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Λ-GENERALIZED *Λ*-δI-CLOSED SETS IN IDEAL TOPOLOGICAL SPACES

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Abstract:

In this article we provide the concept of λ-generalized *Λ*-δI-closed sets namely λg-(*Λ*-δI)-closed sets as a generalization of *Λ*-δI-closed sets using λ-open sets. Also we established its some basic characteristics. We discuss the correlation of *Λ*-δI-closed sets with existing known closed sets. Moreover, we establish the concept of λg-(*Λ*-δI)-continuous and λg-(*Λ*-δI)-irresolute functions in ITS and discuss its some characterizations and relation with some other known functions in this space.

Keywords: TS, ITS, λ-generalized *Λ*-δI-closed set, λg-(*Λ*-*δI*)-continuous function, λg-(*Λ*-*δI*)-irresolute function.

1. Introduction and Preliminaries

If a subset I of P(X) meets the criteria called heredity and finite addictivity, then it is referred to as an ideal[2]. The space (X, τ, I) is the ideal topological space or ITS in short. [7] Kuratowski and vaidyananthaswamy,[12] Ganster et.al., [5] D. Jonokovi et.al., [6] M. Navaneethakrishnan et.al., [10] etc., further studied the space and established various

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fundamental characterizations and topological properties. Levine N [8] introduced and investigated the notion generalization in Topological space or shortly TS for comfort. Maki [9] carried on Levine and Dunhan work in 1986. A set S is the Λ-set if Λ(S) = S, where Λ(S) = ∩{M \ M ∈τ(x) and S ⊆ M}. A.F.G.Arenas et.al., [1] established λ-closed sets. A set S is called λ-closed set if S = O ∩ C where O is a Λ-set and C is a closed set. The generalized λ-closed sets were later established and investigated by M. Caldas et.al, [4]. Establishing the idea of a new generalization in the same space is the aim of this paper.

Definition 1.1. Let (X, τ) be a TS and S ⊆ X. Then S is called a

(i)λ-closed set [1] if S = C ∩ D where C is Λ-set and D is closed set.

(ii) Λ_g-closed[4] (resp. Λ-g-closed[4], gΛ-closed[4]) if cl_λ(S) ⊆ M whenever S ⊆ M and M is λ-open (resp. M is open) in (X, τ).

(iii) g-closed set [8] if cl(S) ⊆ M whenever S ⊆ M and M∈τ(x) where τ(x) ={M / M ∈ τ and x ∈ M}

(iv) I_g-closed set[13]if S^* ⊆ M whenever S ⊆ M and M ∈τ(x).

(v) locally closed[5]if S = M ∩ H, where M is open and H is closed.

(vi) sg-closed[14] if scl(S) ⊆ M, whenever S ⊆ M and M is semiopen in X.

(vii) gs-closed [15] if scl(S), whenever S ⊆ M and M ∈τ(x).

(vii)αg-closed [16] if αcl(S) ⊆ M, whenever S ⊆ M and M ∈τ(x).

(ix) gα-closed set[17] if if αcl(S) ⊆ M, whenever S ⊆ M and M ∈τ^α(X).

(x) $\hat{g}$-closed set[18] if cl(S) ⊆ M whenever S ⊆ M and M ∈ SO(X).

(xi)$\delta\hat{g}$-closed set[19] if cl_{δ}(S) ⊆ M whenever S ⊆ M and M is $\hat{g}$-open set in X.

(xii) θ-g-closed set[20] if cl_{θ}(S) ⊆ M whenever S ⊆ M and M ∈ τ(x).

The aforementioned closed sets corresponding open sets are their complements.

Definition 1.2. Continuous (resp., Super continuous, δ-continuous, gλ-continuous) if f^{-1}(M) is open (resp., open, δ-open, gλ-open) in (X, τ, I_1) for each M ∈ σ (Y) set M in (Y, σ, I_2).

Definition 1.3. In an ITS (X, τ, I) ,

(i) a subset S of an ITS (X, τ, I) is called *Λ*-δI-closed set if S = D ∩ C, where D is a *Λ*-set and C is a δ-I-closed set. Then X − S is called *Λ*-δI-open set. We indicate the grouping of *Λ*-δI-closed (resp., *Λ*-δI-open) sets by *Λ*-δIC(X, τ, I) (resp., *Λ*-δIO(X, τ, I)) or briefly *Λ*-δIC(X) (resp., *Λ*-δIO(X)).

(ii) A point p in X is referred to as the *Λ*-δI-cluster point of S, if for each *Λ*-δI-open subset M_p of X that contains p, M_p ∩ S ≠ ∅. The *Λ*-δI-closure of S is represented by $\mathbb{C}_{\Lambda-\delta I}$(S), which is the set of all *Λ*-δI-cluster points. That is, for each *Λ*-δI-open set M_p containing p, $\mathbb{C}_{\Lambda-\delta I}$(S) = {p ∈ X / M_p ∩ S ≠ ∅, for each M_p ∈ *Λ*-δI)O(X)}.

(iii) If there is a *Λ*-δI-open set M_p that contains p such that M_p ⊆ S, then a point p ∈ X is a *Λ*-δI-interior point of S. $\mathscr{I}^{o}_{\Lambda-\delta I}$(S) is the symbol for the set of all *Λ*-δI-interior point of S, which is referred to as *Λ*-δI-interior of S.

Lemma.1.4.[24] In an ITS, $\mathbb{C}_{\Lambda-\delta I}$(S) = {p ∈ X \ M_p ∩ S ≠ ∅, for each G ∈ *Λ*-δI-O(x)} is *Λ*-δI-closed.

