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NUMERICAL ANALYSIS OF LAMINAR FLOW OF A NANOFLUID PAST A PLATE-FINITE DIFFERENCE APPROACH

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ABSTRACT

This paper utilizes the finite difference method to numerically analyze the laminar flow of a Cu-water nanofluid past a permeable, vertical plate. This study focuses on how the flow characteristics are affected by the Reynolds number (R), solid volume fraction (φ), and magnetic parameter (M). The findings suggest that increasing R improves fluid motion and raises velocity. Improved viscosity and momentum transfer result from a higher solid volume fraction (φ), which affects the flow profile. The velocity distribution changes when a magnetic field (M) is present because it creates a Lorentz force that opposes fluid motion. With ramifications for fluid dynamics and heat transfer engineering applications, this work sheds light on the behavior of nanofluids in magnetohydrodynamic (MHD) flows.

Keywords: Nanofluid, laminar flow, Reynolds number, Solid volume fraction, Magnetic field, MHD, Finite difference method.

Introduction

The enhanced convective heat and mass transfer properties of nanofluids have recently drawn a lot of attention in a variety of fields, including environmental studies, blood flow dynamics, and flow boiling. Further developments in these applications, however, still require a comprehensive grasp of nanoparticle behavior in laminar flow. Ajeeb, Wagd, et al. [1] examined forced convection heat transfer in non-Newtonian MWCNT nanofluids moving through microchannels in a laminar environment. Examined are the effects of magnetic field, Reynolds number, and solid volume fraction on heat transfer and velocity. The results indicate that while the magnetic field affects flow behavior, increasing Reynolds number and nanoparticle concentration improves heat transfer. Saha, Sandip, et al. [2] study how heat moves convectively when a laminar nanofluid flows through a rectangular microchannel using various baffle-corrugation designs. It examines the effects of various baffle geometries on flow behavior and heat transfer efficiency. The results show that optimized baffle configurations improve convective heat transfer, especially at higher Reynolds numbers.

Ben Ltaifa et al.[3] investigates the numerical analysis of mixed convection heat transfer and laminar flow in a rectangular inclined micro-channel filled with Water/Al2O3 nanofluid. The paper focuses on how the inclination angle, Reynolds number, and nanoparticle concentration affect the flow and heat transfer performance. Results indicate that mixed convection significantly enhances heat transfer, with nanoparticle volume fraction improving the thermal conductivity and overall thermal performance.Burggraf, Jonas, et al. [4] focuses on the numerical investigation of laminar flow heat transfer of TiO2-water nanofluid in a heated pipe. It examines the effects of various parameters, such as Reynolds number, nanoparticle concentration, and heat flux, on the flow behavior and thermal performance. The results demonstrate that the presence of TiO2 nanoparticles enhances the heat transfer rate, improving thermal efficiency in the pipe.

Laminar flow is a type of fluid motion where the fluid flows in parallel layers with little to no mixing between them, occurring at low Reynolds numbers. In this flow regime, fluid particles move in a smooth, orderly manner, which results in a predictable and steady velocity profile. It is widely studied in fluid dynamics to understand the behavior of fluids under controlled conditions. Laminar flow is essential for predicting heat transfer, fluid resistance, and the transport of particles or heat. Applications include flow through pipes, lubrication systems, and biomedical flows, where stability and minimal turbulence are crucial. It is also important in microfluidics, where precise control of fluid motion is required. Khan, Umair, et al. [5] investigates MHD mixed convection hybrid nanofluid flow over a permeable, moving, inclined flat plate, considering the effects of thermophoretic and radiative heat flux. The paper examines the combined impact of magnetic fields, inclination angle, and heat transfer mechanisms on the flow and thermal characteristics. Results show that thermophoresis and radiation influence the heat transfer rate, while mixed convection enhances fluid movement and temperature distribution.Huang, Zhong, et al. [6] explores natural convection and radiation heat transfer in power-law fluid food contained in symmetrical open containers. The paper investigates the effects of fluid properties and container geometry on heat transfer performance. Results show that both natural convection and radiative heat transfer play significant roles in thermal distribution within the container, influencing the cooling or heating efficiency of the food.Rauf, Abdul, et al. [7]investigates MHD mixed convection flow of Maxwell hybrid nanofluid, incorporating the effects of Soret, Dufour, and morphology. The paper examines how thermal diffusion (Soret effect), mass diffusion (Dufour effect), and the morphology of the nanofluid influence the heat and mass transfer characteristics. Results highlight the combined effects of these parameters on flow behavior, thermal distribution, and overall convection efficiency in the presence of a magnetic field.

