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Edge Detour Monophonic Number of a Fuzzy Graph

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ABSTRACT

Let M be a set of vertices in a connected non-trivial fuzzy graph $G:(V,\sigma,\mu)$. Then the edge detour monophonic closure of M, denoted by $(dM)_e$, is the set of all edges lying on the fuzzy detour monophonic path between a pair of vertices of M. A set M of vertices in a connected non-trivial fuzzy graph $G:(V,\sigma,\mu)$ is defined to be an edge detour monophonic set of G if $(dM)_e = E(G)$, the edge set of G. An edge detour monophonic set of minimum cardinality is called an edge detour monophonic basis of G and the cardinality of an edge detour monophonic basis in G is the edge detour monophonic number of G, denoted by $dmn_e(G)$. This article introduces the fuzzy graph's edge detour monophonic number. This concept's compliance with a few properties is examined.

Keywords: fuzzy graph, fuzzy strong chord, fuzzy monophonic path, fuzzy detour monophonic path, edge detour monophonic number.

AMS Subject Classification: 05C72, 05C38, 05C90.

1. Introduction:

Zadeh in 1965 [14] developed a mathematical phenomenon for describing the uncertainties prevailing in day-to-day life situations by introducing the concept of fuzzy sets. A fuzzy set is a generalization of a crisp set in which the set's components may or may not be members. Instead of merely considering 0 or 1, fuzzy sets allow their members to have membership degree values between 0 and 1 for better results. The membership degree of a member, which is a single value within the range [0, 1], does not equal probability; rather, it represents the element's degree of belongingness to the fuzzy set. Due to ignorance of the issue, the single membership degree values, however, are unable to handle the uncertainties. Rosenfeld [7] first proposed the idea of fuzzy graph in 1975.

If $\mu(x,y) \leq \sigma(x) \wedge \sigma(y)$ for every $x, y \in \sigma^*$, a fuzzy graph $G:(V,\sigma,\mu)$ is a fuzzy graph. A path P is a sequence of different vertices of length n such that $\mu(u_{i-1}, u_i) > 0, i = 1,2,\ldots,n$ and the degree of membership of a weakest arc is defined as its strength. If$u_0 = u_n, n \geq 3$, the path turns into a cycle, and if the cycle contains more than one weakest arc, it is referred to be a fuzzy cycle. The maximum strength of all paths connecting two vertices x and y is known as the strength of connectedness, and it is shown by the symbol $CONN_G(x,y)$. When an arc in a fuzzy graph is eliminated, its weight must at least equal the connectedness of its end vertices. An $x - y$ path is said to be a strong path if Ponly comprises strong arcs. If a connected fuzzy graph $G:(V,\sigma,\mu)$ has a spanning fuzzy sub graph $F:(V,\sigma,\upsilon)$ a tree where for every arcs (x,y) not in F, $CONN_F(x,y) > \mu(x,y)$ it is referred to as a fuzzy tree. A fuzzy tree with a singular maximum spanning tree that is a star is called a fuzzy star. If a fuzzy sub graph $H:(V,\tau,\pi)$ is

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bipartite and spans the fuzzy graph $G:(V,\sigma,\mu)$ then the weight of the pair (x,y) in G is strictly less than the strength of the pair (x,y) in H for all edges that are not in H. If every vertex of a fuzzy bipartite graph with fuzzy bipartite (V_1,V_2) is a strong neighbor to every node of V_1, the graph is said to be a complete fuzzy bipartite.

A crisp graph's detour monophonic number was first introduced in [13]. In a crisp graph, the chord of a path P is the edge that connects two of its non-adjacent vertices. If a path P lacks chords, it is said to be monophonic path. The term " $x-y$ detour monophonic path" refers to the longest $x-y$ monophonic path. If each vertex v of a graph G lies on an $x-y$ detour monophonic path for some $x,y \in M$, then the set $M$of those vertices is a detour monophonic set. The detour monophonic number of G is the smallest cardinality of a detour monophonic set of G. A set M of vertices is said to be an edge monophonic set of G if every edge of G lies on a monophonic path connecting some pair of vertices in M. The minimum cardinality of G's edge monophonic sets is known as the edge monophonic number. The edge detour monophonic number of crisp graph was introduced in [12]. A set $M$of vertices that has every edge of G lying on a detour monophonic path connecting some pair of vertices in M is known as an edge detour monophonic set of G.The lowest cardinality of G's edge detour monophonic sets is known as its edge detour monophonic number, which is represented by the symbol $edm(G)$.