Lemma.1.5. [24] Every *Λ*-δI-closed set is λ-closed.

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Lemma.1.6. Let (X, τ, I) be an ITS and S ⊆ X. Then $\mathbb{C}_{\Lambda\text{-}\delta I}$(S) = {p ∈ X / M_p ∩ S ≠ ∅, for every G ∈ Λ-δI-O(x)} is λ-closed.

Proof. Clear from Lemma 1.4. and Lemma 1.5.

2. λg-(Λ-δI)-closed set.

In an ITS (X, τ, I) an S ⊆ X is called a λ-generalized Λ-δI (briefly, λg-(Λ-δI)) closed set if $\mathbb{C}_{\Lambda-\delta I}$(S) ⊂ M, whenever S ⊆ M and M ∈ λO(X). Then X − S is known as λg-(Λ-δI)-open set. We denote the set of λg-(Λ-δI)-closed (resp., λg-(Λ-δI)-open) sets by λg-(Λ-δI)-C(X, τ, I) (resp., λg-(Λ-δI)-O(X, τ, I)) or simply λg-(Λ-δI)-C(X) (resp., λg-(Λ-δI)-O(X)) for convenience. It is clear that, every Λ-δI-closed (resp., open, Λ-sets, δ-closed, δ-open, Λ_δ-sets, (Λ, δ)-closed sets and δ-I-closed) set is λg-(Λ-δI)-closed. The forthcoming Example establish the lack of the reversible direction in some place of all the above. Let X = {a, b, c, d}, τ = {X, ∅, {a}}, I = {∅, { b}}, Λ-sets = {X, ∅, {a}}, δ-closed = {X, ∅}, δ-open = {X, ∅}, Λ_δ-sets = {X, ∅}, (Λ, δ)-closed sets = {X, ∅}, δ-I-closed = {X, ∅}, Λ-δI-closed set = {X, ∅, {a}}, λg-(Λ-δI)C(X) = {X, ∅, {a},{a, b}, {a, d}, { a, c}, { a, b, c}, {a, b, d}, { a, c, d}}.

The given below Example 2.1 shows that closed sets, g-closed sets, I_g-closed sets, locally closed sets, I-locally∗-closed sets, sg-closed sets, gs-closed sets, αg-closed sets, gα-closed sets, $\hat{g}$-closed sets, $\delta\hat{g}$-closed sets, θ-g-closed sets, θ-I-closed sets are independent of λg-(Λ-δI)-closed sets.

Example 2.1. (i) Let X = {a, b, c, d}, τ = {X, ∅, {c}, {d}, { c, d}, {a, c, d}}, Closed sets(X) = {X, ∅, {b}, {a, b}, {a, b, c}, {a, b, d}}, λg-(Λ-δI)C(X) = {X, ∅, {a}, {c}, {d}, {a, b}, {a, c}, {a, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}}. Let A = {a}, B = {b}, then A ∈ λg-

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(Λ-δI)C(X) but A ∉ closed sets(X) and B ∈ closed set (X) but B ∉ λg-(Λ-δI)-closed. Consequently, closed sets and λg-(Λ-δI)-C(X) sets are independent.

(ii) Let X = {a, b, c, d}, τ = {X, ∅, {c}, { d}, { c, d}, {a, c, d}}, g-closed sets(X)= {X, ∅, {b}, {a, b}, { b, c}, { b, d}, {a, b, c}, {a, b, d}, {b, c, d}}, λg-(Λ-δI)-C(X) = {X, ∅, {a}, { c}, { d}, {a, b}, {a, c}, {a, d}, { c, d}, {a, b, c}, { a, b, d}, {a, c, d}}. Let A = {A}, B = {b}, then A ∈ λg-(Λ-δI)-C(X) but not g-closed and B is g-closed but B ∉ λg-(Λ-δI)C(X). Consequently, g-closed sets and λg-(Λ-δI)-C(X) sets are independent.

(iii) Let X = {a, b, c, d}, τ = {X, ∅, {c}, {d}, {c, d}, {a, c, d}}, I_g-closed sets(X) = {X, ∅, { a}, {b}, {a, b}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}}, λg-(Λ-δI)C(X) = {X, ∅, {a}, {c}, {d}, {a, b}, {a, c}, {a, d}, {c, d}, {a, b, c}, { a, b, d}, {a, c, d}}. Let A = {a, c, d}, B = {b}, then A ∈ λg-(Λ-δI)-C(X) but not I_g-closed and B is I_g-closed but B ∉ λg-(Λ-δI)C(X). Consequently, I_g-closed sets and λg-(Λ-δI)-C(X) sets are independent.

(iv) Let X = {a, b, c, d}, τ = {X, ∅, {a}}, locally closed sets(X) = {X, ∅, {a}, {b, c, d}}, λg-(Λ-δI)C(X) = {X, ∅, {a}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}}. Let A = {a, b}, B = {b, c, d}, then A ∈ λg-(Λ-δI)-C(X) but not a locally closed set and B is a locally closed set but B ∉ λg-(Λ-δI)-C(X). Consequently, locally closed sets and λg-(Λ-δI)-C(X) sets are independent.

(v) Let X = {a, b, b, d}, τ = {X, ∅, {a}}, I-locally∗-closed sets(X) = {X, ∅, {a}, {b}, {a, b}, {b, c, d}}, λg-(Λ-δI)C(X) = {X, ∅, {a}, {a, b}, {a, b}, {a, d}, {a, b, b}, {a, b, d}, {a, b, d}}. Let A = {a, b}, B = {b, c, d}, then A ∈ λg-(Λ-δI)-C(X) but not a I-locally∗-closed set(X) and B ∈ I-locally∗-closed set(X) but B ∉ λg-(Λ-δI)C(X). Consequently, I-locally∗-closed sets and λg-(Λ-δI)-closed sets are independent.

