

# Prime Difference Mean Cordial Labeling of Path and Cycle Graphs

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## Abstract

Let  $G = (V, E)$  be a graph with  $p$  vertices and  $q$  edges. A graph  $G$  is said to be a primedifference mean cordial labeling if there exists an injective function  $f: V(G) \rightarrow \{0, 1, 2, \dots, q\}$  such that the induced edge labeling  $f^*: E(G) \rightarrow \{0, 1\}$  defined by

$$f^*(e = uv) = \begin{cases} 1 & \text{if } f(u) - f(v) \equiv 0 \pmod{2} \\ 0 & \text{otherwise} \end{cases}$$

satisfying the condition that for every  $v \in V(G)$  with  $\deg(v) \geq 1$ ,  $S_v = \sum\{f^*(e = uv)/uv \in E(G)\}$  is 1 or prime and  $|e_f(i) - e_f(j)| \leq 1, \forall i, j \in \{0, 1\}$  where  $e_f(x)$  denote the number of edges labeled with  $x$ . A graph with prime difference mean cordial labeling is called prime difference mean cordial graph. In this paper prime difference mean cordiality of some graphs are discussed.

**Key words:** prime, mean, cordial, mean cordial, difference mean cordial, prime difference mean cordial, path, cycle.

**Subject Classification:** 05C78

## 1. INTRODUCTION

Graphs considered here are simple, finite, connected and undirected. The vertex set and edge set of a graph are  $V(G)$  and  $E(G)$  respectively. The concept of cordial labeling was introduced by Cahit in the year 1987. The concept of mean labeling was introduced by S. Somasundaram and Raja Ponraj. The concept of prime mean labeling was introduced by K. Palani. The concept of difference mean labeling was introduced by R. Savithri and S. Shenbega

Devi. Motivated from the above concepts, we introduced a new type of labeling called prime difference mean cordial labeling.

**Definition 1.1** Let  $G = (V, E)$  be a graph with  $p$  vertices and  $q$  edges. A graph  $G$  is said to be a prime difference mean cordial labeling if there exists an injective function  $f: V(G) \rightarrow \{0, 1, 2, \dots, q\}$  such that the induced edge labeling  $f^*: E(G) \rightarrow \{0, 1\}$  defined by

$$f^*(e = uv) = \begin{cases} 1 & \text{if } f(u) - f(v) \equiv 0 \pmod{2} \\ 0 & \text{otherwise} \end{cases}$$

satisfying the condition that for every  $v \in V(G)$  with  $deg(v) \geq 1$ ,  $S_v = \sum\{f^*(e = uv)/uv \in E(G)\}$  is 1 or prime and  $|e_f(i) - e_f(j)| \leq 1, \forall i, j \in \{0, 1\}$  where  $e_f(x)$  denote the number of edges labeled with  $x$

**Example 1.2.** A graph that admits a prime difference mean cordial labeling is given below:

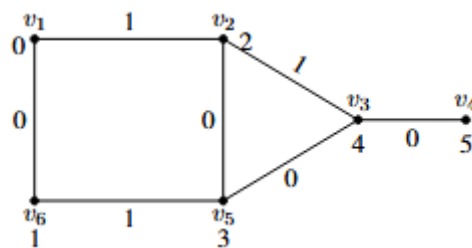


Figure 1

**Theorem 1.3.** The path  $P_n, n \geq 2$  is a prime difference mean cordial graph when  $n \equiv 0 \pmod{4}$ .

**Proof.** Let  $V(P_n) = v_i; 1 \leq i \leq n$  and  $E(P_n) = \{v_i v_{i+1}; 1 \leq i \leq n - 1\}$  be the vertex set and edge set of path  $P_n$ .

