Weighted Composition Of Powers Of Quasi Paranormal Operators

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ABSTRACT

Let T be a bounded linear operator on an infinite dimensional complex Hilbert space.The quasi paranormal operator satisfying $\left\|T^2x\right\|^{2p} \le \left\|T^{3p}x\right\|\left\|Tx\right\|$ for every $x \in H$. We prove the powers of quasi paranormal operators and weighted composition operators also exists on L^2 Space and Hardy Space.

Key words: quasi paranormal operators,weighted hardy space

1. Introduction

In this article, it is discussed and shows the powers of quasi paranormal operator if $\left\|T^2x\right\|^{2p} \le \left\|T^{3p}x\right\|\left\|Tx\right\|$ for every $x \in H$. It comprises the classes of paranormal operators and quasi class 'A'operators. Illustrated the powers of quasi paranormal composition operators and similarly weighted composition of powers of quasi paranormal operators on L^2 space and Hardy Space are described.

2.Characteristics of powers of quasi paranormal operators

Theorem 2.1:

Let the powers of quasi paranormal operator be $T \in B(H)$

iff $\left\|T^{p+1}x\right\|^{2p} \le \left\|T_1^{2p+1}x\right\|\left\|x\right\|$ for every $x \in H$.

Proof

Let the powers of quasi paranormal operator $T \in B(H)$,

$$T^*T^{*2p}\, T^{2p}T - 2\lambda T^*T^{*p}\, T^pT + \lambda^2 T^*T \geq 0$$

$$\Rightarrow T^{*2p+1}\, T^{2p+1} - 2\lambda T^{*p+1}\, T^{p+1}T + \lambda^2 T^*T \geq 0$$

$$\Rightarrow a = T^*T, b = 2T^{*p+1}\, T^{p+1}; C = \lambda T^{*2p+1}\, T^{2p+1}$$

$$\Rightarrow b^2 - 4ac \geq 0$$

$$\Rightarrow \left[-2\lambda T^{*p+1}\, T^{p+1}\right]^2 - 4(T^*T)\left(T^{*2p+1}\, T^{2p+1}\right) \geq 0$$

$$4\left(T^{*p+1}\, T^{p+1}\right)^2 - 4(T^*T)\left(T^{*2p+1}\, T^{2p+1}\right) \geq 0$$

$$4\left(T^{*p+1}\, T^{p+1}\right)^2 \geq 4(T^*T)\left(T^{*2p+1}\, T^{2p+1}\right)$$

$$\left(T^{*p+1}\, T^{p+1}\right)^2 \geq \left\|T^{2p+1}x\right\|^2 \|Tx\|^2$$

$$\left\|T^{p+1}x\right\|^{2p} \leq \left\|T_1^{2p+1}x\right\| \|x\| \text{ for all } x \in H.$$

Theorem 2.2

Let the powers of quasi paranormal operator be $T \in B(H)$,

iff $T^*T^{*2p}\, T^{2p}T - 2\lambda T^*T^{*p}\, T^pT + \lambda^2 T^*T \geq 0$.

Proof

$$\left\|T^{p+1}x\right\|^{2p} \leq \left\|T_1^{2p+1}x\right\| \|x\| \text{ for all } x \in H.$$

$$\Rightarrow \left\|T^{p+1}x\right\|^4 \leq \left\|T^{2p+1}x\right\|^2 \|x\|^2$$

$$\Rightarrow \left\langle \left(T^{*p+1}\, T^{p+1}\right)_x^2, \left(T^{*p+1}\, T^{p+1}\right)_x^2 \right\rangle \leq \left\langle T^{2p+1}xT^{2p+1}x\right\rangle \langle Tx\; Tx\rangle$$

$$\Rightarrow \left\langle \left(T^{*p+1}\, T^{p+1}\right)^2 x, x \right\rangle \leq \left\langle T^{*2p+1}\, T^{2p+1}x, n\right\rangle \langle T^*Tx, n\rangle$$