Magnetohydrodynamics (MHD) studies the behavior of electrically conducting fluids under magnetic fields, combining fluid dynamics and electromagnetism. It has significant applications in astrophysics, fusion energy, cooling systems, and biomedical engineering. MHD enhances heat transfer, controls flow stability, and enables efficient electromagnetic pumping. Its applications extend to aerospace, oceanography, and advanced industrial processes. Chabani, I., et al. [8] numerically analyzes the natural convection of a magnetic hybrid nanofluid in a porous trapezoidal enclosure. The effects of magnetic fields, nanoparticle concentration, and enclosure geometry on heat transfer and fluid flow are examined. Results show that adjusting porosity and magnetic strength significantly influences thermal performance and convective behavior.Anusha, T., et al. [9] examines the magnetohydrodynamic (MHD) flow of a nanofluid with a Marangoni-driven laminar boundary layer over a porous medium. It explores heat and mass transfer characteristics influenced by nanoparticles and external magnetic fields. The research aims to understand the effects of these factors on fluid dynamics and thermal performance.

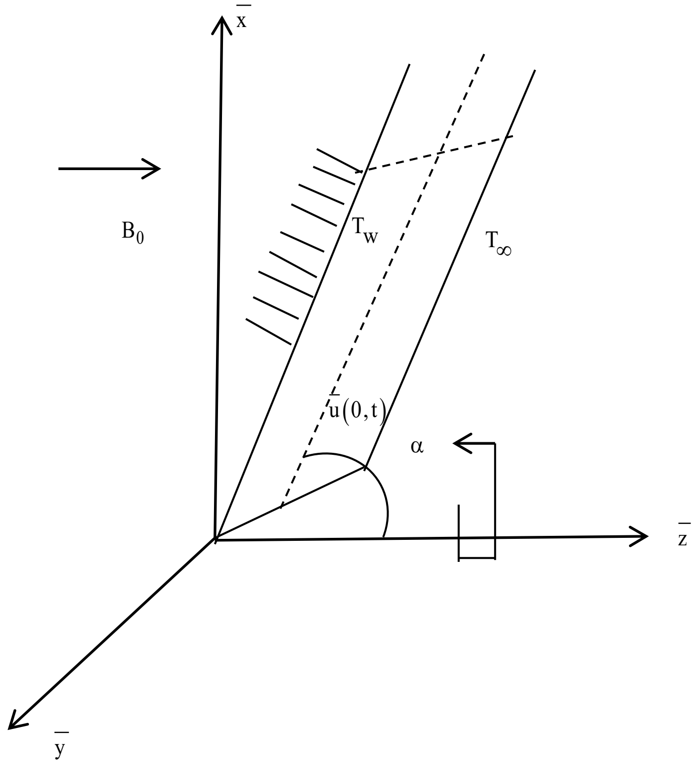
In this paper, focuses on the analysis of both free and forced convection heat transfer in nanofluids, specifically in two-dimensional flow past an inclined vertical plate. exploring how the nanofluid, consisting of nanoparticles suspended in a base fluid, enhances the heat transfer characteristics compared to conventional fluids. The effects of varying parameters such as Reynolds number, solid volume fraction of nanoparticles, and the angle of inclination are examined in detail. Both natural (free) and forced convection mechanisms are analyzed to understand their impact on thermal performance. Special attention is given to the behavior of nanofluids under different flow conditions and their ability to improve heat transfer efficiency. The paper aims to provide a comprehensive understanding of how nanofluids can be optimized for various engineering applications, including heat exchangers and cooling systems.

