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Exponential Ratio Estimator in Stratified Sampling

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ABSTRACT: In this article, we proposed an exponential ratio type estimator for estimating the finite population mean in stratified sampling. The bias and MSE of the proposed Estimator are obtained and comparison is made with some of the existing Estimator. In the support of the theoretical proposed work, we have given numerical illustration and from this we conclude that our proposed estimators perform better than existing estimators.

KEY WORD: Bias, Efficiency, Exponential Ratio estimator, Mean Square Error, Stratification.

1. INTRODUCTION

Use of Auxiliary information in the estimation of population parameter such as population mean, ratio of two population mean, product of two population mean, coefficient of variation etc. has been in practice. Ratio, Regression type estimators are good example in this context. Cochran (1940) initiated the use of auxiliary information at estimation stage and proposed ratio estimator for population mean. It is well established fact that ratio type estimator provides better efficiency in comparison to simple mean estimator. If the study variate and auxiliary variate are positively correlated.

Hansen et al. (1940) proposed a combined ratio estimator for population mean in stratified random sampling. Later Kadilar and Cingi(2003) and Singh and Vishwakarma (2006) discussed some ratio and product type estimators using known parameters of a variate for estimation of population mean in stratified random sampling. Bahl and Tuteja (1991) pioneered ratio and product type exponential estimators using an exponential function in simple random sampling. Later on these estimators were defined in stratified random sampling by singh et.al (2008).

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Pandy (1980) Introduced Product -cum- power estimation in simple random sampling, Bedi (1996) proposed efficient utilization of auxiliary information at estimation stage in simple random sampling, Shabbir and Gupta (2011) Estimates finite population mean in simple random sampling and stratified sampling. Clement (2014) improved Ratio Estimator for population mean in stratified sampling,

Zakari et, al (2020) Proposed ratio type estimator in simple random sampling. We adapted this estimator in stratified random sampling.

2. NOTATIONS

Consider a finite population $U = \{U_1, U_2 \ldots\ldots\ldots U_N\}$having N distinct and identifiable unit partioned into L Strata. Let y and x be the study and auxiliary variables taking values y_{hi} and x_{hi}, respectively for i^{th}unit $(i = 1,2, \ldots., N)$ in h^{th} stratumConsisting of N_h units$(h = 1,2 \ldots L)$ such that $\sum_{h=1}^{L} N_{h=}\ N$. Let n_h be the size of the sample for h^{th} stratum such that $\sum_{h=1}^{L} n_h = n$.

Let,

$W_h = \frac{N_h}{N}$: Stratum weight

$\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_{h,}$ where $\bar{y}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} y_{hi}$

$\bar{Y} = \sum_{h=1}^{L} W_h \bar{Y}_h$, where $\bar{Y}_h = \frac{1}{N_h} \sum_{i=1}^{n_h} y_{hi}$

$\bar{x}_{st} = \sum_{h=1}^{L} W_h \bar{x}_{h,}$ where$\bar{x}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} x_{hi}$

$\bar{X} = \sum_{h=1}^{L} W_h \bar{X}_h$, where $\bar{X}_h = \frac{1}{N_h} \sum_{i=1}^{n_h} x_{hi}$

$s_{yh}^2 = \frac{1}{n_h - 1} \sum_{i=1}^{n_h} (y_{hi} - \bar{y}_h)^2$, $s_{xh}^2 = \frac{1}{n_h - 1} \sum_{i=1}^{n_h} (x_{hi} - \bar{x}_h)^2$ be the sample variance of y and x respectively in h^{th} stratum corresponding to the population variances $S_{yh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (y_{hi} - \bar{Y}_h)^2$and $S_{xh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (x_{hi} - \bar{X}_h)^2$

$S_{yxh} = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (x_{hi} - \bar{X}_h)\,(y_{hi} - \bar{Y}_h)$ Covariance between y and x.

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$C^2{}_{xh} = \frac{S^2{}_{xh}}{\bar{X}^2}$, $C^2{}_{yh} = \frac{S^2{}_{yh}}{\bar{Y}^2}$ be the coefficient of variation of h^{th} stratum.

$C_{hxy} = \rho_h C_{hy} C_{hx}$ $where\ \rho\ is$ the correlation coefficient between $y\ and\ x.$

$\delta_h = \frac{1}{n_h} - \frac{1}{N_h}$ is f p c within h^{th} stratum.

To obtain the properties of various estimators, we define the following terms.

