Between Semi* α Closed Sets and α^* Closed Sets

***Siluvai A**
Research Scholar (Reg.No-18131282091015),
PG and Research Department of Mathematics, G. Venkataswamy Naidu College,
(Affiliated to Manonmaniam Sundaranar University, Abishekabatti, Tirunelveli)
Kovilpatti, Tamilnadu – 628 502, India.
E-mail: siluwin@gmail.com

S. Pious Missier
Head & Associate Professor (Retd), Department of Mathematics,
Don Bosco College of Arts and Science,
(Affiliated to Manonmaniam Sundaranar University, Abishekabatti, Tirunelveli)
Keela Eral, Thoothukudi, Tamil Nadu-628 908, India.
E-mail: spmissier@gmail.com

S. Gabriel Raja
Research Scholar (Reg. No – 19222052091008), G. Venkataswamy Naidu College,
(Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli),
Kovilpatti, Tamil Nadu – 628 502, India.
E-mail: gabrilraja@gmail.com

Abstract: Many topologist defined and studied many strong and weak forms of open and closed sets in general topology. Anbarasi Rodrigo .P and Pious Missier .S [9] have introduced α^* closed sets which is a weaker form of α closed sets. Robert .A and Pious Missier .S [11] have introduced $S^*\alpha$ closed sets which is a weaker form semi*- α - closed sets. In this paper we introduce a new class of sets, namely $\widehat{S_p}^*$- closed sets as the compliment of $\widehat{S_p}^*$- open sets. We intend to analize the characterization of $\widehat{S_p}^*$- Closed Sets. We also study the properties of this particular class of sets which have born interesting results. We also define $\widehat{S_p}^*$ - closure of a subset and investigate the fundamental properties of $\widehat{S_p}^*$- closure. An analysis of the properties of this particular class of sets have borne interesting results. The results are substantiated with appropriate examples.

Key Words: semi α open sets, semi α closed sets, $\widehat{S_p}^*$- Open Sets, $\widehat{S_p}^*$ - interior, $\widehat{S_p}^*$- Closed Sets and $\widehat{S_p}^*$ - closure.

1. INTRODUCTION

In this paper, we introduce the concept of $\widehat{S_P}^*$-closed sets as the complement of $\widehat{S_P}^*$-open sets, we study the characterizations of $\widehat{S_P}^*$-closed sets. We establish that the class of $\widehat{S_P}^*$closed sets is placed between the class of $Semi^*$-α-closed set and α^*-closed sets. We investigate the fundamental properties of $\widehat{S_p}^*$- closed sets. We also define $\widehat{S_P}^*$- closure of a subset. We give explicit expression for $\widehat{S_P}^*$-closure and $\widehat{S_P}^*$- interior of a set.

2. PRELIMINARIES

Definition 2.1 Let A be a subset of a topological space (X,τ). Then

(i) A is semi-open set [4] if there is an open set U in X such that $U \subseteq A \subseteq Cl(U)$ or equivalently if $A \subseteq Cl(Int(A))$.

(ii) A is pre-open set [6] if $A \subseteq Int(Cl(A))$ and pre-closed if $Cl(Int(A)) \subseteq A$.

(iii) A is α-open set [7] if $A \subseteq Int(Cl(Int(A))$ and α-closed if $Cl(Int(Cl(A)) \subseteq A$.

(iv) A is semi pre-open set [13] if A ⊆ *Cl*(*Int*(*Cl*(*A*))) and semi pre -closed if *Int*(*Cl*(*Int*(*A*)) ⊆*A*.

The pre-closure of A, α-closure of A and semi pre closure of A are analogously defined and they are respectively denoted by *PCl*(*A*), α*Cl*(*A*) and *SPCl*(A).