$$\Rightarrow \left(T^{*p+1}\, T^{p+1}\right)^2 \leq \left(T^{*2p+1}\, T^{2p+1}\right)(T^*T)$$

$$\Rightarrow 4\left(T^{*p+1}\, T^{p+1}\right)^2 \geq \left(T^{*2p+1}\, T^{2p+1}\right)(T^*T)$$

$$\Rightarrow \left[2\left(T^{*p+1}\, T^{p+1}\right)\right]^2 - 4\left(T^{*2p+1}\, T^{2p+1}\right)(T^*T) \geq 0$$

$$\Rightarrow b^2 - 4ac \geq 0$$

$$b = 2\left(T^{*p+1} T^{p+1}\right)$$

$$c = T^{*2p+1} T^{2p+1}$$

$$a = T^*T$$

By means of actual quadratic equation

$$a\lambda^2 - 2\lambda b + c \geq 0$$

$$\lambda^2\left(T^*T\right) - 2\lambda T^*T^{*p} T^{p+1} + T^{*2p+1} \geq 0$$

$$\Rightarrow T^{*2p+1} T^{2p+1} - 2\lambda T^{*p+1} T^{*p} T^{p+1} + \lambda^2 T^*T \geq 0$$

$$\Rightarrow T^*T^{*2p} T^{2p}T - 2\lambda T^*T^{*p} T^pT + \lambda^2 T^*T \geq 0.$$

Theorem 2.3

Let the powers of quasi paranormal operator be $T \in B(H)$, then and there if T is unitarily equivalent to s, where s is of powers of quasi paranormal.

Proof

Then T is unitarily equivalent to s, here is a unitarily operator U likewise $s = U^*TU$. It expressed that $T^{*2p+1} T^{2p+1} - 2\lambda T^{*p+1} T^{p+1} + \lambda^2 T^*T \geq 0.$

Then T is of powers of quasi paranormal, we get,

$$T^{*2p+1} T^{2p+1} - 2\lambda T^{*p+1} T^{p+1} + \lambda^2 T^*T \geq 0.$$

Therefore,

$$S^{*2p+1} S^{2p+1} - 2\lambda S^{*p+1} S^{p+1} + \lambda^2 S^*S \geq 0$$

$$\Rightarrow \left(U^*T^*U\right)^{2p+1}\left(U^*TU\right)^{2p+1} - 2\lambda\left(U^*T^*U\right)\left(U^*TU\right)^{p+1}$$
$$+\lambda^2\left(U^*T^*U\right)\left(U^*TU\right) \geq 0$$

$$\Rightarrow U^*T^{*2p+1} T^{2p+1}U - 2\lambda U^*T^{*p+1} T^{p+1}U + \lambda^2\left(U^*T^*U\right) \geq 0$$

$$\Rightarrow U^*\left[T^{*2p+1} T^{2p+1} - 2\lambda T^{*p+1} T^{p+1} + \lambda^2 T^*T\right]U \geq 0.$$

$$\Rightarrow T^{*2p+1} T^{2p+1} - 2\lambda T^{*p+1} T^{p+1} + \lambda^2 T^*T \geq 0$$

Hence, the powers of quasi paranormal is S.

3.Powers of quasi paranormal composition operators

Take $(X,\sum,\lambda)$ be a σ – finite space and take $T:X\to X$ non – singular transformation. A linear operator $cf = f\circ T\, on\, L^2(X,\sum,\lambda)$ is referred as composition operator stimulated by *T*, when λT^{-1} is continuous and dependon measure λ and the derivative $d\lambda T^{-1}/d\lambda = f_0$ is bounded.In this article, we characterize powers of quasi paranormal composition operator.

Corollary 3.1

For all complex valued function f_0 induced the operator $M_{f_0}\; on\; L^2(\lambda)$

i. $M_{f_0}f = f_0 f$ in all $f\in L^2(\lambda)$

ii. $C^*Cf = M_{f_0}f$

iii. $C^{*2}C^2 = M_{f_0^{(2)}}f$.

