 - θ O(\tilde{U}, X), N_0 - θ SO(\tilde{U}, X) and N_0 -RO(\tilde{U}, X) respectively denotes the families of all nano semi-open(N_0 -SO), nano pre-open(N_0 -PO), nano θ -open(N_0 - θ O) nano θ semi-open(N_0 - θ SO) and nano regular-open(N_0 -RO) subsets of (\tilde{U}, X).

Definition 2.7: Let $(\tilde{U}, N_R(x))$ and $(\tilde{V}, N_R(y))$ be two N_0 T Spaces. A function $\eta: (\tilde{U}, N_R(x)) \rightarrow (\tilde{V}, N_R(y))$ is said to be

- (1) Nano continuous [9] if $\eta^{-1}(B_N)$ is N_0 open set (N_0 -OS) in \tilde{U} for every N_0 open set B_N in \tilde{V} .
- (2) Nano semi continuous [9] if $\eta^{-1}(B_N)$ is N_0 semi open set (N_0 -SO) in \tilde{U} for every N_0 open set B_N in \tilde{V} .
- (3) Nano pre continuous [9] if $\eta^{-1}(B_N)$ is N_0 pre open set (N_0 -PO) in \tilde{U} for every N_0 open set B_N in \tilde{V} .
- (4) Nano b-continuous [4] if $\eta^{-1}(B_N)$ is N_0 b-open set (N_0 -bOS) in \tilde{U} for every N_0 open set B_N in \tilde{V} .

Definition 2.8: [3] Let us consider N_0 TS (\tilde{U}, N_0^T) with the set $A_N \subseteq \tilde{U}$. Then the set A_N is known as **Nano- b-open** set (briefly, N_0 b-O set) if $A_N \subseteq N_0 \text{ cl}(N_0 \text{ int}(A_N)) \cup N_0 \text{ int}(N_0 \text{ cl}(A_N))$. The complement of the N_0 b-open set is known as nano- b-closed set (briefly, N_0 b-C set).

Example 2.9: [3] Let $\tilde{U} = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ with $\tilde{U}/\square = \{\{\omega_1\}, \{\omega_3\}, \{\omega_2, \omega_4\}\}$ and $X = \{\omega_1, \omega_2\}$. Then $N_0^T = \{\tilde{U}, \emptyset, \{\omega_1\}, \{\omega_1, \omega_2, \omega_4\}, \{\omega_2, \omega_4\}\}$ and N_0 b-open sets are $\tilde{U}, \emptyset, \{\omega_1\}, \{\omega_2\}, \{\omega_4\}, \{\omega_1, \omega_2\}, \{\omega_1, \omega_3\}, \{\omega_1, \omega_4\}, \{\omega_2, \omega_4\}, \{\omega_1, \omega_2, \omega_3\}, \{\omega_1, \omega_2, \omega_4\}, \{\omega_2, \omega_3, \omega_4\}$.

3. Nano bc-Irresolute Functions

Definition 3.1. A function $\zeta: (\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}}) \rightarrow (\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$ is called nano- $b\mathcal{C}$ -irresolute (briefly, N_0 - $b\mathcal{C}$ -irresolute) if the inverse image of every N_0 - $b\mathcal{C}$ -OS in $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$ is N_0 - $b\mathcal{C}$ -OS in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$.

Example 3.2. Let $\tilde{U}_{\mathcal{N}} = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, $\tilde{U}_{\mathcal{N}}/\mathcal{R} = \{\{\omega_1\}, \{\omega_3\}, \{\omega_2, \omega_4\}\}$ and $\mathcal{X}_{\mathcal{N}} = \{\omega_1, \omega_2\} \subset \tilde{U}_{\mathcal{N}}$. Then the $\mathcal{NT} = \tau_{\mathcal{N}}(\mathcal{X}_{\mathcal{N}}) = \{\tilde{U}_{\mathcal{N}}, \emptyset_{\mathcal{N}}, \{\omega_1\}, \{\omega_2, \omega_4\}, \{\omega_1, \omega_2, \omega_4\}\}$. N_0 - $b\mathcal{C}$ -OS $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}}) = \{\tilde{U}_{\mathcal{N}}, \emptyset_{\mathcal{N}}, \{\omega_3\}, \{\omega_1, \omega_3\}, \{\omega_2, \omega_3, \omega_4\}\}$. Let $\tilde{V}_{\mathcal{N}} = \{\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, \tilde{e}_4\}$, $\tilde{V}_{\mathcal{N}}/\mathcal{R} = \{\{\tilde{e}_2\}, \{\tilde{e}_3\}, \{\tilde{e}_1, \tilde{e}_4\}\}$ and $\mathcal{K}_{\mathcal{N}} = \{\tilde{e}_1, \tilde{e}_3\} \subset \tilde{V}_{\mathcal{N}}$. Then $\varphi_{\mathcal{N}}(\mathcal{K}_{\mathcal{N}}) = \{\tilde{V}_{\mathcal{N}}, \emptyset_{\mathcal{N}}, \{\tilde{e}_3\}, \{\tilde{e}_1, \tilde{e}_4\}, \{\tilde{e}_1, \tilde{e}_3, \tilde{e}_4\}\}$. Then N_0 - $b\mathcal{C}$ -OS $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$

$=\{\tilde{V}_{\mathcal{N}}, \emptyset_{\mathcal{N}}, \{\tilde{e}_2, \tilde{e}_3\}, \{\tilde{e}_1, \tilde{e}_2, \tilde{e}_4\}, \{\tilde{e}_2\}\}$. Define $\zeta: (\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}}) \rightarrow (\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$ by $\zeta(\omega_1) = \tilde{e}_3$, $\zeta(\omega_2) = \tilde{e}_4$, $\zeta(\omega_3) = \tilde{e}_2$, $\zeta(\omega_4) = \tilde{e}_1$. Hence ζ is \mathcal{N}_0 - $b\mathcal{C}$ -irresolute function.