Definition 2.2 Let A be a subset of a topological space (X, τ). Then

(a) A is called a generalized closed set [5] (briefly g-closed) if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
(b) A is called a semi generalized closed set [1] (briefly sg-closed) if $SCl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ).
(c) A is called a pre generalized closed set [6] (briefly pg-closed) if $PCl(A) \subseteq U$ whenever $A \subseteq U$ and U is pre-open in (X, τ).
(d) A is called a semi star generalized closed set [2] (briefly S^*g-closed) if $SCl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ).
(e) A is called a semi generalized star open set [13] (briefly $S_g{}^*$ -open) if there is an open set U in X such that $U \subseteq$ A ⊆ Scl *(U).
(f) A is called a semi star open set [9] (briefly S^*-open) if there is an open set U in X such that $U \subseteq A \subseteq Cl^*(U)$ or equivalently if $A \subseteq Cl^*(Int(A))$.
(g) A is called an alpha star open set [10] (briefly α^*-open) if there is an open set U in X such that $U \subseteq A \subseteq Cl^*(U)$ or equivalently if $A \subseteq Int^*(Cl(Int(A)))$.
(h) A is called a semi star alpha open set [8] (briefly $S^*\alpha$-open) if there is an open set U in X such that $U \subseteq A \subseteq Cl^*(U)$ or equivalently if $A \subseteq Int^*(Cl(Int(A)))$.
(i) A is called a $\widehat{S_P}^*$- open set [12] if here is an open set U such that $U \subseteq A \subseteq PCl^*(U)$.

Theorem 2.3 [12]

(a) Every open set is $\widehat{S_P}^*$- open set
(b) Every α open set is $\widehat{S_P}^*$- open set
(c) Every S^* open set is $\widehat{S_P}^*$- open set
(d) Every $S^*\alpha$ open set is $\widehat{S_P}^*$- open set
(e) Every S_g^* open set is $\widehat{S_P}^*$- open set
(f) Every $\widehat{S_P}^*$- open set is α^* open set
(g) Every $\widehat{S_P}^*$- open set is sg open set

Theorem 2.4 [12] If A is a $\widehat{S_P}^*$- open set in X and B is an open set in X, then $A \cap B$ is a $\widehat{S_P}^*$- open set in X.

3. $\widehat{S_P}^*$- CLOSED SETS

Definition 3.1 A subset A of a Space X is called $\widehat{S_P}^*$- closed set if its complement $(X \backslash A)$ is $\widehat{S_P}^*$- open in X. The class of all $\widehat{S_P}^*$closed sets in (X, τ) is denoted by $\widehat{S_P}^* C(X, \tau)$ or simply $\widehat{S_P}^* C(X)$.

Theorem 3.2 If A is a subset of a topological space X, the following statements are equivalent

(i) A is $\widehat{S_P}^*$closed
(ii) There is a pre-closed F in X such that $PInt^*(F) \subseteq A \subseteq F$
(iii) $PInt^*\big(Cl(A)\big) \subseteq A$

(iv) $PInt^*\big(Cl(A)\big) = PInt^*(A)$

(v) $PInt^*\Big(A \cup Int\big(Cl(A)\big)\Big) = PInt^*(A)$

Proof: (i) ⇒ (ii) Suppose A is $\widehat{S_P}^*$- closed set in X. Then $(X\backslash A)$ is $\widehat{S_P}^*$- open in X. Then by Definition 2.2 (i), there is an open set in X, such that $U \subseteq (X - A) \subseteq PCl^*(U)$. Taking complements, we get $(X\backslash U) \supseteq A \supseteq \big(X\backslash PCl^*(U)\big)$. Since for any subset U of X, $PInt^*(U) = X\backslash PCl^*(U)$, we have $(X\backslash U) \supseteq A \supseteq PInt^*(U)$. That is $F \supseteq A \supseteq PInt^*(X\backslash U)$ whose $F = (X\backslash U)$ is closed in X. This proves (ii)

(ii) ⇒ (iii) By assumption there is a closed set F in X such that $PInt^*(F) \subseteq A \subseteq F$. Then $PCl(A) \subseteq F$ and hence $PInt^*\big(PCl(A)\big) \subseteq PInt^*(F) \subseteq A$. Thus (iii) is proved.