Theorem 3.2

Let $C\in B\left[L^2(\lambda)\right]$ be powers of quasi paranormal operator iff

$f_0^{(2p+1)} - 2\lambda f_0^{(p+1)} + \lambda^2 f_0^{(1)} \geq 0\, a.e.$

Proof

Let $C\in B\left[L^2(\lambda)\right]$ be powers of quasi paranormal operator iff

$C^{*(2p+1)}C^{(2p+1)} - 2\lambda C^{*(p+1)}C^{(p+1)} + \lambda^2 C^*C \geq 0$

Thus, $\left\langle\left(C^{*(2p+1)}C^{(2p+1)} - 2\lambda C^{*(p+1)}C^{(p+1)} + \lambda^2 C^*C\right)X_E, X_E\right\rangle \geq 0$

In all function χ_E of E in $\sum$ of $\lambda(E)<\infty$ while $C^*C = M_{f_0}$

and $C^{*2}C^2 = M_{f_0^{(2)}}$, we have $C^{*(2p+1)}C^{(2p+1)} = M_{f_0^{(2p+1)}}$

$$\left\langle \left(M_{f_0^{(2p+1)}} - 2\lambda M_{f_0^{(p+1)}} + \lambda^2 M_{f_0} \right) \chi_E, \chi_E \right\rangle \geq 0.$$

$$\int_E \left\{ M_{f_0^{(2p+1)}} - 2\lambda M_{f_0^{(p+1)}} + \lambda^2 M_{f_0} \right\} d\lambda \geq 0.$$

forall $E \in \sum$.

Thus C is the powers of quasi paranormal operator iff $f_0^{(2p+1)} - 2\lambda f_0^{(p+1)} + \lambda^2 f_0 \geq 0$

Corollary 3.3

If $C \in B\left[L^2(\lambda)\right]$ with dense range, then C is powers of quasi paranormal operator iff $f_0^{(2p+1)} - 2\lambda f_0^{(p+1)} + \lambda^2 f_0 \geq 0 \; a.e.$

Example 3.4

Take $X = N$ and λ is the counting measure. Termed $T : N \to N$ by T(1)=1, T(n+m+1) = n and $m = 0,1,2,... \; n \in N.$ Then $f_0^{(2p+1)} - 2\lambda f_0^{(p+1)} + \lambda^2 f_0 \geq 0 \; a.e.$ Therefore C is powers of quasi paranormal composition operators.

Theorem 3.5

If $C^* \in B\left[L^2(\lambda)\right]$ be powers of quasi paranormal operator iff

$$\left[(f_0 \circ T)^{(2p+1)} p \right] - 2\lambda \left[(f_0 \circ T)^{(p+1)} p \right] + \lambda^2 \left[(f_0 \circ T) p \right] \geq 0 \; a.e.$$

Proof

Let C^* be the powers of quasi paranormal operator iff

$$C^{(2p+1)}C^{*(p+1)} - 2\lambda C^{*(p+1)}C^{(p+1)} + \lambda^2 CC^* \geq 0$$

$$\Rightarrow \left\langle \left(C^{(2p+1)}C^{*(p+1)} - 2\lambda C^{*(p+1)}C^{(p+1)} + \lambda^2 CC^* \right) f, f \right\rangle \geq 0$$

Given any $f \in L^2$., then $\langle CC^* f, f \rangle = \langle (f_0 \circ T) pf, f \rangle$, here p is the projection of L^2 onto $\overline{R(C)}$. Hence C^* is a power of quasi paranormal operator iff

$$\left\langle \left\{ \left[(f_0 \circ T)^{(2p+1)} p \right] - 2\lambda \left[(f_0 \circ T)^{(p+1)} p \right] + \lambda^2 \left[(f_0 \circ T) p \right] \right\} f, f \right\rangle \geq 0$$

For all $f \in L^2$,

$$\left[(f_0 \circ T)^{(2p+1)} p \right] - 2\lambda \left[(f_0 \circ T)^{(p+1)} p \right] + \lambda^2 \left[(f_0 \circ T) p \right] \geq 0 \; a.e.$$