Theorem 3.3. A function $\zeta: (\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}}) \rightarrow (\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$ is said to be nano- $b\mathcal{C}$ -irresolute if the inverse image of every \mathcal{N}_0 - $b\mathcal{C}$ -CS in $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$ is \mathcal{N}_0 - $b\mathcal{C}$ -CS in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$.

Proof. Let $\zeta: (\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}}) \rightarrow (\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$ be \mathcal{N}_0 - $b\mathcal{C}$ -irresolute function and $\mathcal{T}_{\mathcal{N}}$ be a \mathcal{N}_0 - $b\mathcal{C}$ -CS in $\mathcal{N}TS$ $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Then $(\mathcal{T}_{\mathcal{N}})^c$ is \mathcal{N}_0 - $b\mathcal{C}$ -OS in $\mathcal{N}TS$ $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Since, ζ is \mathcal{N}_0 - $b\mathcal{C}$ -irresolute function, $\zeta^{-1}(\mathcal{T}_{\mathcal{N}})^c$ is \mathcal{N}_0 - $b\mathcal{C}$ -OS in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$. Hence $\zeta^{-1}(\mathcal{T}_{\mathcal{N}})$ is \mathcal{N}_0 - $b\mathcal{C}$ -CS in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$.

Theorem 3.4. Let $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$, $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$ and $(\hat{W}_{\mathcal{N}}, \hat{Z}_{\mathcal{N}})$ be $\mathcal{N}TS$. If $\zeta: (\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}}) \rightarrow (\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$ and $\eta: (\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}}) \rightarrow (\hat{W}_{\mathcal{N}}, \hat{Z}_{\mathcal{N}})$ are two functions. If ζ is \mathcal{N}_0 - $b\mathcal{C}$ -irresolute and η is \mathcal{N}_0 - $b\mathcal{C}$ -cts then $\eta \circ \zeta$ is \mathcal{N}_0 - $b\mathcal{C}$ -cts.

Proof. Let $\mathcal{T}_{\mathcal{N}}$ be a \mathcal{N}_0 -CS in $(\hat{W}_{\mathcal{N}}, \hat{Z}_{\mathcal{N}})$. Since η is \mathcal{N}_0 - $b\mathcal{C}$ -cts, $\eta^{-1}(\mathcal{T}_{\mathcal{N}})$ is a \mathcal{N}_0 - $b\mathcal{C}$ -CS in $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Since ζ is \mathcal{N}_0 - $b\mathcal{C}$ -irresolute, $\zeta^{-1}(\eta^{-1}(\mathcal{T}_{\mathcal{N}}))$ is a \mathcal{N}_0 - $b\mathcal{C}$ -CS in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$. Thus $(\eta \circ \zeta)^{-1}(\mathcal{T}_{\mathcal{N}})$ is \mathcal{N}_0 - $b\mathcal{C}$ -CS in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$, for every \mathcal{N}_0 - $b\mathcal{C}$ -CS $\mathcal{T}_{\mathcal{N}}$ in $(\hat{W}_{\mathcal{N}}, \hat{Z}_{\mathcal{N}})$. Hence the composition $\eta \circ \zeta: (\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}}) \rightarrow (\hat{W}_{\mathcal{N}}, \hat{Z}_{\mathcal{N}})$ is \mathcal{N}_0 - $b\mathcal{C}$ -cts.

Theorem 3.5. Let $\zeta: (\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}}) \rightarrow (\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$ and $\eta: (\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}}) \rightarrow (\hat{W}_{\mathcal{N}}, \hat{Z}_{\mathcal{N}})$ be two maps. If ζ and η are both \mathcal{N}_0 - $b\mathcal{C}$ -irresolute, then $\eta \circ \zeta$ is \mathcal{N}_0 - $b\mathcal{C}$ -irresolute.

Proof. Let $\mathcal{T}_{\mathcal{N}}$ be a \mathcal{N}_0 - $b\mathcal{C}$ -CS in $(\hat{W}_{\mathcal{N}}, \hat{Z}_{\mathcal{N}})$. Since η is \mathcal{N}_0 - $b\mathcal{C}$ -irresolute, $\eta^{-1}(\mathcal{T}_{\mathcal{N}})$ is a \mathcal{N}_0 - $b\mathcal{C}$ -CS in $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Since ζ is \mathcal{N}_0 - $b\mathcal{C}$ -irresolute, $\zeta^{-1}(\eta^{-1}(\mathcal{T}_{\mathcal{N}}))$ is \mathcal{N}_0 - $b\mathcal{C}$ -CS in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$. Thus, $(\eta \circ \zeta)^{-1}(\mathcal{T}_{\mathcal{N}}) = \zeta^{-1}(\eta^{-1}(\mathcal{T}_{\mathcal{N}}))$ is \mathcal{N}_0 - $b\mathcal{C}$ -CS in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$, for every \mathcal{N}_0 - $b\mathcal{C}$ -CS in $(\hat{W}_{\mathcal{N}}, \hat{Z}_{\mathcal{N}})$. Hence $\eta \circ \zeta$ is \mathcal{N}_0 - $b\mathcal{C}$ -irresolute.