We introduced and looked into detour monophonic sets in a fuzzy graph as a result of the edge detour monophonic number of a crisp graph. Let $P: u_1, u_2, \dots, u_i, u_{i+1}, \dots, u_j, u_{j+1}, \dots, u_n$ be a path of a fuzzy graph $G:(V,\sigma,\mu)$. A strong chord of a path P in a fuzzy graph is an edge u_iu_j if $\mu(u_i,u_j) \geq \mu(u_i,u_{i+1}) \wedge \mu(u_{i+1},u_{i+2}) \wedge \dots . \wedge \mu(u_{j-1},u_j)$. A path P in a fuzzy graph is called monophonic path if it is a strong chord less path.

In this study, we introduce the idea of a fuzzy graph's edge detour monophonic number.

2. Edge detour monophonic number of a fuzzy graph:

The crisp graphs edge detour monophonic number was introduced by Santhakumaran and others [12]. In this section, we introduce the concept of edge detour monophonic number of a fuzzy graph.

Definition 2.1 Let M be a set of vertices in a connected non-trivial fuzzy graph $G=(V,\sigma,\mu)$. Then the edge detour monophonic closure of M, denoted by $(dM)_e$, is the set of all edges lying on the fuzzy detour monophonic path between a pair of vertices of M.

Definition 2.2 A set M of vertices in a connected non-trivial fuzzy graph $G=(V,\sigma,\mu)$ is defined to be an edge detour monophonic set of G if $(dM)_e = E(G)$, the edge set of G.

Definition 2.3 An edge detour monophonic set of minimum cardinality is called an edge detour monophonic basis of G and the cardinality of an edge detour monophonic basis in G is the edge detour monophonic number of G, denoted by $dmn_e(G)$.

Example 2.4 For the fuzzy graph given in Figure 2.1, $M_1 = \{v_1, v_2, v_5, v_6\}$ and $M_2 = \{v_2, v_3, v_5, v_6\}$ are the edge detour monophonic bases of G so that $dmn_e(G) = 4$.

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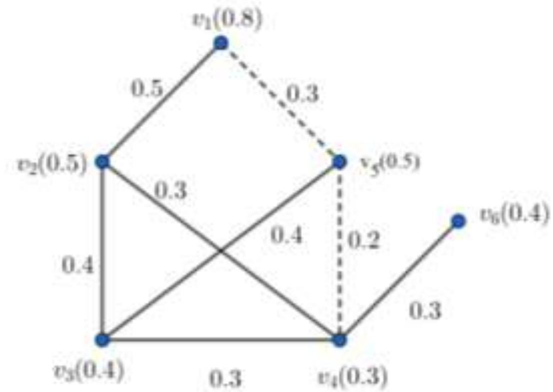

Figure 2.1 The edge detour monophonic number of the fuzzy graph

Proposition 2.5 Assuming $G = (V, \sigma, \mu)$ is a non-trivial fuzzy tree. Then its fuzzy end vertices form the unique edge detour monophonic set.

Proof: Let M represent the collection of all $G's$ fuzzy ends vertices.

Case (i): There are no δ -arcs in G.

G is a fuzzy tree without δ -arcs, and since fuzzy trees don't have β -arcs, each arc of G must be α -strong. It follows that G must be a fuzzy tree for $G^\star$ to be a tree. Since each arc of G is definitely resides on a fuzzy detour monophonic path joining some pair of vertices in M, it follow that the fuzzy end vertices of G are the end vertices of $G^\star$, which are unique. Thus, M is an edge detour monophonic set of G.

The edge incident on u does not lie on any fuzzy detour monophonic path joining pairs of vertices in M if u is a fuzzy end vertices of $G^\star$, therefore it is also the edge detour monophonic set of least cardinality. The number of fuzzy end vertices in G is, therefore $dmn_e(G)$, and M is the edge monophonic set for G.

Case (ii): G has at least one δ -arc.

