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NUMERICAL ANALYSIS OF WAVY FLOW OF A NANOFLUID PAST A PLATE-FINITE DIFFERENCE APPROACH

[1]Penthala Rajitha, [2]S Karunakar Reddy
[1]sar.telangana@gmail.com
[2]karunakarreddy.shadagonda@gmail.com
[1,2] Assistant Professor, Department of Mathematics,
St.Mary's Group of Institutions Hyderabad

Abstract

This paper presents a numerical analysis of the wavy flow of Cu-water nanofluid past a permeable, vertical plate using the finite difference approach. The study investigates the effects of the Reynolds number (R), solid volume fraction (φ), and magnetic parameter (M) on the wavy flow characteristics. The results indicate that increasing R enhances fluid motion, influencing the velocity distribution along the wavy surface. A higher solid volume fraction (φ) improves the effective viscosity and thermal conductivity of the nanofluid, affecting the overall flow behavior. The presence of a magnetic field (M) introduces a Lorentz force that opposes fluid motion, modifying the velocity field and influencing stability. This study provides valuable insights into the behavior of wavy nanofluid flow in magnetohydrodynamic (MHD) environments, with potential applications in engineering fields such as heat exchangers, biomedical devices, and cooling technologies.

Keywords: Nanofluid, Wavy flow, Reynolds number, Solid volume fraction, Magnetic field, MHD, Finite difference method.

Introduction

Nanofluids have recently garnered significant attention in the context of convective heat and mass transfer across diverse fields such as flow boiling, biomedical engineering, and environmental systems. Despite the progress, there remains a pressing need for comprehensive studies that capture the complex behavior of nanoparticles in such convective transport processes. This chapter aims to fill that research gap by offering insights into the computational modeling of nanofluid-based convective heat and mass transfer across various geometries, including inclined plates, stretching sheets, and cylindrical surfaces. The book *Convective Heat and Mass Transfer* emphasizes numerical and theoretical investigations, providing a computational learning framework for researchers and practitioners. For example, Tiwari and Das [1] introduced a dual-component nanofluid model, demonstrating how nanoparticle volume fraction enhances natural convection in a differentially heated cavity.

Xuan and Li [4] emphasized Brownian motion's role in convective enhancement, while Ghasemi and Aminossadati [3] showed how inclination angle and nanoparticle properties affect convection patterns. Sheikholeslami and Ganji [2] used the lattice Boltzmann method (LBM) to simulate MHD nanofluid flow in inclined geometries. Selimefendigil and Öztop [5] employed a two-phase mixture model for analyzing MHD convection.

Li, Shuguang, et al. [6] conducted a theoretical study on Al_2O_3/H_2O and $(Al_2O_3 + Ag)/H_2O$ nanofluids near a solid sphere, exploring how nanoparticle composition affects thermal conductivity, fluid dynamics, and heat transfer efficiency. Raza, Qadeer, et al. [7] performed a numerical analysis on ternary hybrid nanofluids in a two-dimensional rotating porous medium, examining how variations in particle shape, size, and composition influence the thermal performance. Jain, Ruchi, et al. [8] numerically explored the heat and mass transfer characteristics of a Williamson hybrid nanofluid (CuO/CNTs-water) over a permeable stretching/shrinking surface under mixed convection and non-Newtonian rheology. Foundational models such as the thermal conductivity formulation proposed by Graham [9] and the viscosity model by Jang and Choi [10] are employed to describe the influence of nanoparticle size and volume fraction in these simulations. While these models provide valuable analytical support, the applicability to real-time systems remains open-ended, inviting further exploration by advanced researchers.

This chapter further delves into both free and forced convection of nanofluids past an inclined vertical plate in two-dimensional configurations. The suspension of nanoparticles in base fluids like water or ethylene glycol results in significantly enhanced thermal conductivity, thereby improving the efficiency of heat transfer systems. In automotive cooling applications, nanofluids help prevent engine overheating and contribute to better fuel efficiency. In electronics, they enable high-performance cooling of microprocessors. Industrial heat exchangers also benefit from enhanced heat dissipation, leading to energy savings and improved process control. Biomedical technologies such as hyperthermia treatment and targeted drug delivery exploit the controlled thermal responses of nanofluids for precise interventions. Holagh and Ahmed [11] provided a critical review of vertical gas-liquid slug flow, emphasizing its hydrodynamic impact on heat and mass transfer. Ahmad, Shafee, et al. [12] examined MHD natural convection in cavities with localized heating and cooling due to heat sources and sinks, revealing the role of magnetic fields in manipulating thermal and flow characteristics. In a broader context, Das et al. [13] outlined the contrasting requirements for heat transfer in energy systems and cooling technologies using nanofluids—highlighting the need for optimized heat transfer rates depending on the application.

