

# COMPACTNESS IN QUIN TERRANEAN FUZZY TOPOLOGICAL SPACES

N.CHITRA, RESEARCH SCHOLAR<sup>1</sup>, Dr.C.JANAKI,ASSOCIATE PROFESSOR<sup>2</sup>

Department of Mathematics, L.R.G. Government Arts College for Women, Tirupur - 4

## ABSTRACT:

Quin-Terranean fuzzy sets are newly introduced concept of fuzzy witch is an extension of Fermatean fuzzy sets, Pythagorean fuzzy sets, Intuitionistic fuzzy sets and fuzzy sets. Quin-Terranean fuzzy sets are most effective tool to deal the uncertainty. By using the concept of Quin-Terranean fuzzy sets, Quin-Terranean fuzzy topological spaces are defined by us. In this paper we introduce the concept of Quin-Terranean fuzzy compactness. Also we have defined various types of Quin-Terranean fuzzy compactness namely Quin-Terranean fuzzy almost compactness and Quin-Terranean fuzzy near compactness. Quin-Terranean fuzzy almost compactness is described in terms of Quin-Terranean fuzzy regular closed or Quin-Terranean fuzzy regular open. Moreover, we analyze how Quin-Terranean fuzzy compactness interacts with various forms of Quin-Terranean fuzzy continuity. Also we have studied various properties of Quin-Terranean fuzzy compactness. We prove that the image of a Quin Terranean fuzzy compact space under a fuzzy continuous mapping which is also surjective remains Quin Terranean fuzzy compact.

## KEY WORDS :

**Quin-Terranean Fuzzy sets, Quin-Terranean Fuzzy Topology, Quin-Terranean Fuzzy Continuity, Quin-Terranean Fuzzy Compactness.**

## 1. INTRODUCTION

In 1965, Zadeh [14] introduced fuzzy sets, while Chang [3] later developed the theory of fuzzy topological spaces. In [4,5,6,7,8] certain weaker variations of fuzzy compactness were explored for the first time. Atanassov [1,2] subsequently introduced the idea of intuitionistic fuzzy sets, leading Çoker [4,5] to define intuitionistic fuzzy topological spaces. Eş and Çoker [4,5] further examined properties such as fuzzy almost compactness, fuzzy near compactness, and fuzzy light compactness within intuitionistic fuzzy topological spaces. In 2014, Yager [12,13] proposed the concept of Pythagorean fuzzy subsets, a novel class of non-standard fuzzy sets with significant applications in natural and social sciences. Inspired by Chang's work [4,5] some researchers [10] extended this idea to study Pythagorean fuzzy topological spaces. In 2019, Haydares [9] developed by introducing the notion of Pythagorean fuzzy compactness. The concept of Quin Terranean fuzzy sets which are super set of Fermatean fuzzy sets were introduced by Rajarajeswari and Thirunamakanni [11].By using this concept of Quin Terranean fuzzy sets in the paper, the notion of Quin Terranean fuzzy compactness is defined by expanding the concepts of fuzzy compactness and intuitionistic fuzzy compactness.

## 2. PRELIMINARIES

### Definition 2.1

Let  $X$  be a universal set.  $X$  contains a fuzzy set  $\mathcal{F}$ , denoted by  $\mathcal{F} = \{(\tilde{x}, \alpha_{\mathcal{F}}(\tilde{x})) : \tilde{x} \in X\}$  where  $\alpha_{\mathcal{F}} : X \rightarrow [0, 1]$  represents the membership function. The membership of  $\tilde{x} \in X$  in  $\mathcal{F}$  is denoted by  $\alpha_{\mathcal{F}}(\tilde{x}) \in [0, 1]$ . The notation  $\mathcal{FS}(X)$  will represent the set of all fuzzy sets over  $X$ .

### Definition 2.2

In  $X$ , an intuitionistic fuzzy set  $\mathcal{J}$  is defined as  $\mathcal{J} = \{(\tilde{x}, \alpha_{\mathcal{J}}(\tilde{x}), \beta_{\mathcal{J}}(\tilde{x})) : \tilde{x} \in X\}$ , where  $\alpha_{\mathcal{J}} : X \rightarrow [0, 1]$ ,  $\beta_{\mathcal{J}} : X \rightarrow [0, 1]$  are the degrees of membership and non-membership of  $\tilde{x}$  to  $\mathcal{J}$  respectively which satisfies  $0 \leq \alpha_{\mathcal{J}}(\tilde{x}) + \beta_{\mathcal{J}}(\tilde{x}) \leq 1, \forall \tilde{x} \in X$ . Let  $\mathcal{JFS}(X)$  denotes the set of all intuitionistic fuzzy sets over  $X$ .