Theorem 3.6. A function $\zeta: (\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}}) \rightarrow (\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$ is \mathcal{N}_0 - $b\mathcal{C}$ -irresolute if and only if $\zeta(\mathcal{N}_0$ - $b\mathcal{C}$ - $cl(\mathcal{G}_{\mathcal{N}})) \subseteq \mathcal{N}_0$ - $b\mathcal{C}$ - $cl(\zeta(\mathcal{G}_{\mathcal{N}}))$, for every \mathcal{N}_0 -subset $\mathcal{G}_{\mathcal{N}}$ of $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$.

Proof. Suppose ζ is \mathcal{N}_0 - $b\mathcal{C}$ -irresolute. Let $\zeta(\mathcal{G}_{\mathcal{N}}) \subseteq (\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Hence \mathcal{N}_0 - $b\mathcal{C}$ - $cl(\zeta(\mathcal{G}_{\mathcal{N}}))$ is \mathcal{N}_0 - $b\mathcal{C}$ -CS in $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Since ζ is \mathcal{N}_0 - $b\mathcal{C}$ -irresolute, $\zeta^{-1}(\mathcal{N}_0$ - $b\mathcal{C}$ - $cl(\zeta(\mathcal{G}_{\mathcal{N}})))$ is \mathcal{N}_0 - $b\mathcal{C}$ -CS in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$. Since $\zeta(\mathcal{G}_{\mathcal{N}}) \subseteq \mathcal{N}_0$ - $b\mathcal{C}$ - $cl(\zeta(\mathcal{G}_{\mathcal{N}}))$, which implies $\mathcal{G}_{\mathcal{N}} \subseteq \zeta^{-1}(\mathcal{N}_0$ - $b\mathcal{C}$ - $cl(\zeta(\mathcal{G}_{\mathcal{N}})))$. Since \mathcal{N}_0 - $b\mathcal{C}$ - cl

$(\mathcal{G}_{\mathcal{N}})$ is the smallest \mathbb{N}_0 - \mathcal{bC} -CS containing $\mathcal{G}_{\mathcal{N}}$, \mathbb{N}_0 - \mathcal{bC} -cl $(\mathcal{G}_{\mathcal{N}}) \subseteq \zeta^{-1}(\mathbb{N}_0$ - \mathcal{bC} -cl $(\zeta(\mathcal{G}_{\mathcal{N}})))$. Hence $\zeta(\mathbb{N}_0$ - \mathcal{bC} -cl $(\mathcal{G}_{\mathcal{N}})) \subseteq \mathbb{N}_0$ - \mathcal{bC} -cl $(\zeta(\mathcal{G}_{\mathcal{N}}))$.

Conversely, assume that $\zeta(\mathbb{N}_0$ - \mathcal{bC} -cl $(\mathcal{G}_{\mathcal{N}})) \subseteq \mathbb{N}_0$ - \mathcal{bC} -cl $(\zeta(\mathcal{G}_{\mathcal{N}}))$, for every \mathbb{N}_0 -subset $\mathcal{G}_{\mathcal{N}}$ of $(\tilde{\mathcal{U}}_{\mathcal{N}}, \tau_{\mathcal{N}})$. Let $\mathcal{T}_{\mathcal{N}}$ be \mathbb{N}_0 - \mathcal{bC} -CS in $(\tilde{\mathcal{V}}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Now, $\zeta^{-1}(\mathcal{T}_{\mathcal{N}}) \subseteq (\tilde{\mathcal{U}}_{\mathcal{N}}, \tau_{\mathcal{N}})$. Hence $\zeta(\mathbb{N}_0$ - \mathcal{bC} -cl $(\zeta^{-1}(\mathcal{T}_{\mathcal{N}}))) \subseteq \mathbb{N}_0$ - \mathcal{bC} -cl $(\zeta(\zeta^{-1}(\mathcal{T}_{\mathcal{N}}))) = \mathbb{N}_0$ - \mathcal{bC} -cl $(\mathcal{T}_{\mathcal{N}})$, which implies \mathbb{N}_0 - \mathcal{bC} -cl $(\zeta^{-1}(\mathcal{T}_{\mathcal{N}})) \subseteq \zeta^{-1}(\mathbb{N}_0$ - \mathcal{bC} -cl $(\mathcal{T}_{\mathcal{N}})) = \zeta^{-1}(\mathcal{T}_{\mathcal{N}})$ that implies $\zeta^{-1}(\mathcal{T}_{\mathcal{N}})$ is \mathbb{N}_0 - \mathcal{bC} -CS in $(\tilde{\mathcal{U}}_{\mathcal{N}}, \tau_{\mathcal{N}})$, for every \mathbb{N}_0 - \mathcal{bC} -CS $\mathcal{T}_{\mathcal{N}}$ in $(\tilde{\mathcal{V}}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Hence ζ is \mathbb{N}_0 - \mathcal{bC} -irresolute.

