

MHD Flow and the characteristics of Heat and Mass transfer past an Exponential Stretching Sheet with heat source/sink

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ABSTRACT

In this paper, we analyzed the effect of a suction and Soret number on heat and mass transfer MHD flow past an exponentially stretching sheet with the heat source/sink. Using similarity transformation the system of PDEs is changed into a system of non-linear ODEs, which was then solved numerically by Rungakutta fourth order method together with shooting technique. The Numerical results are obtained for the skin friction coefficient, Nusselt and Sherwood numbers for selected values of the governing parameters, such as the suction, magnetic field parameter M , viscous dissipation parameter Ec , heat generation parameter δ , Schmidt number Sc , and the chemical reaction rate parameter K_1 . Besides, it is obtained that the concentration profile decreases with an increment of the Schmidt number. A comparison was made with a previous study available in the literature and we found that it is in a good agreement.

Key words: MHD, Boundary layer flow, Suction, stretching sheet.

1. INTRODUCTION

MHD boundary layer flow of heat and mass transmit over a stretching sheet has wide applications in industrial and manufacturing process. Some of its applications are hot rolling, wire drawing, glass-fiber, and paper production, drawing of plastic films, metal and polymer extraction, and metal spinning. In the process of manufacturing the following activities like simultaneous heating or cooling and kinematics of stretching has a decisive influence on the quantity of the final products (Magayari and Keller [1]). The first person who investigated the similarity solution for laminar boundary layer flow and the transfer of heat over a stretching surface was Crane [2]. Later, many researchers have studied on the stretching sheet problems and presented a detailed analysis by including different aspects, such as heat flux, permeability and unsteadiness characteristics, etc. Carragher and Crane [3], Dutta [4], Grubka and Bobba [5], Elbassebshy [6], Elbashareshy and Bazid [7], Mahapatra and Gupta [8] are among the

researchers who studied on it. Recently Mukhopahyay[9] presented heat transfer and MHD boundary layer flow towards an exponentially stretching sheet embedded in a thermally stratified medium subject to suction. He has shown that increasing the magnetic parameter reduces the fluid velocity.

Viscous dissipation has extensive industrial applications: for example, a considerable temperature rises are observed in polymer processing flows such as injection molding or extraction at high rates. Aerodynamic heating in the thin boundary layer around high-speed aircraft raises the temperature of the skin. The processes of converting mechanical energy of downward flowing water into thermal and acoustical energy are dissipation. Jena et al. [10] considered the diffusion-thermo (Dufour) and thermal diffusion (Soret) impact on MHD viscoelastic fluid flow over a porous vertical stretching sheet subject to variable magnetic field embedded in a porous medium in the attendance of chemical reaction and heat source/sink.

Seini and Makinde [11] have examined an MHD boundary layer flow of a viscous incompressible steady fluid over an exponentially stretching sheet with the impact of a homogeneous chemical reaction and radiation. Their result indicated that raising the values of the transverse magnetic field and radiation parameter reduce heat transfer rate at the surfaces. Heat and mass transfer on a boundary layer of an electrically conducting viscous fluid through a porous media over an exponentially stretching sheet with an impact of a magnetic field was investigated by Swain et al. [12]. In their study, they also considered the effect heat source/sink and thermal radiation. In the context of exothermic and endothermic chemical reactions, heat generation is very valuable. Mass transfer effect on MHD flow past an impulsively started infinite vertical plate was presented by Shankar and Kishan [13]. Furthermore, flow on an MHD boundary layer, heat and mass transfer of an incompressible viscous and radiating fluid due to an exponentially stretching sheet were studied by Devi et al. [14].

