10.48047/jocaaa.2024.33.02.42

Total Dominator Color Class Total Dominating Sets in Braid graph, Comb Graph, Fan Graph and Double Fan Graph

[1] **Dr.A.Vijayalekshmi**,

Associate Professor & Head,
Department of Mathematics,
S.T.Hindu College, Nagercoil-629002,
Tamilnadu, India.
(Affliated to Manonmaniam Sundaranar University, Tirunelveli-627012)
Email: vijimath.a@gmail.com
[2] P.Jaspin Ida,
Research Scholar
Reg.No:20123152092018,
Department of Mathematics,
S.T.Hindu College, Nagercoil-629002,
Tamilnadu, India.
(Affliated to Manonmaniam Sundaranar University, Tirunelveli-627012)

Abstract

Let $G = (V, E)$ be a finite, undirected and connected graph with minimum degree at least one. A proper coloring $\mathcal{C}$ of G is said to be a total dominator color class total dominating set of G if each vertex properly dominates a color class in $\mathcal{C}$ and each color class in $\mathcal{C}$ is properly dominated by a vertex in V(G). A total dominator color class total dominating set D of G is a minimal total dominator color class total dominating set if no proper subset of D is a total dominator color class total dominating set of G. The total dominator color class total domination number is the minimum cardinality taken over all minimal total dominator color class total dominating sets in G and is denoted by $\gamma_{\chi}^{td}(G)$. Here we obtain $\gamma_{\chi}^{td}(G)$ for Braid graph, Comb graph, Fan graph and Double fan graph.

Keywords: Chromatic number, Domination number, Total domination, Dominator color class dominating set, Total dominator color class total domination number.

AMS Subject Classification: 05C15, 05C69.

1. Introduction

All graphs considered in this paper are finite, undirected graphs with minimum degree at least one and we follow standard definitions of graph theory as found in [9]. Let G = (V,E) be a connected graph with no isolated vertices. The open neighborhood N(v) of a vertex $v \in V(G)$ consists of the set of all vertices adjacent to v. The closed neighborhood of v is $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighborhood N(S) is defined to be $\cup_{v \in S} N(v)$, and the closed neighborhood of S is $N[S] = N(S) \cup S$. For any set H of vertices of G, the induced subgraph $\langle H \rangle$ is the maximal subgraph of G with vertex set H. A subset S of V is called a dominating set if every vertex in V−S is adjacent to some vertex in S.

A dominating set S is called a minimal dominating set if no proper subset of S is a dominating set of G. The domination number $\gamma(G)$ is the minimum cardinality taken over all minimal dominating sets of G. A γ-set of G is any minimal dominating set with cardinality γ. A proper coloring of G is an assignment of colors to the vertices of G such that adjacent vertices

10.48047/jocaaa.2024.33.02.42

have different colors. The smallest number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by $\chi(G)$. A total dominator coloring of G is a proper coloring of G with the extra property that every vertex in G properly dominates a color class and is denoted by $\chi_{td}(G)$. This notion was introduced by A.Vijayalekshmi et al [10]. A color class dominating set of G is a proper coloring $\mathcal{C}$ of G with the extra property that every color classes in $\mathcal{C}$ is dominated by a vertex in G. A color class dominating set $\mathcal{C}$ is said to be a minimal color class dominating set if no proper subset of $\mathcal{C}$ is a color class dominating set of G. The color class domination number of G is the minimum cardinality taken over all minimal color class dominating sets of G and is denoted by $\gamma_\chi(G)$. This notion was introduced by A.Vijayalekshmi et al [4].