Mathematical Formulation

The main assumptions governing the flow with heat transfer is as follows:

- The unsteady convection flow of Copper-water Nano-fluids was considered in past vertical moving semi-infinite plate
- This plate was assumed to be non-electrically conducting and move in the positive z-direction with the velocity U_0
- Here transverse magnetic field is applied
- Initially considering the electrically conducting fluid on the plate is at rest.
- Assuming that the induced magnetic field is smaller than the external magnetic field. By taking surface temperature as a constant value T_w and the ambient temperature asT_∞, where $T_w > T_\infty$
- Suppose that the regular fluid and suspended Cu particle are at the thermal equilibrium and no slip occurs between them
- The fluid was assumed to follow the Bossinesq approximation

The Cartesian co-ordinate system and also the geometry of the plate are shown in the following diagram.



1 - Schematic diagram

Governing Equations

The boundary layer equations governing the flow and temperature as per above assumptions are as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho_{nf}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu_{nf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\sigma {B_0}^2 u \tag{2}$$

$$\rho_{nf}\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=\mu_{nf}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

The boundary conditions for this problem are as follows:

$$u(x,y,t)=0, v(x,y,t)=0, T(x,y,t)=T_\infty, t<0$$

$$u(x,y,0)=0, v(x,y,0)=0$$

$$u(0,y,t')=0, v(0,y,t')=0$$

$$u(x,0,t')=x, v(x,0,t')=1$$

$$u(b,y,t')=0, v\left(x,\frac{b}{2},t'\right)=b+x\ , v\left(x,\frac{b}{2},t'\right)=0 \tag{4}$$

Thermo-Physical properties were related as follows:

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s, \alpha_{nf}=\frac{k_{nf}}{(\rho c_p)_{nf}}$$

$$(\rho c_p)_{nf}=(1-\phi)(\rho c_p)_f+\phi(\rho c_p)_s$$

$$(\rho\beta)_{nf}=(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_s$$

Here we introduce the following dimensionless variables as:

$$X=\frac{x}{L}, Y=\frac{y}{L}, t=t'\frac{U_0}{L}, U=\frac{u}{U_0}, V=\frac{v}{U_0} \tag{5}$$

Using equations (6), (7), (8) the equations (2), (3) & (4) can also be written in the following dimensionless form

$$\frac{\partial U(x,y,t)}{\partial t} + U(x,y,t)\frac{\partial U(x,y,t)}{\partial X} + V(x,y,t)\frac{\partial U(x,y,t)}{\partial Y} = \frac{1+(2.5)\emptyset}{(1-\emptyset+\emptyset(\frac{\rho_s}{\rho_f}))R}\left(\frac{\partial^2 U(x,y,t)}{\partial X^2} + \frac{\partial^2 U(x,y,t)}{\partial Y^2}\right) -$$

$$MU(x,y,t) \qquad (6)$$

$$\frac{\partial V(x,y,t)}{\partial t} + U(x,y,t)\frac{\partial V(x,y,t)}{\partial X} + V(x,y,t)\frac{\partial V(x,y,t)}{\partial Y} = \frac{1+(2.5)\emptyset}{(1-\emptyset+\emptyset(\frac{\rho_s}{\rho_f}))R}\left(\frac{\partial^2 V(x,y,t)}{\partial X^2} + \frac{\partial^2 V(x,y,t)}{\partial Y^2}\right) \qquad (7)$$

Here the corresponding boundary condition of equation (5) was written in the dimensionless form as:

$$U(x,y,0) = 0, V(x,y,0) = 0$$

$$U(0,y,t) = 0, V(0,y,t) = 0$$

$$U(x,0,t) = x, V(x,0,t) = 1$$

$$U(b,y,t) = 0, V\left(x,\frac{b}{2},t\right) = b + x \ , V\left(x,\frac{b}{2},t\right) = 0 \qquad \mathbf{(8)}$$

Where the parameters present in the above equations are as follows:

$Pr = \frac{v_f}{\alpha_f}$Prandtl Number

$M = \frac{\sigma {B_0}^2 L}{U_0 \rho_f}$ (Magnetic field parameter)

$R = \frac{LU_0}{\nu}$(Reynolds Number

Solution of the Problem

The differential equations from (6) to (7) are nonlinear and coupled. The boundary conditions described in (8) were applied when solving these equations. The study is carried out in two dimensions and the domain under consideration is an infinite rectangular plate. The plate's width is one unit and its height is two units for computation. Mathematica “Method of lines” tool is used to solve this system.