Define  $f: V(P_n) \rightarrow \{0, 1, 2, \dots, q\}$  as follows:

For  $1 \leq i \leq n$ ,

$$f(v_i) = \begin{cases} \frac{i-1}{2}, & i \equiv 1 \pmod{2} \\ \frac{n+i-2}{2}, & i \equiv 0 \pmod{2} \end{cases}$$

Let the induced edge labelings are as follows:

For  $1 \leq i \leq n$ ,

$$f^*(v_i v_{i+1}) = \begin{cases} 1, & i \equiv 1 \pmod{2} \\ 0, & i \equiv 0 \pmod{2} \end{cases}$$

Here  $e_f(0) = \frac{n-1}{2}$  and  $e_f(1) = \frac{n}{2}$ .

Therefore  $|e_f(1) - e_f(0)| \leq 1$ .

Then  $S_{v_1} = f^*(v_1 v_2) = 1$ .

For  $2 \leq i \leq n - 1$ ,

$$S_{v_i} = f^*(v_i v_{i+1}) + f^*(v_{i-1} v_i) = 1$$

$$S_{v_n} = f^*(v_{n-1} v_n) = 1$$

Therefore for every  $v \in V(P_n), n \geq 2, S_v$  is equal to 1 or prime.

Therefore  $P_n$  admits prime difference mean cordial labeling.

Hence  $P_n$  is a prime difference mean cordial graph.

**Example 1.4.**  $P_8$  is a prime difference mean cordial graph which is given below:

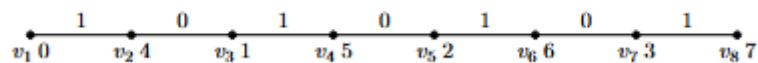


Figure 2

**Theorem 1.5.** The cycle  $C_n$  is a prime difference mean cordial graph when  $n \equiv 0(mod 4)$ .

**Proof**

Let  $V(C_n) = \{v_i: 1 \leq i \leq n\}$  be the vertex set of cycle  $C_n$ .

Let  $E(C_n) = \{v_i v_{i+1}: 1 \leq i \leq n - 1\} \cup \{v_n v_1\}$ .

Define  $f: V(C_n) \rightarrow \{0,1,2, \dots, q\}$  as follows:

For  $1 \leq i \leq n$ ,

$$f(v_i) = \begin{cases} \frac{i-1}{2}, & i \equiv 1(mod 2) \\ \frac{n+i-2}{2}, & i \equiv 0(mod 2) \end{cases}$$

Let the induced edge labelings are as follows:

For  $1 \leq i \leq n$ ,

$$f^*(v_i v_{i+1}) = \begin{cases} 0, & i \equiv 1(mod 2) \\ 1, & i \equiv 0(mod 2) \end{cases}$$

Here  $e_f(0) = \frac{n}{2}$  and  $e_f(1) = \frac{n}{2}$ .

Therefore  $|e_f(1) - e_f(0)| \leq 1$ .

$$S_{v_1} = f^*(v_1 v_2) + f^*(v_1 v_n) = 1$$

For  $2 \leq i \leq n - 1$ ,

$$S_{v_i} = f^*(v_i v_{i+1}) + f^*(v_{i-1} v_i) = 1$$

$$S_{v_n} = f^*(v_n v_1) + f^*(v_{n-1} v_n) = 1$$

Therefore for every  $v \in V(C_n), n \geq 3$ ,  $S_v$  is equal to 1 or prime.

Therefore  $C_n$  is a prime power mean cordial labelling when  $n \equiv 0(mod 4)$ .

Hence  $C_n$  is a prime power mean cordial graph when  $n \equiv 0(mod 4)$ .