Let us assume that the number of $\delta - arcs$ in G be k. Let $\delta_i = u_i v_i$ ($1 \leq i \leq$n) be the δ-arcs of G. Let $M_1 = \{V(\delta_1), V(\delta_2), \ldots., V(\delta_k)\}, k \leq n$ such that if $V(\delta_i) \cap V(\delta_j)$= $\{u_1, u_2, ..u_k\}$ for $i \neq j$. Then include u_1 only in M_1. Let M_2 be the set of all end vertices of G. Then $M = M_1 \cup M_2$ is a subset of every edge detour monophonic set of G. Since M is an edge detour monophonic set of G, $dmn_e(G) = |M_1 \cup M_2|$. We prove that M is unique. On the contrary M is not unique. Then there exists a dmn_e- set of M' of G such that $|M'| = |M_1 \cup M_2|$. Since $M_1 \cup M_2 \subseteq M'$, it follows that $M' = M$. Therefore dmn_e-set of G is unique.

Corollary 2.6 Let $G = (V, \sigma, \mu)$, a fuzzy tree with $n \geq 3$ vertices, be such that $G^*$is a tree. If $G^*$is a star graph, then $dmn_e(G) = n - 1$.

Proof: Assume G^* is a star graph with n vertices, specifically $K_{1,n-1}$. According to Proposition 2.5, the set of all fuzzy end vertices of G constitute an edge detour monophonic basis of G because G is a fuzzy tree without δ -arcs. Therefore, $dmn_e(G) = n - 1$.

Remark 2.7 It should be noted that a connected non-trivial fuzzy graph's edge detour monophonic set need not be unique. For a fuzzy graph, there may be more than one edge detour monophonic set. $M_1 = \{v_1, v_4, v_7\}$ and $M_2 = \{v_2, v_5, v_7\}$ are two edge detour monophonic sets of the fuzzy graph $G: (V, \sigma, \mu)$ shown in Figure 2.2.

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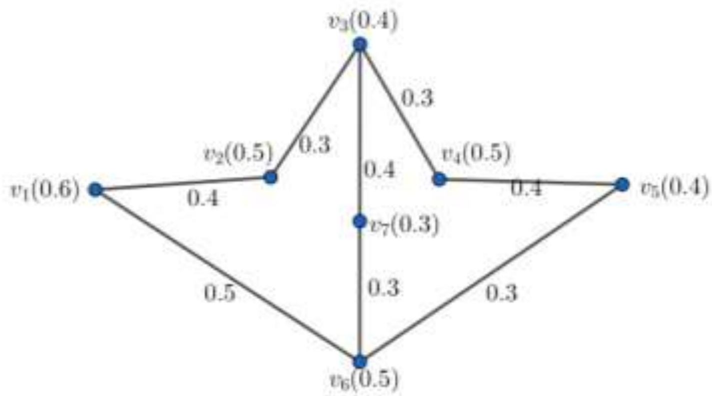

Figure 2.2 Illustration of the edge detour monophonic sets of a fuzzy graph

Proposition 2.8 Any connected non-trivial fuzzy graph $G = (V, \sigma, \mu)$ with n vertices has the property $2 \leq dmn_e(G) \leq n$.

Proof : $dmn_e(G) \geq 2$ means that an edge detour monophonic set requires at least two vertices. Since the set of all vertices in G is an edge detour monophonic set of G. Therefore, it is evident that $dmn_e(G) \leq n$. Therefore, $2 \leq dmn_e(G) \leq n$.

Proposition 2.9 Let $G: (V, \sigma, \mu)$ be a non-trivial connected fuzzy graph on n vertices containing no δ-arcs. If G has exactly one node u of degree $n-1$ in G^*, then the edge detour monophonic number, $dmn_e(G) = n-1$.

Proof: Let u be the only vertex of degree $n-1$ on G. Then, $n \geq 3$. Suppose $M = V(G) - \{u\}$. First, we demonstrate that M is an edge detour monophonic set of G. This demonstrates how M encloses all of G's edges. Think about the following scenarios.

Case (i): Edges incident with u

Make (u, v) an edge incident with u. There is at least one neighbor of u named v' such that v and $v'$are not adjacent, as u is the only vertex of degree $n-1$. The edge (u, v) is on the fuzzy detour monophonic path joining $v'$and v in M, written as $v' - \text{u} - \text{v}$. The edge (u, v) is thus covered by M. It can also be demonstrated that M covers all edges that are incident with u.

