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A NUMERICAL SIMILARITY SOLUTION FOR LAMINAR THERMAL BOUNDARY LAYER OVER A FLAT PLATE WITH A CONVECTIVE SURFACE BOUNDARY CONDITION

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Abstract: This paper considers the typical issue of hydrodynamic and thermal boundary layer over a flat plate in a uniform flow of fluid. It is outstanding that similarity solutions of the energy equation are effective for the boundary states of steady surface temperature and constant heat flux. Be that as it may, no such result has been attempted for the convective surface boundary condition. The paper exhibits that a similarity solutions is possible if the convective heattransfer related with the hot fluid on the lower surface of the plate is corresponding to $x^{-1/2}$. Numerical solutions of the subsequent energy equation are solved by MATLAB inbuilt solver with fixed Prandtl number.

Keywords: Flat plate, bvp4c, Blasius flow, Nusselt number, Absolute heat transferrate.

Introduction:

In fluid mechanics, the issue of laminar hydrodynamic and thermal boundary layers over the flat plate in a uniform flow of fluid is a completely examined. On account of steady surface temperature at the plate, the similarity solution for the thermal boundary layer is likewise embedded and broadly cited in heat transfer text book such as [1]. The problem of a steady forced convection thermal boundary-layer past a flat plate with a prescribed surface heat flux is investigated both analytically and numerically. Recently, several new methods have been presented to overcome the cited difficulties. Some of these procedures includeKeller box scheme[2], Runge–Kutta fourth order method [3], Homotopy Analysis Method (HAM)[4, 5], compact integral form [5].The paper exhibits that a similarity solutions is possible if the convective heat transfer of the fluid heating the plate on its lower surface is relative to $x^{-1/2}$. Numerical results of the subsequent heat similitude condition are accounted for Prandtl quantities of 0.1, 0.72, and 10 and a scope of estimations of the parameter portraying the hot liquid convection process. In case the heat transfer coefficient is a consistent, by then comparative data address represent the local similarity solutions of the problem.

Mathematical analysis

Consider the two dimensional of hydrodynamic and thermal boundary layer flows over a flat plate in a stream of cold fluid at temperature T_∞ moving over the top surface of the plate with a uniform velocity U_∞.Assuming steady, laminar flow, incompressible, with constant fluid properties and negligible viscous dissipation and the bottom surface of the plate is heated by convection from a hot fluid at temperature T_f which provides aheat transfer coefficient h_f. The continuity, momentum, and energy equations describing the flow can be written as

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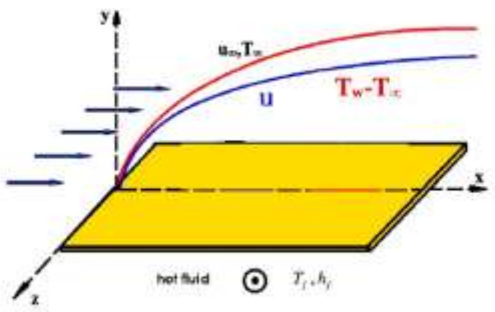

Fig.1. Schematic flow diagram

Continuity equation: $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$ …(1)

Momentum equation: $u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}$ …(2)

Energy equation: $u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}$ …(3)

The boundary conditions at the plate surface and far into the cold fluid, the proper boundary condition are given by

$$u(x,0)=\ v(x,0)=0\quad -k\frac{\partial T}{\partial y}(x,0)=h_f\left(T_f-T(x,0)\right)\quad at\ \ y\to 0$$
… (4)
$$u(x,\infty)\to U_\infty,\quad T(x,\infty)\to T_\infty,\qquad as\ \ y\to\infty$$

Where u, v are the velocity components the x-axis (along the plate) and y- axis (normal to the plate),$T$is the temperature, υ m is the kinematic viscosity of the fluid, and α is the thermal diffusivity of the fluid, respectively

A similarity solution of Eqs.(1)-(4) are obtained by defining the stream functions $u=\frac{\partial\psi}{\partial y}$, $v=-\frac{\partial\psi}{\partial x}$ with independent variable η and dependent variable f .

Next, introduce the similarity transformations

$\psi=\sqrt{U_\infty x\upsilon}\,f(\eta)\,,T=T_\infty+\left(T_f-T_\infty\right)\theta(\eta)$, where $\eta=y\sqrt{\left(\frac{U_\infty}{x\upsilon}\right)}$. Using the dimensionless and similarity variables, eqns.(2) and (4) reduce to the following form:

$$2f'''+ff''=0 \qquad \text{… (5)}$$

$$2\theta''+\Pr f\theta'=0 \qquad \text{… (6)}$$

The boundary conditions becomes in the following form:

$$f=0, f'=0,\ \theta'=-a[1-\theta(0)],\quad at\ \eta\to 0$$
… (7)
$$f'\to 1,\theta\to 0,\qquad as\ \ \eta\to\infty$$