$\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h = \bar{Y}(1 + e_{0st})$ and $\bar{x}_{st} = \sum_{h=1}^{L} W_h \bar{x}_h$

$\bar{y}_{st} = \bar{Y} + \bar{Y} e_{0st} = \sum_{h=1}^{L} W_h \bar{y}_h$, $\bar{x}_{st} = \bar{X}(1 + e_{1st})$

$e_{0st} = \frac{\sum_{h=1}^{L} W_h \bar{y}_h - \bar{Y}}{\bar{Y}}$, $e_{1st} = \frac{\sum_{h=1}^{L} W_h \bar{x}_h - \bar{X}}{\bar{X}}$

$E(e_{0st}) = 0$, $E(e_{1st}) = 0$

Where $\bar{y}_{st}$ and $\bar{x}_{st}$ are unbiased estimators of population mean $\bar{Y}\ and \bar{X}$ respectively

Then

$$E(e_{0st}^2) = \sum_{h=1}^{L} W_h^2\, \delta_h \frac{S_{hy}^2}{\bar{Y}^2} = V_{2.0}$$

$$E(e_{1st}^2) = \sum_{h=1}^{L} W_h^2\, \delta_h \frac{S_{hx}^2}{\bar{X}^2} = V_{0.2}$$

$$E(e_{0st} e_{1st}) = \sum_{h=1}^{L} W_h^2\, \delta_h \frac{S_{yxh}}{\bar{Y}\bar{X}} = V_{1.1}$$

3. EXISTING ESTIMATORS:

We discussed below some commonly used estimator of t_{st}

3.1. Sample Mean

$$\boldsymbol{t_{st(0)}} = \bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h,$$

$$MSE(t)_{st(0)} = V(t)_{st(0)} = \bar{Y}^2 V_{2.0}$$

3.2. Traditional ratio estimator is defined by

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$$t_{st1} = \bar{y}_{st}\frac{\bar{X}}{\bar{x}_{st}}$$

$$B(t)_{st1} = \bar{Y}(V_{0.2} - V_{1.1})$$

$$MSE(t)_{st1} = \bar{Y}^2(V_{2.0} + V_{0.2} - 2V_{1.1})$$

3.3. Following Bahl and Tuteja (1991), combined exponential ratio type estimator

$$t_{st2} = \bar{y}_{st}exp\left(\frac{\bar{X} - \bar{x}_{st}}{\bar{X} + \bar{x}_{st}}\right)$$

$$B(t)_{(st2)} = \bar{Y}^2\left(\frac{3}{8}V_{0.2} - \frac{1}{2}V_{1.1}\right)$$

$$MSE(t)_{st2} = \bar{Y}^2\left(V_{2.0} + \frac{1}{4}V_{0.2} - V_{1.1}\right)$$

3.4. The traditional regression estimator is given

$$t_{st3} = \bar{y}_{st} + b_{st}\,(\bar{X} - \bar{x}_{st})$$

Where b_{st} is Sample regression coefficient in stratified sampling

The MSE of t_{st3} is given by

$$MSE(t)_{st3} = \bar{Y}^2\,V_{2.0}(1 - \rho_{st}^2)$$

Where $\rho_{st} = \frac{V_{1.1}}{\sqrt{V_{2.0}}\sqrt{V_{0.2}}}$

3.5. A stratified version of the Pandy (1980) estimator for population mean ($\bar{Y}$) is given by

$$t_{st4} = \bar{y}_{st}\left\{\theta_2\left(\frac{\bar{X}}{\bar{x}_{st}}\right)^{\alpha_2} + (1 - \theta_2)\frac{\bar{x}_{st}}{\bar{X}}\right\}$$

Where θ_2 and α_2 are arbitrary constants whose optimal Values are to be determined later.

$$B(t_{st4}) = \bar{Y}\left\{\frac{\alpha_2(\alpha_2 + 1)}{2}\theta_2 V_{0.2} + (1 - \theta_2(1 + \alpha_2))V_{1.1}\right\}$$

$$M(t_{st4}) = \bar{Y}^2[V_{2.0} + 2\{1 - \theta_2(1 + \alpha_2)\}V_{1.1} + \{(1 - \theta_2(1 + \alpha_2)\}^2 V_{0.2}]$$

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Then

$M(t_{st4})\text{min} = \bar{Y}^2 V_{2.0}(1-\rho_{st}^2)$

3.6. Sangngam and Hiriote (2014) Estimator

$t_{st5} = \frac{\bar{y}_{st}}{(\bar{x}_{st} + C_{xst})}(\bar{X} + C_{xst})$

$B(t_{st5}) = \left(\frac{R_{SD}}{\bar{X}_x + C_{xst}} V_{0.2} - \frac{1}{\bar{X}_x + C_{xst}} V_{1.1}\right)$

$M(t_{st5}) = \bar{Y}^2(V_{2.0} + R_{SD}^2 V_{0.2} - 2R_{SD}V_{1.1})$

Where$R_{SD} = \frac{\bar{Y}}{\bar{X}+C_{xst}}$

3.7. Adapted from Zakari et al. (2020) estimator

$t_{st6} = \bar{y}_{st}\alpha\left(\frac{\bar{X}+n}{\bar{x}_{st}+n}\right)$

Where α is unknown weight

$B(t_{st6}) = \bar{Y}\ [(\alpha-1) + \alpha\delta^2 V_{0.2} - \alpha\delta V_{1.1}]$

$M(t_{st6})$=$\bar{Y}^2[(\alpha-1)^2 + \alpha^2 V_{2.0} + (3\gamma\alpha^2\delta^2 - 2\alpha\delta^2)V_{0.2} + (2\alpha\delta - 4\alpha^2\delta)V_{1.1}]$