Theorem 3.7. A function $\zeta : (\tilde{\mathcal{U}}_{\mathcal{N}}, \tau_{\mathcal{N}}) \rightarrow (\tilde{\mathcal{V}}_{\mathcal{N}}, \varphi_{\mathcal{N}})$ is \mathbb{N}_0 - \mathcal{bC} -irresolute if and only if $\zeta^{-1}(\mathbb{N}_0$ - \mathcal{bC} -int $(\mathcal{G}_{\mathcal{N}})) \subseteq \mathbb{N}_0$ - \mathcal{bC} -int $(\zeta^{-1}(\mathcal{G}_{\mathcal{N}}))$, for every \mathbb{N}_0 -subset $\mathcal{G}_{\mathcal{N}}$ of $(\tilde{\mathcal{V}}_{\mathcal{N}}, \varphi_{\mathcal{N}})$.

Proof. Suppose ζ is \mathbb{N}_0 - \mathcal{bC} -irresolute. Let $\mathcal{G}_{\mathcal{N}} \subseteq (\tilde{\mathcal{V}}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Hence \mathbb{N}_0 - \mathcal{bC} -int $(\mathcal{G}_{\mathcal{N}})$ is \mathbb{N}_0 - \mathcal{bC} -OS in $(\tilde{\mathcal{V}}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Since ζ is \mathbb{N}_0 - \mathcal{bC} -irresolute, $\zeta^{-1}(\mathbb{N}_0$ - \mathcal{bC} -int $(\mathcal{G}_{\mathcal{N}}))$ is \mathbb{N}_0 - \mathcal{bC} -OS in $(\tilde{\mathcal{U}}_{\mathcal{N}}, \tau_{\mathcal{N}})$. Hence \mathbb{N}_0 - \mathcal{bC} -int $(\zeta^{-1}(\mathbb{N}_0$ - \mathcal{bC} -int $(\mathcal{G}_{\mathcal{N}}))) = \zeta^{-1}(\mathbb{N}_0$ - \mathcal{bC} -int $(\mathcal{G}_{\mathcal{N}}))$. Since, $\mathcal{G}_{\mathcal{N}} \subseteq (\tilde{\mathcal{V}}_{\mathcal{N}}, \varphi_{\mathcal{N}})$, \mathbb{N}_0 - \mathcal{bC} -int $(\mathcal{G}_{\mathcal{N}}) \subseteq \mathcal{G}_{\mathcal{N}}$ always. Hence $\zeta^{-1}(\mathbb{N}_0$ - \mathcal{bC} -int $(\mathcal{G}_{\mathcal{N}})) = \mathbb{N}_0$ - \mathcal{bC} -int $(\zeta^{-1}(\mathbb{N}_0$ - \mathcal{bC} -int $(\mathcal{G}_{\mathcal{N}}))) \subseteq \mathbb{N}_0$ - \mathcal{bC} -int $(\zeta^{-1}(\mathcal{G}_{\mathcal{N}}))$. Hence $\zeta^{-1}(\mathbb{N}_0$ - \mathcal{bC} -int $(\mathcal{G}_{\mathcal{N}})) \subseteq \mathbb{N}_0$ - \mathcal{bC} -int $(\zeta^{-1}(\mathcal{G}_{\mathcal{N}}))$.

Conversely, let $\zeta^{-1}(\mathbb{N}_0$ - \mathcal{bC} -int $(\mathcal{G}_{\mathcal{N}})) \subseteq \mathbb{N}_0$ - \mathcal{bC} -int $(\zeta^{-1}(\mathcal{G}_{\mathcal{N}}))$, for every \mathbb{N}_0 -subset $\mathcal{G}_{\mathcal{N}}$ of $(\tilde{\mathcal{V}}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Let $\mathcal{T}_{\mathcal{N}}$ be a \mathbb{N}_0 - \mathcal{bC} -OS in $(\tilde{\mathcal{V}}_{\mathcal{N}}, \varphi_{\mathcal{N}})$ and hence \mathbb{N}_0 - \mathcal{bC} -int $(\mathcal{T}_{\mathcal{N}}) = \mathcal{T}_{\mathcal{N}}$. By our assumption, $\zeta^{-1}(\mathcal{T}_{\mathcal{N}}) \subseteq \mathbb{N}_0$ - \mathcal{bC} -int $(\zeta^{-1}(\mathcal{T}_{\mathcal{N}}))$. But \mathbb{N}_0 - \mathcal{bC} -int $(\zeta^{-1}(\mathcal{T}_{\mathcal{N}})) \subseteq \zeta^{-1}(\mathcal{T}_{\mathcal{N}})$. Hence $\zeta^{-1}(\mathcal{T}_{\mathcal{N}}) = \mathbb{N}_0$ - \mathcal{bC} -int $(\zeta^{-1}(\mathcal{T}_{\mathcal{N}}))$. Then $\zeta^{-1}(\mathcal{T}_{\mathcal{N}})$ is \mathbb{N}_0 - \mathcal{bC} -OS in $(\tilde{\mathcal{U}}_{\mathcal{N}}, \tau_{\mathcal{N}})$, for every \mathbb{N}_0 -subset $\mathcal{T}_{\mathcal{N}}$ of $(\tilde{\mathcal{V}}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Hence ζ is \mathbb{N}_0 - \mathcal{bC} -irresolute.