### Definition 2.3

Let  $M$  and  $N$  be IFS'S of the form  $M = \{(\tilde{x}, \alpha_M(\tilde{x}), \beta_M(\tilde{x})) : \tilde{x} \in X\}$  and  $N = \{(\tilde{x}, \alpha_N(\tilde{x}), \beta_N(\tilde{x})) : \tilde{x} \in X\}$ . Then,

- i.  $M \subseteq N$  iff  $\alpha_M(\tilde{x}) \leq \alpha_N(\tilde{x})$  and  $\beta_M(\tilde{x}) \geq \beta_N(\tilde{x})$ ,
- ii.  $\bar{M} = \{(\tilde{x}, \beta_M(\tilde{x}), \alpha_M(\tilde{x})) : \tilde{x} \in X\}$ ,
- iii.  $M \cap N = \{(\tilde{x}, \alpha_M(\tilde{x}) \wedge \alpha_N(\tilde{x}), \beta_M(\tilde{x}) \vee \beta_N(\tilde{x})) : \tilde{x} \in X\}$
- iv.  $M \cup N = \{(\tilde{x}, \alpha_M(\tilde{x}) \vee \alpha_N(\tilde{x}), \beta_M(\tilde{x}) \wedge \beta_N(\tilde{x})) : \tilde{x} \in X\}$

### Definition 2.4

Let  $X$  be a nonempty set an intuitionistic fuzzy topology is a family  $\tau$  of intuitionistic fuzzy sets in  $X$  satisfying the following axioms

- i.  $0_X, 1_X \in \tau$ ,
- ii.  $S_1 \cap S_2 \in \tau$  for any  $S_1, S_2 \in \tau$ ,
- iii.  $\cup_{i \in \mathcal{J}} S_i \in \tau$  for any arbitrary family  $\{S_i : i \in \mathcal{J}\} \subseteq \tau$ .

In this instance, the pair  $(X, \tau)$  is referred to as an intuitionistic fuzzy topological space  $\mathcal{JFTS}$  and each IFS in  $\tau$  is called an intuitionistic fuzzy open set ( $\mathcal{JFOS}$ ) in  $X$ .

### Definition 2.5

The intuitionistic fuzzy closed set ( $\mathcal{JFCS}$ ) in  $X$  is the complement  $\bar{M}$  of an intuitionistic fuzzy open set  $M$  in an intuitionistic fuzzy topological space  $(X, \tau)$ .

### Definition 2.6

Let  $(X, \tau)$  be an  $\mathcal{JFTS}$  and  $M = \{(\tilde{x}, \alpha_M(\tilde{x}), \beta_M(\tilde{x})) : \tilde{x} \in X\}$  be an  $\mathcal{JFS}$  in  $X$ . Then the fuzzy closure and fuzzy interior of  $M$  are defined by

$$\begin{aligned} \mathcal{JFCl}(M) &= \cap \{R : R \text{ is and } \mathcal{JFCS} \text{ in } X \text{ and } M \subseteq R\} \\ &\text{and} \\ \mathcal{JFInt}(M) &= \cup \{S : S \text{ is and } \mathcal{JFOS} \text{ in } X \text{ and } S \subseteq M\} \end{aligned}$$

**Proposition 2.7**

Let  $(X, \tau)$  be intuitionistic fuzzy topological space, and  $M$  be an  $\mathcal{IFS}$  on  $X$ . Then the following properties hold:

- i.  $1 - \mathcal{IFCl}(M) = \mathcal{IFInt}(1 - M)$
- ii.  $1 - \mathcal{IFInt}(M) = \mathcal{IFCl}(1 - M)$

**Definition 2.8**

Let  $X$  be a non-empty set. A Pythagorean fuzzy set  $\mathcal{P}$  on  $X$  is defined as  $\mathcal{P} = \{(\tilde{x}, \alpha_{\mathcal{P}}(\tilde{x}), \beta_{\mathcal{P}}(\tilde{x})) : \tilde{x} \in X\}$  where  $\alpha_{\mathcal{P}}(\tilde{x}) : X \rightarrow [0, 1]$  represents the degree of membership of each element  $\tilde{x} \in X$  to the set  $\mathcal{P}$  and  $\beta_{\mathcal{P}}(\tilde{x}) : X \rightarrow [0, 1]$  represents the degree of non membership of each element  $\tilde{x} \in X$  to the set  $\mathcal{P}$  such that  $0 \leq (\alpha_{\mathcal{P}}(\tilde{x}))^2 + (\beta_{\mathcal{P}}(\tilde{x}))^2 \leq 1$ , for all  $\tilde{x} \in X$  (Pythagorean condition).