(iii) ⇒ (iv) By assumption, $PInt^*\big(Cl(A)\big) \subseteq A$ and this implies that $PInt^*\big(Cl(A)\big) \subseteq PInt^*(A)$. Since $A \subseteq PCl(A)$, $PInt^*(A) \subseteq PInt^*\big(PCl(A)\big)$. These $PInt^*(A) = PInt^*\big(cl(A)\big)$. This proves (iv).

(iv) ⇒ (v) Suppose $PInt^*\big(Cl(A)\big) = PInt^*(A)$, then taking the complements, we get $X\backslash PInt^*\big(Cl(A)\big) = X\backslash\big(PInt^*(A)\big)$ and $PCl^*\big(Int(X - A)\big) = PCl^*(X - A)$. Hence, $(X - A)$ is $\widehat{S_P}^*$-open which implies that A is $\widehat{S_P}^*$-closed.

Theorem 3.3 Arbitrary intersection of $\widehat{S_P}^*$- closed sets is also $\widehat{S_P}^*$- closed.

Proof: Let $\{A_\alpha\}$ be a collection of $\widehat{S_P}^*$closed sets in X. Since each A_α is $\widehat{S_P}^*$-closed in X, $(X\backslash A_\alpha)$ is $\widehat{S_P}^*$-open in X. Since arbitrary union of $\widehat{S_P}^*$- open sets is open, $U(X\backslash A_\alpha)$ is $\widehat{S_P}^*$- open. That is $(X\backslash \cap A_\alpha)$ is $\widehat{S_P}^*$open. Hence $\cap A_\alpha$ is $\widehat{S_P}^*$- closed.

Corollary 3.4 If A is $\widehat{S_P}^*$- closed and U is $\widehat{S_P}^*$-open in X, then $(A\backslash U)$ is $\widehat{S_P}^*$-closed in X.

Proof: Since U is $\widehat{S_P}^*$- open, $(X\backslash U)$ is $\widehat{S_P}^*$-closed. Hence, $A \cap (X\backslash U) = (A\backslash U)$ is $\widehat{S_P}^*$- closed.

Remark 3.5 Union of two $\widehat{S_P}^*$- closed sets need not be $\widehat{S_P}^*$- closed as seen from the following example

Example 3.6 Consider the topological space (X, τ) where $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Here $\{a\}$ and $\{b\}$ are $\widehat{S_P}^*$-closed. But their union $\{a, b\}$ is not $\widehat{S_P}^*$-closed.

Theorem 3.7 If A is $\widehat{S_P}^*$-closed in X and B is closed in X, then $A \cup B$ is $\widehat{S_P}^*$-closed in X.

Proof: Since A is $\widehat{S_P}^*$-closed, $(X\backslash A)$ is $\widehat{S_P}^*$open. Also $(X\backslash B)$ is open. Moreover, $(X\backslash A) \cap (X\backslash B) = X\backslash(A \cup B)$ is $\widehat{S_P}^*$-open. Hence, $A \cup B$ is $\widehat{S_P}^*$-closed set in X.

Theorem 3.8 Every closed set is $\widehat{S_P}^*$-closed.

Proof: Follows from Theorem 2.3 (a)

Remark 3.9 The converse of the above theorem need not be true as can be seen from the following example.

Example 3.10 Consider a topological space (X, τ) where $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Here the subsets {a}, {b}, and {c} are $\widehat{S_P}^*$-closed but not closed.

Theorem 3.11 Every α-closed set in a topological space (X, τ) is $\widehat{S_P}^*$- closed.

Proof: Follows from Theorem 2.3 (b)

Remark 3.12 The converse of the above theorem is not true as seen from the following example

Example 3.13 Consider a topological space (X,τ), where $X=\{a,b,c,d\}$ and $\tau=\{X,\phi,\{a\},\{b\},\{a,b\},\{a,b,c\}\}$. Here the subsets $\{a\},\{b\},\{a,c\}$ are $\widehat{S_P}^*$- closed but not α-closed.

Theorem 3.14 Every $semi^*\alpha$-closed set in a topological space (X,τ) is $\widehat{S_P}^*$-closed set.
Proof: Follows from Theorem 2.3 (d)

Remark 3.15 Converse of the above theorem is not true as can be seen from the following example.

