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# WEIGHTED COMPOSITION OF POWERS OF QUASI PARANORMAL OPERATORS

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## ABSTRACT

Let TTT be a bounded linear operator defined on an infinite-dimensional complex Hilbert space. An operator TTT is said to be **quasi-paranormal** if it satisfies the inequality

‖T2x‖2≤‖T3x‖·‖Tx‖for all x∈H.\|T^2x\|^2 \leq \|T^3x\| \cdot \|Tx\| \quad \text{for all } x \in \mathcal{H}.‖T2x‖2≤‖T3x‖·‖Tx‖for all x∈H.

In this study, we establish that the **powers of quasi-paranormal operators** not only retain essential properties within their class but also remain well-defined and meaningful when extended to both the L2L^2L2 space and the Hardy space. Moreover, we demonstrate that **weighted composition operators**, when generated by quasi-paranormal operators, also exhibit analogous boundedness and structural behavior in these function spaces.

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**Key words: quasi paranormal operators,weighted hardy space**

## 1. Introduction

This article presents a detailed discussion on the powers of quasi-paranormal operators under the condition that TnT^nTn is quasi-paranormal for every natural number nnn. The class of such operators encompasses both the class of paranormal operators and quasi class A operators. The study further illustrates the behavior and structural properties of quasi-paranormal composition operators, along with their weighted counterparts. Special attention is given to their action on L2L^2L2 spaces and Hardy spaces, where the boundedness and functional characteristics of these operators are analyzed in depth.

## 2. Characteristics of Powers of Quasi-Paranormal Operators

Theorem 2.1

Let TTT be a bounded linear operator. Then TTT is said to be of powers of quasi-paranormal operators if and only if

‖T2x‖2≤‖T3x‖·‖Tx‖for all x∈H.\|T^2x\|^2 \leq \|T^3x\| \cdot \|Tx\| \quad \text{for all } x \in \mathcal{H}.‖T2x‖2≤‖T3x‖·‖Tx‖for all x∈H.

Proof:

Assume TTT is a quasi-paranormal operator. By definition, the inequality

10.48047/jocaaa.2023.31.04.34

‖T2x‖2≤‖T3x‖·‖Tx‖\|T^2x\|^2 \leq \|T^3x\| \cdot \|Tx\|‖T2x‖2≤‖T3x‖·‖Tx‖

holds for all x∈Hx \in \mathcal{H}x∈H. Since this property is preserved for each power TnT^nTn, the operator satisfies the quasi-paranormal condition at all levels, thus proving the result.

Theorem 2.2

Let TTT be a bounded linear operator. Then TTT is of powers of quasi-paranormal operators if and only if

‖Tnx‖2≤‖Tn+1x‖·‖Tn−1x‖for all x∈H, n∈N.\|T^nx\|^2 \leq \|T^{n+1}x\| \cdot \|T^{n-1}x\| \quad \text{for all } x \in \mathcal{H},\ n \in \mathbb{N}.‖Tnx‖2≤‖Tn+1x‖·‖Tn−1x‖for all x∈H, n∈N.

Proof:

Given that TTT satisfies the quasi-paranormal inequality for each power nnn, we apply the condition

‖Tnx‖2≤‖Tn+1x‖·‖Tn−1x‖\|T^nx\|^2 \leq \|T^{n+1}x\| \cdot \|T^{n-1}x\|‖Tnx‖2≤‖Tn+1x‖·‖Tn−1x‖

for all x∈Hx \in \mathcal{H}x∈H, which follows from iteratively applying the quasi-paranormal definition. This inequality can be connected through functional calculus and supports the operator belonging to this class.

Theorem 2.3

Let TTT be a bounded linear operator. Then TTT is of powers of quasi-paranormal operators if and only if it is unitarily equivalent to another operator SSS, where SSS is itself of powers of quasi-paranormal operators.

Proof:

Suppose TTT is unitarily equivalent to SSS, i.e., there exists a unitary operator UUU such that

T=U∗SU.T = U^*SU.T=U∗SU.

Since the property of being quasi-paranormal is preserved under unitary equivalence, and SSS is assumed to satisfy the powers of quasi-paranormal condition, then TTT also inherits this property. Hence, TTT is of powers of quasi-paranormal operators.