**Definition 2.9**

Let be a universe of discourse. A Quin-Terranean fuzzy set  $\mathcal{QT}$  in  $X$  is defined as  $\mathcal{QT} = \{(\tilde{x}, \alpha_{\mathcal{QT}}(\tilde{x}), \beta_{\mathcal{QT}}(\tilde{x})) : \forall \tilde{x} \in X\}$  where  $\alpha_{\mathcal{QT}}(\tilde{x}) : X \rightarrow [0, 1]$  represents the degree of dependence of the element  $\tilde{x}$  and  $\beta_{\mathcal{QT}}(\tilde{x}) : X \rightarrow [0, 1]$  represents the degree of non-dependence of the element  $\tilde{x}$  such that  $0 \leq (\alpha_{\mathcal{QT}}(\tilde{x}))^{4/5} + (\beta_{\mathcal{QT}}(\tilde{x}))^{5/4} \leq 1, \forall \tilde{x} \in X$  (Quin-Terranean condition) For any Quin-Terranean fuzzy set  $\mathcal{QT}$  and  $\forall \tilde{x} \in X$ , the degree of hesitance of  $\tilde{x}$  to  $X$  is defined as  $\pi_{\mathcal{QT}}(\tilde{x}) = (1 - (\alpha_{\mathcal{QT}}(\tilde{x}))^{4/5} + (\beta_{\mathcal{QT}}(\tilde{x}))^{5/4})^{4/5}$ .

**Definition 2.10**

Let  $\mathcal{QT}_1 = \{(\tilde{x}, \alpha_{\mathcal{QT}_1}(\tilde{x}), \beta_{\mathcal{QT}_1}(\tilde{x})) : \tilde{x} \in X\}$  and  $\mathcal{QT}_2 = \{(\tilde{x}, \alpha_{\mathcal{QT}_2}(\tilde{x}), \beta_{\mathcal{QT}_2}(\tilde{x})) : \tilde{x} \in X\}$  be two Quin-Terranean fuzzy subsets of a set  $X$ . Then,

- i. The complement of  $\mathcal{QT}_1$  is defined by

$$\mathcal{QT}_1^c = (\beta_{\mathcal{QT}_1}(\tilde{x}), \alpha_{\mathcal{QT}_1}(\tilde{x}))$$

- ii. The intersection of  $\mathcal{QT}_1$  and  $\mathcal{QT}_2$  is defined by

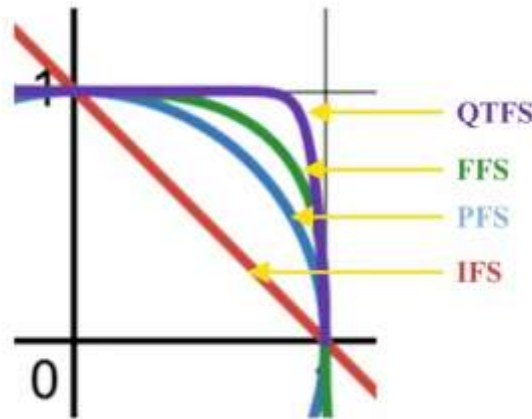
$$\mathcal{QT}_1 \cap \mathcal{QT}_2 = \{(\tilde{x}, \min \{\alpha_{\mathcal{QT}_1}(\tilde{x}), \alpha_{\mathcal{QT}_2}(\tilde{x})\}, \max \{\beta_{\mathcal{QT}_1}(\tilde{x}), \beta_{\mathcal{QT}_2}(\tilde{x})\})\}$$

- iii. The union of  $\mathcal{QT}_1$  and  $\mathcal{QT}_2$  is defined by

$$\mathcal{QT}_1 \cup \mathcal{QT}_2 = \{(\tilde{x}, \max \{\alpha_{\mathcal{QT}_1}(\tilde{x}), \alpha_{\mathcal{QT}_2}(\tilde{x})\}, \min \{\beta_{\mathcal{QT}_1}(\tilde{x}), \beta_{\mathcal{QT}_2}(\tilde{x})\})\}$$

- iv. We say  $\mathcal{QT}_1$  is a subset of  $\mathcal{QT}_2$  or  $\mathcal{QT}_2$  contains  $\mathcal{QT}_1$  and write  $\mathcal{QT}_1 \subseteq \mathcal{QT}_2$  if  $\alpha_{\mathcal{QT}_1}(\tilde{x}) \leq \alpha_{\mathcal{QT}_2}(\tilde{x})$  and  $\beta_{\mathcal{QT}_1}(\tilde{x}) \geq \beta_{\mathcal{QT}_2}(\tilde{x})$ .