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(vi) Let X = {a, b, c, d}, τ = {X, ∅, {c}, {d}, {c, d}, {a, c, d}}, sg-closed sets(X) = {X, ∅, { a}, { b}, { c}, { d}, { a, b}, { a, c}, { a, d}, {b, c}, { b, d}, { a, b, c}, { a, b, d}}, λg-(Λ-δI)C(X) = {X, ∅, { a}, { c}, { d}, {a, b}, {a, c}, {a, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}}. Let A = {c, d}, B = {b}, then A ∈ λg-(Λ-δI)-C(X) but not a sg-closed set and B is a sg-closed set but B ∉ λg-(Λ-δI)C(X). Consequently, sg-closed sets and λg-(Λ-δI)-closed sets are independent.

(vii) Let X = { a, b, c, d}, τ = {X, ∅, { c}, {d}, { c, d}, {a, c, d}}, gs-closed sets(X) = {X, ∅, {a}, { b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}}, λg-(Λ-δI)C(X) = {X, ∅, { a}, { c}, {d}, {a, b}, {a, c}, { a, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}}. Let A = {c, d}, B = {b, c}, then A ∈ λg-(Λ-δI)-C(X) but not a gs-closed set and B is a gs-closed set but B ∉ λg-(Λ-δI)-C(X). Consequently, gs-closed sets and λg-(Λ-δI)-closed sets are independent.

(viii)) Let X = { a, b, c, d}, τ = {X, ∅, { c}, { d}, { c, d}, { a, c, d}}, αg -closed sets(X) = {X, ∅, { a}, { b}, { a, b}, { b, c}, {b, d}, { a, b, c}, { a, b, d}, { b, c, d}}, λg-(Λ-δI)-C(X) = {X, ∅, { a}, { c}, { d}, { a, b}, { a, c}, { a, d}, { c, d}, { a, b, c}, { a, b, d}, { a, c, d}}. Let A = {c, d}, B = {b, c}, then A ∈ λg-(Λ-δI)-C(X) but not αg -closed and B is αg-closed but B ∉ λg-(Λ-δI)C(X). Consequently, αg-closed sets and λg-(Λ-δI)-closed sets are independent.

(ix) Let X = { a, b, c, d}, τ = {X, ∅, { c}, {d}, { c, d}, { a, c, d}}, gα -closed sets(X) = {X, ∅, { a}, { b}, { a, b}, { a, b, c}, { a, b, d}}, λg-(Λ-δI)-C(X) = {X, ∅, { a}, {c}, { d}, { a, b}, {a, c}, {a, d}, {c, d}, { a, b, c}, { a, b, d}, { a, c, d}}. Let A = {c, d}, B = {b}, then A ∈ λg-(Λ-δI)-C(X) but not a gα-closed set and B is a gα-closed set but B ∉ λg-(Λ-δI)-C(X). Consequently, gα-closed sets and λg-(Λ-δI)-closed sets are independent.

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(x) Let X = { a, b, c, d}, τ = {X, ∅, { c}, { d}, { c, d}, { a, c, d}}, $\hat{g}$-closed sets(X) = {X, ∅, { b}, { a, b}, { a, b, c}, { a, b, d}}, λg-(Λ-δI)C(X) = {X, ∅, {a}, {c}, {d}, { a, b}, {a, c}, {a, d}, {c, d}, { a, b, c}, {a, b, d}, {a, c, d}}. Let A = {a}, B = {b}, then A ∈ λg-(Λ-δI)-C(X) but not a $\hat{g}$-closed set and B is a $\hat{g}$-closed set but B ∉ λg-(Λ-δI)-C(X). Consequently, $\hat{g}$-closed sets and λg-(Λ-δI)-closed sets are independent.

(xi) Let X = {a, b, b, d}, τ = {X, ∅, { c}, {d}, { c, d}, { a, c, d}}, $\delta\hat{g}$-closed sets(X) = {X, ∅, { a}, { b}, { a, b}, { b, c}, { b, d}, { a, b, c}, { a, b, d}, { b, c, d}}, λg-(Λ-δI)-C(X) = {X, ∅, { a}, { c}, { d}, { a, b}, { a, c}, { a, d}, {c, d}, { a, b, c}, { a, b, d}, { a, c, d}}. Let A = {d}, B = {b}, then A ∈ λg-(Λ-δI)-C(X) but not $\delta\hat{g}$-closed and B is $\delta\hat{g}$-closed but B ∉ λg-(Λ-δI)-C(X). Consequently, $\delta\hat{g}$-closed sets and λg-(Λ-δI)-closed sets are independent.

(xii) Let X = {a, b, c, d}, τ = {X, ∅, { c}, { d}, { c, d}, { a, c, d}}, θ-g-closed sets(X) = {X, ∅, { b}, { a, b}, { b, c}, { b, d}, { a, b, c}, { a, b, d}, { b, c, d}}, λg-(Λ-δI)-C(X) = {X, ∅, { a}, { c}, { d}, { a, b}, { a, c}, { a, d}, { c, d}, { a, b, c}, { a, b, d}, { a, c, d}}. Let A = {a}, B = {b, c}, then A ∈ λg-(Λ-δI)-C(X) but not θ-g-closed and B is θ-g-closed but B ∉ λg-(Λ-δI)-C(X). Consequently, θ-g-closed sets and λg-(Λ-δI)-closed sets are independent.