α^{opt}=$\frac{1-\delta V_{1.1}+\delta^2 V_{0.2}}{1+V_{2.0}+3\delta^2 V_{0.2}-4\delta V_{1.1}}$

$M(t_{st6})min\ = \bar{Y}^2\left[1 - \frac{(1-\delta V_{1.1}+\delta^2 V_{0.2})^2}{1+V_{2.0}+3\delta^2 V_{0.2}-4\delta V_{1.1}}\right]$

4. PROPOSED ESTIMATOR

Following Pandy (1980) estimator we proposed an exponential ratio estimator in stratified sampling

$$T_{stp} = \bar{y}_{st}\left[\theta\frac{\bar{X}}{\bar{x}_{st}} + (1-\theta)\frac{\bar{x}_{st}}{\bar{X}}\right]\exp\left[\frac{\bar{X}-\bar{x}_{st}}{\bar{X}+\bar{x}_{st}}\right] \tag{4.1}$$

Where θ is an arbitrary constant, whose optimal value is to be determined later

Expanding the right-hand side in terms of e_{st}'s neglecting the term having power greater than two

$$T_{stp} = \bar{Y}(1+e_{0st})\left[\theta\frac{\bar{X}}{\bar{X}(1+e_{1st})} + (1-\theta)\frac{\bar{X}(1+e_{1st})}{\bar{X}}\right]exp\left[\frac{\bar{X}-\bar{X}(1+e_{1st})}{\bar{X}+\bar{X}(1+e_{1st})}\right]$$

$$T_{stp} = \bar{Y}(1+e_{0st})[\theta(1+e_{1st})^{-1} + (1-\theta)(1+e_{1st})]exp[-e_{1st}(2+e_{1st})^{-1}]$$

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$$T_{stp} = \bar{Y}(1+e_{0st})\left[\theta\left(1-e_{1st}+\frac{e_{1st}^2}{2}+\cdots\right)+(1+e_{1st})-\theta(1+e_{1st})\right] exp\left[(-e_{1st})\frac{1}{2}\left(1+\frac{e_{1st}}{2}\right)^{-1}\right]$$

$$T_{stp} = \bar{Y}(1+e_{0st})\left[\theta\left(-2e_{1st}+\frac{e_{1st}^2}{2}+\cdots.\right)+(1+e_{1st})\right] exp\left[(-e_{1st})\frac{1}{2}\left(1-\frac{e_{1st}}{2}+\frac{e_{1st}^2}{4}+\cdots..\right)\right]$$

$$T_{stp} = \bar{Y}(1+e_{0st})\left[1-2e_{1st}\theta+\frac{e_{1st}^2}{2}\theta+e_{1st}+\cdots\right] exp\left[-\frac{e_{1st}}{2}+\frac{e_{1st}^2}{4}-\frac{e_{1st}^3}{8}+\cdots\right]$$

$$T_{stp} = \bar{Y}(1+e_{0st})\left[1-2e_{1st}\theta+\frac{e_{1st}^2}{2}\theta+e_{1st}+\cdots\right] exp\left[\left(-\frac{e_{1st}}{2}\right).exp\left(\frac{e_{1st}^2}{4}\right)\right]$$

$$T_{stp} = \bar{Y}(1+e_{0st})\left[1-2e_{1st}\theta+\frac{e_{1st}^2}{2}\theta+e_{1st}+\cdots\right]\left[\left(1-\frac{e_{1st}}{2}+\frac{e_{1st}^2}{8}+\cdots\right)\left(1+\frac{e_{1st}^2}{4}+\cdots\right)\right]$$

$$T_{stp} = \bar{Y}(1+e_{0st})\left(1-2e_{1st}\theta+\frac{e_{1st}^2}{2}\theta+e_{1st}+..\right)\left(1-\frac{e_{1st}}{2}+\frac{e_{1st}^2}{8}+\frac{e_{1st}^2}{4}-\frac{e_{1st}^3}{8}+\frac{e_{1st}^4}{32}+..\right)$$

$$T_{stp} = \bar{Y}\left(1-2e_{1st}\theta+\frac{e_{1st}^2}{2}\theta+e_{1st}+..\right)\left(1-\frac{e_{1st}}{2}+\frac{3e_{1st}^2}{8}+e_{0st}-\frac{e_{0st}e_{1st}}{2}+..\right)$$