The notation  $1_X$  is used throughout this paper for the Quin-Terranean fuzzy subsets  $(1,0)$  and  $0_X$  for the Quin-Terranean fuzzy subsets  $(0,1)$  that is  $\alpha_{1_X} = 1$  and  $\beta_{1_X} = 0, \alpha_{0_X} = 0$  and  $\beta_{0_X} = 1$ .



**Definition 2.11**

Let  $X$  be a fixed non-empty set and let  $\tau$  be a collection of Quin-Terranean fuzzy subsets of  $X$ . If,

- i.  $0_X, 1_X \in \tau$ ,
- ii. for any  $QT_1, QT_2 \in \tau$ , we have  $QT_1 \cap QT_2 \in \tau$ ,
- iii. for any  $\{QT_i\}_{i \in J} \subseteq \tau$ , we have  $\bigcup_{i \in J} QT_i \in \tau$ .

Each Quin-Terranean fuzzy subset in  $\tau$  is referred to as a Quin-Terranean Fuzzy Open Set (*QTFOS*) in  $X$  and the pair  $(X, \tau)$  is referred to as a Quin-Terranean Fuzzy Topological Space (*QTFTS*). Closed Quin-Terranean fuzzy subsets are the complement of open Quin-Terranean fuzzy subsets. Since a Quin-Terranean fuzzy subset can be any fuzzy subset or intuitionistic fuzzy subset or Pythagorean fuzzy subset of a set, we can see that any fuzzy topological space or intuitionistic fuzzy topological space or Pythagorean fuzzy topological space is a Quin-Terranean fuzzy topological space. Conversely, it is clear that a Quin-Terranean fuzzy topological space does not necessarily have to be a Pythagorean fuzzy topological space or intuitionistic fuzzy topological space or fuzzy topological space. It is possible that even an open Quin-Terranean fuzzy subset is not a Pythagorean fuzzy subset or an intuitionistic fuzzy subset or a fuzzy subset which can be seen from the following example.

**Example**

Let  $X = \{c_1, c_2\}$  be a set with two elements. We define a family of Quin Terranean fuzzy subsets  $\tau$  as follows  $\tau = \{1_X, 0_X, QT_1, QT_2\}$ , where

$$QT_1 = \{\langle c_1, \alpha_{QT_1}(c_1) = 0.996, \beta_{QT_1}(c_1) = 0.5 \rangle, \langle c_2, \alpha_{QT_1}(c_2) = 0.7, \beta_{QT_1}(c_2) = 0.9 \rangle\}$$

$$QT_2 = \{\langle c_1, \alpha_{QT_2}(c_1) = 0.8, \beta_{QT_2}(c_1) = 0.9 \rangle, \langle c_2, \alpha_{QT_2}(c_2) = 0.1, \beta_{QT_2}(c_2) = 0.9 \rangle\}$$

Note that although  $(X, \tau)$  is a Quin Terranean fuzzy topological space, it is neither intuitionistic nor Pythagorean nor Fermatean.  $(X, \tau)$  is a Quin Terranean fuzzy topological space. Since,

$$QT_1 \cup QT_2 = \{\langle c_1, \alpha_{QT_1 \cup QT_2}(c_1) = 0.996, \beta_{QT_1 \cup QT_2}(c_1) = 0.5 \rangle, \langle c_2, \alpha_{QT_1 \cup QT_2}(c_2) = 0.7, \beta_{QT_1 \cup QT_2}(c_2) = 0.9 \rangle\}$$

We observe that  $0.7 + 0.9 > 1$ ;  $0.7^2 + 0.9^2 = 1.3 > 1$

Also  $0.7^3 + 0.9^3 = 0.343 + 0.729 = 1.072 > 1$ .

Hence,  $QT_1 \cup QT_2$  is not an Intuitionistic Fuzzy set, not a Pythagorean Fuzzy set and not a Fermatean Fuzzy set. Therefore  $(X, \tau)$  is not an Intuitionistic Fuzzy topological space, not a Pythagorean Fuzzy topological space and also not a Fermatean Fuzzy topological space.