Case (ii): Edges not incident with u.

Take into account any (x, y) edge that is not incident with u. It is obvious that it is on the fuzzy detour monophonic path (x, y) itself, where $x, y \in M$. M , therefore covers the (x, y) edge. M also covers all edges that do not incident with u.

Thus it is clear from both cases that M is an edge detour monophonic set of G and so $dmn_e(G) \leq |M| = n-1$. Next we show that M is an edge detour monophonic basis of G. Let $T \subset V(G)$ be such that $|T| \leq n-2$. Then there exists at least two vertices say v and $w \in V(G)$ such that $v, w \notin T$. Again consider the following sub cases.

Sub Case (a): $u \in T$

Then, since $v, w \notin T$, the edge (u, v) or (u, w) cannot lie on a fuzzy monophonic joining two vertices of T. Thus, T can not lie on edge detour monophonic set of G.

Sub Case (b): $u \notin T$

Then $u \notin v$ or $u \notin w$ so that the edges (u, v) or (u, w) cannot lie on any fuzzy detour monophonic joining two vertices of T. So, T is not an edge detour monophonic set of G.

Thus in any case, T is not an edge detour monophonic set of G. Hence M is the unique edge detour monophonic set of G having minimum cardinality and so $dmn_e(G) = |M| = n-1$.

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Remark 2.10 The converse of the Proposition 2.9 need not true. For the fuzzy graph $G=(V,\sigma,\mu)$ given in Figure 2.3, $M=\{v_1,v_2,v_4,v_5,v_6\}$ is the edge detour monophonic basis of G, and therefore, $dmn_e(G)=5=6-1$. But no vertex of G has degree 5 in G^*.

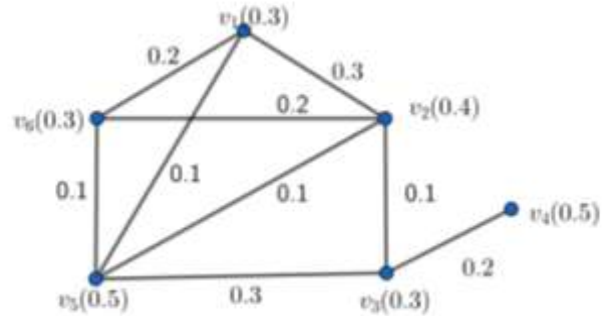

Figure 2.3 A fuzzy graph with edge detour monophonic number = 5

Proposition 2.11 If a connected non-trivial fuzzy graph $G=(V,\sigma,\mu)$ on n vertices containing no δ-arcs has more than one vertex of degree $n-1$, then every edge detour monophonic set of G contain all vertices of degree $n-1$.

Proof: Let G be a fuzzy graph with n vertices and multiple $n-1$ degree vertices. Assume that any two vertices of G with degrees $n-1$ each are x and y. Then, in order to build an edge (x,y) that does not lie on any fuzzy detour monophonic path joining two vertices of G other than x and y, x must be adjacent to y and y must be adjacent to x. So it follows that x and y are present in every edge detour monophonic set of G. It follows that every edge detour monophonic set of G contains all vertices of degree $n-1$ since x and y are arbitrary.

Proposition 2.12 Let $G=(V,\sigma,\mu)$ be a connected fuzzy graph on n vertices containing no δ-arcs such that $G^\star=K_n(n\geq 2)$. Then the edge detour monophonic number $dmn_e(G)=n$.

Proof: Each arc (u,v) is a fuzzy detour monophonic path for every $u,v\in V(G)$ since all of the arcs in G are strong and because $G^\star=K_n(n\geq 2)$, and no arc lies on the fuzzy detour monophonic path between any two other vertices in G. If $T\subset M$ such that vertex u of M is not in T, incident on u is not any fuzzy detour monophonic pairs of vertices in T, and so T is not an edge detour monophonic cover of G, then $M=V(G)$ is an edge detour monophonic set of G of lowest cardinality. The unique edge detour monophonic basis of G is therefore $M=V(G)$, and $dmn_e(G)=n$.