Theorem 3.8. A function $\zeta : (\tilde{\mathcal{U}}_{\mathcal{N}}, \tau_{\mathcal{N}}) \rightarrow (\tilde{\mathcal{V}}_{\mathcal{N}}, \varphi_{\mathcal{N}})$ is \mathbb{N}_0 - \mathcal{bC} -irresolute if and only if \mathbb{N}_0 - \mathcal{bC} -cl $(\zeta^{-1}(\mathcal{G}_{\mathcal{N}})) \subseteq \zeta^{-1}(\mathbb{N}_0$ - \mathcal{bC} -cl $(\mathcal{G}_{\mathcal{N}}))$, for every \mathbb{N}_0 -subset $\mathcal{G}_{\mathcal{N}}$ of $(\tilde{\mathcal{V}}_{\mathcal{N}}, \varphi_{\mathcal{N}})$.

Proof. Suppose ζ is \mathbb{N}_0 - \mathcal{bC} -irresolute. Let $\mathcal{G}_{\mathcal{N}} \subseteq (\tilde{\mathcal{V}}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Hence \mathbb{N}_0 - \mathcal{bC} -cl $(\mathcal{G}_{\mathcal{N}})$ is \mathbb{N}_0 - \mathcal{bC} -CS in $(\tilde{\mathcal{V}}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Since ζ is \mathbb{N}_0 - \mathcal{bC} -irresolute, $\zeta^{-1}(\mathbb{N}_0$ - \mathcal{bC} -cl $(\mathcal{G}_{\mathcal{N}}))$ is \mathbb{N}_0 - \mathcal{bC} -CS in $(\tilde{\mathcal{U}}_{\mathcal{N}}, \tau_{\mathcal{N}})$. Hence \mathbb{N}_0 - \mathcal{bC} -cl $(\zeta^{-1}(\mathbb{N}_0$ - \mathcal{bC} -cl $(\mathcal{G}_{\mathcal{N}}))) = \zeta^{-1}(\mathbb{N}_0$ - \mathcal{bC} -cl $(\mathcal{G}_{\mathcal{N}}))$. Since, $\mathcal{G}_{\mathcal{N}} \subseteq \mathbb{N}_0$ - \mathcal{bC} -cl $(\mathcal{G}_{\mathcal{N}})$, then $\zeta^{-1}(\mathcal{G}_{\mathcal{N}}) \subseteq \zeta^{-1}(\mathbb{N}_0$ - \mathcal{bC} -cl $(\mathcal{G}_{\mathcal{N}}))$. Now, \mathbb{N}_0 - \mathcal{bC} -cl $(\zeta^{-1}(\mathcal{G}_{\mathcal{N}})) \subseteq \mathbb{N}_0$ - \mathcal{bC} -cl $(\zeta^{-1}(\mathbb{N}_0$ - \mathcal{bC} -cl $(\mathcal{G}_{\mathcal{N}}))) = \zeta^{-1}(\mathbb{N}_0$ - \mathcal{bC} -cl $(\mathcal{G}_{\mathcal{N}}))$ which implies \mathbb{N}_0 - \mathcal{bC} -cl $(\zeta^{-1}(\mathcal{G}_{\mathcal{N}})) \subseteq \zeta^{-1}(\mathbb{N}_0$ - \mathcal{bC} -cl $(\mathcal{G}_{\mathcal{N}}))$, for every \mathbb{N}_0 -subset $\mathcal{G}_{\mathcal{N}}$ of $(\tilde{\mathcal{V}}_{\mathcal{N}}, \varphi_{\mathcal{N}})$.

Conversely, let $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-cl}(\zeta^{-1}(\mathcal{G}_{\mathcal{N}})) \subseteq \zeta^{-1}(\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-cl}(\mathcal{G}_{\mathcal{N}}))$, for every $\mathfrak{N}\mathfrak{b}\mathfrak{C}$ -subset $\mathcal{G}_{\mathcal{N}}$ of $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Let $\mathcal{T}_{\mathcal{N}}$ be a $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-CS}$ in $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$ and hence $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-cl}(\mathcal{T}_{\mathcal{N}}) = \mathcal{T}_{\mathcal{N}}$. By our assumption, $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-cl}(\zeta^{-1}(\mathcal{T}_{\mathcal{N}})) \subseteq \zeta^{-1}(\mathcal{T}_{\mathcal{N}})$. But $\zeta^{-1}(\mathcal{T}_{\mathcal{N}}) \subseteq \mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-cl}(\zeta^{-1}(\mathcal{T}_{\mathcal{N}}))$. Hence $\zeta^{-1}(\mathcal{T}_{\mathcal{N}}) = \mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-cl}(\zeta^{-1}(\mathcal{T}_{\mathcal{N}}))$. Then $\zeta^{-1}(\mathcal{T}_{\mathcal{N}})$ is $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-CS}$ in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$, for every $\mathfrak{N}\mathfrak{b}\mathfrak{C}$ -subset $\mathcal{T}_{\mathcal{N}}$ of $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Hence ζ is $\mathfrak{N}\mathfrak{b}\mathfrak{C}$ -irresolute.