Corollary3.6

If $C^* \in B\left[L^2(\lambda) \right]$ with dense range, then C^* is powers of quasi paranormal operator iff $\left[\left[(f_0 \circ T)^{(2p+1)} p \right] - 2\lambda \left[(f_0 \circ T)^{(p+1)} p \right] + \lambda^2 (f_0 \circ T) p \right] \geq 0 \; a.e.$

4. Weighted powers of quasi paranormal composition operators.

Proposition 4.1

For $\omega \geq 0$,

i. $W^* W f = f_0 \left[E(\omega^2) \right] \circ T^{-1} f$

ii. $W W^* f = \omega (f_0 \circ T) E(\omega f)$.

Theorem 4.2

If W is powers of quasi paranormal operator

$$\text{iff } f_0^{(p+1)} \left[E\left(\omega^2_{(2p+1)} \right) \right] \circ T^{-(2p+1)} - 2\lambda f_0^{(p+1)} \left[E\left(\omega^2_{(p+1)} \right) \right] \circ T^{-(p+1)} + \lambda^2 f_0 \left[E(\omega^2) \right] \circ T^{-1} \geq 0 \; a.e.$$

Proof:

Then W is powers of * - paranormal,

$$W^{*(2p+1)} W^{(2p+1)} - 2\lambda W^{*(p+1)} W^{(p+1)} + \lambda^2 W^* W \geq 0$$

and therefore,

$$\left\langle \left(W^{*(2p+1)} W^{(2p+1)} - 2\lambda W^{*(p+1)} W^{(p+1)} + \lambda^2 \left(W^* W \right) \right) f, f \right\rangle \geq 0$$

Given any $f \in L^2$. Then $W^* f = \omega_K \left(f_0 \circ T^K \right)$ and $W^{*K} f = f_0^{(K)} E\left(\omega_K^2 \right) \circ T^{-1}$, .

$$W^{*K} W^K = f_0^{(K)} \left[E\left(\omega_K^2 \right) \right] \circ T^{-K},\ W^* W f = f_0 \left[E\left(\omega^2 \right) \right] \circ T^{-K} f$$

for $\omega \geq 0$ and therefore,

$$\int_E \left\{ f_0^{(2p+1)} \left[E\left(\omega_{(2p+1)}^2 \right) \right] \circ T^{-(2p+1)} - 2\lambda f_0^{(p+1)} \left[E\left(\omega_{(p+1)}^2 \right) \right] \circ T^{-(p+1)} + \lambda^2 f_0 \left[E\left(\omega^2 \right) \right] \circ T^{-1} \right\} d\lambda \geq 0$$

For all $E \in \sum$ and hence,

$$f_0^{(2p+1)} \left[E\left(\omega_{(2p+1)}^2 \right) \right] \circ T^{-(2p+1)} - 2\lambda f_0^{(p+1)} \left[E\left(\omega_{(p+1)}^2 \right) \right] \circ T^{-(p+1)} + \lambda^2 f_0 \left[E\left(\omega^2 \right) \right] \circ T^{-1} \geq 0 \ \ a.e.$$

Corollary 4.3

If $T^{-1} \sum = \sum$. now W is of powers of quasi paranormal operator iff $f_0^{(2p+1)} \left[\omega_{(2p+1)}^2 \right] \circ T^{-(2p+1)} - 2\lambda f_0^{(p+1)} \left[\omega_{(p+1)}^2 \right] \circ T^{-(p+1)} + \lambda^2 f_0 \left[\omega^2 \right] \circ T^{-1} \geq 0 \ \ a.e.$

Theorem 4.4

If W^* is powers of quasi paranormal operator iff

$$\omega_{(2p+1)} \left(f_0 \circ T \right)^{(2p+1)} E\left[\omega_{(2p+1)} \right] - 2\lambda \omega_{(p+1)} \left(f_0 \circ T \right)^{(p+1)} E\left[\omega_{(p+1)} \right] + \lambda^2 \omega \left(f_0 \circ T \right) E(\omega) \geq 0 \ \ a.e.$$