Discussion of the Results

The graphs in Figures 2 to 16 plot the effects of several parameters on velocities (u and v) including Reynolds number (Re), solid volume fraction (φ), magnetic parameter (M), plate inclination angle (γ). For water, the Prandtl number (Pr) was set at 0.7. Horizontal velocity (v) is displayed at the x = 1 level, while vertical velocity (u) were computed at the y = 1/2 level.

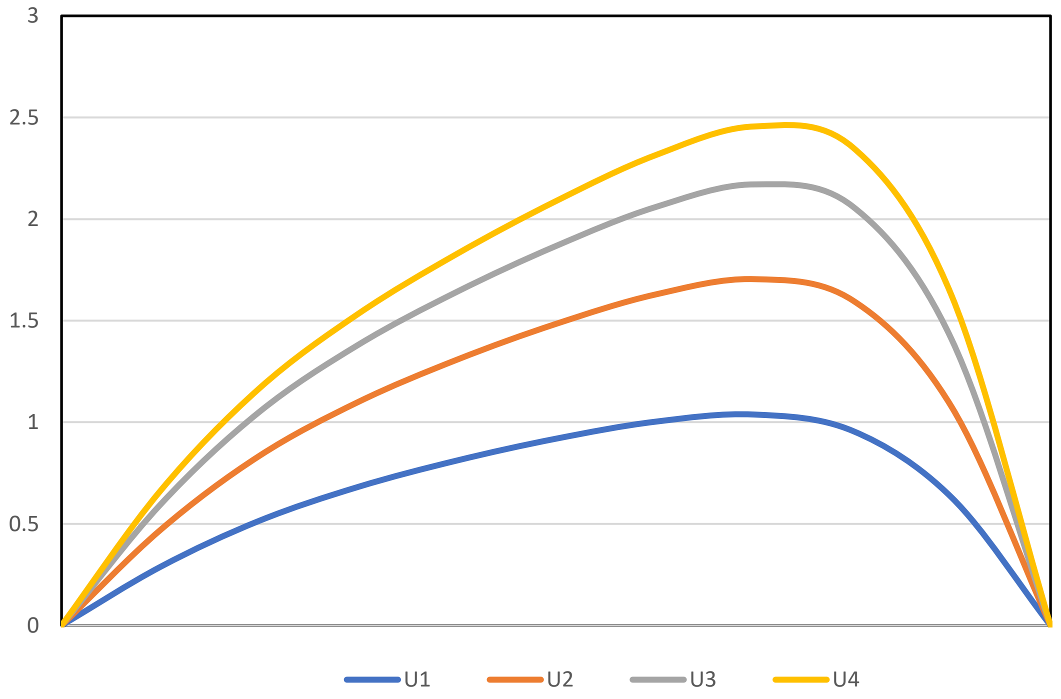


Fig.2. Profile of vertical velocity u with Re