4. CONTRA $\mathfrak{N}\mathfrak{b}\mathfrak{C}$ - IRRESOLUTE FUNCTIONS

Definition 4.1. A function $\zeta : (\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}}) \rightarrow (\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$ is said to be contra $\mathfrak{N}\mathfrak{b}\mathfrak{C}$ - irresolute if $\zeta^{-1}(\mathcal{T}_{\mathcal{N}})$ is a $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-CS}$ in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$, for every $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-OS}$ $\mathcal{T}_{\mathcal{N}}$ in $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$.

Example 4.2. Let $\tilde{U}_{\mathcal{N}} = \{\mathfrak{a}_1, \mathfrak{a}_2, \mathfrak{a}_3, \mathfrak{a}_4\}$, $\tilde{U}_{\mathcal{N}}/\mathcal{R} = \{\{\mathfrak{a}_1\}, \{\mathfrak{a}_3\}, \{\mathfrak{a}_2, \mathfrak{a}_4\}\}$ and $\mathcal{X}_{\mathcal{N}} = \{\mathfrak{a}_1, \mathfrak{a}_2\} \subseteq \tilde{U}_{\mathcal{N}}$. Then $\mathcal{N}\mathcal{T} = \tau_{\mathcal{N}}(\mathcal{X}_{\mathcal{N}}) = \{\tilde{U}_{\mathcal{N}}, \emptyset_{\mathcal{N}}, \{\mathfrak{a}_1\}, \{\mathfrak{a}_2, \mathfrak{a}_4\}, \{\mathfrak{a}_1, \mathfrak{a}_2, \mathfrak{a}_4\}\}$. $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-OS}(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}}) = \{\tilde{U}_{\mathcal{N}}, \emptyset_{\mathcal{N}}, \{\mathfrak{a}_3\}, \{\mathfrak{a}_1, \mathfrak{a}_3\}, \{\mathfrak{a}_2, \mathfrak{a}_3, \mathfrak{a}_4\}\}$. Let $\tilde{V}_{\mathcal{N}} = \{\tilde{\mathfrak{e}}_1, \tilde{\mathfrak{e}}_2, \tilde{\mathfrak{e}}_3\}$, $\tilde{V}_{\mathcal{N}}/\mathcal{R} = \{\{\tilde{\mathfrak{e}}_2\}, \{\tilde{\mathfrak{e}}_1, \tilde{\mathfrak{e}}_3\}\}$ and $\mathcal{K}_{\mathcal{N}} = \{\tilde{\mathfrak{e}}_1, \tilde{\mathfrak{e}}_3\} \subseteq \tilde{V}_{\mathcal{N}}$. Then $\varphi_{\mathcal{N}}(\mathcal{K}_{\mathcal{N}}) = \{\tilde{V}_{\mathcal{N}}, \emptyset_{\mathcal{N}}, \{\tilde{\mathfrak{e}}_1, \tilde{\mathfrak{e}}_3\}\}$. Then $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-CS}(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}}) = \{\tilde{V}_{\mathcal{N}}, \emptyset_{\mathcal{N}}, \{\tilde{\mathfrak{e}}_2\}, \{\tilde{\mathfrak{e}}_1, \tilde{\mathfrak{e}}_2\}, \{\tilde{\mathfrak{e}}_2, \tilde{\mathfrak{e}}_3\}\}$. Define $\zeta : (\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}}) \rightarrow (\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$ by $\zeta(\mathfrak{a}_1) = \tilde{\mathfrak{e}}_1$, $\zeta(\mathfrak{a}_2) = \tilde{\mathfrak{e}}_3$, $\zeta(\mathfrak{a}_3) = \tilde{\mathfrak{e}}_2$, $\zeta(\mathfrak{a}_4) = \tilde{\mathfrak{e}}_3$. Then $\zeta^{-1}(\tilde{\mathfrak{e}}_2) = \{\mathfrak{a}_2\}$, $\zeta^{-1}(\tilde{\mathfrak{e}}_1, \tilde{\mathfrak{e}}_3) = \{\mathfrak{a}_1, \mathfrak{a}_3\}$, $\zeta^{-1}(\{\tilde{\mathfrak{e}}_2, \tilde{\mathfrak{e}}_3\}) = \{\mathfrak{a}_2, \mathfrak{a}_3, \mathfrak{a}_4\}$. The inverse image of every $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-CS}$ in $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$ is $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-OS}$ in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$. Hence ζ is contra $\mathfrak{N}\mathfrak{b}\mathfrak{C}$ - irresolute function.

Theorem 4.3. A function $\zeta : (\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}}) \rightarrow (\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$ is contra $\mathfrak{N}\mathfrak{b}\mathfrak{C}$ - irresolute if and only if the inverse image of $\zeta^{-1}(\mathcal{T}_{\mathcal{N}})$ is $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-OS}$ in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$, for every $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-CS}$ $\mathcal{T}_{\mathcal{N}}$ in $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$.