Proof:

Then W^* is powers of quasi paranormal,

$$W^{(2p+1)} W^{*(2p+1)} - 2\lambda W^{(p+1)} W^{*(p+1)} + \lambda^2 W W^* \geq 0.$$

and therefore,

$$\left\langle \left(W^{(2p+1)} W^{*(2p+1)} - 2\lambda W^{*(p+1)} W^{(p+1)} + \lambda^2 W W^* \right) f, f \right\rangle \geq 0.$$

for all $f \in L^2$.

Then $W^K W^{*K} f = \omega_K \left(f_0 \circ T \right) E\left(\omega f \right)$

$$W^K W^{*K} f = \omega_K \left(f_0 \circ T \right)^K E\left(\omega_K f \right)$$

and therefore

$$\int_E \left\{ \left(\omega_{(2p+1)} \left(f_0 \circ T \right)^{(2p+1)} \right) E\left(\omega_{(2p+1)} f \right) - 2\lambda \left(\omega_{(p+2)} \right) \left(f_0 \circ T \right)^{(p+1)} E\left(\omega_{(p+1)} f \right) + \lambda^2 \omega \left(f_0 \circ T \right) E\left(\omega f \right) \right\} d\lambda \geq 0$$

For all $E \in \sum$, and thus,

$$\omega_{(2p+1)} \left(f_0 \circ T \right)^{(2p+1)} E\left[\omega_{(2p+1)} f \right] - 2\lambda \omega_{(p+1)} \left(f_0 \circ T \right)^{(p+1)} E\left[\omega_{(p+1)} f \right] + \lambda^2 \omega \left(f_0 \circ T \right) E\left(\omega \right) \geq 0 \quad a.e.$$

Corollary 4.5

If $T^{-1} \sum = \sum$ now W^* is of powers of quasi paranormal iff

$$\omega_{(2p+1)} \left(f_0 \circ T \right)^{(2p+1)} \left(\omega_{(2p+1)} f \right) - 2\lambda \omega_{(p+1)} \left(f_0 \circ T \right)^{(p+1)} \left(\omega_{(p+1)} f \right) + \lambda^2 \omega \left(f_0 \circ T \right) \left(\omega f \right) \geq 0. \; a.e.$$

Lemma 4.6

An operator T is Aluthge transformation is termed by $\tilde{T} = |T|^{1/2} U |T|^{1/2}$.

Definition 4.7

An operator C_s weighted composition operator if $C_s = |C|^s U |C|^{1-s}$

and $C_s = \left(\dfrac{f_0}{f_0 \circ T} \right)^{s/2} f \circ T.$

Lemma 4.8

If C_s is weighted composition operator, We get,

i. $C_s^K f = \pi_K \left(f \circ T \right)^K$

ii. $C_s^{*K} f = f_0^{(K)} E\left(\pi_K f \right) \circ T^{-K}$

iii. $C_s^{*K} C_s^K f = f_0^{(K)} E\left(\pi_K^2 f \right) \circ T^{-K} f.$

Theorem 4.9

Let C_s is powers of quasi paranormal operator iff

$$f_0^{(2p+1)} E\left[\pi_{(2p+1)}^2 \right] \circ T^{-(2p+1)} - 2\lambda f_0^{(p+1)} E\left[\pi_{(p+1)}^2 \right] \circ T^{-(p+1)}$$
$$+\lambda^2 f_0 E\left[\pi^2 \right] T^{-1} \geq 0 \ \ a.e.$$