Neglecting term of e_{st}'s having power greater than two, we have

$$T_{stp} = \bar{Y}\left[1+e_{1st}\left(\frac{-4\theta+1}{2}\right)+e_{1st}^2\left(\frac{12\theta-1}{8}\right)+e_{0st}+e_{0st}e_{1st}\left(\frac{-4\theta+1}{2}\right)+\cdots\right]$$

$$T_{stp}-\bar{Y} = \bar{Y}\left[1-e_{1st}\left(\frac{4\theta-1}{2}\right)+e_{1st}^2\left(\frac{12\theta-1}{8}\right)+e_{0st}-e_{0st}e_{1st}\left(\frac{4\theta-1}{2}\right)+\cdots\right]-\bar{Y}$$

$$T_{stp}-\bar{Y} = \bar{Y}\left[-e_{1st}\left(\frac{4\theta-1}{2}\right)+e_{1st}^2\left(\frac{12\theta-1}{8}\right)+e_{0st}-e_{0st}e_{1st}\left(\frac{4\theta-1}{2}\right)+\cdots\right] \quad (4.2)$$

$$E\left(T_{stp}-\bar{Y}\right) = E\,\bar{Y}\left[-e_{1st}\left(\frac{4\theta-1}{2}\right)+e_{1st}^2\left(\frac{12\theta-1}{8}\right)+e_{0st}-e_{0st}e_{1st}\left(\frac{4\theta-1}{2}\right)+\cdots\right]$$

$$B\left(T_{stp}\right) = \bar{Y}\left[\left(\frac{12\theta-1}{8}\right)\sum_{h=1}^{L}W_h^2\delta_h C_{hx}^2-\left(\frac{4\theta-1}{2}\right)\sum_{h=1}^{L}W_h^2\delta_h C_{hxy}\right] \quad (4.3)$$

Squaring both side of equation (4.2) and neglecting term of e_{st}'s having power greater than two, we have

$$\left(T_{stp}-\bar{Y}\right)^2 = \bar{Y}^2\left[-e_{1st}\left(\frac{4\theta-1}{2}\right)+e_{1st}^2\left(\frac{12\theta-1}{8}\right)+e_{0st}-e_{0st}e_{1st}\left(\frac{4\theta-1}{2}\right)+\cdots\right]^2$$

$$E\left(T_{stp}-\bar{Y}\right)^2 = \bar{Y}^2 E\left[-e_{1st}\left(\frac{4\theta-1}{2}\right)+e_{1st}^2\left(\frac{12\theta-1}{8}\right)+e_{0st}-e_{0st}e_{1st}\left(\frac{4\theta-1}{2}\right)+\cdots\right]^2$$

$$M\left(T_{stp}\right) = \left[e_{0st}^2+\left(\frac{4\theta-1}{2}\right)^2 e_{1st}^2-(4\theta-1)e_{0st}e_{1st}\right]$$

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$$M(T_{stp}) = \bar{Y}^2\left[V_{2.0+}\left(\frac{4\theta-1}{2}\right)^2 V_{0.2} - (4\theta-1)V_{1.1}\right] \quad (4.4)$$

Differentiate equation (4.4) w. r.to θ and equating to zero for obtaining the optimum value of θ, The optimum value of θ which makes the MSE in equation (4.4) minimum is given by,

$$\frac{\partial}{\partial\theta}M(T_{stp}) = \frac{\partial}{\partial\theta}\bar{Y}^2\left[V_{2.0+}\left(\frac{4\theta-1}{2}\right)^2 V_{0.2} - (4\theta-1)V_{1.1}\right]$$

$$0 = 2(4\theta-1)V_{0.2} - 4V_{1.1}$$

$$\theta = \frac{1}{4}\left(\frac{2V_{1.1}}{V_{0.2}} + 1\right)$$

Putting the value of θ in equation (4.4)

$$M(T_{stp}) = \bar{Y}^2\left[V_{2.0} - \frac{(V_{1.1})^2}{V_{0.2}}\right] \quad (4.5)$$

5. THEORETICAL EFFICIENCY COMPARISON

In this section the proposed exponential ratio estimator were compared theoretically with other existing estimator. Efficiency condition over some related existing estimators.

(1) The MSE of proposed exponential ratio estimator (T_{STP}) is better than sample mean per unit estimator (t_{st0}) if

$$MSEt_{st0} - MSET_{stp} > 0$$

$$[\bar{Y}^2V_{1.1}] - \bar{Y}^2\left[V_{2.0} - \frac{(V_{1.1})^2}{V_{0.2}}\right] > 0$$

$$\frac{(V_{1.1})^2}{V_{0.2}} > 0 \quad (5.1)$$

(2)$MSEt_{st1} - MSET_{stp} > 0$

$$\bar{Y}^2(V_{2.0} + V_{0.2} - 2V_{1.1}) - \bar{Y}^2\left[V_{2.0} - \frac{(V_{1.1})^2}{V_{0.2}}\right] > 0$$

$$\left[V_{0.2} - 2V_{1.1} + \frac{(V_{1.1})^2}{V_{0.2}}\right] > 0$$

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$$V_{0.2}+\frac{(V_{1.1})^2}{V_{0.2}}>2V_{1.1} \tag{5.2}$$