Proof. Let $\mathcal{T}_{\mathcal{N}}$ be $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-CS}$ in $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Then $\tilde{V}_{\mathcal{N}} - \mathcal{T}_{\mathcal{N}}$ is $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-OS}$ in $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Since ζ is contra $\mathfrak{N}\mathfrak{b}\mathfrak{C}$ - irresolute, $\zeta^{-1}(\tilde{V}_{\mathcal{N}} - \mathcal{T}_{\mathcal{N}})$ is $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-CS}$ in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$. Therefore $\zeta^{-1}(\mathcal{T}_{\mathcal{N}})$ is $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-OS}$ in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$. Conversely, assume that the inverse image of every $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-CS}$ in $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$ is $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-OS}$ in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$. Let $\mathcal{G}_{\mathcal{N}}$ be $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-OS}$ in $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Then $\tilde{V}_{\mathcal{N}} - \mathcal{G}_{\mathcal{N}}$ is $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-CS}$ in $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. By assumption, $\zeta^{-1}(\tilde{V}_{\mathcal{N}} - \mathcal{G}_{\mathcal{N}})$ is $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-OS}$ in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$.

Now, $\zeta^{-1}(\tilde{V}_{\mathcal{N}} - \mathcal{G}_{\mathcal{N}}) = \tilde{V}_{\mathcal{N}} - \zeta^{-1}(\mathcal{G}_{\mathcal{N}})$ is $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-OS}$ in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$ which implies that $\zeta^{-1}(\mathcal{G}_{\mathcal{N}})$ is $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-CS}$ in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$. Thus $\zeta^{-1}(\mathcal{G}_{\mathcal{N}})$ is $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-CS}$ in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$, for every $\mathfrak{N}\mathfrak{b}\mathfrak{C}\text{-OS}$ $\mathcal{G}_{\mathcal{N}}$ in $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Hence ζ is contra $\mathfrak{N}\mathfrak{b}\mathfrak{C}$ - irresolute.

Remark 4.4. The composition of any two contra $\mathfrak{N}\mathfrak{b}\mathfrak{C}$ - irresolute functions is need not be a contra $\mathfrak{N}\mathfrak{b}\mathfrak{C}$ - irresolute function. It is shown in the following illustration.

Example 4.5. Let $\tilde{U}_{\mathcal{N}} = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, $\tilde{U}_{\mathcal{N}} / \mathcal{R} = \{\{\omega_1\}, \{\omega_4\}, \{\omega_2, \omega_3\}\}$ and $\mathcal{X}_{\mathcal{N}} = \{\omega_1, \omega_2\} \subseteq \tilde{U}_{\mathcal{N}}$. Then $\mathcal{NT} = \tau_{\mathcal{N}}(\mathcal{X}_{\mathcal{N}}) = \{\tilde{U}_{\mathcal{N}}, \emptyset_{\mathcal{N}}, \{\omega_1\}, \{\omega_2, \omega_3\}, \{\omega_1, \omega_2, \omega_3\}\}$. Let $\tilde{V}_{\mathcal{N}} = \{\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, \tilde{e}_4\}$, $\tilde{V}_{\mathcal{N}} / \mathcal{R} = \{\{\tilde{e}_1\}, \{\tilde{e}_2, \tilde{e}_3, \tilde{e}_4\}\}$ and $\mathcal{K}_{\mathcal{N}} = \{\tilde{e}_2, \tilde{e}_3\} \subseteq \tilde{V}_{\mathcal{N}}$. Then $\varphi_{\mathcal{N}}(\mathcal{K}_{\mathcal{N}}) = \{\tilde{V}_{\mathcal{N}}, \emptyset_{\mathcal{N}}, \{\tilde{e}_2, \tilde{e}_3, \tilde{e}_4\}\}$. Let $W_{\mathcal{N}} = \{\delta_1, \delta_2, \delta_3, \delta_4\}$, $W_{\mathcal{N}} / \mathcal{R} = \{\{\delta_2\}, \{\delta_3\}, \{\delta_1, \delta_4\}\}$ and $\hat{Z}_{\mathcal{N}} = \{\delta_1, \delta_2\}$. Then $\psi_{\mathcal{N}}(\hat{Z}_{\mathcal{N}}) = \{W_{\mathcal{N}}, \emptyset_{\mathcal{N}}, \{\delta_2\}, \{\delta_1, \delta_4\}, \{\delta_1, \delta_2, \delta_4\}\}$. Define $\zeta: (\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}}) \rightarrow (\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$ by $\zeta(\omega_1) = \tilde{e}_1$, $\zeta(\omega_2) = \tilde{e}_2$, $\zeta(\omega_3) = \tilde{e}_4$, $\zeta(\omega_4) = \tilde{e}_3$ and $\eta: (\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}}) \rightarrow (\hat{W}_{\mathcal{N}}, \hat{Z}_{\mathcal{N}})$ by $\eta(\tilde{e}_1) = \delta_3$, $\eta(\tilde{e}_2) = \delta_4$, $\eta(\tilde{e}_3) = \delta_2$, $\eta(\tilde{e}_4) = \delta_1$. Here ζ and η are contra \mathbb{N}_0 -bc-irresolute functions but their composition need not be a contra \mathbb{N}_0 -bc-irresolute function.