A dominator color class dominating set of G is a proper coloring $\mathcal{C}$ of G with the extra property that each vertex v in G is dominated by a color class $\mathcal{C}_i \in \mathcal{C}$ and each color class $\mathcal{C}_i \in \mathcal{C}$ is dominated by a vertex in G. The dominator color class domination number of G is the minimum cardinality taken over all dominator color class dominating sets in G and is denoted by γ_χ^d (G). This notion was introduced by A.Vijayalekshmi et al [5]. A proper coloring $\mathcal{C}$ of G is said to be a total dominator color class total dominating set of G if each vertex properly dominates a color class in $\mathcal{C}$ and each color class in $\mathcal{C}$ is properly dominated by a vertex in V(G). A total dominator color class total dominating set D of G is a minimal total dominator color class total dominating set if no proper subset of D is a total dominator color class total dominating set of G. The total dominator color class total domination number is the minimum cardinality taken over all minimal total dominator color class total dominating sets in G and is denoted by $\gamma_\chi^{td}(G)$. This notion was introduced by A.Vijayalekshmi et al [6]. The join $G_1 + G_2$ of graphs G_1 and G_2 with disjoint vertex set V_1 and V_2 and edge sets E_1 and E_2 respectively, is the graph union $G_1 \cup G_2$ together with each vertex in V_1 is adjacent to every vertices in V_2. The Braid graph B(n) is formed by the pair of paths $P_n^{'}$ and $P_n^{''}$ by joining i^{th} vertex of $P_n^{'}$ with $(i+1)^{th}$ vertex of $P_n^{''}$ and i^{th} vertex of $P_n^{''}$ with $(i+2)^{th}$ vertex of $P_n^{'}$ with the new edges. The Comb graph $P_n \odot K_1$ is a graph formed by attaching a single pendant edge to each vertex of a path graph P_n. A Fan graph F_n is formed by joining each vertex of a path graph to a single central vertex. A Double Fan graph $F(m, n)$ is formed by joining two fan graphs, each consisting of a path and a central vertex connected to all vertices of the path.

2.Main Results

Proposition:2.1

For $n \geq 2, \gamma_\chi^{td}(P_n \odot K_1) = 2n$

Proof:

Let $V(P_n \odot K_1) = \{u_i, v_i / 1 \leq i \leq n\}$ with $deg(v_i) = 1$, for every i=1,2,…,n and $V(P_n) = \{u_i / 1 \leq i \leq n\}$ assigned distinct colors say 2i and (2i-1)($1 \leq i \leq n$) to the vertices $\{u_i\}$ and $\{v_i\}$, we obtained $\gamma_\chi^{td} -$ coloring of $P_n \odot K_1$. Thus $\gamma_\chi^{td}(P_n \odot K_1) = 2n$

10.48047/jocaaa.2024.33.02.42

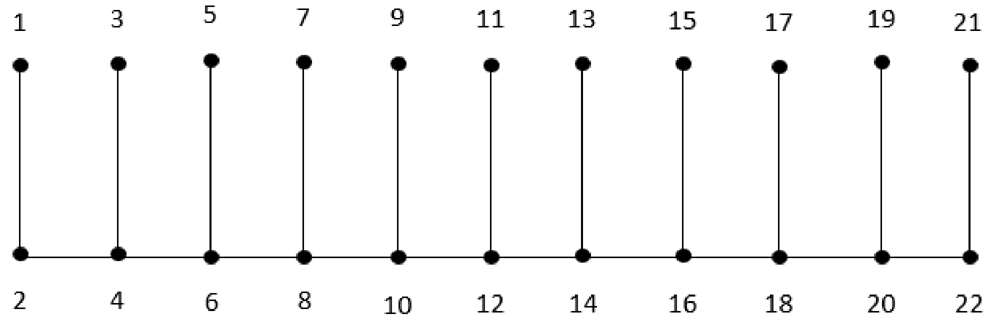


Figure 2.1 $\gamma_{\chi}^{td}(P_{11} \odot K_1) = 22$

Proposition:2.2

For $n \geq 2, \gamma_{\chi}^{td}(F_n) = 3$

Proof:

Let $V(F_n) = \{u, v_i / 1 \leq i \leq n\}$ with $deg(u) = n$ and $deg(v_1) = deg(v_n) = 2$ and $deg(v_i) = 3$, for every $i = 1,2,\ldots,n$ assigned distinct colors say 1,2 and 3 to the vertices $\{u\}$, $\{v_{2i}/1 \leq i \leq n\}$ and $\{v_{2i-1}/1 \leq i \leq n\}$ respectively, we obtain $\gamma_{\chi}^{td} -$ coloring of F_n. So $\gamma_{\chi}^{td}(F_n) = 3$