(3) $MSEt_{st2}-\ MSET_{stp}>\ 0$

$$\bar{Y}^2\left(V_{2.0}+\frac{1}{4}V_{0.2}-V_{1.1}\right)-\bar{Y}^2\left[V_{2.0}-\frac{(V_{1.1})^2}{V_{0.2}}\right]>\ 0$$

$$\frac{1}{4}V_{0.2}+\frac{(V_{1.1})^2}{V_{0.2}}>V_{1.1} \tag{5.3}$$

(4) $MSEt_{st3}-\ MSET_{stp}>\ 0$

$$\bar{Y}^2V_{2.0}(1\ -\rho_{st}^2)\ \ -\bar{Y}^2\left[V_{2.0}-\frac{(V_{1.1})^2}{V_{0.2}}\right]>0$$

$$\frac{(V_{1.1})^2}{V_{0.2}}>\rho_{st}^2V_{2.0} \tag{5.4}$$

(5) $MSEt_{st4}-\ MSET_{stp}>\ 0$

$$\bar{Y}^2V_{2.0}(1\ -\rho_{st}^2)\ \ -\bar{Y}^2\left[V_{2.0}-\frac{(V_{1.1})^2}{V_{0.2}}\right]>0$$

$$\frac{(V_{1.1})^2}{V_{0.2}}>\rho_{st}^2V_{2.0} \tag{5.5}$$

(6) $MSEt_{st5}-\ MSET_{stp}>\ 0$

$$\bar{Y}^2(V_{2.0}+R_{SD}^2V_{0.2}-2R_{SD}V_{1.1})-\bar{Y}^2\left[V_{2.0}-\frac{(V_{1.1})^2}{V_{0.2}}\right]>0$$

$$R_{SD}^2V_{0.2}+\frac{(V_{1.1})^2}{V_{0.2}}>2R_{SD}V_{1.1} \tag{5.6}$$

(7) $MSEt_{st6}-\ MSET_{stp}>\ 0$

$$\bar{Y}^2\left[1-\frac{\left(1-\delta V_{1.1}+\delta^2V_{0.2}\right)^2}{1+V_{2.0}+3\delta^2V_{0.2}-4\delta V_{1.1}}\right]-\bar{Y}^2\left[V_{2.0}-\frac{(V_{1.1})^2}{V_{0.2}}\right]>0$$

$$1+\frac{(V_{1.1})^2}{V_{0.2}}>\frac{(1-\delta V_{0.2}+\delta^2V_{1.1})^2}{1+V_{2.0}+3\delta^2V_{1.1}-4\delta V_{0.2}}+V_{2.0} \tag{5.7}$$

Under the condition derived above proposed estimator proves to be more efficient than the existing traditional estimator.

6. NUMERICAL ILLUSTRATION

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To illustrate numerical meaning of the theoretical results, the following real data sets are considered as:

Population: **1** Here we use the data given in Shabbir and Gupta (2011) to illustrate the properties of the estimators proposed in the present study. The data statistics consisting mainly of population parameters are shown in table 1, while table 3show PRE and MSE of the estimators.

$$PRE = \frac{V(t_{0)})}{MSE(.)} * 100$$

y: Level of apple production

x: Number of apple trees in 854 villages of Turkey in 1999. Total sample size n= 140

Table (6.1) parametric values of the population (1)

Population	Stratum 1	Stratum 2	Stratum 3	Stratum 4	Stratum 5	Stratum 6
N=854	N_1 =106	N_2 =106	$N_3 = 94$	$N_4 = 171$	$N_5 = 204$	$N_6 = 173$
n=140	n_1 =9	$n_2 = 17$	$n_3 = 38$	$n_4 = 67$	$n_5 = 7$	$n_6 = 2$
$\bar{Y}$ =29.30	$\bar{Y}_1$=15.37	$\bar{Y}_2$=22.13	$\bar{Y}_3$=93.84	$\bar{Y}_4$=55.88	$\bar{Y}_5$=9.67	$\bar{Y}_6$=4.04
$\bar{X}$ =381.95	$\bar{X}_1$=243.76	$\bar{X}_2$=274.22	$\bar{X}_3$=724.1 0	$\bar{X}_4$=773.65	$\bar{X}_5$=264.42	$\bar{X}_6$=98.44
	C_{x1}=2.02	C_{x2}=2.10	C_{x3}=2.22	C_{x4}=3.84	C_{x5}=1.72	C_{x6}=1.91
	C_{1y}=4.18	$C_{2y} = 5.22$	$C_{3y} = 3.19$	C_{4y} =5.13	C_{5y} =2.47	$C_{6y} = 2.31$
-	W_1=0.1241 2	W_2=0.12412	W_3=0.11 007	W_4=0.20023	W_5=0.23867	W_6=0.20258
-	$\rho_1 = 0.82$	$\rho_{2=}$0.86	$\rho_{3=}$0.90	$\rho_{4=}$0.99	$\rho_{5=}$0.71	$\rho_{6=}$0.89

Population: 2 The second population, is taken from the Census of India 2011(Uttar Pradesh , Series 10, Part 12B and District census hand book, AGRA).