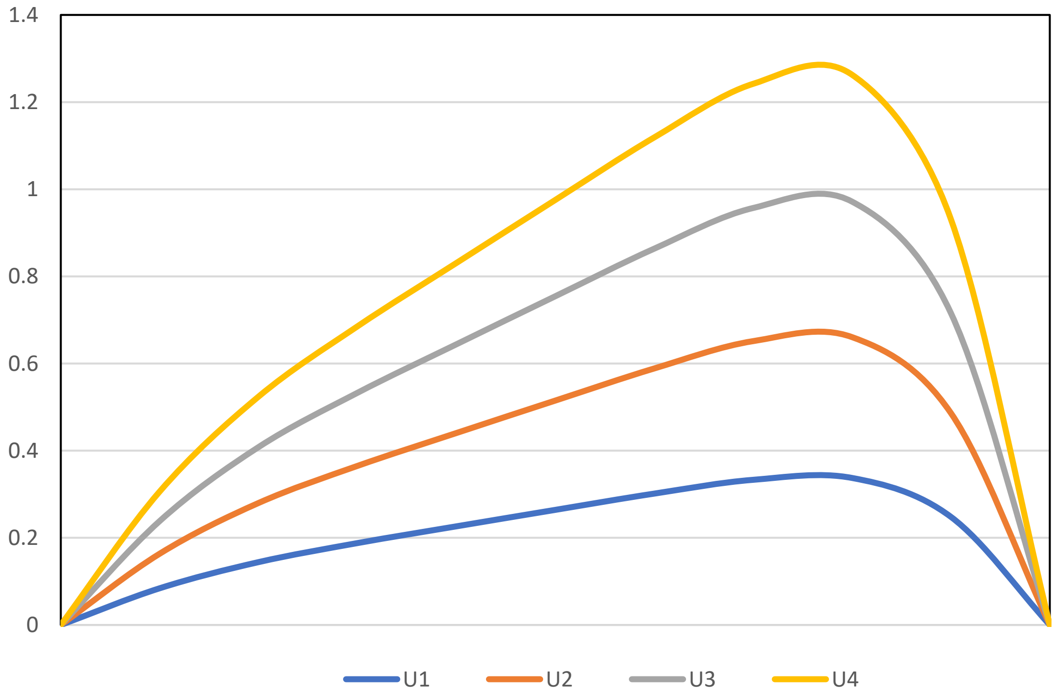


Fig.3 Profile of vertical velocity u with φ

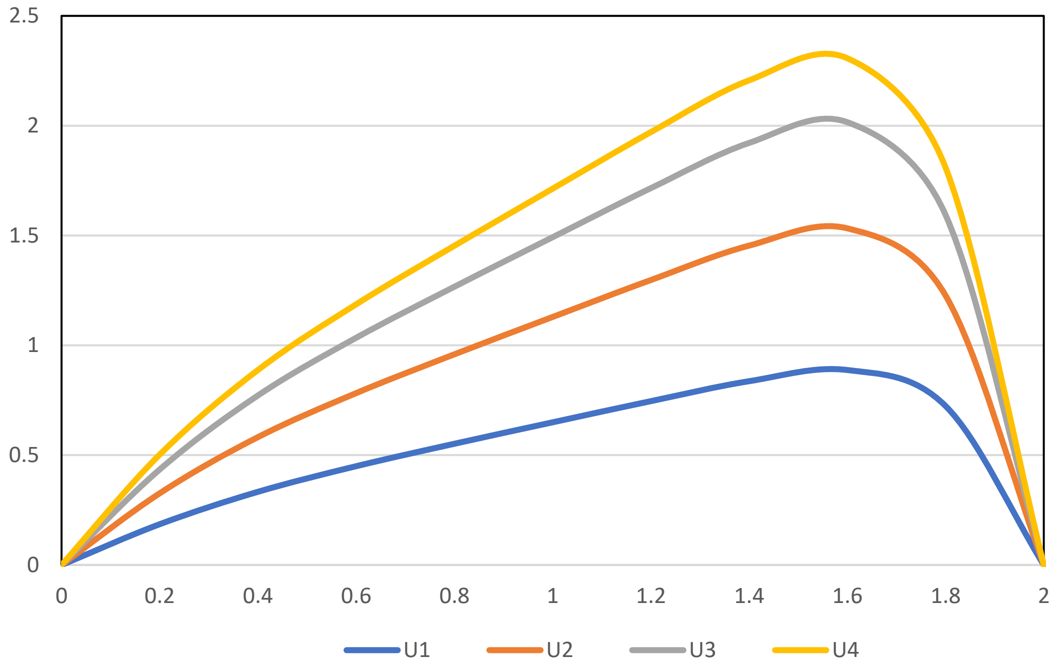


Fig.4 Profile of vertical velocity u with M

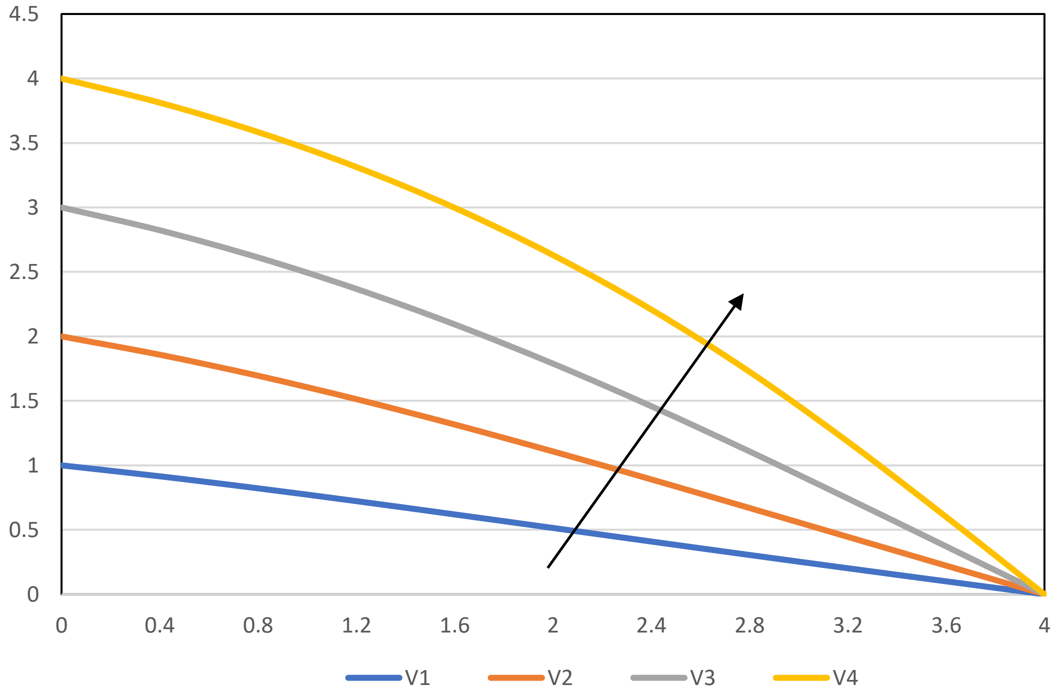


Fig.7 Profile of v with Re

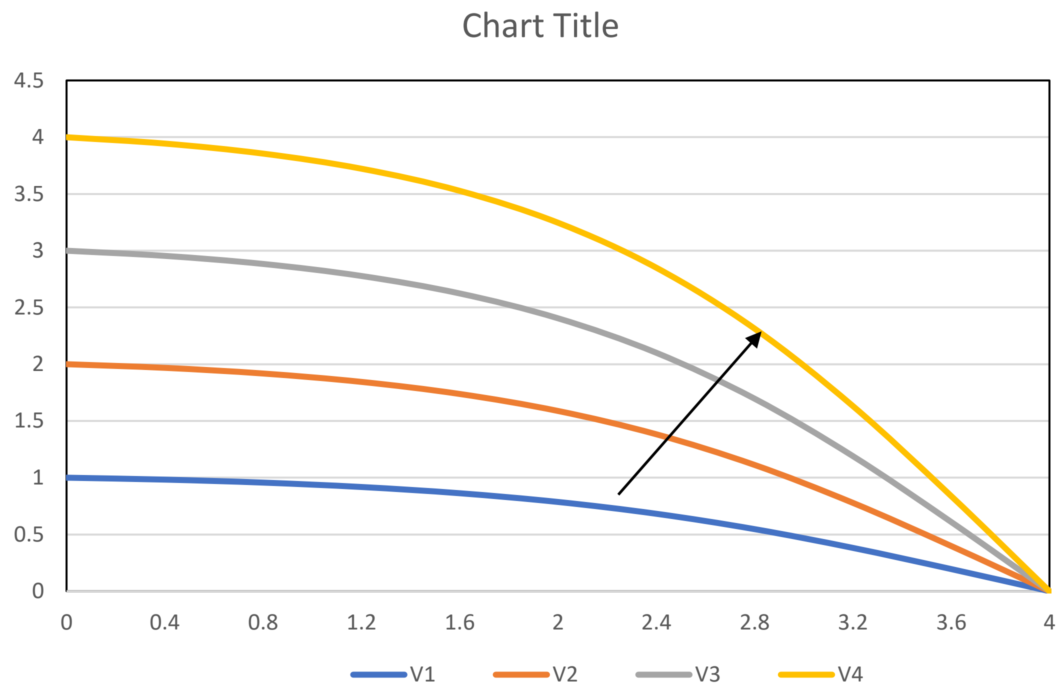


Fig.8. Profile of v with φ

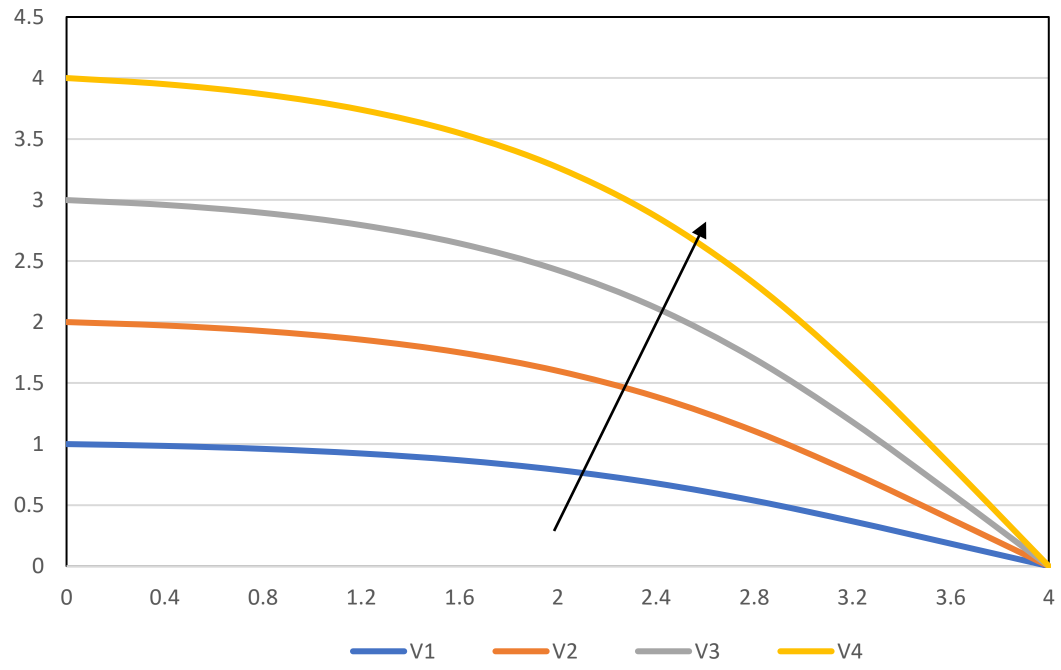


Fig.9. Profile of v with M

The vertical velocity (u) profiles are displayed from figures 2-6. The vertical flow is found more in natural convection than in forced convection for all variations. Movement of the plate accelerates the flow is more in natural convection. Thus the movement of the boundary regulates the flow in forced convection case. From Fig.2 it is evident that the flow was directly proportional to the inertial force in natural convection and the flow is inversely proportional to the inertial force in forced convection. Fig.3 confirms that the solid volume fraction is always inversely proportional to the flow in both convections. But the flow retards gradually. From Fig.4 Lorentz force dominates the flow in both convections. Heat source is not much significant on flow in both convections, but it showed negative impact when it was increased (Fig.5). It was happened due to the presence of metal particle which is acted as heat absorbers. So the Cu-water Nanofluid will be preferred in heat generating systems. Fig.6 indicates the flow is more when it inclines to the base more for both convections.