**3.Powers of quasi paranormal composition operators**

10.48047/jocaaa.2023.31.04.34

Take $\left(X,\sum,\lambda\right)$ be a $\sigma$ – finite space and take $T:X\rightarrow X$ non – singular transformation. A linear operator $cf=f\circ T\, on\, L^2\left(X,\sum,\lambda\right)$ is referred as composition operator stimulated by *T*, when $\lambda T^{-1}$ is continuous and dependon measure $\lambda$ and the derivative $d\lambda T^{-1}/d\lambda=f_0$ is bounded.In this article, we characterize powers of quasi paranormal composition operator.

## Corollary 3.1

For all complex valued function $f_0$ induced the operator $M_{f_0}\ on\ L^2\left(\lambda\right)$

i. $M_{f_0}f=f_0 f$ in all $f\in L^2\left(\lambda\right)$

ii. $C^*Cf=M_{f_0}f$

iii. $C^{*2}\,C^2=M_{f_0^{(2)}}f.$

## Theorem 3.2

Let $C\in B\left[L^2\left(\lambda\right)\right]$ be powers of quasi paranormal operator iff

$$f_0^{(2p+1)}-2\lambda f_0^{(p+1)}+\lambda^2 f_0^{(1)}\geq 0\, a.e.$$

## Proof

Let $C\in B\left[L^2\left(\lambda\right)\right]$ be powers of quasi paranormal operator iff

$$C^{*(2p+1)}\,C^{(2p+1)}-2\lambda C^{*(p+1)}\,C^{(p+1)}+\lambda^2 C^*C\geq 0$$

Thus, $\left\langle\left(C^{*(2p+1)}\,C^{(2p+1)}-2\lambda C^{*(p+1)}\,C^{(p+1)}+\lambda^2 C^*C\right)X_E,X_E\right\rangle\geq 0$

In all function $\chi_E$ of E in $\sum$ of $\lambda\left(E\right)<\infty$ while $C^*C=M_{f_0}$

and $C^{*2}C^2=M_{f_0^{(2)}}$, we have $C^{*(2p+1)}C^{(2p+1)}=M_{f_0^{(2p+1)}}$

$$\left\langle\left(M_{f_0^{(2p+1)}}-2\lambda M_{f_0^{(p+1)}}+\lambda^2 M_{f_0}\right)\chi_E,\chi_E\right\rangle\geq 0.$$

$$\int_E\left\{M_{f_0^{(2p+1)}}-2\lambda M_{f_0^{(p+1)}}+\lambda^2 M_{f_0}\right\}d\lambda\geq 0.$$

forall $E\in\sum.$

Thus C is the powers of quasi paranormal operator iff $f_0^{(2p+1)}-2\lambda f_0^{(p+1)}+\lambda^2 f_0\geq 0$

10.48047/jocaaa.2023.31.04.34

**Corollary 3.3**

If $C \in B\left[L^2(\lambda)\right]$ with dense range, then C is powers of quasi paranormal operator iff $f_0^{(2p+1)} - 2\lambda f_0^{(p+1)} + \lambda^2 f_0 \geq 0 \; a.e.$

**Example 3.4**

Take $X = N$ and $\lambda$ is the counting measure. Termed $T : N \to N$ by T(1)=1, T(n+m+1) = n and $m = 0,1,2,... \; n \in N$. Then $f_0^{(2p+1)} - 2\lambda f_0^{(p+1)} + \lambda^2 f_0 \geq 0 \; a.e.$ Therefore C is powers of quasi paranormal composition operators.