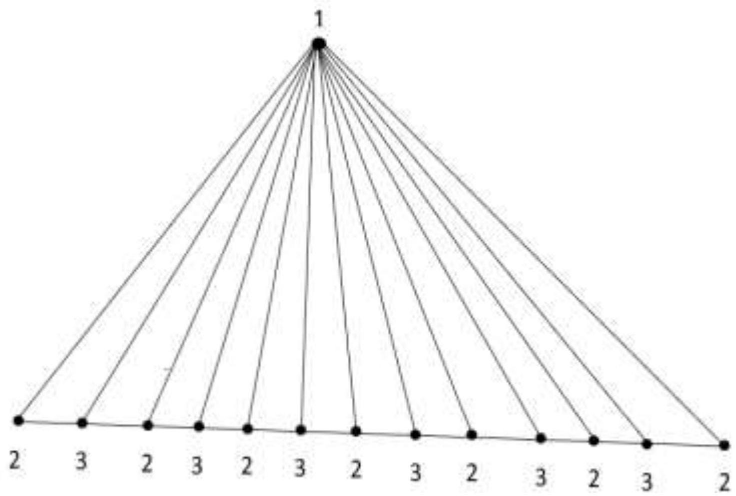


Figure 2.2 $\gamma_{\chi}^{td}(F_{13}) = 3$

Proposition:2.3

For $n \geq 2, \gamma_{\chi}^{td}(F(m,n)) = 3$

Proof:

Let $V(F(m,n)) = \{u, v, v_i/1 \leq i \leq n\}$ with $deg(u) = deg(v) = n$ and $deg(v_1) = deg(v_n) = 3$ and $deg(v_i) = 4$, for every $i = 1,2,\ldots,n$ assigned distinct colors say 1,2 and 3 to the vertices {u,v}, $\{v_{2i}/1 \leq i \leq n\}$ and $\{v_{2i-1}/1 \leq i \leq n\}$ respectively, we obtain $\gamma_{\chi}^{td} -$ coloring of $F(m,n)$. So $\gamma_{\chi}^{td}(F(m,n)) = 3$