An MHD flow on a boundary layer and the characteristics of heat transfer of a non-Newtonian viscoelastic fluid over a flat plate with a linear velocity in the attendance of non-uniform heat source and thermal radiation were inspected by Abel and Mahesha [15]. In addition, the MHD flow of a Newtonian liquid and the properties of heat and mass transfer over an exponentially stretching sheet with radiation effect have been examined by Kameswaran et al. [16]. Their result showed that the Prandtl number and radiation parameter have an inverse effect on temperature

profile. Moreover, Khalili et al. [17] examined MHD boundary layer flow past an exponentially stretching with chemical reaction, radiation and heat sink. They have observed that the reaction rate parameter affected the concentration profiles significantly and the concentration thickness of boundary layer decreases when the reaction rate parameter increases.

A vast body of knowledge encompassing analytical and numerical studies explaining various aspects is now available on the stretching flow [18]–[27]. To the best of the author's knowledge, the effect of chemical reaction and viscous dissipation on MHD flow past an exponentially stretching sheet with a heat sink is not studied adequately in a comprehensive way. Hence, this problem is investigated. The aim of this investigation is to discuss such a flow problem. Similarity variables have been used to transform the governing PDEs equations into a nonlinear ordinary differential equations. So as to reveal the impact of various governing parameters on the velocity, a temperature, concentration, coefficient of skin friction, Nusselt number and Sherwood number a parametric analysis is accomplished and discussed in detail.

Nomenclature:	
V_0, k_0, Q_0	are constants
B_0	magnetic field strength
C	concentration of the fluid in the boundary layer
C_0	reference concentration
C_f	skin friction coefficient
C_p	specific heat at constant pressure
C_ω	concentration near the surface
C_∞	concentration far away from the surface
D	mass diffusivity coefficient through temperature gradient
Ec	viscous dissipation parameter
f'	dimensionless velocity of the fluid
j_ω	mass flux
k	thermal conductivity of the fluid
L	reference length
M	magnetic field parameter
Nu	local Nusselt number
Pr	Prandtl number
q_ω	surface heat flux

Re	Reynolds number
S	suction
Sc	Schmidt number
Sh	local Sherwood number
T	temperature of the fluid
T_ω	temperature at the surface of the sheet
T_∞	ambient temperature of the fluid
T_0	reference temperature
U_ω	velocity of the stretching surface
U_0	characteristic velocity
Greek symbols:	
ρ	density of the fluid
θ	dimensionless temperature of the fluid
τ_ω	wall shear stress
ϕ	dimensionless concentration of the fluid
η	similarity variable
ν	kinematic viscosity of the fluid
δ	heat generation parameter
σ	electrical conductivity of the fluid

k_1 : chemical reaction rate parameter
μ : coefficient of viscosity
Subscripts:

ω : condition at the wall
∞ : condition at infinity

2. MATHEMATICAL FORMULATION

In this article, a laminar, two dimensional steadyflow of an incompressible viscous, electrically conducting fluid over a continuous exponentially stretching surface is considered. The origin of the system is positioned at the slit from which the sheet is drawn. In this coordinate the of frame the x – axis is taken along the path of the continuous stretching plane.

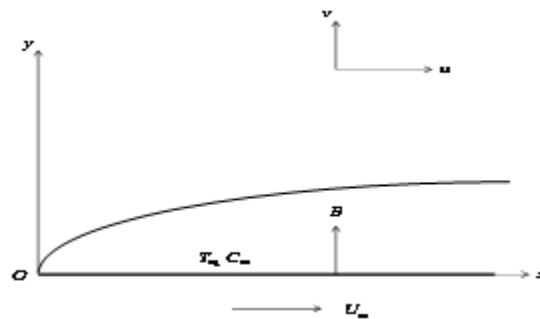


Figure 1. Schematics of the problem.

The y – axis is measured normal to the surface of the sheet. The sheet velocity is supposed to vary as an exponential function of the distance x from the slit. The temperature and concentration far away from the fluid as displayed in Figure 1 are denoted by T_∞ and C_∞ , respectively. The concentration differences and ambient temperature sheet are also supposed to be exponential functions of the distance x from the slit. A variable magnetic field of strength pertained normal to the sheet is denoted by $B(x)$. The variable chemical reaction is imagined to be $k_p(x)$ and variable heat sink parameter $Q^*(x)$, where k_0 and Q_0 are constants. The parameters $T_w, C_w,$ and B_0 stands for the temperature at the surface of the sheet, concentration at the surface of the sheet and magnetic field strength respectively.