**Definition 2.12**

Let  $f_{QT}: X \rightarrow Y$  be a function and let  $X$  and  $Y$  be two non-empty sets. Let  $O$  and  $P$  be (respectively) Quin Terranean fuzzy subsets of  $X$  and  $Y$ . The membership and non-membership functions of image of  $O$  with respect to  $f_{QT}$  is denoted as  $f_{QT}[O]$  and are defined by

$$\alpha_{f_{QT}[O]}(\tilde{y}) = \begin{cases} \sup_{z \in f_{QT}^{-1}(\tilde{y})} \alpha_O(z) & \text{if } f_{QT}^{-1}(\tilde{y}) \neq \emptyset \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\beta_{f_{QT}[O]}(\tilde{y}) = \begin{cases} \inf_{z \in f_{QT}^{-1}(\tilde{y})} \beta_O(z) & \text{if } f_{QT}^{-1}(\tilde{y}) \neq \emptyset \\ 0 & \text{otherwise,} \end{cases}$$

respectively, the membership and non-membership functions of the pre-image of  $P$  with respect to  $f_{QT}$  and is denoted by  $f_{QT}^{-1}[P]$  are given by  $\alpha_{f_{QT}^{-1}[P]}(\tilde{x}) = \alpha_{[P]}(f_{QT}(\tilde{x}))$  and  $\beta_{f_{QT}^{-1}[P]}(\tilde{x}) = \beta_{[P]}(f_{QT}(\tilde{x}))$  respectively.

**Proposition 2.13**

Let  $f_{QT}: X \rightarrow Y$  be a function and let  $X$  and  $Y$  be two non-empty sets. Then, we have

- (1) If  $f_{QT}^{-1}[P^c] = [f_{QT}^{-1}P]^c$  for any Quin Terranean fuzzy subset  $P$  of  $Y$ .
- (2)  $[f_{QT}O]^c \subseteq f_{QT}[O^c]$  for any Quin Terranean fuzzy subset  $O$  of  $X$ .
- (3) if  $P_1 \subseteq P_2$ , then  $f_{QT}^{-1}[P_1] \subseteq f_{QT}^{-1}[P_2]$  where  $P_1$  and  $P_2$  are Quin Terranean fuzzy subsets of  $Y$ .
- (4) if  $O_1 \subseteq O_2$ , then  $f_{QT}[O_1] \subseteq f_{QT}[O_2]$  where  $O_1$  and  $O_2$  are Quin Terranean fuzzy subsets of  $X$ .
- (5)  $f_{QT}[f_{QT}^{-1}[P]] \subseteq P$  for any Quin Terranean fuzzy subset  $P$  of  $Y$ .
- (6)  $O \subseteq f_{QT}^{-1}[f_{QT}[O]]$  for any Quin Terranean fuzzy subset  $O$  of  $X$ .

**Definition 2.14**

Let  $f_{QT}: X \rightarrow Y$  be a function and let  $(X, \tau_1)$  and  $(Y, \tau_2)$  be two Quin Terranean fuzzy topological spaces. If for any Quin Terranean fuzzy subset  $L$  of  $X$  and for any neighbourhood  $N$  of  $f_{QT}[L]$

There exists an neighbourhood  $M$  of  $L$  such that  $f_{QT}[M] \subseteq N$ , then  $f_{QT}$  is said to be Quin Terranean fuzzy continuous.

**Theorem 2.15**

Let  $X$  and  $Y$  be two Quin Terranean fuzzy topological spaces with  $\tau_1$  and  $\tau_2$ , respectively. If  $f_{QT}^{-1}[P]$  is an open Quin Terranean fuzzy subset of  $X$  for any open Quin Terranean fuzzy subset  $P$  of  $Y$ , then  $f_{QT}: X \rightarrow Y$  is Quin Terranean fuzzy continuous.

### 3. QUIN TERRANEAN FUZZY COMPACTNESS

Here, we extend A.HaydarEş[9] original idea of Connectedness In Pythagorean Fuzzy Topological Spaces to the case of Quin Terranean fuzzy topological space.

#### Definition 3.1

Let  $(X, \tau)$  be a Quin Terranean fuzzy topological space.

- i. A fuzzy open cover of  $X$  is defined as a family  $\{(\alpha_{S_i}, \beta_{S_i}) : i \in \mathcal{J}\}$  Quin Terranean fuzzy open subsets in  $X$  that satisfies the condition  $\cup\{(\alpha_{S_i}, \beta_{S_i}) : i \in \mathcal{J}\} = 1_X$ . A finite subcover of  $\{(\alpha_{S_i}, \beta_{S_i}) : i \in \mathcal{J}\}$  of  $X$ , which is also a fuzzy open cover of  $X$ , is a finite subfamily of a Quin Terranean fuzzy open cover  $\{(\alpha_{S_i}, \beta_{S_i}) : i \in \mathcal{J}\}$  of  $X$ .
- ii. A family  $\{(\alpha_{R_i}, \beta_{R_i}) : i \in \mathcal{J}\}$  of Quin Terranean fuzzy closed subsets in  $X$  is said to satisfies the Finite Intersection Property (FIP), if and only if every finite subfamily  $\{(\alpha_{R_i}, \beta_{R_i}) : i = 1, 2, \dots, n\}$  of the family meets the condition that their intersection is non-empty  
 $\cap\{(\alpha_{R_i}, \beta_{R_i}) : i = 1, 2, \dots, n\} \neq \emptyset$ .
- iii. A Quin Terranean fuzzy topological space  $(X, \tau)$  is said to be **Quin Terranean fuzzy compact** if and only if for every Quin Terranean fuzzy open cover of  $X$ , there exists a finite subcover that still covers  $X$ .