The classical study by Kuznetsov and Nield [14] on natural convective boundary layer flow of nanofluids along a vertical flat plate utilized the model by Buongiorno [15], which introduced seven slip mechanisms. Among these, Brownian diffusion and thermophoresis are particularly significant in the absence of turbulence, making them central to current nanofluid research. Recent studies have expanded the understanding of radiative and magnetic effects. Nazir, M. Waqas, et al. [16] investigated MHD natural convection in a porous triangular cavity, accounting for radiative heat flux and internal heat generation. Their results inform thermal management strategies in applications such as energy storage and industrial systems. Ali, Qasim, et al. [17] applied the Prabhakar fractional model to study kerosene oil-based hybrid nanofluids containing ferric oxide and zinc oxide nanoparticles, revealing the influence of fractional dynamics on heat and mass transport.

Natural convection of nanofluids over a vertical plate is a widely studied phenomenon due to its relevance in enhancing thermal performance. The fluid rises due to buoyancy forces, and the suspended nanoparticles enhance thermal conductivity, improving heat dissipation. Parameters such as nanoparticle type, solid volume fraction, and surface heating conditions all play a role in optimizing performance for real-world applications such as solar panels, automotive cooling, and electronics. Khan and Aziz [18] studied this phenomenon for nanofluids over a vertical plate with a uniform heat flux, emphasizing the enhancement of heat transfer due to nanoparticle suspension. Hamad and Pop [19] examined unsteady MHD free convection over a permeable vertical flat plate, highlighting how increased solid volume fraction and localized heat sources can substantially improve heat transfer efficiency. Sundar et al. [20] provided experimental validation using Fe_3O_4-based magnetic nanofluids in pipe flows, directly supporting the practical utility of these fluids in thermal engineering.

Mathematical Formulation

Assumptions of the flow with heat transfer in the geometry are as follows:

- The study considers unsteady convective flow of a Copper–water nanofluid past a vertical, semi-infinite moving plate.

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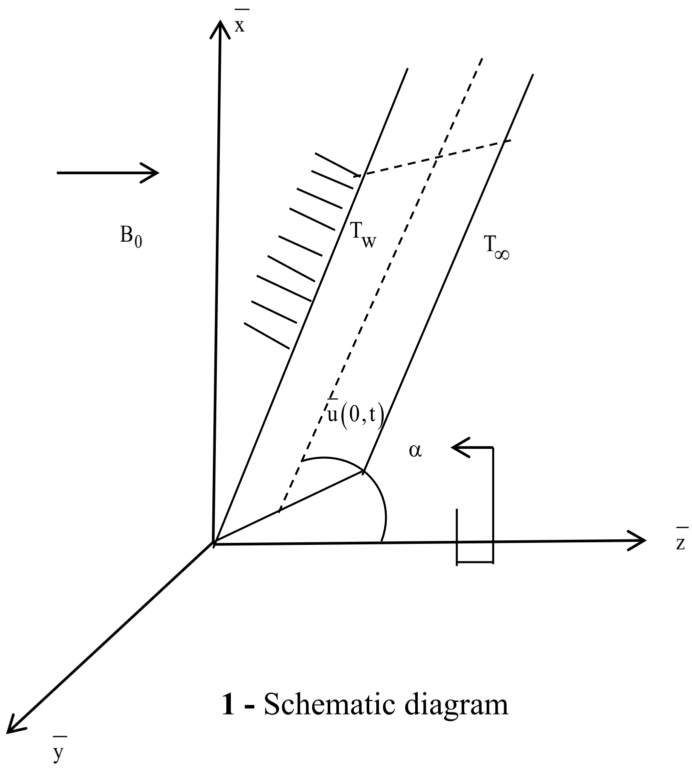
- The plate moves in the positive z-direction with a constant velocity U0.
- The nanofluid consists of water as the base fluid and copper (Cu) nanoparticles.
- The fluid is electrically conducting, and a uniform transverse magnetic field is applied perpendicular to the flow.
- The induced magnetic field is assumed to be negligible compared to the applied magnetic field.
- Initially, the fluid and the plate are at rest.
- The surface temperature of the plate is maintained at a constant Tw , and the ambient fluid temperature is $T\infty$, such that $Tw>T\infty$.
- Thermal equilibrium is assumed between the base fluid and suspended nanoparticles; no slip occurs between them.
- The fluid is incompressible and follows the Boussinesq approximation (density variation only in the buoyancy term).
- A Cartesian coordinate system is used, with the z-axis along the vertical direction of the plate.