**Theorem 3.5**

If $C^* \in B\left[L^2(\lambda)\right]$ be powers of quasi paranormal operator iff

$$\left[(f_0 \circ T)^{(2p+1)} p\right] - 2\lambda\left[(f_0 \circ T)^{(p+1)} p\right] + \lambda^2\left[(f_0 \circ T) p\right] \geq 0 \; a.e.$$

**Proof**

Let $C^*$ be the powers of quasi paranormal operator iff

$$C^{(2p+1)} C^{*(p+1)} - 2\lambda C^{*(p+1)} C^{(p+1)} + \lambda^2 C C^* \geq 0$$

$$\Rightarrow \left\langle \left(C^{(2p+1)} C^{*(p+1)} - 2\lambda C^{*(p+1)} C^{(p+1)} + \lambda^2 C C^*\right) f, f \right\rangle \geq 0$$

Given any $f \in L^2$., then $\langle CC^* f, f \rangle = \langle (f_0 \circ T) pf, f \rangle$, here p is the projection of $L^2$ onto $\overline{R(C)}$. Hence $C^*$ is a power of quasi paranormal operator iff

$$\left\langle \left\{ \left[(f_0 \circ T)^{(2p+1)} p\right] - 2\lambda\left[(f_0 \circ T)^{(p+1)} p\right] + \lambda^2\left[(f_0 \circ T) p\right] \right\} f, f \right\rangle \geq 0$$

For all $f \in L^2$,

$$\left[(f_0 \circ T)^{(2p+1)} p\right] - 2\lambda\left[(f_0 \circ T)^{(p+1)} p\right] + \lambda^2\left[(f_0 \circ T) p\right] \geq 0 \; a.e.$$

**Corollary3.6**

10.48047/jocaaa.2023.31.04.34

If $C^* \in B\left[L^2(\lambda)\right]$ with dense range, then $C^*$ is powers of quasi paranormal operator iff

$$\left[\left[\left(f_0 \circ T\right)^{(2p+1)} p\right] - 2\lambda\left[\left(f_0 \circ T\right)^{(p+1)} p\right] + \lambda^2\left(f_0 \circ T\right) p\right] \geq 0 \ a.e.$$

## 4. Weighted Powers of Quasi-Paranormal Composition Operators

**Proposition 4.1**

Let WWW be a weighted composition operator. Then:

1. W(f)=w·(f∘ϕ)W(f) = w \cdot (f \circ \phi)W(f)=w·(f∘ϕ), where www is a measurable weight function and ϕ\phiϕ is a self-map on the domain.
2. WWW is linear and bounded on the corresponding function space if www is bounded and ϕ\phiϕ is analytic.

**Theorem 4.2**

Let WWW be a weighted composition operator. Then WWW is of powers of quasi-paranormal operators if and only if:

∥W2f∥2≤∥W3f∥·∥Wf∥for all f∈H.\|W^2 f\|^2 \leq \|W^3 f\| \cdot \|W f\| \quad \text{for all } f \in \mathcal{H}.∥W2f∥2≤∥W3f∥·∥Wf∥for all f∈H.

**Proof:**

Assume WWW satisfies the quasi-paranormal inequality. Then:

∥W2f∥2≤∥W3f∥·∥Wf∥\|W^2 f\|^2 \leq \|W^3 f\| \cdot \|W f\|∥W2f∥2≤∥W3f∥·∥Wf∥

holds for all fff. Since this inequality is preserved across powers of WWW, it follows that WWW is of powers of quasi-paranormal operators.

**Corollary 4.3**

If WWW satisfies the above condition on a dense subspace of H\mathcal{H}H, then WWW is of powers of quasi-paranormal operators.

**Theorem 4.4**

Let W∗W^*W∗ be the adjoint of a weighted composition operator WWW. Then W∗W^*W∗ is of powers of quasi-paranormal operators if and only if:

∥(W∗)2f∥2≤∥(W∗)3f∥·∥W∗f∥for all f∈H.\|(W^*)^2 f\|^2 \leq \|(W^*)^3 f\| \cdot \|W^* f\| \quad \text{for all } f \in \mathcal{H}.∥(W∗)2f∥2≤∥(W∗)3f∥·∥W∗f∥for all f∈H.

**Proof:**

Assuming W∗W^*W∗ satisfies the above inequality, the operator is closed under powers in the quasi-paranormal class. The property remains valid for all f∈Hf \in \mathcal{H}f∈H, proving the result.

10.48047/jocaaa.2023.31.04.34

**Corollary 4.5**

If W∗W^*W∗ satisfies the quasi-paranormal condition on a dense subset, then W∗W^*W∗ is of powers of quasi-paranormal operators.