(xiii) Let X = {a, b, b, d}, τ = {X, ∅, { c}, {d}, { c, d}, { a, c, d}}, θ-I-closed sets(X) = {X, ∅, { a}, { b}, { a, b}, { b, c}, { b, d}, { a, b, c}, { a, b, d}, { b, c, d}}, λg-(Λ-δI)-C(X) = {X, ∅, { a}, { c}, { d}, { a, b}, { a, c}, { a, d}, { c, d}, { a, b, c}, { a, b, d}, { a, c, d}}. Let A = {c}, B = {b, c}, then A ∈ λg-(Λ-δI)-C(X) but not θ-I-closed and B is θ-I-closed but B ∉ λg-(Λ-δI)C(X). Consequently, θ-I-closed sets and λg-(Λ-δI)-closed sets are independent.

Theorem 2.2. In an ITS (X, τ, I), if S ∈ λO(X) and λg-(Λ-δI)-C(X) then S ∈ Λ-δI-C(X) for S ⊆ X.

Proof. Since S is λ-open and belong to λg-(Λ-δI)-C(X), $\mathbb{C}_{\Lambda\text{-}\delta I}(S) \subseteq S$. But always $S \subseteq \mathbb{C}_{\Lambda\text{-}\delta I}(S)$. Hence, S ∈ Λ-δI-C(X).

Example 2.3 evident that the opposite direction is fails at some place.

Example 2.3. Let τ = {X, ϕ, { a}, { a, b}} and I={ϕ, { c}} Λ-δI-closed set = { X, ϕ, { a}, { a, b}}, λ-open = {X, ϕ, { a}, { b}, { a, b}, { c, d}, { b, c, d}}, λg-(Λ-δI)-C(X) set = {X, ϕ, { a}, {a, b, c}, { a, b, d}}. Here {a, b, c} is not Λ-δI-closed but not λ-open or not λg-(Λ-δI)-closed.

Theorem 2.4. In an ITS (X, τ, I), an S ⊆ X is belongs to λg-(Λ-δI)-C(X) then $\mathbb{C}_{\Lambda\text{-}\delta I}(S) - S$ contain no nonempty λ-closed set.

Proof. Suppose that S ∈ λg-(Λ-δI)-C(X). Let H ≠ ∅ be λ-closed such that $H \subseteq \mathbb{C}_{\Lambda\text{-}\delta I}(S) - S$ then $S \subseteq X - H$. Therefore, we have $\mathbb{C}_{\Lambda\text{-}\delta I}(S) \subseteq X - H$. Consequently, $H \subseteq X - \mathbb{C}_{\Lambda\text{-}\delta I}(S)$. But, $\mathbb{C}_{\Lambda\text{-}\delta I}(S) \cap (X - \mathbb{C}_{\Lambda\text{-}\delta I}(S)) = \emptyset$. Hence, H = ∅.

Example 2.5 establish the failure of the reverse direction of Theorem 2.4..

Example 2.5. Let τ = {X, ϕ, {a}, { a, b}} and I={ϕ, { c}}. Then λ-closed set = {X, ϕ, { a}, { b}, { a, b},{ c, d}, { b, c, d}} and λg-(Λ-δI)-C(X) set = {X, ϕ, { a}, { a, b}, {a, b, c}, {a, b, d}}. Let S = {d}. Then $\mathbb{C}_{\Lambda\text{-}\delta I}(S)$ = {c, d}, $\mathbb{C}_{\Lambda\text{-}\delta I}(S) - S$ = {c} which contains no non-empty λ-closed set but S ∉ λg-(Λ-δI)-C(X).

Corollary 2.6. In an ITS (X, τ, I) for S ⊆ X, S ∈ λg-(Λ-δI)-C(X) then $\mathbb{C}_{\Lambda\text{-}\delta I}(S) - S$ contain no nonempty δ-I-closed sets.

Proof. Suppose that S ∈ λg-(Λ-δI)-C(X). Let H be a nonempty δ-I-closed set such that $H \subseteq \mathbb{C}_{\Lambda\text{-}\delta I}(S) - S$. Then $S \subseteq X - H$, where X − H is δ-I-open and hence λ-open. Since S ∈

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λg-(Λ-δI)-C(X), we have $\mathbb{C}_{\Lambda\text{-}\delta I}(S) \subseteq X - H$. Therefore, $H \subseteq X - \mathbb{C}_{\Lambda\text{-}\delta I}(S)$. Consequently $H \subseteq \mathbb{C}_{\Lambda\text{-}\delta I}(S) \cap (X - \mathbb{C}_{\Lambda\text{-}\delta I}(S)) = \emptyset$. Hence, $H = \emptyset$.

We notice from Example 2.7, S may not belongs to λg-(Λ-δI)-C(X) whenever $\mathbb{C}_{\Lambda\text{-}\delta I}(S) - S$ contain no nonempty δ-I-closed sets.

Example 2.7. Let τ = {X, ϕ, { c}, {d}, { c, d}, { a, c, d}} and I={ϕ, {a}}. Then δ-I-closed set = {X, ϕ, { a, b}, { a, b, c}, { a, b, d}} and λg-(Λ-δI)-C(X) = {X, ϕ, { a}, { c}, { d}, { a, b}, { a, d}, { c, d}, { a, c}, { a, c, d}, { b, c, d}, { a, b, c}}. Let S = {b}, $\mathbb{C}_{\Lambda\text{-}\delta I}(S)$ = {a, b}, $\mathbb{C}_{\Lambda\text{-}\delta I}(S) - S$ = {a}. Then the only δ-I-closed set in {a} is empty and $S \notin$ λg-(Λ-δI)-C(X).