1 - Schematic diagram

Governing Equations

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The boundary layer equations governing the flow and temperature as per above assumptions are as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad \textbf{(1)}$$

$$\rho_{nf}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu_{nf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\sigma {B_0}^2 u \quad \textbf{(2)}$$

$$\rho_{nf}\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=\mu_{nf}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \quad \textbf{(3)}$$

The boundary conditions for this problem are as follows:

$$u(x,y,0)=0, v(x,y,0)=0$$

$$u(0,y,t')=0, v(0,y,t')=0$$

$$u(x,0,t')=x, v(x,0,t')=1$$

$$u(b,y,t')=0, v\left(x,\frac{b}{2},t'\right)=b+x \ , v\left(x,\frac{b}{2},t'\right)=0 \quad \textbf{(4)}$$

Thermo-Physical properties were related as follows:

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s, \alpha_{nf}=\frac{k_{nf}}{(\rho c_p)_{nf}}$$

$$(\rho c_p)_{nf}=(1-\phi)(\rho c_p)_f+\phi(\rho c_p)_s$$

$$(\rho\beta)_{nf}=(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_s$$

Here we introduce the following dimensionless variables as:

$$X=\frac{x}{L}, Y=\frac{y}{L}, t=t'\frac{U_0}{L}, U=\frac{u}{U_0}, V=\frac{v}{U_0} \quad \textbf{(5)}$$

Using equations (6), (7), (8) the equations (2), (3) & (4) can also be written in the following dimensionless form

$$\frac{\partial U(x,y,t)}{\partial t}+U(x,y,t)\frac{\partial U(x,y,t)}{\partial X}+V(x,y,t)\frac{\partial U(x,y,t)}{\partial Y}=\frac{1+(2.5)\emptyset}{(1-\emptyset+\emptyset(\frac{\rho_s}{\rho_f}))R}\left(\frac{\partial^2 U(x,y,t)}{\partial X^2}+\frac{\partial^2 U(x,y,t)}{\partial Y^2}\right)-MU(x,y,t) \quad (6)$$

$$\frac{\partial V(x,y,t)}{\partial t}+U(x,y,t)\frac{\partial V(x,y,t)}{\partial X}+V(x,y,t)\frac{\partial V(x,y,t)}{\partial Y}=\frac{1+(2.5)\emptyset}{(1-\emptyset+\emptyset(\frac{\rho_s}{\rho_f}))R}\left(\frac{\partial^2 V(x,y,t)}{\partial X^2}+\frac{\partial^2 V(x,y,t)}{\partial Y^2}\right) \quad \textbf{(7)}$$

Here the corresponding boundary condition of equation (5) was written in the dimensionless form as:

$$U(x,y,0)=0, V(x,y,0)=0$$

$$U(0,y,t)=0, V(0,y,t)=\cos\left(\frac{\pi}{6}y\right)$$

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$$U(x,0,t) = x, V(x,0,t) = 0$$

$$U(b,y,t) = 0, V\left(x,\frac{b}{2},t\right) = b + x \ , V\left(x,\frac{b}{2},t\right) = 0, V(b,y,t) = \frac{b}{2} + \cos\left(\frac{\pi}{6}\right)y \quad \textbf{(8)}$$

Where the parameters present in the above equations are as follows:

$Pr = \frac{\nu_f}{\alpha_f}, (Pr\,a\,ndtlNumber)$,

$M \quad = \frac{\sigma {B_0}^2 L}{U_0 \rho_f}, (Magneticfieldparameter)$,

$Re = \frac{L U_0}{\nu}, (Re\,y\,noldsNumber)$,

Solution of the Problem

The Numerical Solution and Heat Transfer Quantification for the The system of nonlinear**,** coupled differential equations represented by equations (9)–(11) is solved subject to the boundary conditions specified in equation (12).The analysis is performed in two dimensions, considering a computational domain defined over an infinite rectangular plate. For numerical computation, the plate is assumed to have a unit width and a height of two units.The system is solved using Mathematica10.4 via the NDSolve function, which enables the numerical integration of differential equations over the specified domain. The dimensionless Nusseltnumber **(**Nu**)**, which quantifies the rate of local heat transfer at the plate surface, is defined as:

$$Nu = -\frac{k_{nf}}{k_f}\theta'(0)$$

where:

k_{nf} is the thermal conductivity of the nanofluid,

$k_f$is the thermal conductivity of the base fluid,

$\theta'(0)$denotes the gradient of the dimensionless temperature at the wall.

Discussion of the Results

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The graphs in Figures 2 to 7 illustrate the influence of key physical parameters—Reynolds number (Re), solid volume fraction (φ), magnetic parameter (M), on the flow velocities (u and v) and temperature (θ) for both forced and natural convection regimes.

For the simulations, the vertical velocity (u) and temperature (θ) profiles are observed at the mid-height of the domain (y = 1/2), while the horizontal velocity (v) is evaluated along the vertical section (x = 1). The results highlight contrasting behaviors in natural and forced convection: natural convection shows stronger vertical motion due to buoyancy forces. Variations in parameters like φ and M generally lead to reduced velocities due to increased resistance (via nanoparticle loading or Lorentz forces), whereas the Reynolds number can either accelerate or retard flow depending on the convection mode. These insights help in tailoring nanofluid flow conditions for optimized thermal systems.