**Example 1.6.**  $C_8$  is a prime difference mean cordial graph which is given below:

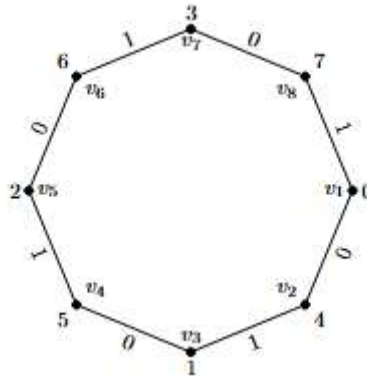


Figure 3

**Theorem 1.7.** Total graph of path  $P_n, T(P_n)$  is a prime difference mean cordial graph for all  $n \geq 3$ .

**Proof:** Let  $V(T(P_n)) = \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n - 1\}$  be the vertex set of the total graph of path  $T(P_n)$ .

$$E(T(P_n)) = \{v_i v_{i+1} : 1 \leq i \leq n - 1\} \cup \{v_i u_i : 1 \leq i \leq n - 1\} \cup \{v_{i+1} u_i : 1 \leq i \leq n - 1\} \cup \{u_i u_{i+1} : 1 \leq i \leq n - 2\}.$$

Define  $f: V(T(P_n)) \rightarrow \{0,1,2, \dots, q\}$  as follows:

$$f(v_i) = 2i - 2, 1 \leq i \leq n$$

$$f(u_i) = 2i - 1, 1 \leq i \leq n - 1$$

Let the induced edge labelings are as follows:

$$f^*(v_i v_{i+1}) = 1, 1 \leq i \leq n - 1$$

$$f^*(v_i u_i) = 0, 1 \leq i \leq n - 1$$

$$f^*(v_{i+1} u_i) = 0, 1 \leq i \leq n - 1$$

$$f^*(u_i u_{i+1}) = 1, 1 \leq i \leq n - 2$$

Here  $e_f(1) = 2n - 3$  and  $e_f(0) = 2n - 2$ .

Therefore  $|e_f(1) - e_f(0)| \leq 1$ .

$$S_{v_1} = f^*(v_1 v_2) + f^*(v_1 u_1) = 1 + 0 = 1$$

For  $2 \leq i \leq n - 1$ ,

$$S_{v_i} = f^*(v_i v_{i+1}) + f^*(v_{i-1} v_i) + f^*(v_i u_i) + f^*(u_{i-1} v_i) = 2$$

$$S_{v_n} = f^*(v_{n-1} v_n) + f^*(u_{n-1} v_n) = 1$$

$$S_{u_1} = f^*(u_1 u_2) + f^*(v_1 u_1) + f^*(u_1 v_2) = 1$$

For  $2 \leq i \leq n - 2$ ,

$$S_{u_i} = f^*(u_i u_{i+1}) + f^*(u_{i-1} u_i) + f^*(u_i v_i) + f^*(u_i v_{i+1}) = 2$$

$$S_{u_{n-1}} = f^*(u_{n-2} u_{n-1}) + f^*(u_{n-1} v_{n-1}) + f^*(u_n v_n) = 1$$

Therefore for every  $v \in V(T(P_n))$ ,  $n \geq 3$ ,  $S_v$  is equal to 1 or prime.

Therefore  $T(P_n)$  is a prime power mean cordial labelling.

Hence  $T(P_n)$  is a prime power mean cordial graph.

**Example 1.6.**  $T(P_6)$  is a prime difference mean cordial graph which is given below:

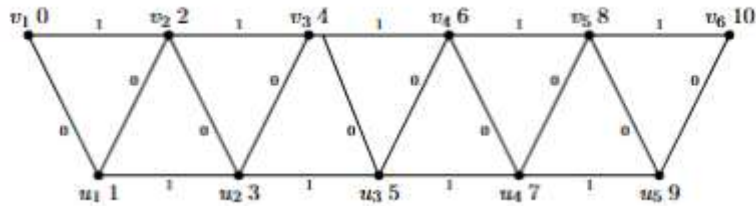


Figure 4

**Theorem 1.8.** Total graph of cycle  $C_n$ ,  $T(C_n)$  is a prime difference mean cordial graph for all  $n$ .

**Proof:** Let  $V(T(C_n)) = \{v_i, u_i : 1 \leq i \leq n\}$  be the vertex set of the total graph of path  $T(C_n)$ .

Let  $E(T(C_n)) = \{v_i v_{i+1}, u_i u_{i+1}, v_i u_i, v_{i+1} u_i : 1 \leq i \leq n\}$ .