Under the above assumptions, the governing equation of the momentum, heat and mass transfer transports subject to viscous dissipation and heat generation can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho} u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q^*(x)}{\rho c_p} (T - T_\infty) + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2 \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_p (C - C_\infty) \quad (4)$$

The velocity components in the x and y axis is symbolized by u and v respectively and the kinematic viscosity ν , the fluid density ρ , the temperature of the fluid T , the thermal conductivity of the fluid k , the specific heat at constant pressure c_p , the boundary layer fluid concentration C , the mass diffusivity coefficient D , and the chemical reaction rate parameter is represented by k_1 .

The boundary conditions associated for the velocity, temperature and concentration profiles are

$$\left. \begin{aligned} u &= U_\omega = U_0 e^{\frac{x}{2L}}, v = -V_\omega(x) \\ T &= T_\omega = T_\infty + T_0 e^{\frac{x}{2L}} \\ C &= C_\omega = C_\infty + C_0 e^{\frac{x}{2L}} \end{aligned} \right\} \text{at } y = 0 \quad (5)$$

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty, \text{ as } y \rightarrow \infty \quad (6)$$

where, U_ω is the uniform velocity of the sheet and L is the reference length. Introducing the following dimensionless quantities, the mathematical analysis of the problem is simplified by establishing the following similarity transformations:

$$\left. \begin{aligned} \eta &= \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} y, u = U_0 e^{\frac{x}{2L}} f'(\eta), \\ v &= -\sqrt{\frac{\nu U_0}{2L}} e^{\frac{x}{2L}} \left(f(\eta) + \eta f'(\eta) \right), \\ T &= T_\infty + T_0 e^{\frac{x}{2L}} \theta(\eta), \\ C &= C_\infty + C_0 e^{\frac{x}{2L}} \phi(\eta), \\ B(x) &= B_0 e^{\frac{x}{2L}}, \\ K_p(x) &= K_0 e^{\frac{x}{2L}}, \\ Q^*(x) &= Q_0 e^{\frac{x}{2L}}, \\ V_w(x) &= V_0 e^{\frac{x}{2L}}. \end{aligned} \right\} \quad (7)$$

Where η is the similarity variable, $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ is the dimensionless temperature, and $\phi(\eta)$ is the dimensionless concentration, u and v are also defined using the stream function as:

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}$$

which satisfies equation (1). Then, by using similarity transformation quantities, the governing equations (2)-(4) are transformed into the ordinary differential equation as follows:

$$f''' + ff'' - 2f'^2 - Mf' = 0 \tag{8}$$

$$\theta'' + Pr(f\theta' - f'\theta + \delta\theta + Ec f'^2) = 0 \tag{9}$$

$$\phi'' + Sc(f\phi' - f'\phi - k_1\phi) = 0 \tag{10}$$

with boundary conditions:

$$\left. \begin{aligned} f(0) = S, f'(0) = 1, \theta(0) = 1, \phi(0) = 1 \text{ at } \eta = 0 \\ f'(\eta) = 0, \theta(\eta) = 0, \phi(\eta) = 0 \text{ as } \eta \rightarrow \infty \end{aligned} \right\} \tag{11}$$

The parameters involved in the above equations are the Eckert number Ec , magnetic parameter M , Prandtl number Pr , heat generation parameter δ , Schmidt number Sc , chemical reaction rate parameter k_1 , and the Suction parameter S . These parameters are defined by:

$$\left. \begin{aligned} S = \frac{V_0}{\sqrt{\frac{\nu U_0}{2L}}}, M = \frac{2\sigma B_0^2 L}{\rho U_0} \\ Ec = \frac{U_\omega^2}{c_p(T_\omega - T_\infty)}, \delta = \frac{2LQ_0}{\rho c_p U_0} \\ k_1 = \frac{2k_0 L}{U_0} \\ Sc = \frac{\nu}{D} \end{aligned} \right\} \tag{12}$$