Where prime denotes diff. With resp. to η. Where $a = \frac{h_f}{k}\sqrt{\frac{\upsilon x}{U_\infty}}$ for the energy equation to have a similarity solution the quantity a must be constant and not a function of x as in a.this condition can be met if the heat transfer coefficient h_f is proportional to $x^{-1/2}$ and assume $h_f = cx^{-1/2}$ (c is constant). Now a becomes $a = \frac{c}{k}\sqrt{\frac{\upsilon}{U_\infty}}$ .The local surface heat flux q''_x and the total heat transfer rate q can be expressed as

$$q''_x = -kW\left(T_f - T_\infty\right)\left(\frac{U_\infty}{\upsilon x}\right)^{1/2}\theta'(0) \qquad \ldots (8)$$

$$q = -2kW\left(T_f - T_\infty\right)\left(\frac{U_\infty L}{\upsilon}\right)^{1/2}\theta'(0) \ (a \text{ is a constant}) \qquad \ldots (9)$$

$$q = -kW\left(T_f - T_\infty\right)\left(\frac{U_\infty}{\upsilon}\right)^{1/2}\int_0^L x^{-1/2}\theta'(0)dx \ (a \text{ is a function of } x) \qquad \ldots (10)$$

Where L and W are the plate length and width of the plate.

Solution of the problem

The coupled nonlinear ordinary differential Eqs. (5)-(6) along with boundary conditions (7) are incorporated with the help of MATLAB tool bvp5c. To get this, the set of ordinary differential equations are first transformed to first order ordinary differential equations by using the successive substitutions

$f = f_1, f' = f_2, f'' = f_3,\ \theta = f_4, \theta' = f_5,$

$f_1' = f_2, f_2' = f_3, f_3' = -\left(0.5 * f_1 * f_3\right)$

$f_4' = f_5,\ f_5' = -\Pr\left(\left(f_1 * f_5\right)\right)$

The boundary conditions take the following structure

$$\begin{cases} f_1(0) = 0, f_2(0) = 0, f_5(0) = -a*[1 - f_4(0)], \ \eta \to 0 \\ f_2(\eta_\infty) = 1, f_4(\eta_\infty) = 0, \quad \eta \to \infty \end{cases}$$

The asymptotic boundary condition (7) at the margin was stable to 10^{-6}.In this approach, the choice of $\eta_\infty = 6$, in agreement with standard practice in the boundary layer analysis.

Results and Discussion

Table 1 shows the results of the problem with an increase of a, both $\theta(0)$ and $-\theta'(0)$ increase for fixed Prandtl numbers of 0.1and 0.72.As indicated by the results, as $a \to \infty$, the solutions approaches the established answer for the constant surface temperature. This can be seen from the boundary condition (7) which diminishes to $\theta(0) = 1 \ \ as\ \eta \to \infty$. Table 1 gives by getting $\theta(0)$ and $-\theta'(0)$ for selected values of a for fixed values of Prandtl number. For

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each Prandtl number, the Nusselt number and the absolute heat transfer rate are increases with an increase of a.

Table 1 : Values of $\theta(0)$ and $-\theta'(0)$ for different values of a

	$\Pr = 0.1$ **as** $\eta_\infty \to 6$		$\Pr = 0.72$ **as** $\eta_\infty \to 6$	
a	$\theta(0)$	$-\theta'(0)$	$\theta(0)$	$-\theta'(0)$
0.05	0.209735	0.039513	0.143993	0.0428
0.1	0.346745	0.065326	0.251738	0.074826
1	0.84147	0.15853	0.770868	0.229132
5	0.963689	0.181556	0.943888	0.28056
10	0.981509	0.184913	0.971134	0.288659
20	0.990668	0.186639	0.985356	0.292886

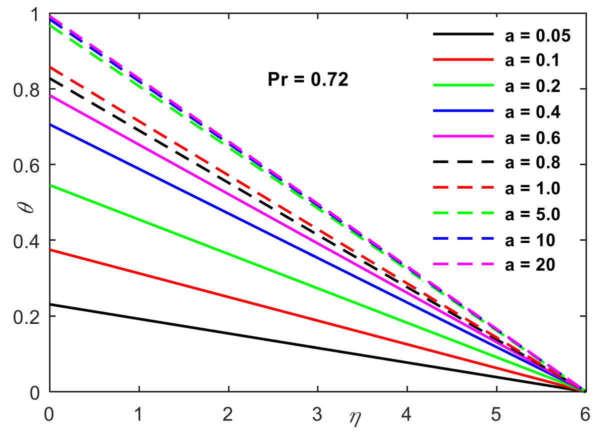

Fig. 2. Temperature profile for different values of parameter a.

Fig. 2 shows the Mat lab bvp4c produced numerical solutions for a fixed Prandtl number of 0.72 and for a change of values of the parameter$a$. For each curvethe vertical intercept gives the plate surface temperature. The plate surface temperature increases with an increase of a.

Conclusions:

The temperature and heat transfer characteristics of the Blasius flow have been examined for a fixed Prandtl number. The following observations are found:

- With an increase of a ,Temperature profile, Nusselt number and the absolute heat transfer rate are increases

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