Example 3.16 Consider a topological space (X,τ), where $X=\{a,b,c\}$, $\tau=\{X,\phi,\{a\},\{b\}\}$. Then the subsets $\{a\}$ and $\{b\}$ are $\widehat{S_P}^*$-closed sets but not $semi^*\alpha$-closed.

Theorem 3.17 Every $semi^*$-closed set is a $\widehat{S_P}^*$-closed set.
Proof: Follows from Theorem 2.3 (c)

Remark 3.18 Converse of the above theorem is not true as can be seen from the following example.

Example 3.19 Consider a topological space (X,τ) where $X=\{a,b,c,d\}$, $\tau=\{X,\phi,\{a,b\}\}$. Then subsets $\{c\}$ and $\{d\}$ are $\widehat{S_P}^*$-closed but not $semi^*$-closed set.

Theorem 3.20 Every Sg^*-closed set is $\widehat{S_P}^*$-closed set.
Proof: Follows from Theorem 2.3 (e)

Remark 3.21 Converse of the above theorem is not true as seen from the following example.

Example 3.22 Consider a topological space (X,τ) where $X=\{a,b,c\}$, $\tau=\{X,\phi,\{a\},\{b\}\}$. Then the subsets $\{a\}$ and $\{b\}$ are $\widehat{S_P}^*$-closed sets but not Sg^*-closed set.

Theorem 3.23 Every $\widehat{S_P}^*$-closed set in a topological space (X,τ) is α^*-closed set.
Proof: Follows from the Theorem 2.3 (f)

Remark 3.24 Converse of the above theorem is not true as seen from the following example.

Example 3.25 Consider a topological space (X,τ) where $X=\{a,b,c\}$, $\tau=\{X,\phi,\{a\},\{a,b\}\}$. Then subsets $\{a\}$ and $\{a,b\}$ are α^*-closed sets but not $\widehat{S_P}^*$-closed sets.

Theorem 3.26 Every $\widehat{S_P}^*$-closed set in a topological space is a semi-generalized closed set.
Proof: Follows from the Theorem 2.3 (g)

Remark 3.27 The converse of the above theorem is not true as seen from the following example.

Example 3.28 Consider a topological space (X,τ) where $X=\{a,b,c\}$, $\tau=\{X,\phi,\{a,b\}\}$. Then the subsets $\{a,c\}$ and $\{b,c\}$ are semi-generalized closed sets but not $\widehat{S_P}^*$-closed sets.

Remark 3.29 The concept of $\widehat{S_P}^*$-closed set is independent of the concepts of pre-closed sets from the following example.

Example 3.30 Consider a topological space (X,τ) where $X = \{a,b,c\}$, $\tau = \{X,\phi,\{a,b\}\}$, the subset $\{a\}$ is a pre-closed set but not a $\widehat{S_P}^*$- closed set. However, in the topological space (X,τ), where $X = \{a,b,c\}$, $\tau = \{X,\phi,\{b,c\},\{a,c\}\}$, the subset $\{a\}$ is a $\widehat{S_P}^*$- closed set but not a pre-closed set. Hence pre-closed set and $\widehat{S_P}^*$- closed set are independent.

Remark 3.31 The concept of $\widehat{S_P}^*$- closed set is independent of the concepts of g-closed sets from the following example.

Example 3.32 Consider a topological space (X,τ) where $X = \{a,b,c,d\}$, $\tau = \{X,\ \phi,\ \{a\},\ \{b\},\ \{a,b\},\{a,b,c\}\}$, the subset $\{a\}$ is not a g-closed set but $\widehat{S_P}^*$- closed set and subset $\{a,b\}$ is a g-closed set but not a $\widehat{S_P}^*$- closed set.

Remark 3.33 The concept of $\widehat{S_P}^*$-closed set is independent of the concepts of pg-closed sets from the following example.