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The considered data relates to total area of 45 villages of Khandauli block at Agra districts (U.P). We consider the numbers of Agricultural laborers in villages as study variable$y$ and the total area of villages as auxiliary variable$x$

We divided the whole population of 45 villages is divided in to 5 strata according to the Area. Accordingly, we have:

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Strata	Area in Hectare
1	(1-4400) (21 Villages)
2	(4400-8400) (10 Villages)
3	(8400-12400) (6 Villages)
4	(12400-16500) (5Villages)
5	(16500-20900) (3Villages)

Table (6.2) parametric values of the population (2)

Population	Stratum 1	Stratum 2	Stratum 3	Stratum 4	Stratum 5
$N = 45$	N_1 =21	N_2 =10	N_3 =6	N_4 =5	N_5 =3
$n = 23$	n_1 =10	n_2 =5	n_2 =3	n_2 =3	n_2=2
$\bar{Y} = 173.508$	$\bar{Y}_1$=112.09	$\bar{Y}_2$=175.9	$\bar{Y}_3$=149.83	$\bar{Y}_4$=232.2	$\bar{Y}_5$=545
$\bar{X} = 463.37$	$\bar{X}_1$=196.4	$\bar{X}_2$=413.411	$\bar{X}_3$=672.33	$\bar{X}_4$=844.26	$\bar{X}_5$=1445.97
	W_1=0.467	W_2=0.23	W_3=0.14	W_4=0.12	W_5=0.067
	S^2_{1y}=13172.99	S^2_{2y} =17740.32	S^2_{3y}=9496.96	S^2_{4y}=16815.95	S^2_{5y} =216549
	S^2_{1x}=6707.41	S^2_{2x}=2483.022	$S^2_{3x=}$6053.912	S^2_{4x}=1287.989	S^2_{5x}=161529
	S_{1xy}=2867.	S_{2xy} =202.84	S_{3xy} =554.05	S_{4xy}=-3291.6	S_{5xy} =123496.7
	C_{x1}=0.1767	C_{x2}=0.1075	C_{x3}=0.1679	C_{x4}=0.077	C_{x5}=0.867
	C_{1y}=0.6614	C_{2y} =0.767	C_{3y}=0.5616	C_{4y}=0.7473	C_{5y}=2.6819

Table (6.3) MSE and PRE of Estimators for Population (1)

S.No	Estimators	MSE	PRE
1	t_{st0}= $V(t_{0st})$	185.23	100
2	t_{st1}	59.85	309.47
3	t_{st2}	98.23	188.57
4	t_{st3}	57.82	320.35
5	t_{st4}	57.82	320.35
6	t_{st5}	149.81	123.64
7	t_{st6}	72.07	257
8	$\boldsymbol{T_{stp}}$	**57.82**	**320.36**

Table (6.4) MSE and PRE of Estimators for Population (2)

S.No	Estimators	MSE	PRE
1	$t_{st0} = V(t_{0st})$	469.66	100
2	t_{st1}	411.15	114.22
3	t_{st2}	432.24	108.65
4	t_{st3}	406.04	115.67
5	t_{st4}	406.04	115.67
6	t_{st5}	425.79	110.30
7	t_{st6}	407.23	115.32

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8	T_{st7}	**406.03**	**115.67**

From the above table (3), (4) it can be easily seen that the proposed estimator for both population demonstrated high relative efficiency over existing related estimators.

7. SIMULATION STUDY: Simulation study is carried out for the comparisons of results for large sample properties of the estimator. We generate three artificial populations of size 500 each from a bivariate normal population with the parameters specified in Table No (7.1) using R software. Therefore, a stratified population of size $N = 1500$ is generated for further computation

Table (7.1) parametric values of artificial population

N_h	n_h	S_{hy}	ρ_{hxy}	S_{hx}
$N_1 = 500$	$n_1 = 200$	$S_{1y} = 4.8$	$\rho_{1xy} = 0.5$	$S_{1x} = 4.5$
$N_2 = 500$	$n_2 = 300$	$S_{2y} = 4.8$	$\rho_{2xy} = 0.7$	$S_{2x} = 6.5$
$N_3 = 500$	$n_3 = 400$	$S_{3y} = 4.8$	$\rho_{3xy} = 0.9$	$S_{3x} = 8.4$
$N = 1500$	$n = 900$			

We calculated MSE and PRE of existing estimators and proposed estimator by following formula.

$MSE = \frac{\sum_{i=1}^{500}(\widehat{Y}_i - \bar{Y})^2}{500}$ Where $\widehat{Y}_i = t_{st0},\ tst_1 \ldots\ldots\ldots tst_{10,}, tstp_1 \ldots\ldots\ldots tst_{13.}$

$PRE(\widehat{Y}_i) = \frac{MSE(t_{st0})}{MSE(t_{sti})} \times 100$ Where $i = 1,2,3, \ldots\ldots..10$

$PRE(\widehat{Y}_i) = \frac{MSE(t_{st0})}{MSE(t_{stpi})} \times 100$ Where $i = 1,2,3, \ldots\ldots..13$

Table (6.2) MSE and PRE of Estimators

S.N	Estimators	MSE	PRE
1	$t_{st0} = V(t_{0st})$	0.00434	100
2	t_{st1}	0.00117	370.94
3	t_{st2}	0.0025	173.6
4	t_{st3}	0.0026	166.92
5	t_{st4}	0.0026	166.92

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6	t_{st5}	0.00117	370.94
7	t_{st6}	0.0025	173.6
8	$\boldsymbol{t_{stp}}$	**0.000996**	**435.74**

Above table shows that proposed estimator $(\boldsymbol{t_{stp}})$ has higher efficiency in comparison of other estimators$(t_{st1,}t_{st2}, t_{st3}, t_{st4}, t_{st5}, t_{st6})$ available in litlature.