10.48047/jocaaa.2024.33.02.42

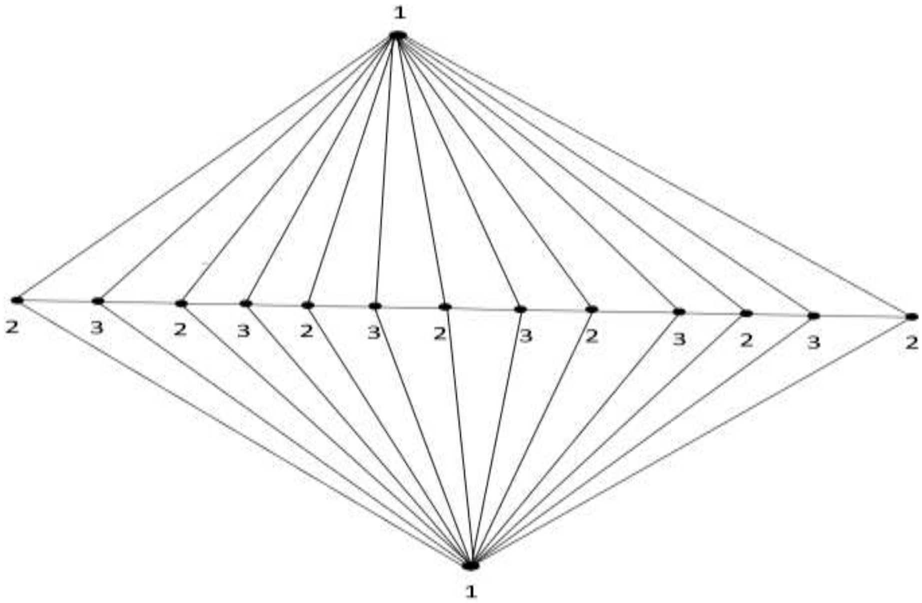


Figure 2.3 $\gamma_\chi^{td}(F(m,n)) = 3$

Theorem:2.4

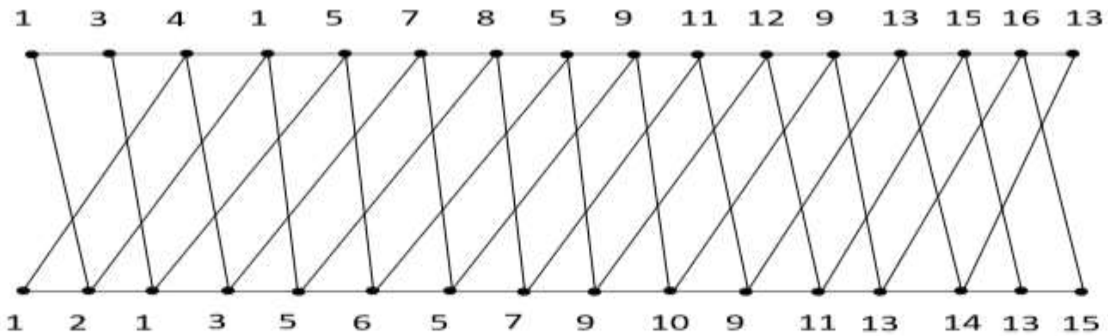
For a Braid graph G, $\gamma_\chi^{td}(B(n)) = \begin{cases} n & \text{if } n \equiv 0(\text{mod}4) \\ n+1 & \text{if } n \equiv 1,3(\text{mod}4) \\ n+2 & \text{if } n \equiv 2(\text{mod}4) \end{cases}$

Proof:

Let $V(B(n)) = \{u_i, v_i / i = 1,2,3,\dots,n\}$, where u_i and v_i are non-adjacent vertices with $\deg(u_i) = \deg(v_i) = 4$ for $i \neq 1,2$ and $i \neq (n-1), n$ and $\deg(u_2) = \deg(v_{n-1}) = 3$ and $\deg(u_1) = \deg(v_n) = 2$. We consider 3 cases.

Case (1): When $n \equiv 0(\text{mod}4)$

Assign distinct colors $4i, (4i-2)$ $(1 \leq i \leq \frac{n}{4})$ to the vertices say $\{u_{4i-1}\}$ and $\{v_{4i-2}\}$ respectively. Assign colors say 1 and (n-1) to the vertices $\{u_1, u_4, v_1, v_3\}$ and $\{v_n\}$ respectively. Assign color say $(4i+1)$ $(1 \leq i \leq \left\lfloor\frac{n}{5}\right\rfloor)$ to the vertices $\{u_{4i+1}, u_{4i+3}, v_{4i+1}, v_{4i-1}\}$. Assign color say $(4i-1)$ $(1 \leq i \leq \frac{n}{4})$ to the vertices $\{u_{4i-2}, v_{4i}\}$ respectively. We obtain a γ_χ^{td}-coloring of B(n). So $\gamma_\chi^{td}(B(n)) = n$



10.48047/jocaaa.2024.33.02.42

Figure 2.4 $\gamma_{\chi}^{td}(B(16)) = 16$

Case (2): When $n \equiv 1,3(\text{mod}4)$

We consider two subcases.

Subcase (2.1): When $n \equiv 1(\text{mod}4)$

Assign distinct colors say 1, (n-3), n and (n+1) to the vertices $\{u_1, u_4, v_1, v_3\}$, $\{u_{n-3}, v_{n-3}, v_{n-2}\}$, $\{u_n, v_n\}$ and $\{u_{n-1}\}$ respectively. Assign distinct colors say $4i(1 \leq i \leq \frac{n}{4})$ and $(4i-2)$ $(1 \leq i \leq \frac{n}{4})$ to the vertices say $\{u_{4i-1}\}$ and $\{v_{4i-2}\}$ respectively. Assign colors say $(4i-1)(1 \leq i \leq \left\lceil\frac{n}{6}\right\rceil)$ and $(4i+1)(1 \leq i \leq \left\lceil\frac{n}{9}\right\rceil)$ to the vertices $\{u_{4i-2}, v_{4i}\}$ and $\{u_{4i+1}, u_{4i+3}, v_{4i+1}, v_{4i+3}\}$ respectively, we get a γ_{χ}^{td}-coloring of B(n).