**Lemma 4.6**

Let TTT be a bounded linear operator. The **Aluthge transformation** of TTT, denoted Δ(T)\Delta(T)Δ(T), is defined by:

Δ(T)=∣T∣1/2U∣T∣1/2\Delta(T) = |T|^{1/2} U |T|^{1/2}Δ(T)=∣T∣1/2U∣T∣1/2

where T=U∣T∣T = U|T|T=U∣T∣ is the polar decomposition.

**Definition 4.7**

An operator WWW is called a **weighted composition operator** if

W(f)=w⋅(f∘ϕ),W(f) = w \cdot (f \circ \phi),W(f)=w⋅(f∘ϕ),

where www is a weight function and ϕ\phiϕ is an analytic self-map.

**Lemma 4.8**

If WWW is a weighted composition operator, then:

1. WWW is bounded if w∈L∞w \in L^\inftyw∈L∞ and ϕ\phiϕ is analytic.
2. WWW maps analytic functions to analytic functions.
3. WWW preserves inner product structure if additional conditions are met.

**Theorem 4.9**

Let WWW be a weighted composition operator. Then WWW is of powers of quasi-paranormal operators if and only if:

∥Wnf∥2≤∥Wn+1f∥⋅∥Wn−1f∥for all f∈H.\|W^n f\|^2 \leq \|W^{n+1} f\| \cdot \|W^{n-1} f\| \quad \text{for all } f \in \mathcal{H}.∥Wnf∥2≤∥Wn+1f∥⋅∥Wn−1f∥for all f∈H.

**Proof:**

By the quasi-paranormal definition extended to higher powers, the inequality holds iteratively, ensuring WWW belongs to the class of powers of quasi-paranormal operators.

**Corollary 4.10**

If the above condition is satisfied for WWW, then WWW is of *-quasi-paranormal type.

**Theorem 4.11**

If WWW is a weighted composition operator, then WWW is of powers of quasi-paranormal operators if and only if:

∥W2f∥2≤∥W3f∥⋅∥Wf∥for all f.\|W^2 f\|^2 \leq \|W^3 f\| \cdot \|W f\| \quad \text{for all } f.∥W2f∥2≤∥W3f∥⋅∥Wf∥for all f.

10.48047/jocaaa.2023.31.04.34

**Proof:**

The condition follows directly from the quasi-paranormal inequality.

**Theorem 4.12**

Let WWW be a weighted composition operator. Then WWW is of powers of quasi-paranormal operators if the condition:

‖Wnf‖2≤‖Wn+1f‖·‖Wn−1f‖\|W^n f\|^2 \leq \|W^{n+1} f\| \cdot \|W^{n-1} f\|‖Wnf‖2≤‖Wn+1f‖·‖Wn−1f‖

holds for all n∈Nn \in \mathbb{N}n∈N and all f∈Hf \in \mathcal{H}f∈H.

**Theorem 4.13**

Let WWW be a weighted composition operator. Then WWW is of powers of quasi-paranormal operators if and only if:

‖W2f‖2≤‖W3f‖·‖Wf‖for all f.\|W^2 f\|^2 \leq \|W^3 f\| \cdot \|W f\| \quad \text{for all }
f.‖W2f‖2≤‖W3f‖·‖Wf‖for all f.

**Proof:**

Given that WWW satisfies the defining inequality of quasi-paranormal operators and that this behavior is preserved under operator powers, the result follows.

**Corollary 4.14**

If the condition holds on a dense subspace of H\mathcal{H}H, then WWW is a power of a quasi-paranormal operator.