#### Example 3.2

Let  $X = \{1, 2\}$  and define a sequence of Quin Terranean fuzzy subsets  $\{M_i : i = 1, 2, 3, \dots\}$  where, as given below.

$$\alpha_{M_i}(1) = \frac{i}{i+1}, \beta_{M_i}(1) = \frac{1}{i+2}, \alpha_{M_i}(2) = \frac{i+1}{i+2}, \beta_{M_i}(2) = \frac{1}{i+3}$$

In this case, the family  $\tau = \{M_i : i = 1, 2, 3, \dots\} \cup \{0_X, 1_X\}$  is a Quin Terranean fuzzy topological space on  $X$ . Since there is no finite subcover for the fuzzy open cover  $\{M_i : i = 1, 2, 3, \dots\}$ , i.e.,  $(X, \tau)$  is not Quin Terranean fuzzy compact.

#### Theorem 3.3

Let  $(X, \tau_1)$  and  $(Y, \tau_2)$  be two Quin Terranean fuzzy topological spaces and suppose there is a Quin Terranean fuzzy continuous surjective mapping  $f_{QT} : X \rightarrow Y$ . If  $(X, \tau_1)$  is Quin Terranean fuzzy compact, then  $(Y, \tau_2)$  is also Quin Terranean fuzzy compact.

#### Proof :

Let  $\{S_i : i \in \mathcal{J}\}$ , where  $S_i = (\alpha_{S_i}, \beta_{S_i})$  for each  $i \in \mathcal{J}$ , be a collection of Quin Terranean fuzzy open sets that form a cover of  $Y$ . According to the definition of Quin Terranean fuzzy continuity,  $\{f_{QT}^{-1}(S_i) : i \in \mathcal{J}\}$  is a Quin Terranean fuzzy open cover of  $X$ , as well. Given that  $X$  is a Quin Terranean fuzzy compact, then there exists a finite subfamily  $\{S_i : i = 1, 2, 3, \dots, n\}$  such that  $\cup_{i=1}^n f_{QT}^{-1}(S_i) = 1_X$ . Since the map  $f_{QT}$  is surjective, we have

$$\bigcup_{i=1}^n f_{Q\mathcal{T}}^{-1}(S_i) = \bigcup_{i=1}^n f_{Q\mathcal{T}}(f_{Q\mathcal{T}}^{-1}(S_i)) \subseteq \bigcup_{i=1}^n f_{Q\mathcal{T}}(f_{Q\mathcal{T}}^{-1}(S_i)) = \bigcup_{i=1}^n S_i = 1_Y.$$

Therefore,  $(Y, \tau_2)$  satisfies the conditions for Quin Terranean fuzzy compactness and hence  $(Y, \tau_2)$  is Quin Terranean fuzzy compact.

#### Definition 3.4

Let  $(X, \tau)$  be a Quin Terranean fuzzy topological space and  $M = (\alpha_M, \beta_M)$  be a Quin Terranean fuzzy subset in  $X$ . The fuzzy interior of  $M$  and its fuzzy closure are then defined by

$$Cl_{Q\mathcal{T}}(M) = \bigcap \{R : R \text{ is a Quin Terranean fuzzy closed subset in } X \text{ and } M \subseteq R\}$$

and

$$int_{Q\mathcal{T}}(M) = \bigcup \{S : S \text{ is a Quin Terranean fuzzy closed subset in } X \text{ and } S \subseteq M\}.$$

#### Definition 3.5

- (i). A Quin Terranean fuzzy topological space  $(X, \tau)$  is said to be Quin Terranean fuzzy almost compact if, for every Quin Terranean fuzzy open cover of  $X$ , there exists a finite subcollection whose closures still cover  $X$ .
- (ii). A Quin Terranean fuzzy topological space  $(X, \tau)$  is defined as Quin Terranean fuzzy nearly compact if, for every Quin Terranean fuzzy open cover of  $X$ , there exists a finite subcollection whose closures' interiors cover  $X$ .

The following implications are apparent in Quin Terranean fuzzy topological spaces.

**Quin Terranean fuzzy compactness  $\rightarrow$  Quin Terranean fuzzy near compactness  $\rightarrow$  Quin Terranean fuzzy almost compactness.**

However, the opposite is not true.