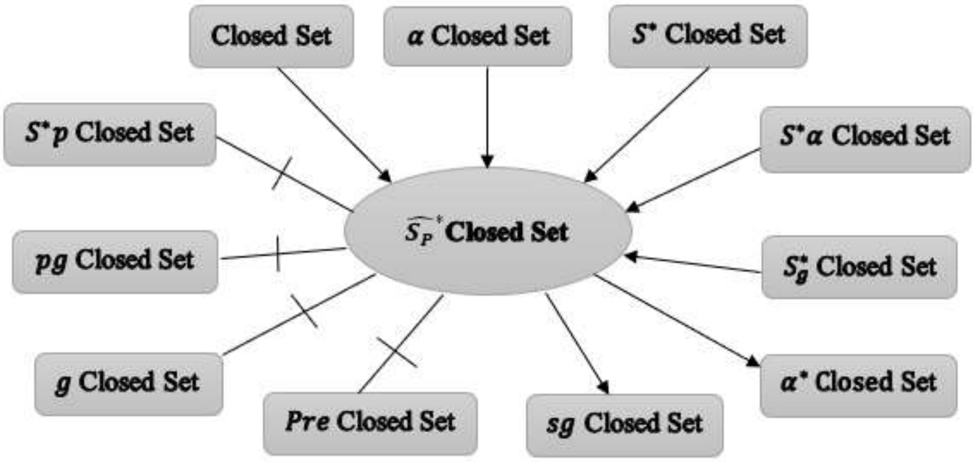
Example 3.34 Consider a topological space (X,τ) where $X = \{a,b,c\}$, $\tau = \{X,\phi,\{a\},\{b\}\}$. Then the subsets $\{b\}$ and $\{a\}$ are $\widehat{S_P}^*$- closed sets and not pre-generalized-closed sets. However, in the topological space (X,τ) where $X = \{a,b,c\}$, $\tau = \{X,\phi,\{a,b\}\}$, the subsets $\{b,c\}$, $\{a,c\}$, $\{b\}$ and $\{a\}$ are pre-generalized-closed sets but not $\widehat{S_P}^*$- closed sets. Hence pg closed set and $\widehat{S_P}^*$- closed set are independent.

Remark 3.35 The concept of $\widehat{S_P}^*$-closed set is independent of the concepts of $semi^*$-pre-closed sets from the following example.

Example 3.36 Consider a topological space (X,τ) where $X = \{a,b,c\}$, $\tau = \{X,\phi,\{a,b\}\}$, the subset $\{b\}$ is a $semi^*$-pre-closed set but not a $\widehat{S_P}^*$- closed set. However, in the topological space (X,τ), where $X = \{a,b,c\}$, $\tau = \{X,\phi,\{a\},\{b\}\}$, the subset $\{b\}$ is a $\widehat{S_P}^*$- closed set but not a $semi^*$-pre-closed set. Hence $semi^*$-pre-closed set and $\widehat{S_P}^*$- closed set are independent.

Theorem 3.37 If A is closed in X and $B \subset X$ is such that $Int^*(A) \subseteq B \subset PCl(A)$, then B is $\widehat{S_P}^*$-closed in X.

Proof: Since A is $\widehat{S_P}^*$closed, $(X\backslash A)$ is open. Further $PInt^*(A) \subseteq B \subseteq PCl(A)$ implies $\left(X\backslash Cl(A)\right) \subseteq (X\backslash B) \subseteq \left(X\backslash PInt^*(A)\right)$. Hence $Int(X\backslash A) \subseteq (X\backslash B) \subseteq PCl(X\backslash A)$. Therefore, by Definition 2.1.1, $(X\backslash B)$ is $\widehat{S_P}^*$open. Hence B is $\widehat{S_P}^*$-closed.



4. $\widehat{S_P}^*$ CLOSURE

Definition 4.1 If A is a subset of a topological space X, then $\widehat{S_P}^*$ closure of A is defined as the intersection of all $\widehat{S_P}^*$ closed in X containing A and is denoted by $\widehat{S_P}^* Cl(A)$.

Theorem 4.2 If A is any subset of a topological space X, then the following statements hold;

(i) $\widehat{S_P}^* Cl(A)$ is $\widehat{S_P}^*$-closed in X. In fact, it is the smallest $\widehat{S_P}^*$-closed set in X containing A.