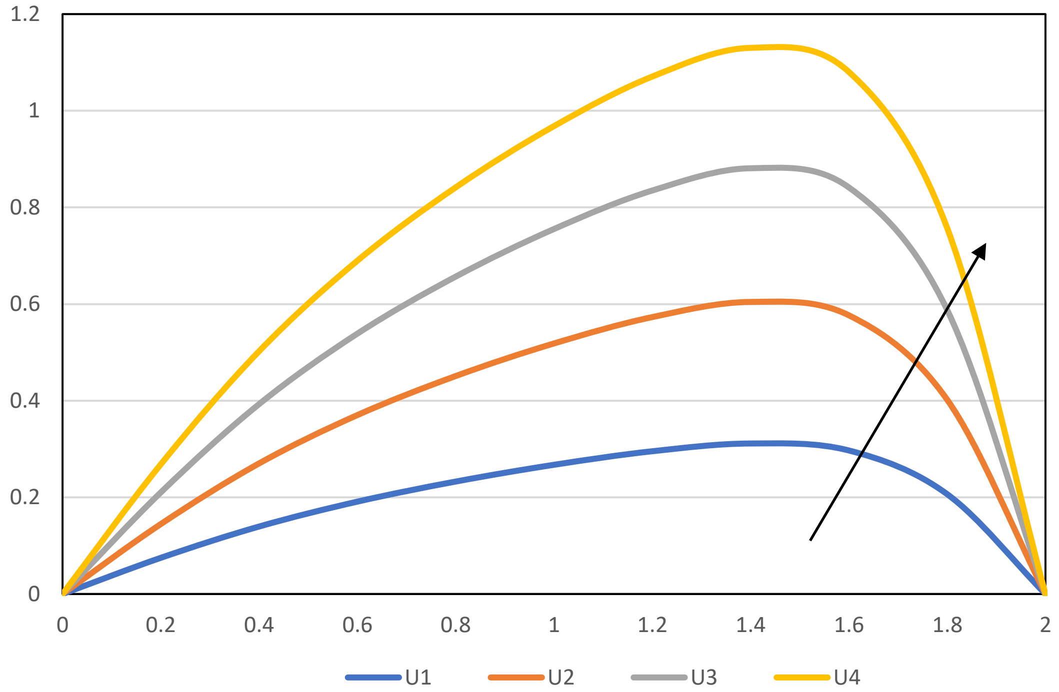

Fig.2. Profile of vertical velocity u with Re

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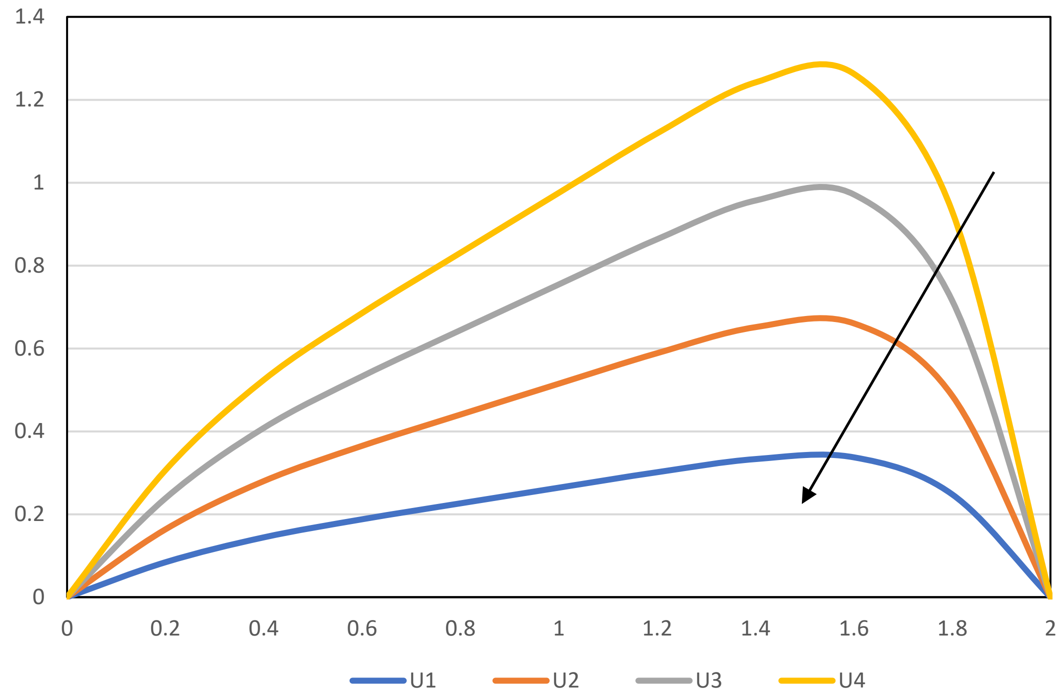