The physical quantities which are involved are the skin friction coefficient, the local Nusselt number, and the local Sherwood number. These quantities can be defined as

$$C_f = \frac{2\tau_\omega}{\rho U_\omega^2} \tag{13}$$

$$Nu = \frac{xq_\omega}{k(T_\omega - T_\infty)} \tag{14}$$

$$Sh = \frac{xj_\omega}{D(C_\omega - C_\infty)} \tag{15}$$

where τ_ω , q_ω , and j_ω are the shear stress, heat flux and mass flux at the surface respectively, and they are defined by

$$\tau_\omega = -\mu \left(\frac{\partial u}{\partial y} \right)_{y=0} = \frac{-\mu U_0}{L} \sqrt{\frac{Re}{2}} e^{\frac{3x}{2L}} f''(0) \quad (16)$$

$$q_\omega = -k \left(\frac{\partial T}{\partial y} \right)_{y=0} = \frac{-k(T_\omega - T_\infty)}{L} \sqrt{\frac{Re}{2}} e^{\frac{x}{2L}} \theta'(0) \quad (17)$$

$$j_\omega = -D \left(\frac{\partial C}{\partial y} \right)_{y=0} = \frac{-D(C_\omega - C_\infty)}{L} \sqrt{\frac{Re}{2}} e^{\frac{x}{2L}} \phi'(0) \quad (18)$$

Using equations (16)-(18), equation (13)-(15) can be transformed in to

$$\frac{\sqrt{\frac{Re}{2}} C_f}{\sqrt{\frac{x}{L}}} = -f''(0) \quad (19)$$

$$\frac{Nu}{\sqrt{\frac{x}{L}} \sqrt{\frac{Re}{2}}} = -\theta'(0) \quad (20)$$

$$\frac{Sh}{\sqrt{\frac{x}{L}} \sqrt{\frac{Re}{2}}} = -\phi'(0) \quad (21)$$

Where μ is the coefficient of viscosity and $Re = \frac{U_0 L}{\nu}$ is the Reynolds number.

3. SOLUTION BY OHAM

The non-linear ordinary differential equations (8)-(10) which is transformed together with boundary conditions (11) are solved by Runge-Kutta fourth order method along with shooting technique. Based on this method on the given problem the following assumptions are made

$$\left. \begin{aligned} f &\rightarrow f(1), f' \rightarrow f(2), f'' \rightarrow f(3), \theta \rightarrow f(4), \\ \theta' &\rightarrow f(5), \phi \rightarrow f(6), \phi' \rightarrow f(7) \end{aligned} \right\}$$

And rewriting equations (8)-(10) with their boundary conditions in the form of

$$f''' = -ff'' + 2f'^2 + Mf' \quad (22)$$

$$\theta'' = -Pr(f\theta' - f'\theta + \delta\theta + Ec f'^2) \quad (23)$$

$$\phi'' = -Sc(f\phi' - f'\phi - k_1\phi) \quad (24)$$

The appropriate boundary conditions are:

$$\left. \begin{aligned} f_a(1) = S, f_a(2) = 1, f_a(4) = 1, f_a(6) = 1, \\ f_b(2) = 0, f_b(4) = 0, f_b(6) = 0 \end{aligned} \right\} \quad (25)$$

The initial and boundary condition points are indicated by a and b i.e. $a = 0, b = \infty$. It supposed the step size $\Delta\eta = 0.01$ and the accuracy convergence criteria to be a five decimal value before solving the problem by the explained method using Matlab program,.