Example 2.13 Consider the connected non-trivial fuzzy graph $G=(V,\sigma,\mu)$ given in Figure 2.4. G contains no δ-arcs. Here each arc is strong and hence $M=\{v_1,v_2,v_3,v_4,v_5,v_6\}$ is the unique edge detour monophonic basis of G and so $dmn_e(G)=6$. But note that the given fuzzy graph is not a complete fuzzy graph.

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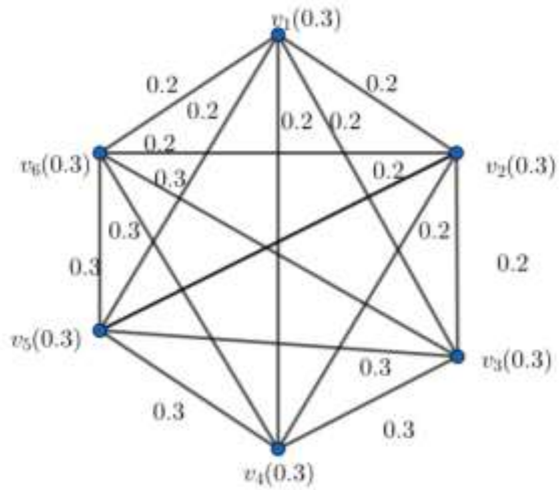

Figure 2.4 A fuzzy graph which is complete in G^*

Remark 2.14 Converse of the Proposition 2.12 need not be true. That is a connected non-trivial fuzzy graph $G = (V, \sigma, \mu)$ on n vertices having edge detour monophonic number n need not be a complete graph in $G^\star$.

For example, the connected fuzzy graph $G = (V, \sigma, \mu)$ given in Figure 2.5 has $M = V(G)$ as the unique edge detour monophonic basis of G and so $dmn_e(G) = 6$. But $G^\star$ is not complete graph.

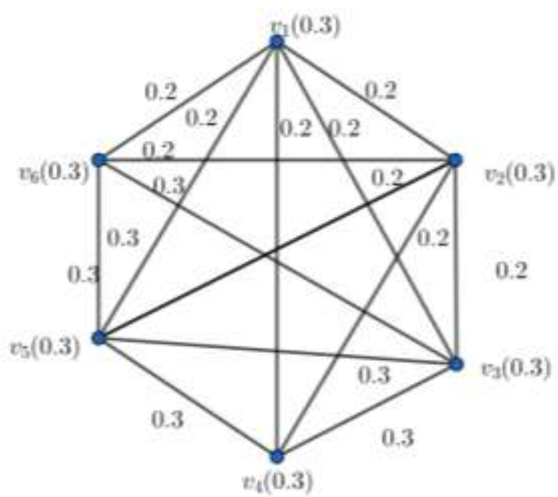

Figure 2.5 A fuzzy graph which is not a complete graph in $G^\star$

Proposition 2.15 If a connected non-trivial fuzzy graph $G = (V, \sigma, \mu)$ on n vertices containing no δ-arcs has more than one vertex of degree $n - 1$, then the edge detour monophonic number of G, $dmn_e(G) = n$.

Proof: Keep in mind that if all of G's vertices have degrees $n - 1$ or above, then $G^\star = K_n$ and, according to Proposition 2.12, $dmn_e(G) = n$. If not, assume that G's vertices $v_1, v_2, \ldots., v_k$ $(2 \leq k \leq n - 2)$ have degrees $n - 1$ each. According to Proposition 2.12, each of these vertices is a part of G's (say M) edge detour monophonic basis. If it's possible, assume $|M| < n$.

If v is a vertex in G, then let $v \notin M$. v must then have a degree lower than $n - 1$. Without it, v would be in M. We demonstrate that no edge having the form $(v, v_i), (1 \leq i \leq k)$ lies on a fuzzy

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detour monophonic path joining a pair of vertices of M.

Case (i): Both vertices of M are of degree $n-1$ each. Since among the vertices $v_{i,}(1 \leq i \leq k)$ of degree $n-1$, any two of them are always adjacent, an edge of the form (v, v_i), $(1\leq i \leq k)$ cannot lie on a fuzzy detour monophonic path joining a pair of vertices v_j and $v_l, (j \neq l), (1 \leq j, l \leq k)$ of M.

Case (ii): One vertex of M is of degree $n-1$ and the other is of degree less than $n-1$.