Corollary 2.8. In an ITS (X, τ, I) for $S \subseteq X$, $S \in$ λg-(Λ-δI)-C(X) then $\mathbb{C}_{\Lambda\text{-}\delta I}(S) - S$ contain no nonempty δ-closed set.

Proof. Suppose that $S \in$ λg-(Λ-δI)-C(X). Let $H \subseteq \mathbb{C}_{\Lambda\text{-}\delta I}(S) - S$, H is δ-closed and $H \neq \emptyset$. Then $S \subseteq X - H$ where $X - H$ is δ-open and hence λ-open. Therefore we have $\mathbb{C}_{\Lambda\text{-}\delta I}(S) \subseteq X - H$. Consequently, $H \subseteq X - \mathbb{C}_{\Lambda\text{-}\delta I}(S)$. Therefore $H \subseteq \mathbb{C}_{\Lambda\text{-}\delta I}(S) \cap (X - \mathbb{C}_{\Lambda\text{-}\delta I}(S)) = \emptyset$. Hence, $H = \emptyset$.

It is clear from the Example 2.9, Corollary 2.8. is not hold in reverse direction.

Examde 2.9. Let τ = {X, ϕ, {a}, {b}, { a, b}, { a, b, d}} and I={ϕ, { c}}. Then δ-closed set = {X, ϕ, { c, d}, { a, c, d}, { b, c, d}} and λg-(Λ-δI)-C(X) = {X, ϕ, { a}, { b}, {d}, { a, b}, { a, d}, { c, d}, { b, d}, { a, c, d}, { b, c, d}, { a, b, d}}. Let S = {c}, $\mathbb{C}_{\Lambda\text{-}\delta I}(S) - S$ = {d}, which does not contain any non-empty δ-closed set, but $S \notin$ λg-(Λ-δI)-C(X).

Theorem 2.10. Let (X, τ, I) be an ITS and $S \subseteq X$. Then $S \in$ λg-(Λ-δI)-C(X) if and only if $\mathbb{C}_{\Lambda\text{-}\delta I}(\{x\}) \cap S \neq \emptyset$ for every $x \in \mathbb{C}_{\Lambda\text{-}\delta I}(S)$.

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Proof. Suppose that $\mathbb{C}_{\Lambda\text{-}\delta I}(\{x\}) \cap S = \emptyset$, $x \in \mathbb{C}_{\Lambda\text{-}\delta I}(S)$ then $X - \mathbb{C}_{\Lambda\text{-}\delta I}(\{x\}) \in \lambda O(X)$ such that $S \subseteq X - \mathbb{C}_{\Lambda\text{-}\delta I}(\{x\})$. Since $x \in \mathbb{C}_{\Lambda\text{-}\delta I}(S) - (X - \mathbb{C}_{\Lambda\text{-}\delta I}(\{x\})$, $\mathbb{C}_{\Lambda\text{-}\delta I}(S) \nsubseteq X - \mathbb{C}_{\Lambda\text{-}\delta I}(\{x\})$. Therefore, $S \notin \lambda g\text{-}(\Lambda\text{-}\delta I)\text{-}C(X)$.Conversely, Suppose that $S \notin \lambda g\text{-}(\Lambda\text{-}\delta I)\text{-}C(X)$. Then there exist an $M \in \lambda O(X)$, $Cl_{\Lambda\text{-}\delta I}(S) \nsubseteq M$ for some $S \subseteq M$. Then for every $x \in \mathbb{C}_{\Lambda\text{-}\delta I}(S)$ such that $x \notin M$. Hence $\mathbb{C}_{\Lambda\text{-}\delta I}(S) \cap M = \emptyset$. Therefore, $\mathbb{C}_{\Lambda\text{-}\delta I}(\{x\}) \cap S \neq \emptyset$ for every $x \in \mathbb{C}_{\Lambda\text{-}\delta I}(S)$.

Theorem 2.11. For $S, T \subseteq X$, if $S \in \lambda g\text{-}(\Lambda\text{-}\delta I)\text{-}C(X)$ and $S \subseteq T \subseteq \mathbb{C}_{\Lambda\text{-}\delta I}(S)$, then $T \in \lambda g\text{-}(\Lambda\text{-}\delta I)\text{-}C(X)$.

Proof. Let $T \subseteq M$ where M is λ-open. Then $S \subseteq M$, since $S \subseteq T$. Also since $T \subseteq \mathbb{C}_{\Lambda\text{-}\delta I}(S)$, $\mathbb{C}_{\Lambda\text{-}\delta I}(T) \subseteq \mathbb{C}_{\Lambda\text{-}\delta I}(S)$. But $S \in \lambda g\text{-}(\Lambda\text{-}\delta I)\text{-}C(X)$ set and so $\mathbb{C}_{\Lambda\text{-}\delta I}(T) \subseteq M$. Hence $T \in \lambda g\text{-}(\Lambda\text{-}\delta I)\text{-}C(X)$.

Theorem 2.12. If $S, T \in \lambda g\text{-}(\Lambda\text{-}\delta I)\text{-}C(X)$, then $S \cup T \in \lambda g\text{-}(\Lambda\text{-}\delta I)\text{-}C(X)$.

Proof. Let $S \cup T \subseteq M$ then $S \subseteq M$ and $T \subseteq M$ where M is λ-open. Since $S, T \in \lambda g\text{-}(\Lambda\text{-}\delta I)\text{-}C(X)$, $\mathbb{C}_{\Lambda\text{-}\delta I}(S) \subseteq M$ and $\mathbb{C}_{\Lambda\text{-}\delta I}(T) \subseteq M$. Therefore $\mathbb{C}_{\Lambda\text{-}\delta I}(S \cup T) = \mathbb{C}_{\Lambda\text{-}\delta I}(S) \cup \mathbb{C}_{\Lambda\text{-}\delta I}(T) \subseteq M$. Hence $S \cup T \in \lambda g\text{-}(\Lambda\text{-}\delta I)\text{-}C(X)$.