The horizontal velocity (v) profiles are displayed from figures 7-11. The horizontal velocity is found more in natural convection than in forced convection for all variations. The flow was found to be more with the base of the plate and retards from the base. From Fig.7 viscous force dominates the inertial force and hence the velocity enhances as Re increases. From Fig. 8 the presence of metal particle enhances the velocity in the Brownian motion of the particle.It was further observed that variation of velocity is significant in forced convection than in natural convection. Fig. 9 exhibits the reduction of velocity with Lorentz force in both the convections. But the variation of M is not clearly significant in both convections.

Conclusions

- Rate of heat transfer is more when plate tends to be horizontal in free convection but no significance observed in forced convections.
- Inclination angle enhances the vertical flow but not significant in horizontal flow.

References

[1]. Li, Shuguang, et al. "Heat and mass transfer characteristics of Al2O3/H2O and (Al2O3+ Ag)/H2O nanofluids adjacent to a solid sphere: A theoretical study." Numerical Heat Transfer, Part A: Applications (2024): 1-19.

[2]. Raza, Qadeer, et al. "Numerically analyzed of ternary hybrid nanofluids flow of heat and mass transfer subject to various shapes and size factors in two-

dimensional rotating porous channel." Case Studies in Thermal Engineering 56 (2024): 104235.

[3]. Jain, Ruchi, et al. "Numerical study of heat and mass transfer of williamson hybrid nanofluid (CuO/CNT's-water) past a permeable stretching/shrinking surface with mixed convective boundary condition." Case Studies in Thermal Engineering 59 (2024): 104313.

[4]. Graham A.L, on the viscosity of suspension of solid spheres, Applied Science and Research, Vol 37 (3), 1981, pp: 275-286.

[5]. Jang S.P and Choi S.U.S, Effects of various parameters on Nano-fluid thermal conductivity, Journal of Heat Transfer, Vol 129(5), 2007, pp: 617– 623.

[6]. Holagh, Shahriyar G., and Wael H. Ahmed. "Critical review of vertical gas-liquid slug flow: An insight to better understand flow hydrodynamics' effect on heat and mass transfer characteristics." International Journal of Heat and Mass Transfer 225 (2024): 125422.

[7]. Ahmad, Shafee, et al. "Numerical analysis of heat and mass transfer of MHD natural convection flow in a cavity with effects of source and sink." Case Studies in Thermal Engineering 53 (2024): 103926.

[8]. Das Sarit Kumar, CHOI Stephen US and Patel Hrishikesh E, Heat Transfer in Nano-fluids–A Review, Heat Transfer Engineering, Vol 27(10), 2006, pp: 3-19.

[9]. Kuznetsov AV and Nield DA, Natural convective boundary layer flow of a Nano-fluid past a vertical plate, International Journal of Thermal Sciences, Vol 288, 2010, pp: 243-247.

[10]. Buongiorno J, Convective transport in Nano-fluids, ASME Journal of Heat Transfer, Vol 128, 2006, pp: 240–250.

[11]. Nazir, M. Waqas, et al. "Effects of radiative heat flux and heat generation on magnetohydodynamics natural convection flow of nanofluid inside a porous triangular cavity with thermal boundary conditions." Numerical Methods for Partial Differential Equations 40.2 (2024): e22768.

[12]. Ali, Qasim, et al. "Prabhakar fractional model for natural convection flow of kerosene oil based hybrid nanofluid containing ferric oxide and zinc oxide nanoparticles." Case Studies in Thermal Engineering 60 (2024): 104648.

[13]. Khan WA and Aziz A, Natural convection flow of a Nano-fluid over a vertical plate with Uniform surface heat flux, International Journal of Thermal Sciences, Vol 50, 2011, pp: 1207-1214.

[14]. Hamad MAA and Pop I, Unsteady MHD free convection flow past a vertical permeable flat plate in a rotating frame of reference with constant heat source in a Nano-fluid, Heat Mass Transfer*,* Vol 47, 2011, pp: 1517– 1524.