4. RESULTS AND DISCUSSIONS

The transformed ODEs of momentum, energy and concentration Eqs. (8)-(10) subjected to the boundary conditions (11) were analytically solved by using RungeKutta fourth order method along with shooting technique. We obtained the graph of velocity, temperature, and concentration profile for diverse values of governing parameters. The results obtained are displayed through tables and figures. The coefficient of skin friction for various values of the magnetic parameter and for a fixed value of $S=Ec=Pr=\delta = k1=Sc=0$ is displayed in Table 1. We have observed that an increase of the magnetic parameter M raises the skin friction coefficient; as a result of the opposition to the flow caused by the induced Lorentz force. The heat transfer coefficients are revealed in Table 2 for different Prandtl number Pr . It is clear that the Nusselt number raises with an increase of Prandtl numbers. In both cases, the present result is in a good agreement with the previously published results.

Table 1. A comparison of the skin friction coefficient $-f''(0)$ for different values of M and for fixed values of $S=Ec=Pr=\delta = k1=Sc=0$.

M	$-f''(0)$		
	Kameswaran et al. [16]	Sai et al. [28]	Present
0	1.28181	1.29038	1.281861
1	1.62918	1.63038	1.629190
2	1.91262	1.91285	1.912625
3	2.15874	2.15879	2.158743
4	2.37937	2.37938	2.379382

Table 2. Values of Nusselt number $\theta'(0)$ in comparison between current study and previous study for varies values of Pr and for fixed values of $S=Ec=M=\delta = k1=Sc=0$.

Pr	$-\theta'(0)$		
	Devi et al. [29]	Khalili et al. [17]	Present
1	0.954811	0.954955	0.954956
2	1.471454	1.471421	1.471422
3	1.869609	1.869044	1.869045
5	2.500128	2.500109	2.500106

Fig. 2 displays the suction parameter S consequence on the velocity profile. We observed that the velocity profile decreases with an increase in the suction parameter. The impact of the suction parameter on the temperature profile is illustrated by Fig. 3. It is watched that the temperature profile is reduced with an enhancement of the suction parameter S . The thickness of the thermal boundary layer lessened with an increase of the suction parameter. The influence of the suction parameter S on the concentration profile is displayed using Fig. 4. Thus, we observed that an increment o the suction parameter S reduces the concentration profile.

Fig. 5 shows the influence of a viscous dissipation parameter Ec on the temperature field. It is seen that the thickness of the thermal boundary layer highers with an enlargement of viscous dissipation parameter Ec . Fig. 6 exemplifies the heat generation parameter δ influence on the concentration profile. It is watched that the heat generation parameter δ reduces the concentration profile. The Prandtl number influence on the temperature profile is displayed by Fig. 7. Hence, large values of Pr reduces the temperature distribution. It is due to the fact that Pr has an inverse relationship with the thermal conductivity of a fluid, as a consequence, the thermal boundary layer thickness also decreases. The Schmidt number Sc and chemical reaction $k1$ impact on the concentration profile is shown by Fig. 8 and Fig. 9, respectively. The result has displayed that, an increment of both parameters reduces the concentration profile. In the case of Schmidt number Sc , increasing the size of Sc means decreasing the diffusivity of the fluid, this causes the fluid less concentrated. The influences of magnetic field parameter M on the velocity

field and temperature profile are shown by Fig. 10 and Fig. 11, respectively. It is obtained the temperature profile increases while the velocity profile decreases with an increment of M . Since a Lorentz force is created due to the occurrence of M that slows down the motion of the fluid, as a result, the distribution of velocity is reduced and the thermal boundary layer thickness is increased.

5. CONCLUSIONS

In this article, the effect of different flow parameters on the dimensionless velocity profile, temperature profile, the concentration profile is considered. The governing problem is solved numerically using RK fourth order method along with shooting technique. It is observed that:

- (i) Increment of S has a reducing effect on the velocity, temperature and concentration profiles.
- (ii) An increase of the viscous dissipation parameter Ec increases the temperature profile,
- (iii) Increasing the heat generation parameter δ enhances the temperature profile.
- (iv) As the values of Sc and k_1 increases the concentration profile decreases.
- (v) The Prandtl number Pr has a reducing effect on the temperature profile..