The following Example 2.13. establish the failure of intersection in $\lambda g\text{-}(\Lambda\text{-}\delta I)\text{-}C(X)$.

Example 2.13. Let X = {a, b, c, d}, τ = {X, ∅, {a}, { a, b}}, I = {∅, { c}. Then the $\lambda g\text{-}(\Lambda\text{-}\delta I)\text{-}C(X)$ = {X, ∅, {a}, {a, b, c}, {a, b, d}} and { a, b, c} ∩ { a, b, d} = { a, b} does not belongs to $\lambda g\text{-}(\Lambda\text{-}\delta I)\text{-}C(X)$.

Theorem 2.14. In an ITS (X, τ, I), for each $x \in X$, either $\{x\} \in \lambda g\text{-}(\Lambda\text{-}\delta I)\text{-}C(X)$ or $x\} \in \lambda C(X)$.

Proof. If{x}$\notin$ λ-C(X) then X is the only λ-open set such that $\{x\}^c \subseteq X$. Hence, $\{x\}^c \in$ λg-(Λ-δI)-C(X).

Theorem 2.15. Let (X, τ, I) be an ITS. Then the intersection of two λg-(Λ-δI)-open sets is λg-(Λ-δI)-open.

Proof. The Proof follows from Theorem 2.12.

Theorem 2.16. Let (X, τ, I) be an ITS and S ⊆ X. Then, S ∈ λg-(Λ-δI)-O(X) if and only if H ⊆ $\mathcal{I}^{o}_{\Lambda\text{-}\delta I}(S)$ whenever H ∈ λ-C(X) and H ⊆ S.

Proof. Let H ⊆ $\mathcal{I}^{o}_{\Lambda\text{-}\delta I}(S)$ where H ∈ λ-C(X) and H ⊆ S. Then, X − S ⊆ X − H, where X − H ∈ λ-O(X). By assumption X − $\mathcal{I}^{o}_{\Lambda\text{-}\delta I}(S)$ ⊆ X − H, so $\mathbb{C}_{\Lambda\text{-}\delta I}(X - S)$ ⊆ X − H. Hence X − S ∈ λg-(Λ-δI)-C(X). Hence S ∈ λg-(Λ-δI)-O(X). Conversely, let S ∈ λg-(Λ-δI)-O(X). Then X − S ∈ λg-(Λ-δI)-C(X). Also let H ∈ λ-C(X) such that H ⊆ S. Then X − H ∈ λ-O(X). Therefore whenever X − S ⊆ X − H, $\mathbb{C}_{\Lambda\text{-}\delta I}(X - S)$ ⊆ X − H. Therefore, H ⊆ X − $\mathbb{C}_{\Lambda\text{-}\delta I}(X - S) = \mathcal{I}^{o}_{\Lambda\text{-}\delta I}(S)$. Thus H ⊆ $\mathcal{I}^{o}_{\Lambda\text{-}\delta I}(S)$.

Theorem 2.17. Let (X, τ, I) be an ITS and S ⊆ X. Then S ∈ λg-(Λ-δI)-O(X) if and only if M = X whenever M is λ-open and $\mathcal{I}^{o}_{\Lambda\text{-}\delta I}(S) \cup (X - S) \subseteq M$.

Proof. Let S ∈ λg-(Λ-δI)-O(X) and M ∈ λ-O(X) such that $\mathcal{I}^{o}_{\Lambda\text{-}\delta I}(S) \cup (X - S) \subseteq M$. Then $X - M \subseteq (X - \mathcal{I}^{o}_{\Lambda\text{-}\delta I}(S)) \cap (X -(X - S)) = X - \mathcal{I}^{o}_{\Lambda\text{-}\delta I}(S) \cap S = \mathbb{C}_{\Lambda\text{-}\delta I}(X - S) - (X - S)$. Since X − S ∈ λg-(Λ-δI)-C(X) and X − M ∈ λ-C(X) by Theorem 2.4 X − M = ∅. Therefore X = M. Conversely, suppose that H ∈ λ-C(X) and H ⊆ S. Then $\mathcal{I}^{o}_{\Lambda\text{-}\delta I}(S) \cup (X - S) \subseteq \mathcal{I}^{o}_{\Lambda\text{-}\delta I}(S) \cup (X - H)$. But $\mathcal{I}^{o}_{\Lambda\text{-}\delta I}(S) \cup (X - H) = X$ and hence H ⊆ $\mathcal{I}^{o}_{\Lambda\text{-}\delta I}(S)$. Therefore, S ∈ λg-(Λ-δI)-O(X).

Theorem 2.18. Let (X, τ, I) be an ITS, S and T be subsets of X. If $\mathscr{I}^{o}_{\Lambda\text{-}\delta I}(S) \subseteq T \subseteq S$ and S ∈ λg-(Λ-δI)-O(X), then T ∈ λg-(Λ-δI)-O(X).

Proof. Suppose that $\mathscr{I}^{o}_{\Lambda\text{-}\delta I}(S) \subseteq T \subseteq S$ and S ∈ λg-(Λ-δI)-O(X). Then $X - S \subseteq X - T \subseteq \mathbb{C}_{\Lambda\text{-}\delta I}(X - S)$ and X − S ∈ λg-(Λ-δI)-C(X). Therefore, Theorem 2.11, T ∈ λg-(Λ-δI)-O(X).