Note that each node $v_i, (1 \leq i \leq k)$ is always adjacent to some vertex (say) u of M of degree less than $n-1$, which is different from v_i, $(1 \leq i \leq k)$. Thus the edge $(v, v_i), (1 \leq i \leq k)$ cannot lie on a fuzzy detour monophonic path joining a vertex v_i and the vertex u of M.

Case (iii): Both vertices of M are of degree less than $n-1$.

let b and c any two vertices of M different from $v_i, (1 \leq i \leq k)$. Since each $v_i, (1 \leq i \leq k)$ is adjacent to both b and c and since $d_m(b, v) \leq 2$, the edge (v, v_i) cannot lie on a fuzzy detour monophonic path joining b and c.

Thus in all three cases it can be seen that the edges $(v, v_i), (1 \leq i \leq k)$ do not lie on any fuzzy detour monophonic path joining a pair of vertices of M, which contradicts to the fact that M is an edge detour monophonic basis of M. Hence $|M| = n$ and so $dmn_e(G) = n$.

Remark 2.16 The converse of the Proposition 2.15 need not be true. For the fuzzy graph G containing no δ- arcs given in Figure 2.6, $M = \{v_1, v_2, v_3, v_4, v_5, v_6\} = V(G)$ is an edge detour monophonic basis of G so that $dmn_e(G) = 6 = n$ and G has no vertex of degree $n - 1 = 6 - 1 = 5$.

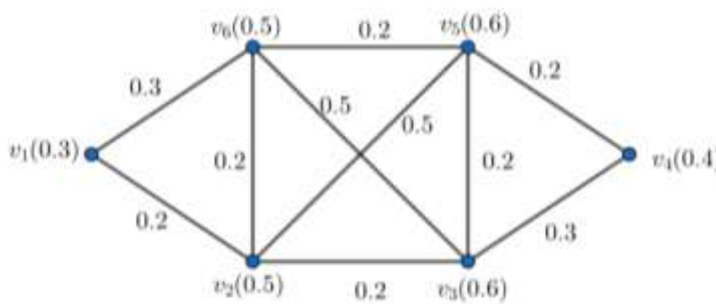

Figure 2.6 A fuzzy graph with $dmn_e(G) = 6$ but no vertex of degree 5 in G^*

Proposition 2.17 The edge detour monophonic number of a non-trivial complete fuzzy graph $G: (V, \sigma, \mu)$ on n vertices is n.

Proof: Since each arc in a complete fuzzy graph G is strong and since the underlying crisp graph $G^\star = K_n$, it follows from Proposition 2.11 that $dmn_e(G) = n$.

Remark 2.18 It is clear from Propositions 2.8 and 2.17 that for a complete fuzzy graph on n vertices, the detour monophonic number of G coincides with its edge detour monophonic number. ie, $dmn(G) = dmn_e(G) = n$.

For example, the complete fuzzy graph $G: (V, \sigma, \mu)$ given in Figure 2.7 has $M = V(G)$ as both detour monophonic basis and edge detour monophonic basis of G since each arc is strong and thus does not lie on any detour monophonic joining any two vertices of G.

Therefore, $dmn(G) = dmn_e(G) = 5$.

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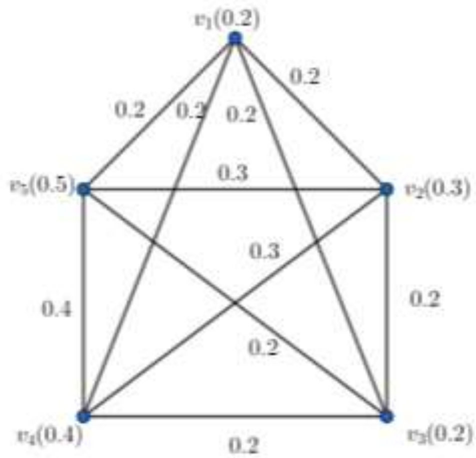

Figure 2.7 A fuzzy graph with $dmn(G) = dmn_e(G) = 5$

Proposition 2.19 Let $G = K_{\sigma_1,\sigma_2} = (V_1 \cup V_2, \sigma, \mu)$ be a complete bipartite fuzzy graph on n vertices. Then

1. $dmn_e(K_{\sigma_1,\sigma_2}) = 2$, if $|V_1| = |V_2| = 1$
2. $dmn_e(K_{\sigma_1,\sigma_2}) = |V_2|$, if $|V_1| = 1$ and $|V_2| \geq 2$
3. $dmn_e(K_{\sigma_1,\sigma_2}) = \min\{r, s\}$, if $|V_1| = r$ and $|V_2| = s$ where $r, s \geq 2$.