#### Example 3.6

Let  $X = \{1, 2\}$  and define a sequence of Quin Terranean fuzzy subsets  $\{S_i : i = 1, 2, 3, \dots\}$  where, as given below.

$$\alpha_{S_i}(1) = 1 - \frac{1}{i}, \beta_{S_i}(1) = \frac{1}{i+1}, \alpha_{S_i}(2) = 1 - \frac{1}{i+1}, \beta_{S_i}(2) = \frac{1}{i+2}$$

In this case, the family  $\tau = \{S_i : i = 1, 2, 3, \dots\} \cup \{0_X, 1_X\}$  is a Quin Terranean fuzzy topological space on  $X$ . Given that  $Cl(S) = 1$  and  $int(S) = 1$  it follows that the topological space  $(X, \tau)$  satisfies the conditions for Quin Terranean fuzzy near compactness. It is not a Quin Terranean fuzzy compact, however, because the Quin Terranean fuzzy open cover  $\{S_i : i = 1, 2, 3, \dots\}$  has no finite subcover.

#### Theorem 3.7

A Quin Terranean fuzzy topological space  $(X, \tau)$  is said to be Quin Terranean fuzzy almost compact if and only if for every collection  $\{S_i : i \in J\}$  of Quin Terranean fuzzy open subsets in  $X$ , where each  $S_i = (\alpha_{S_i}, \beta_{S_i})$  and the collection satisfies the finite intersection property, it follows that:

$$\bigcap_{i \in J} Cl(S_i) \neq 0_X.$$

**Proof :**

Let  $\{(\alpha_{S_i}, \beta_{S_i}) : i \in \mathcal{J}\}$  be any family of Quin Terranean fuzzy open subsets in  $X$  that satisfies the finite intersection property. Suppose the  $\bigcap_{i \in \mathcal{J}} Cl(\alpha_{S_i}, \beta_{S_i}) = 0_X$ . Then we have  $\bigcup_{i \in \mathcal{J}} int(\alpha_{S_i}, \beta_{S_i}) = 1_X$ . Since  $X$  is Quin Terranean fuzzy almost compact, there exists a finite subfamily  $\{(\alpha_{S_i}, \beta_{S_i}) : i=1, 2, 3, \dots, n\}$  such that  $\bigcup_{i=1}^n Cl(int(\alpha_{S_i}, \beta_{S_i})) = 1_X$ .

Therefore, we obtain  $\bigcup_{i=1}^n Cl([Cl(\beta_{S_i}, \alpha_{S_i})]^c) = 1_X$ . Additionally,  $\bigcap_{i=1}^n int(Cl(\alpha_{S_i}, \beta_{S_i})) = 0_X$ . However, since  $S_i \subseteq int(Cl(S_i))$ , it follows that  $\bigcap_{i=1}^n S_i = 0_X$  which contradicts the finite intersection property of this family.

Conversely, let  $\{S_i : i \in \mathcal{J}\}$  be a Quin Terranean fuzzy open cover of  $X$ . Suppose that no finite subcollection of  $\{S_i : i \in \mathcal{J}\}$  whose closures do not cover of  $X$ . Now, the family  $\{[Cl(\beta_{S_i}, \alpha_{S_i})]^c : i \in \mathcal{J}\}$  consists of Quin Terranean fuzzy open subsets  $int(\alpha_{S_i}, \beta_{S_i})$  in  $X$ , this collection satisfies the finite intersection property. By assumption, we have  $\bigcap_{i \in \mathcal{J}} Cl([Cl(\beta_{S_i}, \alpha_{S_i})]^c) \neq 0_X$ . Thus,  $\bigcup_{i \in \mathcal{J}} int(Cl(S_i)) \neq 1_X$  which contradicts the fact that  $\bigcup_{i \in \mathcal{J}} S_i = 1_X$ . since for each  $i \in \mathcal{J}$ , we have  $S_i \subseteq int(Cl(S_i))$ .

### Definition 3.8

A Quin Terranean fuzzy subset  $M$  is defined as a Quin Terranean fuzzy regular open set if  $M = int(Cl(M))$ . Similarly, a Quin Terranean fuzzy subset  $N$  is called a Quin Terranean fuzzy regular closed set if  $N = Cl(int(N))$ .