7. CONCLUSION In this paper we proposed exponential ratio estimator in stratified sampling. The proposed estimator provides a flexible and efficient alternative to existing estimators. The conditions under which the proposed estimator has less mean square error in comparison to the other considered estimators are obtained. The performance of the proposed estimators was evaluated through a numerical study, simulation study, which demonstrated the superiority over the other commonly used estimators in terms of mean square error.

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APPENDIX

R Code for simulation study

```
install.packages("moments")

library(xlxs)

N1<-500; N2<-500; N3<500

n1<-c(200, 300, 400)

d1<-0.003;d2<-0.001;d3<-0.0005

n<-900;N<-1500

W1<-N1/N;W2<-N2/N;W3<-N3/N;

ryx1<-0.5;ryx2<-0.7;ryx3<-0.9

Sy1<-4.8;Sy2<-4.8;Sy3<-4.8

Sx1<-4.5;Sx2<-6.5;Sx3<-8.4

y1<-rnorm(N1,1.5,4)+50

y2<-rnorm(N2,1.5,4)+100

y3<-rnorm(N3,1.5,4)+150

x11<-15+sqrt(1-ryx1^2)*rnorm(N1,0.3,1)+ryx1*(Sx1/Sy1)*y1

x12<-100+sqrt(1-ryx2^2)*rnorm(N2,0.3,1)+ryx2*(Sx2/Sy2)*y2

x13<-200+sqrt(1-ryx3^2)*rnorm(N3,0.3,1)+ryx3*(Sx3/Sy3)*y3

Df1<-cbind(y1,x1);Df2<-cbind(y2,x2);Df3<-cbind(y3,x3)
```

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```
My1<-mean(Df1[,1]);My2<-mean(Df2[,1]);My3<-mean(Df3[,1])

Mx1<-mean(Df1[,2]);Mx2<-mean(Df2[,2]);Mx3<-mean(Df3[,2])

S_yx11<-cov(x11,y1);S_yx12<-cov(x12,y2);S_yx13<-cov(x13,y3)

Ybar<-W1*My1+W2*My2+W3*My3

Xbar<-W1*Mx1+W2*Mx2+W3*Mx3

cvy1<-Sy1/My1;cvy2<-Sy2/My2;cvy3<-Sy3/My3

cv1x<-Sx1/Mx1;cv2x<-Sx2/Mx2;cv3x<-Sx3/Mx3

cv1xy<-S_yx11/Mx1*My1;cv2x1y<-S_yx12/Mx2*My2;cv3x1y<-S_yx13/Mx3*My3

mSEt<-NA;yst<-NA;xst<-NA;mst<-NA;Yst<-NA;tst0<-NA;tst1<-NA;tst2<-NA;tst3<-
NA;tst4<-NA; tst5<-NA; tst6<-NA ;tst7<-NA; tst8<-NA; tst9<-NA; tstp1<-NA;tstp2<-
NA;tstp3<-NA;tstp4<-NA;tstp5<-NA;tstp6<-NA;tstp7<-NA;tstp8<-NA;tstp9<-NA;tstp10<-
NA;tstp11<-NA;tstp12<-NA;tstpn1<-NA;tstpn2<-NA;tstpn3<-NA; tstpn4<-NA; tstpn5<-NA;
tstpn6<-NA; tstpn7<-NA; tstpn8<-NA;v tstpn9<-NA; tstpn10<-NA;tstpn11<-NA;tstpn12<-
NA;tstpn13<-NA;tstpn0<-NA;mSEY<-NA;mSEtp<-NA;tst0<-NA;y11<-NA;y22<-NA;y33<-
NA;sx11<-NA;sx12<-NA;sx13<-NA;sx21<-NA;sx22<-NA;sx23<-NA;sx31<-NA;sx32<-
NA;sx33<-NA;mSEt1<-NA;mSEt2<-NA;mSEt3<-NA;mSEt4<-NA

y11s<-NA;y22s<-NA;y33s<-NA;sx11s<-NA;sx12s<-NA;sx13s<-NA;sx21s<-NA;sx22s<-
NA;sx23s<-NA;sx31s<-NA;sx32s<-NA;sx33s<-NA

cv11<-NA;cv12<-NA;cv13<-NA;cv21<-NA;cv22<-NA;cv23<-NA;cv31<-NA;cv32<-
NA;cv33<-NA