Fig.3 Profile of vertical velocity u with φ

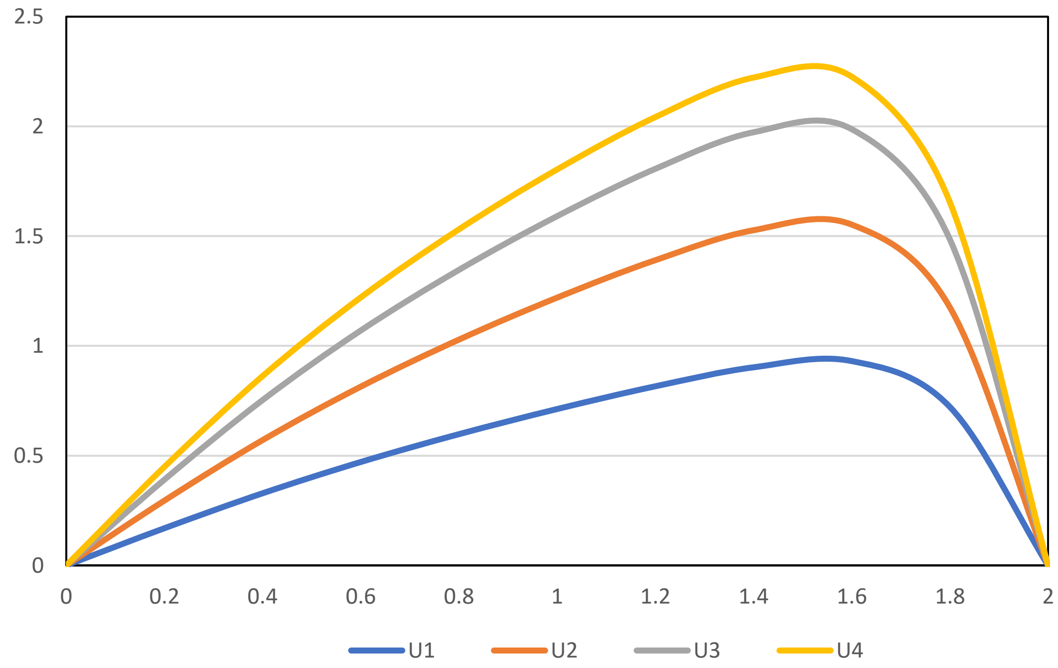

Fig.4 Profile of vertical velocity u with M

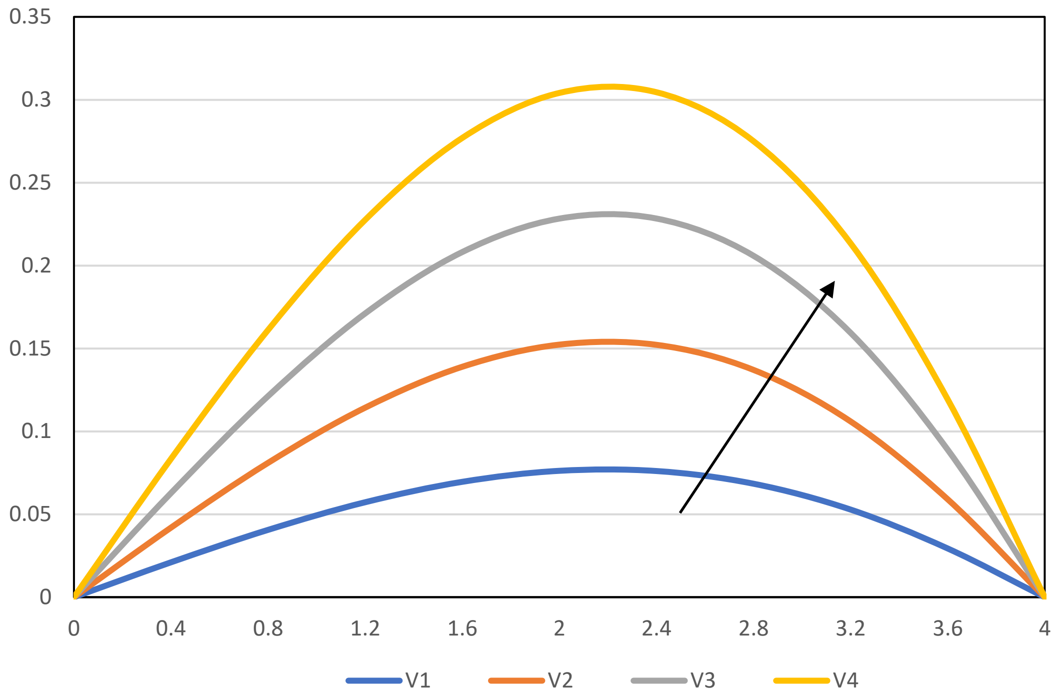

Fig.5 Profile of v with Re

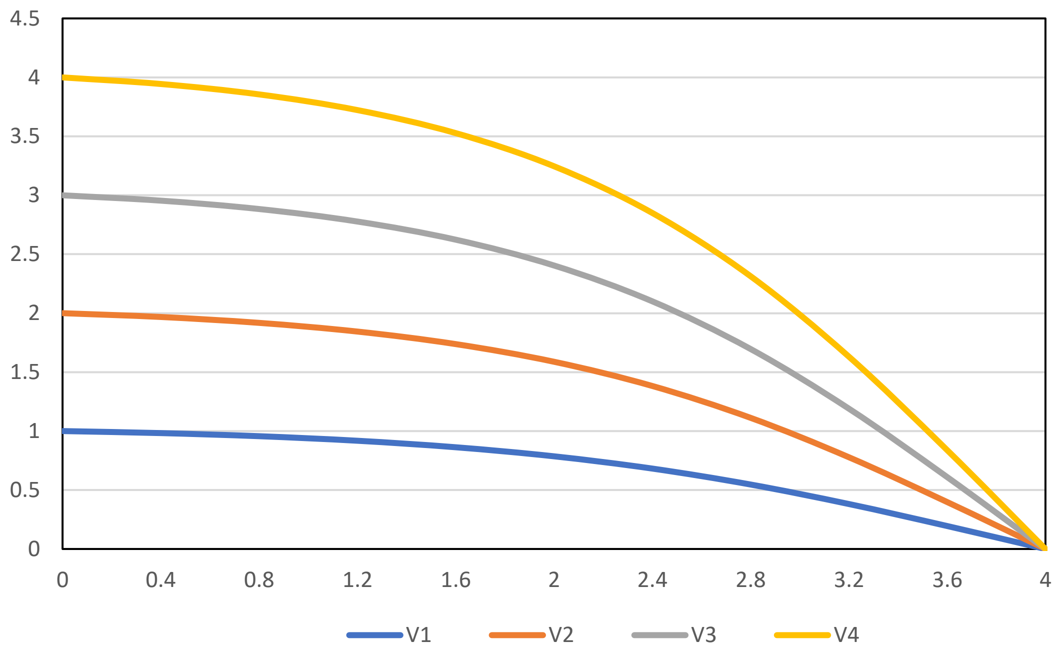

Fig.6. Profile of v with φ

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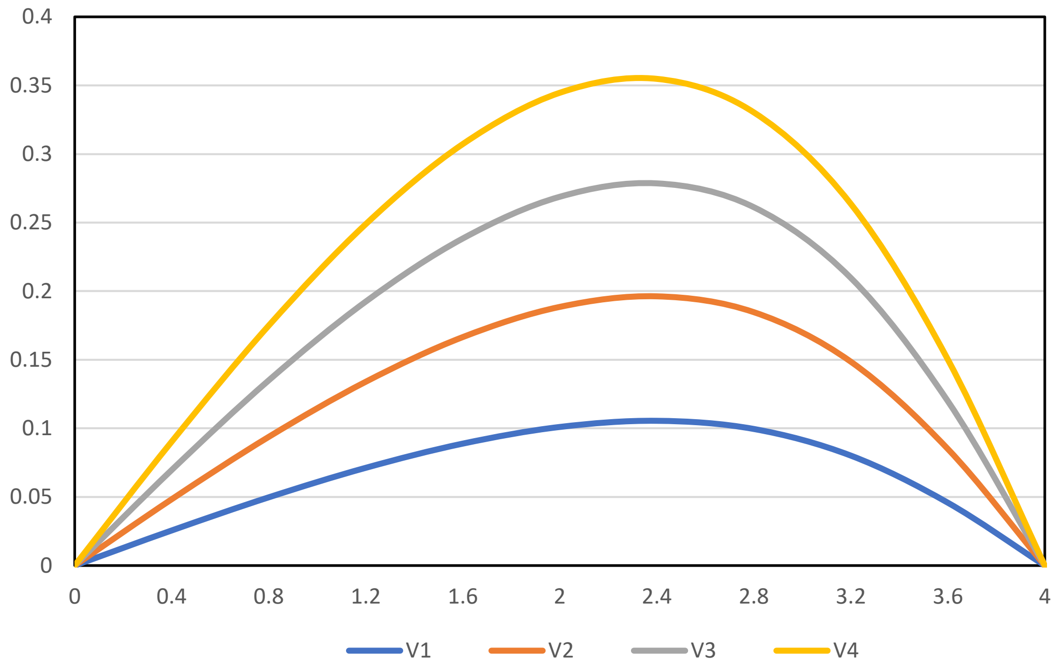

Fig.7. Profile of v with M

The vertical velocity profiles (u) illustrated in Figures 2 to 4 demonstrate how different physical parameters influence the flow behavior in both natural and forced convection. In Figure 2, as the Reynolds number (Re) increases, the vertical velocity rises significantly in natural convection due to the dominance of buoyancy and inertial forces, which accelerate the flow upward. In contrast, in forced convection, a higher Reynolds number leads to a decrease in vertical velocity, showing that stronger inertial forces suppress vertical motion and instead enhance horizontal flow. This inverse behavior highlights the different driving mechanisms between the two convection types.

Figure 3 shows the effect of solid volume fraction (φ), where the vertical velocity decreases gradually with an increase in φ for both convection modes. This is attributed to the increased density and viscosity caused by suspended nanoparticles, which dampen the motion. Figure 4 further reveals that the magnetic parameter (M), representing the influence of a magnetic field, consistently suppresses the vertical velocity due to the Lorentz force. This force resists fluid motion, especially in conductive fluids like nanofluids. Notably, natural convection maintains a higher vertical velocity across all variations, indicating its stronger reliance on thermal buoyancy compared to the mechanically driven forced convection.

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Conclusions

- Rate of heat transfer is more when plate tends to be horizontal in free convection but no significance observed in forced convections.
- Inclination angle enhances the vertical flow but not significant in horizontal flow.

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