(ii) A is $\widehat{S_P}^*$-closed if and only if $\widehat{S_P}^* Cl(A) = A$

Proof: (i) Since $\widehat{S_P}^* Cl(A)$ is the intersection of all $\widehat{S_P}^*$-closed subsets of X containing A, by Theorem 3.3, it is $\widehat{S_P}^*$- closed and it is contained in each $\widehat{S_P}^*$-closed set containing A. This proves (i)

(ii) A is $\widehat{S_P}^*$-closed implies $\widehat{S_P}^* Cl(A) = A$ is obvious. On the other hand, suppose $\widehat{S_P}^* Cl(A) = A$, by (i) $\widehat{S_P}^* Cl(A)$ is $\widehat{S_P}^*$-closed and hence $A\ is\ \widehat{S_P}^*$-closed.

Theorem 4.3 Let $A \subseteq X$ and $x \in X$, then $x \in \widehat{S_P}^* Cl(A)$ if and only if every $\widehat{S_P}^*$- open set in X containing x intersect A.

Proof: Suppose $x \notin \widehat{S_P}^* Cl(A)$. Then $X \backslash \widehat{S_P}^* Cl(A)$ is a $\widehat{S_P}^*$- open set containing x that does not intersect A. On the other hand, suppose there is $\widehat{S_P}^*$-open set U containing x that does not intersect A. Then $(X \backslash U)$ is $\widehat{S_P}^*$- closed set containing A. Therefore, by the definition of $\widehat{S_P}^*$- closure, $\widehat{S_p}^* Cl(A) \subseteq (X \backslash U)$. Since $x \in U$, $x \notin (X \backslash U)$ and hence $x \notin \widehat{S_P}^* Cl(A)$. Thus $x \in \widehat{S_P}^* Cl(A)$ if and only if there is a $\widehat{S_P}^*$- open set containing x that does not intersect A. This proves the theorem.

Theorem 4.4 In any topological space (X, τ), the following results hold

(i) $\widehat{S_P}^* Cl(\phi) = \phi$

(ii) $\widehat{S_P}^* Cl(X) = X$

(iii) $A \subseteq \widehat{S_P}^* Cl(A)$

(iv) If A and B are subset of X, $A \subseteq B$ implies $\widehat{S_P}^* Cl(A) \subseteq \widehat{S_P}^* Cl(B)$

(v) $\widehat{S_P}^* Cl\left(\widehat{S_P}^* Cl(A)\right) = \widehat{S_P}^* Cl(A)$

(vi) $\widehat{S_P}^* Cl(A) \cup \widehat{S_P}^* Cl(B) \subseteq \widehat{S_P}^* Cl(A \cup B)$

(vii) $\widehat{S_P}^* Cl(A \cap B) \subseteq \widehat{S_P}^* Cl(A) \cap \widehat{S_P}^* Cl(B)$

(viii) $Cl\left(\widehat{S_P}^* Cl(A)\right) = Cl(A)$

(ix) $\widehat{S_P}^* Cl\big(Cl(A)\big) = Cl(A)$

Proof: (i) to (iv) follow from definition 3.1

(v) By theorem 3.2 (ii), $\widehat{S_p}^* Cl\left(\widehat{S_p}^* Cl(A)\right) = \widehat{S_p}^* Cl(A)$

(vi) Since $A \subseteq A \cup B$, from (iv) above we have $\widehat{S_p}^* Cl(A) \subseteq \widehat{S_p}^* Cl(A \cup B)$. This proves (vi)

(vii) Since $A \cap B \subseteq A$, from (iv) $\widehat{S_p}^* Cl(A \cap B) \subseteq \widehat{S_p}^* Cl(A)$, similarly $\widehat{S_p}^* Cl(A \cap B) \subseteq \widehat{S_p}^* Cl(B)$. This proves (vii)

(viii) from (iii) above, we have $A \subseteq \widehat{S_p}^* Cl(A)$ and hence $Cl(A) \subseteq cl\left(\widehat{S_p}^* Cl(A)\right)$. Also from (xii) $\widehat{S_p}^* Cl(A) \subseteq Cl(A)$. And hence $Cl\left(\widehat{S_p}^* Cl(A)\right) \subseteq Cl\big(Cl(A)\big) = Cl(A)$. Therefore, $Cl\left(\widehat{S_p}^* Cl(A)\right) = Cl(A)$ which proves (viii).