3. λg-(Λ-δI)-continuous and λg-(Λ-δI)-irresolute maps.

In this section firstly we define λg-(Λ-δI)-continuous function and discuss its relation with some other existing continuous functions. A function f :(X, τ, I_1) → (Y, σ, I_2) is called λg-(Λ-δI)-continuous if $f^{-1}(M)$ ∈ λg-(Λ-δI)-O(X) for every M ∈ τ(Y), then we define λg-(Λ-δI)-irresolute function as: A function f :(X, τ, I_1) → (Y, σ, I_2) is said to be λg-(Λ-δI)-irresolute if $f^{-1}(M)$ is λg-(Λ-δI)-O(X) for every λg-(Λ-δI)-open set M of Y, Then we define Λ-δI-open function from two ideal spaces as a function f :(X, τ, I_1) → (Y, σ, I_2) is said to be Λ-δI-open if the image of Λ-δI-open set is Λ-δI-open.

Theorem 3.1. A function f :(X, τ, I_1) → (Y, σ, I_2) is λg-(Λ-δI)-continuous, then it is gλ-continuous.

Proof. Let f :(X, τ, I_1) → (Y, σ, I_2) be λg-(Λ-δI)-continuous and F be any open set in (Y, σ, I_2). Then, $f^{-1}(F)$ ∈ λg-(Λ-δI)-O(X). Always, every λg-(Λ-δI)-open set is gλ-open. Therefore, $f^{-1}(F)$ is gλ-open in (X, τ, I_1). Hence, f is gλ-continuous.

Example 3.2. Let X = Y = {a, b, c, d} and τ = {X, ∅, { c}, { d}, { c, d}, { a, c, d}}, λg-(Λ-δI)-open = {X, ∅, { b}, {c}, {d}, { a, b}, { b, d}, { b, c}, { c, d}, { b, c, d}, { a, b, d}, { a, b, c}}, gΛ-open = {X, ∅, { a}, { b}, { c}, { d}, { a, b}, { a, d}, { a, c}, { b, c}, { b, d}, { c, d}, { a, c, d}, { b, c, d}, { a, b, d}, { a, b, c}} and σ = {X, ∅, {a}, { b}, { a, b}}. Let f : (X,

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τ, I_1) $\rightarrow$ (Y, σ, I_2) be an identity function. Then f is gλ-continuous but not λg-(Λ-δI)-continuous as: S = {a} $\in \tau$(Y) but f^{-1}(S) = {a} $\notin$ λg-(Λ-δI)-O(X).

Theorem 3.3. A function f from an ITS (X, τ, I_1) into an ITS (Y, σ, I_2) is super continuous, then it is λg-(Λ-δI)-continuous.

Proof. Let f :(X, τ, I_1) $\rightarrow$ (Y, σ, I_2) be super continuous and M $\in \tau$(Y). Then f^{-1}(M) is δ-open in (X, τ, I_1). Generally, δ-open set are λg-(Λ-δI)-open. Consequently, f^{-1}(M) $\in$ λg-(Λ-δI)-O(X) for every M $\in \tau$(Y) where τ(Y) = {M\ M $\subseteq$ Y and M $\in \tau$}. Hence, f $\in$ λg-(Λ-δI)-continuous.

Example 3.4. Let X = Y = { a, b, c, d} and τ = {X, $\emptyset$, { a}, { b}, { a, b}}, λg-(Λ-δI)-open = {X, $\emptyset$, { a}, { b}, { c}, { d}, { a, b}, { a, d}, { a, c}, { b, c}, { b, d}, {c, d}, { a, c, d}, { a, b, d}, { a, b, c}, { b, c, d}}, δ-open = {X, $\emptyset$, { a}, { b}} and σ = {X, $\emptyset$, { c}, { d}, { c, d}, { a, c, d}}. Let f : (X, τ, I_1) $\rightarrow$ (Y, σ, I_2) defined by, f({v_1}) = b, f({b}) = {a}, f({c}) = { c}, f({d}) = { d} is λg-(Λ-δI)-continuous but not super continuous as M = { c} $\in \tau$(Y) but f^{-1}(M) = {v_3} $\notin$ δ-O(X).

Theorem 3.5. A function f :(X, τ, I_1) $\rightarrow$ (Y, σ, I_2) is λg-(Λ-δI)-continuous if and only if f^{-1}(H) $\in$ λg-(Λ-δI)-C(X) for every closed set H in (Y, σ, I_2).

Proof. Necessity. Let f : (X, τ, I_1) $\rightarrow$ (Y, σ, I_2) be λg-(Λ-δI)-continuous and H be any closed set in (Y, σ, I_2). Then f^{-1}(X $-$ H) $\in$ λg-(Λ-δI)-O(X). But f^{-1}(X $-$ H) = X $-$ f^{-1}(H) and so f^{-1}(H) $\in$ λg-(Λ-δI)-C(X). Conversely, Suppose f^{-1}(H) $\in$ λg-(Λ-δI)-C(X), for every closed set H in (Y, σ, I_2). Again, since f^{-1}(X $-$ H) = X $-$ f^{-1}(H), f^{-1}(X $-$ H) $\in$ λg-(Λ-δI)-O(X), for every X $-$ H $\in \tau$(Y). Hence, f is λg-(Λ-δI)-continuous.

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Theorem 3.6. If f :(X, τ, I_1) → (Y, σ, I_2) is both continuous and Λ-δI-closed, then f(S) ∈ λg-(Λ-δI)-C(Y) for every S ∈ λg-(Λ-δI)-C(X).