So $\gamma_{\chi}^{td}(B(n)) = n + 1$

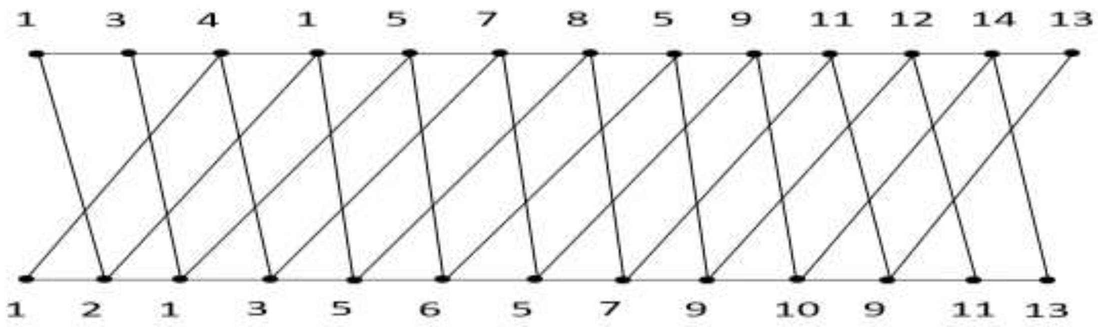


Figure 2.5 $\gamma_{\chi}^{td}(B(13)) = 14$

Subcase (2.1): When $n \equiv 3(\text{mod}4)$

Assign distinct colors say 1,(n-5), (n-2), n and (n+1) to the vertices say $\{u_1, u_4, v_1, v_3\}$, $\{u_{n-2}, v_{n-2}, v_n\}$,$\{u_{n-5}, v_{n-5}, u_{n-2}\}$,$\{u_n\}$ and $\{u_{n-1}\}$ respectively. Assign distinct colors say $4i(1 \leq i \leq \frac{n}{4})$ and $(4i-2)(1 \leq i \leq \left\lceil\frac{n}{4}\right\rceil)$ to the vertices say $\{u_{4i-1}\}$ and $\{v_{4i-2}\}$ respectively. Assign color say $(4i-1)(1 \leq i \leq \frac{n}{4})$ to the vertices $\{u_{4i-2}, v_{4i}\}$ and $(4i+1)(1 \leq i \leq \left\lceil\frac{n}{9}\right\rceil)$ to the vertices $\{u_{4i+1}, u_{4i+3}, v_{4i+1}, v_{4i+3}\}$ respectively, we attain a γ_{χ}^{td}- coloring of B(n).

So $\gamma_{\chi}^{td}(B(n)) = n + 1$

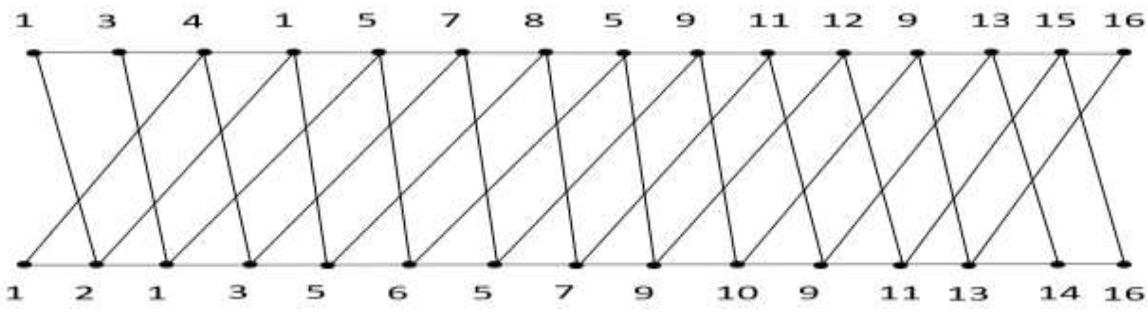


Figure 2.6 $\gamma_{\chi}^{td}(B(15)) = 16$

10.48047/jocaaa.2024.33.02.42

Case (3): When $n \equiv 2(\mathrm{mod}4)$

Since $n - 2 \equiv 0(\mathrm{mod}4)$, the γ_χ^{td} −coloring of B(n) is obtained by the γ_χ^{td} −coloring of B(n-2) followed by γ_χ^{td} − coloring of B(2).