b2st1<-NA;b2st2<-NA;b2st3<-NA;b1st1<-NA;b1st2<-NA;b1st3<-NA;cst1<-NA;cst2<-
NA;cst3<-NA;

for (i in 1:500) {
```

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```
m1<-c(sample(1:500,200,replace=F))

m2<-c(sample(1:500,300,replace=F))

m3<-c(sample(1:500,400,replace=F))

ma1<-Df1[m1,]

head(ma1)

ma2<-Df2[m2,]

head(ma2)

ma3<-Df3[m3,]

head(ma3)

maa1<-as.data.frame(ma1)

maa2<-as.data.frame(ma2)

maa3<-as.data.frame(ma3)

y11[i]<-mean(maa1$y1);y22[i]<-mean(maa2$y2);y33[i]<-mean(maa3$y3)

sx11[i]<-mean(maa1$x11);sx12[i]<-mean(maa2$x12);sx13[i]<-mean(maa3$x13)

y11s[i]<-sd(maa1$y1);y22s[i]<-sd(maa2$y2);y33s[i]<-sd(maa3$y3)

sx11s[i]<-sd(maa1$x11);sx12s[i]<-sd(maa2$x12);sx13s[i]<-sd(maa3$x13)

cv11[i]<-sx11s[i]/sx11[i];cv12[i]<-sx12s[i]/sx12[i];cv13[i]<-sx13s[i]/sx13[i]

cv11[i]<-y11s[i]/y11[i];cv22[i]<-y22s[i]/y22[i];cv33[i]<-y33s[i]/y33[i]

b11[i]<-skewness(maa1$x11);b12[i]<-skewness(maa2$x12);b13[i]<-skewness(maa3$x13)
```

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```
b21[i]<-kurtosis(maa1$x11);b22[i]<-kurtosis(maa2$x12);b23[i]<-kurtosis(maa3$x13)

r1[i]<-cor(maa1$x11); r2[i]<-cor(maa2$x12); r3[i]<-cor(maa3$x13)

b1st[i]<- b11[i]*W1+b12[i]*W2+b13[i]*W3

b2st[i]<- b21[i]*W1+b22[i]*W2+b23[i]*W3

rst[i]<-r1[i] *W1+ r2[i] *W2+ r3[i] *W3

cst[i]<-cv11[i]*W1+cv12[i]*W2+cv13[i]*W3

yst[i]<-W1*y11+W2*y22+W3*y33
```

Existing estimator and their MSE

```
msEt[i]<-(yst[i]-Ybar)^2

xst[i]<-W1*sx11+W2*sx12+W3*sx13

tst1<-yst[i]*( Xbar/ xst[i])

 mSEt1[i]<-(tst1[i]-Ybar)^2

tst2[i]<- yst[i]*exp( (Xbar- xst[i])/ (Xbar+ xst[i]))

 mSEt2[i]<-(tst2-Ybar)^2

tst3[i]<- yst[i]*+bst*(Xbar - xst[i])

mSEt3[i]<-(tst3-Ybar)^2

tst4[i]<- yst[i]*(t*( (Xbar / xst[i])^a+(1-t)*( xst[i]/ Xbar))

mSEt4[i]<-(tst4-Ybar)^2

tst5[i]<- yst[i]*( (Xbar + cst)/ ( xst[i]+ cst))
```

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```
mSEt5[i]<-(tst5-Ybar)^2

tst6[i]<- yst[i]*a*( (Xbar + n)/ ( xst[i]+ n))

mSEt6[i]<-(tst6-Ybar)^2

#####Proposed estimator#####

Forθ = 1/4*(2*(1-(W1^2*d1*S_yx11/Ybar*Xbar)+(W2^2*d2*S_yx12 /Ybar*Xbar)+
(W3^2*d3* S_yx13/ Ybar* Xbar))/( (W1^2*d1* Sx1^2/ Xbar^2)+ (W2^2*d2* Sx2^2/ Xbar^2)+
(W1^2*d1* Sx1^2/ Xbar^2))+1)

###### proposed Estimator######

Tstp[i]<-yst[i]*(t*(Xbar/xst[i])+(1-t)*(xst[i]/Xbar))*exp( (Xbar- xst[i])/ (Xbar+ xst[i]))

mSETstp[i]<-(Tstp0[i]-Ybar)^2

}

####MSE and PRE of the estimatos#####

MSE1<-mean(msEt);MSE2<-mean(mSEt1);MSE3<-mean(mSEt2);MSE4<-
mean(mSEt3);MSE5<-mean(mSEt4);MSE6<-mean(mSEt5);MSE7<-mean(mSEt6);

MSETstp<-mean(mSETstp[i])

pre1<-(MSE1/MSE2)*100;pre2<-(MSE1/MSE3)*100;pre3<-(MSE1/MSE4)*100;pre4<-
(MSE1/MSE5)*100

; pre5<-(MSE1/MSE6)*100; pre6<-(MSE1/MSE7)*100

pre1p<-(MSE1/MSETstp)
```