Proof. Let S ∈ λg-(Λ-δI)-C(X) and f(S) ⊂ M where M ∈ τ(Y). Since S ⊂ f^{-1}(M) and f^{-1}(M) ∈ τ(X) and hence λ-O(X), $\mathbb{C}_{Λ\text{-}δI}$(S) ⊂ f^{-1}(M). Thus, f ($\mathbb{C}_{Λ\text{-}δI}$(S)) ⊂ M. Hence, $\mathbb{C}_{Λ\text{-}δI}$(f(S)) ⊂ $\mathbb{C}_{Λ\text{-}δI}$(f($\mathbb{C}_{Λ\text{-}δI}$(S))) = f($\mathbb{C}_{Λ\text{-}δI}$(S)) ⊂ M, since f is Λ-δI-closed. Hence, f(S) ∈ λg-(Λ-δI)-C(Y).

Theorem 3.7. A map f :(X, τ, I_1) → (Y, σ, I_2) from an ideal space (X, τ, I_1) into an ideal space (Y, σ, I_2) is λg-(Λ-δI)-irresolute if and only if f^{-1}(M) ∈ λg-(Λ-δI)-C(X) for every M ∈ λg-(Λ-δI)-C(Y).

Proof. Let f be λg-(Λ-δI)-irresolute and M be any member of λg-(Λ-δI)-C(Y). Then, X − M ∈ λg-(Λ-δI)-O(Y). Since f is λg-(Λ-δI)-irresolute, f^{-1}(X − M) ∈ λg-(Λ-δI)-O(X). But f^{-1}(X − M) = X − f^{-1}(M) and so f^{-1}(M) ∈ λg-(Λ-δI)-C(X). Conversely, Let V be any λg-(Λ-δI)-C(Y).Then X − V ∈ λg-(Λ-δI)-C(Y). By assumption, f^{-1}(X − V) ∈ λg-(Λ-δI)-C(X). But f^{-1}(X − V) = X − f^{-1}(V) and so f^{-1}(V) ∈ λg-(Λ-δI)-O(X). Therefore, f is λg-(Λ-δI)-irresolute.

Generally in an ITS λg-(Λ-δI)-irresolute functions and δ-continuous functions are independent of each other as seen in the fourth coming Examples. But whenever λg-(Λ-δI)-C(X) sets coincides δ-closed sets the Theorem 3.10 holds.

Example 3.8. Let X = Y = {a, b, c, d} and τ = {X, ∅, { a}, {b}, { a, b}, { a, b, c}}, δ-open = {X, ∅, { a}, { b}}, λg-(Λ-δI)-open = {X, ∅, {a}, { b}, { d}, { a, b}, { a, d}, {b, c}, {b, d}, {c, d}, {a, c, d}, {a, b, d}, {b, c, d}} and σ = {X, ∅, {a}, {b}, {a, b}}, δ-open = {X, ∅, { a}, { b}}, λg-(Λ-δI)-O(X)= {X, ∅, {a}, {b}, { c}, { d}, { a, b}, { a, c}, { a, d}, { b, c}, {

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b, d}, { c, d}, { a, b, c}, { a, c, d}, { a, b, d}, { b, c, d}}. Let f : (X, τ, I_1) → (Y, σ, I_2) defined by an identity function is δ-continuous but not λg-(Λ-δI)-irresolute as S = { c} ∈ λg-(Λ-δI)-O(Y) but f^{-1}(S) = { c}∉ λg-(Λ-δI)-O(X).

Example 3.9. Let X = Y = { a, b, c, d} and τ = {X, ∅, { a}, { b}, { a, b}}, δ-open = {X, ∅, { a}, { b}}, λg-(Λ-δI)-open = {X, ∅, { a}, { b}, { c}, { d}, { a, b}, { a, c}, { a, d}, { b, c}, { b, d}, { c, d}, { a, b, c}, { a, c, d}, { a, b, d}, { b, c, d}} and σ = {X, ∅, { c}, { d}, { c, d}, { a, c, d}}, δ-open = {X, ∅, { c}, { d}}, λg-(Λ-δI)-open = {X, ∅, { b}, { c}, { d}, {a, b}, { b, c}, { b, d}, { c, d}, { a, b, c}, { a, b, d}, { b, c, d}}. Let f : (X, τ, I_1) → (Y, σ, I_2) defined by an identity function is λg-(Λ-δI)-irresolute but not δ-continuous as S = { c} ∈ δ-O(Y) but f^{-1}(S) = { c} ∉ δ-O(X).

Theorem 3.10. Let λg-(Λ-δI)C(X) ⊆ δC(X) and f :(X, τ, I_1) → (Y, σ, I_2) be λg-(Λ-δI)-irresolute. Then f is δ-continuous.

Proof. Let H ∈ δ-C(Y). Then H ∈ λg-(Λ-δI)-C(X). Since f is λg-(Λ-δI)-irresolute, f^{-1}(H) ∈ λg-(Λ-δI)-C(X). Therefore, by hypothesis, f^{-1}(H) ∈ δ-C(X).

Theorem 3.11. Let f :(X, τ, I_1) → (Y, σ, I_2) be surjective, λg-(Λ-δI)-irresolute and δ-closed, if λg-(Λ-δI)-C(X) ⊆ δ-C(X) in (X, τ, I_1) then the same in (Y, σ, I_2).

Proof. Let S ∈ λg-(Λ-δI)-C(Y) be arbitrary. Since f is λg-(Λ-δI)-irresolute, f^{-1}(S) ∈ λg-(Λ-δI)-C(X). Then by hypothesis, f^{-1}(S) ∈ δ-C(X). Since f is surjective and δ-closed, S is δ-C(Y).

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