So $\gamma_\chi^{td}(B(n)) = \gamma_\chi^{td}(B(n-2)) + \gamma_\chi^{td}(B(2)) = n - 2 + 4 = n + 2$

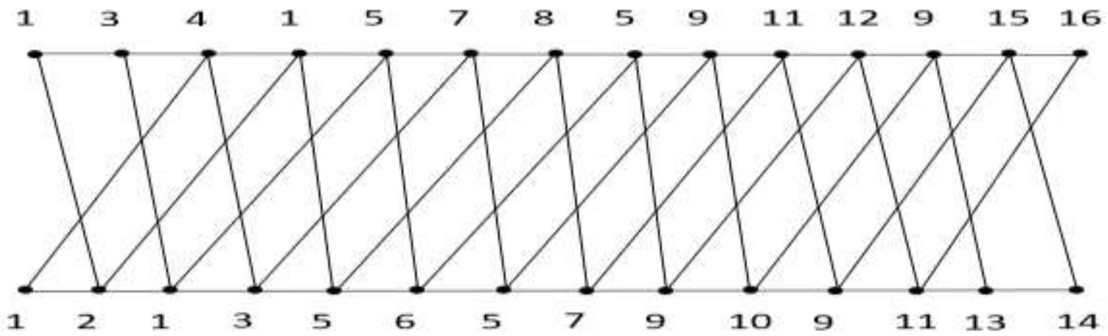


Figure 2.7 $\gamma_\chi^{td}(B(14)) = 16$

References

[1] A.Vijayalekshmi, Total Dominator Colorings in Graphs, International Journal of Advancements in Research & Technology, 1(4) (2012), 1-10.

[2] A.Vijayalekshmi and S.Anusha, Total Dominator Chromatic Number on Various Classes of Graphs, International Journal of Scientific Research and Reviews, 8(4)(2019), 32-40.

[3] A.Vijayalekshmi and J. Virjin Alangara Sheeba, Total Dominator Chromatic Number of Paths Cycles and Ladder graphs, International Journal of Contemporary Mathematical Sciences, 13(5)(2018), 199–204.

[4] A.Vijayalekshmi and A.E.Prabha, Introduction of color class dominating sets in Graphs, Malaya Journal of Matematik, 8(4)(2020), 2186–2189.

[5] A.Vijayalekshmi and P. Niju, An Introduction of Dominator color class dominating sets in Graphs, Malaya Journal of Matematik, 9(1)(2021), 1015–1018.

[6] A.Vijayalekshmi and S. Abisha, An introduction of total dominator color class total dominating sets in Graphs, Malaya Journal of Matematik, vol.9, No.1, 1225-1228, 2021.

[7] A.Vijayalekshmi and S. Anusha, Dominator Chromatic Number on Various Classes of Graphs, International Journal of Scientific Research and Reviews, 9(3)(2020), 91– 101.

[8] A.Vijayalekshmi and A.E.Prabha, Color Class Dominating Sets in various classes of graphs, Malaya Journal of Matematik, Vol. 9, No.1, pp. 195-198, 2021.

[9] F. Harrary, Graph theory, Addition-Wesley Reading Mass,1969.

[10] M.I.Jinnah and A.Vijayalekshmi, Total Dominator Colorings in Graphs, Ph.D Thesis, University of Kerala, 2010.

10.48047/jocaaa.2024.33.02.42

[11] A.E.Prabha and A.Vijayalekshmi, Color Class Dominating Sets in ladder and grid graphs, Malaya Journal of Matematik, Vol. 9, No.1, pp. 993-995, 2021.

[12] Terasa W. Haynes, Stephen T.Hedetniemi, Peter Slater, Domination in Graphs, Advanced Topics, New York, 1998.

[13] Terasa W. Haynes, Stephen T. Hedetniemi, Peter Slater, Domination in Graphs, Marcel Dekker, New York, 1998.