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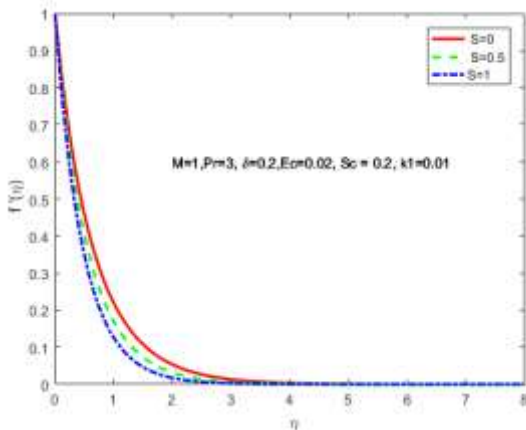


Fig. 2 The influence of S on the velocity field.

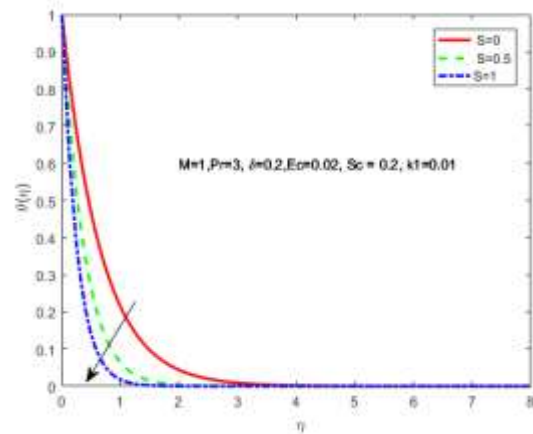


Fig. 3 The influence of S on the temperature field.

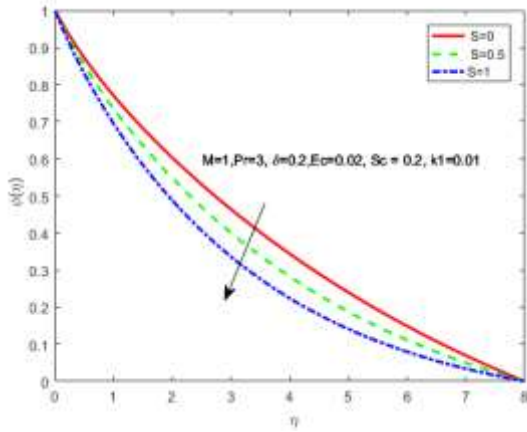


Fig. 4 The influence of S on the concentration distribution..

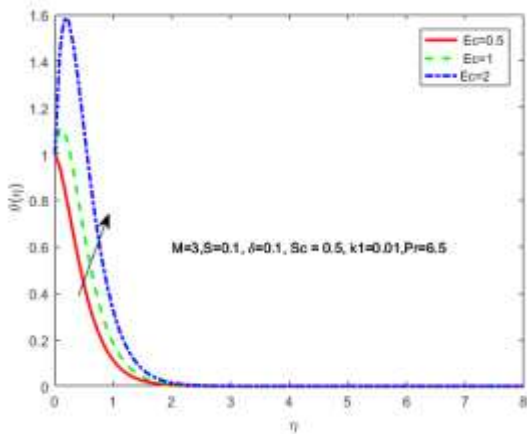


Fig. 5 Temperature profile with an effect of Ec .