Proof:

1. Follows from Proposition 2.17.
2. Follows from Corollary 2.6.
3. Let $r, s \geq 2$. First assume that $r < s$. Let $V_1 = \{u_1, u_2, \dots, u_r\}$ and $V_2 = \{w_1, w_2, \dots, w_s\}$ be a partition of K_{σ_1,σ_2}. Let $M = V_i$. We prove M is an edge detour monophonic set of K_{σ_1,σ_2}.

1. Any edge (u_i, w_j) $(1 \leq i \leq r, 1 \leq j \leq s)$ lies on the fuzzy detour monophonic $u_i - w_j - u_k$ for any $k \neq i$ so that M is an edge detour monophonic set of K_{σ_1,σ_2}.

Claim: M is an edge detour monophonic basis of K_{σ_1,σ_2}.

That is, we prove that M is an edge detour monophonic set of K_{σ_1,σ_2} having minimum cardinality. Let T be any set of vertices such that $|T| < |M| = r$. We show that T is not an edge detour monophonic set of K_{σ_1,σ_2}. Consider the following cases.

Case (i): If $T \subset V_1$, then there exists a vertex $u_i \in V_1$ such that $u_i \notin T$. Then for any edge (u_i, w_j) $(1 \leq j \leq n)$, the only fuzzy detour monophonic path containing (u_i, w_j) are $u_i - w_j - u_k$ $(k \neq i)$ and $w_j - u_i - w_l$ $(l \neq j)$ and so (u_i, w_j) cannot lie on edge detour monophonic set of K_{σ_1,σ_2}.

Case (ii): $T \subset V_2$, then by a similar argument T is not an edge detour monophonic set of K_{σ_1,σ_2}.

Case (iii): If $T \subset M \cup V_2$ such that T contains at least one vertex M and V_2, then since $|T| < |M|$, there exists vertices $u_i \in V_1$ and $w_j \in V_2$ such that $u_i \notin T$ and $w_j \notin T$. Then clearly the edge (u_i, w_j) does not lie on a fuzzy detour monophonic path joining two vertices of T so that T is not an edge detour monophonic set .

Thus in any case, T is not an edge detour monophonic set of K_{σ_1,σ_2}. Hence M is an edge detour monophonic basis of K_{σ_1,σ_2} so that $dmn_e(G) = |M| = r$. Now if $r = s$, we prove similarity that $M = V_1$ *or* V_2 is an edge detour monophonic basis of K_{σ_1,σ_2}.

Example 2.20 Consider the complete bipartite fuzzy graph G given in Figure 2.8

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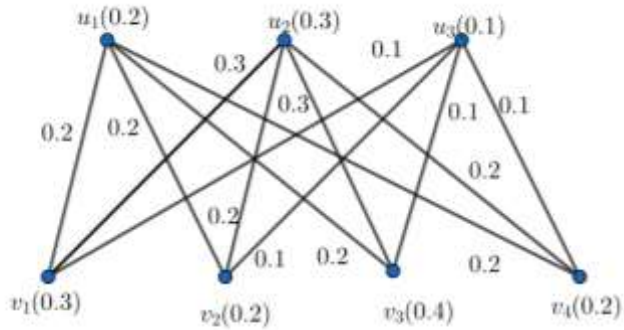

Figure 2.8 A complete bipartite fuzzy graph with $dmn_e(G) = 3$

Here $dmn_e(G) = 3$ and $M = \{u_1, u_2, u_3\}$ is an edge detour monophonic basis of G since $(dM)_e = E(G)$. Hence, $dmn_e(G) = 3 = \min\{3,4\}$.

Conclusion

In this article we studied the edge detour monophonic number of a fuzzy graph. Some general properties satisfied by this concept are studied. The edge detour monophonic number of fuzzy graph of some standard graphs also determined. We can extend this concept to other distance related parameters in fuzzy graph.

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