(ix) follows from the fact that $Cl(A)$ is closed and hence by Theorem 3.8, $Cl(A)$ is $\widehat{S_p}^*$closed and by involving Theorem 3.2, we have $\widehat{S_p}^* Cl\big(Cl(A)\big)$.

Theorem 4.5 If A is a subset of X, then

(i) $\widehat{S_p}^* Cl(X \backslash A) = X \backslash \widehat{S_p}^* Int(A)$

(ii) $\widehat{S_p}^* Int(X \backslash A) = X \backslash \widehat{S_p}^* Cl(A)$

Proof: (i) Let $x \in X \backslash \widehat{S_p}^* Int(A)$ then $x \notin \widehat{S_p}^* Int(A)$ which implies from the definition $\widehat{S_p}^*$interior, that x does not belong to any $\widehat{S_p}^*$open subset of A. Let F be a $\widehat{S_p}^*$closed set containing $(X \backslash A)$. Then its complement, $(X \backslash F)$ is a $\widehat{S_p}^*$open set contained in A. Therefore, $x \in (X \backslash F)$ and so $x \in F$. Thus x lies in every $\widehat{S_p}^*$closed set containing $(X \backslash A)$. Hence by definition of $\widehat{S_p}^*$closure, $x \in \widehat{S_p}^* Cl(X \backslash A)$ and hence $\left(X \backslash \widehat{S_p}^* Int(A)\right) \subseteq \widehat{S_p}^* Cl(X \backslash A)$. Then by the definition of $\widehat{S_p}^*$closure, x belongs to every $\widehat{S_p}^*$closed set containing $(X \backslash A)$. Hence x does not belong to any $\widehat{S_p}^*$open subset of A. Therefore, $x \notin \widehat{S_p}^* Int(A)$. This implies $x \in \left(X \backslash \widehat{S_p}^* Int(A)\right)$. Thus $\widehat{S_p}^* Cl(X \backslash A) \subseteq X \backslash \widehat{S_p}^* Int(A)$. This proves (i).

(ii) On replacing A by $(X \backslash A)$, we get $\widehat{S_p}^* Cl(A) = \left(X \backslash \widehat{S_p}^* Int(A)\right)$.

Theorem 4.6 If A is a subset of a topological space X.

(i) $\widehat{S_p}^* Cl(A) = A \cup PInt^*\big(Cl(A)\big)$

(ii) $\widehat{S_p}^* Int(A) = A \cap PCl^*\big(Int(A)\big)$

Proof: Consider $PInt^*\left(Cl\left(A \cup PInt^*\big(Cl(A)\big)\right)\right) = PInt^*\big(Cl(A)\big) \cup Cl\left(PInt^*\big(Cl(A)\big)\right) = PInt^*\big(Cl(A)\big) \subseteq A \cup PInt^*\big(cl(A)\big)$. Then $A \cup PInt^*\big(Cl(A)\big)$ is a $\widehat{S_p}^*$closed set containing A. Hence by Theorem 3.2, $\widehat{S_p}^* Cl(A) \subseteq A \cup PInt^*$ we have $PInt^*\big(Cl(A)\big) \subseteq PInt^*\left(Cl\left(\widehat{S_p}^* Cl(A)\right)\right) \subseteq \widehat{S_p}^* Cl(A)$. Therefore, $A \cup PInt^*\big(Cl(A)\big) \subseteq \widehat{S_p}^* Cl(A)$. This proves (i)

(ii) Replacing $(X \backslash A)$ in (i), we have $\widehat{S_p}^* Cl(X \backslash A) = (X \backslash A) \cup PInt^*\big(Cl(X \backslash A)\big)$. Clearly, $X \backslash \widehat{S_p}^* Int(A) = (X \backslash A) \cup (X \backslash PCl^*\big(int(A)\big) = X \backslash \left(A \cap PCl^*\big(Int(A)\big)\right)$. Taking the complements, we get (ii).