**5. Powers of Quasi-Paranormal Composition Operators on Weighted Hardy Space**

Let Hω\mathcal{H}_\omegaHω denote the space of formal power series

f(z)=∑n=0∞anznf(z) = \sum_{n=0}^{\infty} a_n z^nf(z)=n=0∑∞anzn

such that

‖f‖2=∑n=0∞|an|2ωn<∞,\|f\|^2 = \sum_{n=0}^{\infty} |a_n|^2 \omega_n < \infty,‖f‖2=n=0∑∞|an|2ωn<∞,

where {ωn}\{\omega_n\}{ωn} is a sequence of positive weights. This defines a **weighted Hardy space** of analytic functions on the unit disc D\mathbb{D}D. The inner product on Hω\mathcal{H}_\omegaHω is given by:

⟨f,g⟩=∑n=0∞anbn¯ωn,\langle f, g \rangle = \sum_{n=0}^{\infty} a_n \overline{b_n} \omega_n,⟨f,g⟩=n=0∑∞an bnωn,

for f(z)=∑anznf(z) = \sum a_n z^nf(z)=∑anzn and g(z)=∑bnzng(z) = \sum b_n z^ng(z)=∑bnzn.

Let ϕ:D→D\phi: \mathbb{D} \to \mathbb{D}ϕ:D→D be an analytic function. The **composition operator** CϕC_\phiCϕ is defined by

Cϕ(f)=f∘ϕ,C_\phi(f) = f \circ \phi,Cϕ(f)=f∘ϕ,

acting on Hω\mathcal{H}_\omegaHω.

10.48047/jocaaa.2023.31.04.34

**Lemma 5.1**

Let α\alphaα be the weight order and z0∈Dz_0 \in \mathbb{D}z0∈D. Then:

i. Define Kz0(z)=∑n=0∞(z0)nωnznK_{z_0}(z) = \sum_{n=0}^{\infty} \frac{(z_0)^n}{\omega_n} z^nKz0 (z)=∑n=0∞ωn(z0)nzn, which acts as the reproducing kernel.

ii. The function Kz0K_{z_0}Kz0 is a point evaluation for Hω\mathcal{H}_\omegaHω, and f(z0)=⟨f,Kz0⟩,for all f∈Hω.f(z_0) = \langle f, K_{z_0} \rangle, \quad \text{for all } f \in \mathcal{H}_\omega.f(z0)=⟨f,Kz0⟩,for all f∈Hω.

**Theorem 5.2**

If CϕC_\phiCϕ is of powers of quasi-paranormal operators on Hω\mathcal{H}_\omegaHω, then ∥Cϕ2f∥2≤∥Cϕ3f∥⋅∥Cϕf∥for all f∈Hω.\|C_\phi^2 f\|^2 \leq \|C_\phi^3 f\| \cdot \|C_\phi f\| \quad \text{for all } f \in \mathcal{H}_\omega.∥Cϕ2f∥2≤∥Cϕ3f∥⋅∥Cϕf∥for all f∈Hω.

**Proof:**

Assume CϕC_\phiCϕ satisfies the quasi-paranormal inequality. Then for all f∈Hωf \in \mathcal{H}_\omegaf∈Hω:

∥Cϕ2f∥2≤∥Cϕ3f∥⋅∥Cϕf∥.\|C_\phi^2 f\|^2 \leq \|C_\phi^3 f\| \cdot \|C_\phi f\|.∥Cϕ2f∥2≤∥Cϕ3f∥⋅∥Cϕf∥.

Applying this recursively:

∥Cϕ3f∥2≤∥Cϕ4f∥⋅∥Cϕ2f∥,∥Cϕ4f∥2≤∥Cϕ5f∥⋅∥Cϕ3f∥,\|C_\phi^3 f\|^2 \leq \|C_\phi^4 f\| \cdot \|C_\phi^2 f\|, \quad \|C_\phi^4 f\|^2 \leq \|C_\phi^5 f\| \cdot \|C_\phi^3 f\|,∥Cϕ3f∥2≤∥Cϕ4f∥⋅∥Cϕ2f∥,∥Cϕ4f∥2≤∥Cϕ5f∥⋅∥Cϕ3f∥,

and so on. Therefore, CϕC_\phiCϕ is confirmed to be of powers of quasi-paranormal operators on Hω\mathcal{H}_\omegaHω.

**6. Conclusion**

This study has established the existence and structural behavior of powers of quasi-paranormal composition operators, along with their weighted counterparts, on both L2L^2L2 spaces and weighted Hardy spaces. The results demonstrate that under appropriate analytic conditions, these operators retain quasi-paranormal properties through their powers, contributing to a deeper understanding of operator classes in functional and Hilbert space analysis.

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