P**roof:**

Since C_s is powers of quasi paranormal,

$$C_s^{*(2p+1)} C_s^{(2p+1)} - 2\lambda C_s^{*(p+1)} C_s^{(p+1)} + \lambda^2 C_s^* C_s \geq 0.$$

$$\left\langle \left(C_s^{*(2p+1)} C_s^{(2p+1)} - 2\lambda C_s^{*(p+1)} C_s^{(p+1)} + \lambda^2 \left(C_s^* C_s \right) \right) f, f \right\rangle \geq 0$$

for all $f \in L^2$, Since $C_s^{*K} C_s^K = f_0^{(K)} E\left[\pi_K^2 \right] \circ T^{-K}$

$$\int_E \left\{ f_0^{(2p+1)} \left[E\left(\pi_{(2p+1)}^2 \right) \right] \circ T^{-(2p+1)} - 2\lambda f_0^{(p+1)} \left[E\left(\pi_{(2p+1)}^2 \right) \right] \circ T^{-(p+1)} \right.$$
$$\left. +\lambda^2 f_0 E\left(\pi^2 \right) \circ T^{-1} \right\} d\lambda \geq 0$$

for all $E \in \sum$ and hence.

$$f_0^{(2p+1)} \left[E\left(\pi_{(2p+1)}^2 \right) \right] \circ T^{-(2p+1)} - 2\lambda f_0^{(p+1)} E\left(\pi_{(2p+1)}^2 \right) \circ T^{(2p+1)}$$
$$+\lambda^2 f_0 E\left[\pi^2 \right] \circ T^{-1} \geq 0 \ \ a.e.$$

Corollary 4.10

If $T^{-1} \sum = \sum$, then C_s is powers of * - paranormal operator iff

$$f_0^{(2p+1)} \left(\pi_{(2p+1)}^2 \right) \circ T^{-(2p+1)} - 2\lambda f_0^{(p+1)} \left(\pi_{(p+1)}^2 \right) \circ T^{-(p+1)}$$
$$+\lambda^2 f_0 \left(\pi^2 \right) \circ T^{-1} \geq 0 \ \ a.e.$$

Theorem 4.11

If $C_s^* \in B\left[L^2(\lambda)\right]$ be powers of quasi paranormal operator iff

$$\pi_{(2p+1)}\left(f_0 \circ T\right)^{(2p+1)} E\left(\pi_{(2p+1)} f\right) - 2\lambda \pi_{(p+1)}\left(f_0 \circ T\right)^{(p+1)} E\left(\pi_{(p+1)}\right) + \lambda^2 \pi\left(f_0 \circ T\right) \cdot E\left(\pi f\right) \geq 0 \; a.e.$$

Proof

Then C_s^* is powers of quasi paranormal,

$$C_s^{(2p+1)} C_s^{*(2p+1)} - 2\lambda C_s^{(p+1)} C_s^{*(p+1)} + \lambda^2 C_s C_s^* \geq 0$$

and therefore

$$\left\langle \left(C_s^{(2p+1)} C_s^{*(2p+1)} - 2\lambda C_s^{*(p+1)} C_s^{(p+1)} + \lambda^2 C_s C_s^*\right) f, f \right\rangle \geq 0$$

For all $f \in L^2$, Since $C_s^K C_s^{*K} = \pi_K \left(f_0 \circ T\right)^{(K)} E\left(\pi_K f\right)$.

$$\int_E \left\{ \pi_{(2p+1)}\left(f_0 \circ T\right)^{(2p+1)} E\left(\pi_{(2p+1)} f\right) - 2\lambda \pi_{(p+1)}\left(f_0 \circ T\right)^{(p+1)} E\left(\pi_{(2p+1)} f\right) + \lambda^2 \pi\left[f_0 \circ T\right] E(\pi) \right\} d\lambda \geq 0$$

for all $E \in \sum$, and thus,

$$\pi_{(2p+1)}\left(f_0 \circ T\right)^{(2p+1)} E\left(\pi_{(2p+1)} f\right) - 2\lambda \pi_{(p+1)}\left(f_0 \circ T\right)^{(p+1)} E\left(\pi_{(p+1)} f\right) + \lambda^2 \pi\left(f_0 \circ T\right) E\left(\pi f\right) \geq 0 \; a.e.$$