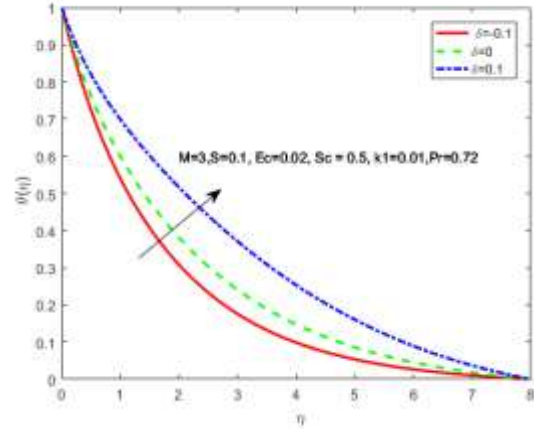


Fig.6 Temperature profile with a change of δ .

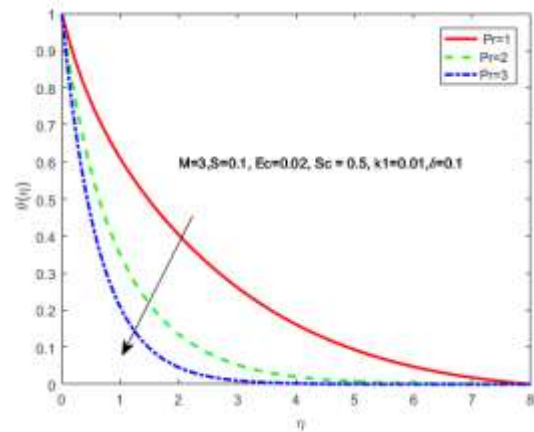


Fig.7 Temperature profile with variation of Pr .

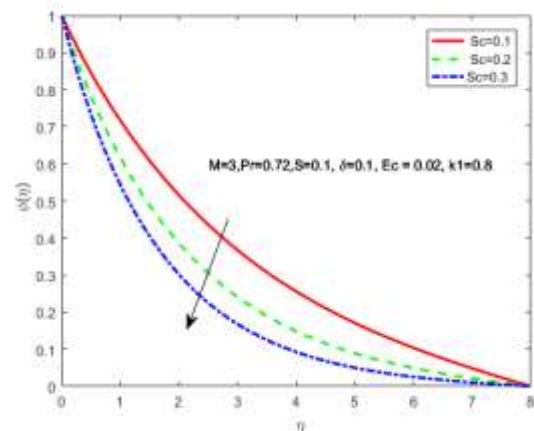


Fig.8 The impact of Sc on the concentration profile.

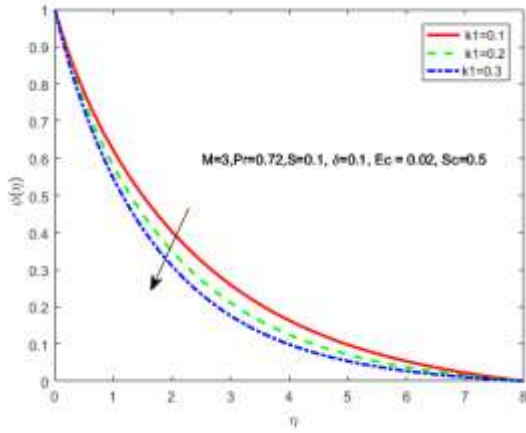


Fig.9 The impact of $K1$ on the concentration profile.

Fig. 10 The behavior of velocity profile with a variation of M .

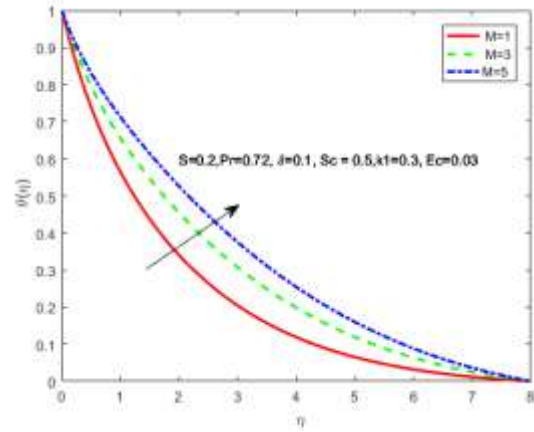


Fig. 11 the impact of M on the temperature.