### Theorem 3.9

In a Quin Terranean fuzzy topological space  $(X, \tau)$  the following statements are equivalent:

- i.  $(X, \tau)$  is Quin Terranean fuzzy almost compact.
- ii. For every collection  $\{S_i : i \in \mathcal{J}\}$ , where  $S_i = (\alpha_{S_i}, \beta_{S_i})$  for each  $i \in \mathcal{J}$ , consisting of Quin Terranean fuzzy regular closed subsets, such that  $\bigcap_{i \in \mathcal{J}} S_i = 0_X$  then there exists a finite subcollection  $\{S_i : i = 1, 2, 3, \dots, n\}$  such that  $\bigcap_{i=1}^n int(S_i) = 0_X$ .
- iii. For every collection  $\{S_i : i \in \mathcal{J}\}$  of Quin Terranean fuzzy regular open sets the finite intersection property holds:  
 $\bigcap_{i \in \mathcal{J}} Cl(S_i) \neq 0_X$  where each  $i \in \mathcal{J}$  is given by  $S_i = (\alpha_{S_i}, \beta_{S_i})$ .
- iv. Every collection of Quin Terranean fuzzy regular open sets that covers  $X$  contains a finite subcollection whose closures also cover  $X$ .

### Proof:

The proof of this theorem follows a reasoning approach analogous to that of Theorem 3.7.

### Definition 3.10

Let  $(X, \tau_1)$  and  $(Y, \tau_2)$  be two Quin Terranean fuzzy topological spaces and let  $f_{QT} : X \rightarrow Y$  be a function. Then  $f_{QT}$  is called :

- i. Quin Terranean fuzzy almost continuous if and only if the preimage of every Quin Terranean fuzzy regular open set in  $Y$  is a Quin Terranean fuzzy open set in  $X$ .
- ii. Quin Terranean fuzzy weakly continuous if and only if for every Quin Terranean fuzzy open set  $M$  in  $Y$ ,

$$f_{QT}^{-1}(M) = \text{int}(f_{QT}^{-1}(Cl(M)))$$

**Theorem 3.11**

Let  $(X, \tau_1)$  and  $(Y, \tau_2)$  be two Quin Terranean fuzzy topological spaces and let  $f_{QT}: X \rightarrow Y$  be a surjective function that is Quin Terranean fuzzy almost continuous. If  $(X, \tau_1)$  is Quin Terranean fuzzy almost compact, then  $(Y, \tau_2)$  is also Quin Terranean fuzzy almost compact.

**Proof:**

Let  $\{S_i : i \in J\}$  be a Quin Terranean fuzzy open cover of  $Y$ , where each  $S_i = (\alpha_{S_i}, \beta_{S_i})$ ,  $i \in J$ . Since  $f_{QT}$  is Quin Terranean fuzzy almost continuous, the collection  $f_{QT}^{-1}(\text{int}(Cl(M))) : i \in J$  forms a Quin Terranean fuzzy open cover of  $X$ .

Since  $X$  is Quin Terranean fuzzy almost compact, there exists a finite subcollection  $\{S_i : i = 1, 2, 3, \dots, n\}$  such that  $\bigcup_{i=1}^n Cl(f_{QT}^{-1}(S_i)) = 1_X$ . Since  $f_{QT}$  is surjective and Quin Terranean fuzzy almost continuous, we get

$$f_{QT}(\bigcup_{i=1}^n Cl(f_{QT}^{-1}(\text{int}(Cl(S_i)))) = (\bigcup_{i=1}^n Cl(f_{QT}^{-1}(\text{int}(Cl(S_i)))) = f_{QT}(1_X) = 1_Y$$

Thus,  $\bigcup_{i=1}^n Cl(S_i) = 1_Y$ . Therefore,  $(Y, \tau_2)$  is also Quin Terranean fuzzy almost compact.

**Theorem 3.12**

Let  $(X, \tau_1)$  and  $(Y, \tau_2)$  be two Quin Terranean fuzzy topological spaces and let  $f_{QT}: X \rightarrow Y$  be a surjective function that is Quin Terranean fuzzy weakly continuous. If  $(X, \tau_1)$  is Quin Terranean fuzzy compact, then  $(Y, \tau_2)$  is also Quin Terranean fuzzy almost compact.

**Proof:**

The reasoning in this proof is analogous to that of Theorem 3.11.

## 4 CONCLUSIONS

In this paper, we present the concept of Quin Terranean fuzzy compactness, which generalizes the ideas of both fuzzy topological spaces and intuitionistic fuzzy topological spaces. Also we have defined various types of Quin-Terranean fuzzy compactness namely Quin-Terranean fuzzy almost compactness and Quin-Terranean fuzzy near compactness and studied their properties. We observe that every Quin Terranean fuzzy compactness is Quin Terranean fuzzy near compactness. Also every Quin Terranean fuzzy near compactness is Quin Terranean fuzzy almost compactness. The findings of this study can be further extended to explore the properties of Quin Terranean fuzzy connectedness.

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## COMPETING INTERESTS

The author has stated that there are no competing interests.

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