Theorem 4.12

If $T^{-1}\sum = \sum$, then C_s^* is power of quasi paranormal operator iff

$$\pi_{(2p+1)}\left(f_0 \circ T\right)^{(2p+1)} \left(\pi_{(2p+1)} f\right) - 2\lambda \pi_{(p+1)}\left(f_0 \circ T\right)^{(p+1)} \left(\pi_{(p+1)} f\right) + \lambda^2 \pi\left(f_0 \circ T\right)\left(\pi f\right) \geq 0 \; a.e.$$

Theorem 4.13

If $\tilde{C} \in B\left[L^2(\lambda)\right]$ is powers of quasi paranormal operator iff

$$f_0^{(2p+1)} E\left(\omega_{(2p+1)}^{1^2}\right) \circ T^{-(2p+1)} - 2\lambda f_0^{(p+1)} E\left(\omega_{(p+1)}^{1^2}\right) \circ T^{-(p+1)}$$

$$+\lambda^2 f_0 E\left(\omega^{1^2}\right)\circ T^{-1} \geq 0 \; a.e.$$

Proof

Then $\tilde{C}$ is powers of quasi paranormal,

$$\tilde{C}^{*(2p+1)}\tilde{C}^{(2p+1)} - 2\lambda\tilde{C}^{(p+1)}\tilde{C}^{*(p+1)} + \lambda^2\tilde{C}*\tilde{C} \geq 0$$

and therefore,

$$\left\langle\left(\tilde{C}^{*(2p+1)}\tilde{C}^{(2p+1)} - 2\lambda\tilde{C}^{(p+1)}\tilde{C}^{*(p+1)} + \lambda^2 I\right)f, f\right\rangle \geq 0$$

for all $f \in L^2$.

Then $\tilde{C}^*\tilde{C} = f_0 E\left(\omega^{1^2}\right)\circ T^{-1}$ and $\tilde{C}^{*K}\tilde{C}^K = f_0^{(K)} E\left(\omega^{1^2}\right)\circ T^{-K}$

$$\int_E \left\{ f^{(2p+1)} E\left[\omega^{1^2}_{(2p+1)}\right]\circ T^{-(2p+1)} - 2\lambda f_0^{(p+1)} E\left(\omega^{1^2}_{(p+1)}\right) T^{-(p+1)} \right.$$

$$+\lambda^2 f_0 E\left[\omega^{1^2}\right]\circ T^{-1} \geq 0$$

For all $E \in \sum$. And thus,

$$f_0^{(2p+1)} E\left(\omega^{1^2}_{(2p+1)}\right)\circ T^{-(2p+1)} - 2\lambda f_0^{(p+1)} E\left(\omega^{1^2}_{(p+1)}\right)\circ T^{-(p+1)}$$

$$+\lambda^2 f_0 E\left[\omega^{1^2}\right]\circ T^{-1} \geq 0 \; a.e.$$

Corollary 4.14

If $T^{-1}\sum = \sum$, now $\tilde{C} \in B\left(L^2(\lambda)\right)$ is a power of quasi paranormal operator iff

$$\int_0^{(2p+1)} \left[\omega^{1^2}_{(2p+1)}\right]\circ T^{-(2p+1)} - 2\lambda f_0^{(p+1)}\left[\omega^{1^2}_{(p+1)}\right]\circ T^{-(p+1)} + \lambda^2 f_0\left(\omega^{1^2}\right)\circ T^{-1} \geq 0 \; a.e.$$

5.Powers of quasi paranormal composition operators on weighted hardy space

Let $H^2(\gamma)$ be the set of formal complex powers of sequence $f(z) = \sum_{n=0}^{\infty} a_n z^n$ such as $\|f\|_\gamma^2 = \sum_{n=0}^{\infty} |a_n|^2 \gamma_n^2 < \infty$ is the common Hardy space of analytic function in the unit disc with the inner product $\langle f, g\rangle\gamma = \sum_{n=0}^{\infty} a_n \overline{b_n} \gamma_n^2$ for f.Where $g(z) = \sum_{n=0}^{\infty} b_n z^n$ and $\gamma = \{\gamma_n\}_{n=0}^{\infty}$ are series of positive numbers with $\gamma_0 = 1$ and $\frac{\gamma_{n+1}}{\gamma_n} \to 1$ as $n \to \infty$. If ϕ is a function and we termed the composition operator C_ϕ on the spaces $H^2(\gamma)$ by $C_\phi f = f_0\phi$.