Theorem 4.6. Let $\zeta: (\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}}) \rightarrow (\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$ and $\eta: (\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}}) \rightarrow (\hat{W}_{\mathcal{N}}, \hat{Z}_{\mathcal{N}})$ be two functions.

- (i) If η is \mathbb{N}_0 -bc-cts and ζ is contra \mathbb{N}_0 -bc-irresolute, then $\eta \circ \zeta$ is contra \mathbb{N}_0 -bc-cts
- (ii) If η is contra \mathbb{N}_0 -bc-irresolute and ζ is contra \mathbb{N}_0 -bc-irresolute, then $\eta \circ \zeta$ is \mathbb{N}_0 -bc-irresolute.
- (iii) If η is \mathbb{N}_0 -bc-irresolute and ζ is contra \mathbb{N}_0 -bc-irresolute, then $\eta \circ \zeta$ is contra \mathbb{N}_0 -bc-irresolute.

Proof. (i) Let $\mathcal{T}_{\mathcal{N}}$ be \mathbb{N}_0 -OS in $(\hat{W}_{\mathcal{N}}, \hat{Z}_{\mathcal{N}})$. Since η is \mathbb{N}_0 -bc-cts, $\eta^{-1}(\mathcal{T}_{\mathcal{N}})$ is \mathbb{N}_0 -bc-OS in $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Since ζ is contra \mathbb{N}_0 -bc-irresolute, $\zeta^{-1}(\eta^{-1}(\mathcal{T}_{\mathcal{N}}))$ is \mathbb{N}_0 -bc-CS in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$. That is, $\zeta^{-1}(\eta^{-1}(\mathcal{T}_{\mathcal{N}})) = (\eta \circ \zeta)^{-1}(\mathcal{T}_{\mathcal{N}})$ is \mathbb{N}_0 -bc-CS in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$. Thus $(\eta \circ \zeta)^{-1}(\mathcal{T}_{\mathcal{N}})$ is \mathbb{N}_0 -bc-CS in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$, for each $\mathcal{T}_{\mathcal{N}}$ is \mathbb{N}_0 -OS in $(\hat{W}_{\mathcal{N}}, \hat{Z}_{\mathcal{N}})$. Hence $\eta \circ \zeta$ is contra \mathbb{N}_0 -bc-cts.

(ii) Let $\mathcal{T}_{\mathcal{N}}$ be \mathbb{N}_0 -bc-OS in $(\hat{W}_{\mathcal{N}}, \hat{Z}_{\mathcal{N}})$. Since η is contra \mathbb{N}_0 -bc-irresolute, $\eta^{-1}(\mathcal{T}_{\mathcal{N}})$ is \mathbb{N}_0 -bc-CS in $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Since ζ is contra \mathbb{N}_0 -bc-irresolute, $\zeta^{-1}(\eta^{-1}(\mathcal{T}_{\mathcal{N}}))$ is \mathbb{N}_0 -bc-OS in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$. That is, $\zeta^{-1}(\eta^{-1}(\mathcal{T}_{\mathcal{N}})) = (\eta \circ \zeta)^{-1}(\mathcal{T}_{\mathcal{N}})$ is \mathbb{N}_0 -bc-OS in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$. Thus $(\eta \circ \zeta)^{-1}(\mathcal{T}_{\mathcal{N}})$ is \mathbb{N}_0 -bc-OS in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$, for each $\mathcal{T}_{\mathcal{N}}$ is \mathbb{N}_0 -bc-OS in $(\hat{W}_{\mathcal{N}}, \hat{Z}_{\mathcal{N}})$. Hence $\eta \circ \zeta$ is \mathbb{N}_0 -bc-irresolute.

(iii) Let $\mathcal{T}_{\mathcal{N}}$ be \mathbb{N}_0 -bc-OS in $(\hat{W}_{\mathcal{N}}, \hat{Z}_{\mathcal{N}})$. Since η is \mathbb{N}_0 -bc-irresolute, $\eta^{-1}(\mathcal{T}_{\mathcal{N}})$ is \mathbb{N}_0 -bc-OS in $(\tilde{V}_{\mathcal{N}}, \varphi_{\mathcal{N}})$. Since ζ is contra \mathbb{N}_0 -bc-irresolute, $\zeta^{-1}(\eta^{-1}(\mathcal{T}_{\mathcal{N}}))$ is \mathbb{N}_0 -bc-CS in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$. That is, $\zeta^{-1}(\eta^{-1}(\mathcal{T}_{\mathcal{N}})) = (\eta \circ \zeta)^{-1}(\mathcal{T}_{\mathcal{N}})$ is \mathbb{N}_0 -bc-CS in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$. Thus $(\eta \circ \zeta)^{-1}(\mathcal{T}_{\mathcal{N}})$ is \mathbb{N}_0 -bc-CS in $(\tilde{U}_{\mathcal{N}}, \tau_{\mathcal{N}})$, for each $\mathcal{T}_{\mathcal{N}}$ is \mathbb{N}_0 -bc-OS in $(\hat{W}_{\mathcal{N}}, \hat{Z}_{\mathcal{N}})$. Hence $\eta \circ \zeta$ is contra \mathbb{N}_0 -bc-irresolute.

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