Define  $f: V(T(C_n)) \rightarrow \{0, 1, 2, \dots, q\}$  as follows:

$$f(u_i) = 2i - 2, 1 \leq i \leq n$$

$$f(v_i) = 2i - 1, 1 \leq i \leq n$$

Let the induced edge labelings are as follows:

$$f^*(v_i v_{i+1}) = 1, 1 \leq i \leq n - 1$$

$$f^*(v_n v_1) = 1$$

$$f^*(u_i u_{i+1}) = 1, 1 \leq i \leq n - 1$$

$$f^*(u_n u_1) = 1$$

$$f^*(v_i u_i) = 0, 1 \leq i \leq n$$

$$f^*(v_i u_{i+1}) = 1, 1 \leq i \leq n - 1$$

Here  $e_f(1) = 2n$  and  $e_f(0) = 2n$ .

Therefore  $|e_f(0) - e_f(1)| \leq 1$ .

$$S_{v_1} = f^*(v_1 v_n) + f^*(v_1 v_2) + f^*(v_1 u_2) = 2$$

For  $2 \leq i \leq n - 1$ ,

$$S_{v_i} = f^*(v_{i-1}v_i) + f^*(v_iv_{i+1}) + f^*(v_iu_i) + f^*(v_iu_{i+1}) = 2$$

$$S_{v_n} = f^*(v_{n-1}v_n) + f^*(v_nv_1) + f^*(v_nu_1) = 2$$

$$S_{u_1} = f^*(u_1v_1) + f^*(u_1u_2) + f^*(u_1v_n) + f^*(u_1u_n) = 2$$

For  $2 \leq i \leq n - 1$ ,

$$S_{u_i} = f^*(u_{i-1}u_i) + f^*(u_iu_{i+1}) + f^*(u_iv_i) + f^*(v_{i-1}u_i) = 2$$

$$S_{u_{n-1}} = f^*(u_{n-2}u_{n-1}) + f^*(u_{n-1}v_{n-1}) + f^*(u_nv_n) = 1$$

$$S_{u_n} = f^*(u_nv_n) + f^*(u_nu_1) + f^*(v_{n-1}u_n) + f^*(u_{n-1}u_n) = 2$$

Therefore for every  $v \in V(T(C_n)), n \geq 3$ ,  $S_v$  is equal to 1 or prime.

Therefore  $T(C_n)$  is a prime power mean cordial labelling.

Hence  $T(C_n)$  is a prime power mean cordial graph.

**Example 1.6.**  $T(C_6)$  is a prime difference mean cordial graph which is given below:

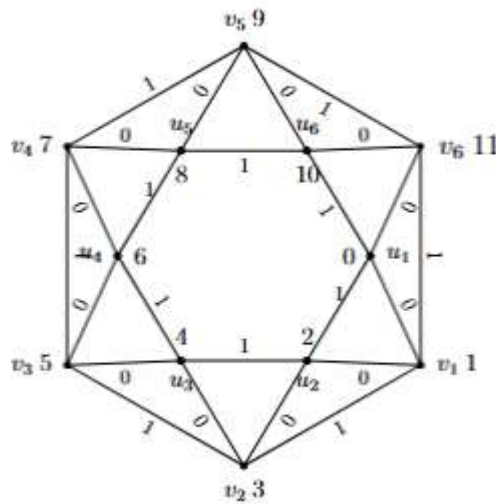


Figure 5

### Conclusion

In this paper we introduced the concept of prime difference mean cordial labeling and the prime difference mean cordial labeling of graphs such as path, cycle and total graph of path and cycle were found out.

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