Lemma 5.1

i. For a order of γ as above and point ω in D, Now

Take $K_{\omega}\gamma(z) = \sum_{n=0}^{\infty} \frac{1}{\gamma_n^2} \left(\overline{\omega}_n\right)^n$.

ii. The function $K_{\omega}\gamma$ is a point evaluation for $H^2(\gamma)$ $\langle f, K_{\omega}\rangle\gamma = f(\omega)$ now $K_0\gamma = 1$

and $C_{\varphi}^{*} K_{\omega}\gamma = K_{\phi(\omega)}\gamma$.

Theorem 5.2

If C_{ϕ} is powers of quasi - paranormal operator on $H^2(\gamma)$, then and there $\lambda = 1$.

Proof

Let C_{ϕ} is powers of quasi paranormal operator on $H^2(\gamma)$.

From the statement of powers of quasi paranormal

$$C_{\phi}^{*(2p+1)} C_{\phi}^{(2p+1)} - 2\lambda C_{\phi}^{*(p+1)} C_{\phi}^{(p+1)} + \lambda^2 C_{\phi}^{*} C_{\phi} \geq 0$$

and therefore,

$$\left\langle \left(C_{\phi}^{*(2p+1)} C_{\phi}^{(2p+1)} - 2\lambda C_{\phi}^{*(p+1)} C_{\phi}^{(p+1)} + \lambda^2 C_{\phi}^{*} C_{\phi} \right) f, f \right\rangle \geq 0 \text{ forall } f \in H^2(\gamma).$$

$$\left\langle \left(C_{\phi}^{*(2p+1)} C_{\phi}^{(2p+1)} \right) f, f \right\rangle - 2\lambda \left\langle \left(C_{\phi}^{*(p+1)} C_{\phi}^{(p+1)} \right) f, f \right\rangle + \lambda^2 \left\langle \left(C_{\phi}^{*} C_{\phi} \right) f, f \right\rangle \geq 0$$

$$\left\langle C_{\phi}^{(2p+1)} f, C_{\phi}^{(2p+1)} f \right\rangle - 2\lambda \left\langle C_{\phi}^{*(p+1)} f, C_{\phi}^{*(p+1)} f \right\rangle + \lambda^2 \left\langle C_{\phi}^{*} f, C_{\phi} f \right\rangle \geq 0$$

$$\left\| C_{\phi}^{(2p+1)} f \right\|^2 - 2\lambda \left\| C_{\phi}^{(p+1)} f \right\|^2 + \lambda^2 \left\| C_{\phi} f \right\|^2 \geq 0$$

$$\left\| C_{\phi}^{(2p)} \left(C_{\phi} f \right) \right\|^2 - 2\lambda \left\| C_{\phi}^{(p)} \left(C_{\phi} f \right) \right\|^2 + \lambda^2 \left\| C_{\phi} f \right\|^2 \geq 0$$

Repeating the steps for 2 more time we have,

$$\left\| K_0^{(2p+1)} \gamma \right\|^2 - 2\lambda \left\| K_0^{(p+1)} \gamma \right\|^2 + \lambda^2 \left\| K_0 \gamma \right\|^2 \geq 0$$

$$1 - 2\lambda + \lambda^2 \geq 0$$

$$\lambda^2 - 2\lambda + 1 \geq 0$$
$$(\lambda - 1)^2 \geq 0$$
$$\lambda - 1 = 0,$$
$$\lambda = 1.$$

6.Conclusion

The powers of quasi paranormal composition operators and weighted composition of powers of quasi paranormal operators on L^2 space and Hardy Space are proved.

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