Theorem 4.7 Let A and B be subsets of a topological space (X, τ).

(i) If A is open in X, then $A \cap \widehat{S_p}^* Cl(B) \subseteq \widehat{S_p}^* Cl(A \cap B)$

(ii) If A is $\widehat{S_p}^*$closed in X, then $\widehat{S_p}^* Cl(A \cap B) \subseteq A \cap \widehat{S_p}^* Cl(B)$

(iii) If A is open and $\widehat{S_p}^*$closed in X, then $\widehat{S_p}^* Cl(A \cap B) = A \cap \widehat{S_p}^* Cl(B)$

Proof: (i) Let $x \in A \cap \widehat{S_p}^* Cl(B)$ and U be a $\widehat{S_p}^*$open set in X containing x. Since A is open, by Theorem 2.4, $U \cap A$ is $\widehat{S_p}^*$open set in X containing x. Since $x \in \widehat{S_p}^* Cl(B)$, $(U \cap A) \cap B \neq \phi$. Thus every $\widehat{S_p}^*$open set containing x intersects $A \cap B$. Hence, we get $x \in \widehat{S_p}^* Cl(A \cap B)$ which proves (i).

(ii) Since A is $\widehat{S_p}^*$closed, by Theorem 3.2, $\widehat{S_p}^*Cl(A) = A$. Using this, with Theorem 4.4 (vii), we have $\widehat{S_p}^*Cl(A \cap B) \subseteq \widehat{S_p}^*Cl(A) \cap \widehat{S_p}^*Cl(B) = A \cap \widehat{S_p}^*Cl(B)$.

(iii) Follows from (i) and (ii)

REFERENCES

[1] Bhattacharyya, P. and Lahiri, B.K., *Semi-generalized closed sets in Topology*, Ind. Jr. Math., 29(1987), 375-382.

[2] Davis, A.S., *Indexed systems of neighborhoods for general topological spaces*, Amer.Math. Monthly, 68 (1961), 886-893.

[3] Dunham, W., *A new closure operator for Non-T_1topologies*, *Kyungpook Math. J*. 22 (1) (1982), 55-60.

[4] Levine, N., *Generalized Closed Sets in Topology, Rend. Circ. Mat. Palermo* 19(2), (1970), 89-96.

[5] Maheswari, S. N. and Prasad, R., *Some new separation axioms, Annales de la Soc. Sci. de Bruxelles*, T. 89 III (1975), 395-402.

[6] Mashhour, A.S., Abd El-Monsef, M.E. and El-Deeb, S.N., *On Precontinuous and Weak Precontinuous mappings*, Proc. Math. Phys. Soc. Egypt 53(1982), 47-53.

[7] Njastad, O., Some Classes of Nearly Open Sets, *Pacific J. Math*., 15 (3) (1965), 961-970.

[8] Pious Missier S and P. Anbarasi Rodrigo, *Strongly**- continuous functions in Topological Spaces*, IOSR Journal of Mathematics, Volume 10, Issue 4 Ver.I (Jul-Aug) PP 55-60

[9] Pious Missier S and P. Anbarasi Rodrigo, *Some Notions of Nearly Open Sets in Topological Spaces*, International Journal of Mathematical Archive, 4(12), 2013, 1-7

[10] Pious Missier. S and Arul Jesti .J, *A new notion of open sets in topological spaces*, International Journal of Mathematical Archives(IJMA), 3(11), 2012, 3990-3996.

[11] Pious Missier .S, Robert A, *Between α- closed sets and semi α-closed sets*, International Journal of Modern Engineering Reserch,4(6), 2014, 34-41.

[12] Pious Missier .S, Siluvai .A and Gabriel Raja .S, *A new class of star generalized open sets weaker than semi*open sets*, International Journal of food and nutritional sciences, Vol.11, issue12, 2022.

[13] Shanin, N.A., *On separation in topological spaces*, *C.R.(Doklady) Acad. Sci. URSS(N.S.)*, 38, (1943), 110-113.