

Complex Neutrosophic Soft Graphs.

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Abstract

In this study, we introduce and build on the concept of complex neutrosophic soft graphs (CNSGs), which provide a sophisticated framework for modeling vagueness, uncertainty, and indeterminacy in complex systems. A strong complex neutrosophic soft graph (SCNSG) is defined as one in which, under soft set parameters, both edges and vertices have complex-valued memberships and high levels of neutrosophic uncertainty. We provide a new metric that considers the cumulative strength and uncertainty of connections within graph attributes, as well as the complements of a complex neutrosophic soft graph and a strong complex neutrosophic soft graph. In uncertain networks, complement operations reveal information on inverse behaviors and dual linkage. We expand on the concept of isomorphism and propose criteria for structural equivalence under complicated neutrosophic incorrect mappings. Examples are presented to show how the proposed concepts can be put into practice. The theoretical findings presented in this study will lead to additional research into complex uncertain networks, notably in information fusion, cognitive modeling, and decision support systems.

Keywords: Soft Graph; Neutrosophic Soft Graph; Complex Neutrosophic Soft Graph.

1. INTRODUCTION

Zadeh [22] proposed fuzzy set theory to tackle uncertainty. Numerous researchers have subsequently investigated fuzzy logic and fuzzy set theory in an attempt to solve a wide range of real-world problems involving uncertain situations. Smarandache's [15] notion of neutrosophic sets (NSs) is a powerful mathematical tool for dealing with inconsistent, ambiguous, and incomplete data in the real world. Neutrosophic sets are distinguished by a truth-membership function (T), an indeterminacy-membership function (I), and a falsity-membership function (F), each of which can take values in the standard or non-standard interval of 0 to 1.

Akram [4, 5, 6, 7] differentiates between two soft computing approaches for conveying ambiguity and uncertainty: fuzzy sets and soft sets. They used soft computing models to investigate graph ambiguity and uncertainty. Several features of regular fuzzy soft graphs were investigated. He then discussed intuitionistic fuzzy soft graphs, their applications, and neutrosophic soft graphs. Soft graphs have parameters for each vertex and edge. Satham Hussain [14] suggests that fuzzy sets and neutrosophic soft sets are excellent methods for representing data with information. This method resulted in neutrosophic soft graphs. This allows decision-makers to adjust their judgments to their own areas of competence.

Satham Hussain [13] introduced the notions of neutrosophic graphs and strong neutrosophic graphs, which served as a framework for further in-depth research. This novel theoretical framework incorporates features of both neutrosophic graphs and graphs. In this case, the total of the membership

values for truth, indeterminacy, and falsity is between 0 and 2, because truth and falsity are considered dependent variables. Neutrosophic graphs are considered equivalents in this theory. This theory is being developed to help with applications in operations research and tackling issues in social networks. A particularly important issue in the latter arena is the rise of phony or fraudulent profiles. The neutrosophic graph model provides a fresh technique for addressing such difficulties. The study examined the qualities of strong, complete, and self-complementary neutrosophic graphs and provided examples to demonstrate the suggested notions.

A combination of neutrosophic and soft sets, the complicated neutrosophic soft set model was proposed by Said Broumi [12]. It presents and illustrates the fundamental operations of set theory along with other concepts related to this model structure. To demonstrate the use of this paradigm, a decision-making problem including subjective and ambiguous information is given. This illustration shows how well the model works to deal with ambiguity and support well-informed decision-making. Moreover, it provides opportunities for further investigation to enhance the approach and examine its consequences in other domains, including artificial intelligence and decision science.

This study includes complex neutrosophic soft graphs, strong complex neutrosophic soft graphs, and constant complex neutrosophic soft graphs. Following careful examination, we identified several amazing properties of the complement and self-complement of the complicated neutrosophic soft graph. These discoveries offer up new paths for study in the discipline, perhaps leading to improved applications in network theory and decision-making processes. Our research emphasizes the importance of these graph types in simulating ambiguity and vagueness in a variety of real-world circumstances.

To clearly highlight the value of the complex neutrosophic soft graph model, we give a method-specific comparison with existing neutrosophic-based techniques:

- **Standard Neutrosophic Graph:** Standard neutrosophic graphs differ from classical graphs in that they give three membership degrees to each vertex and edge—truth (T), indeterminacy (I), and falsity (F), with each value lying between $[0, 1]$. Their capacity to handle complex ambiguity or many criteria is limited, but they are essential for modelling simple uncertain interactions.
- **Neutrosophic Soft Graphs:** Neutrosophic soft graphs enable parameter-based modelling by combining the concept of a soft set with neutrosophic graphs. Each vertex or edge is based on a variety of soft properties, increasing flexibility without addressing the vagueness issue.
- **Neutrosophic Graphs:** Neutrosophic graphs combine the features of neutrosophic logic and set theory. In addition to the T, I, and F components, each connection is described by imprecise degrees (truth-membership and falsity-membership constrained by a degree of hesitation or ambiguity). This model is particularly useful for making judgments since it gives a more accurate depiction of various real-world uncertainties, such as hesitancy or insufficient knowledge.
- **Complex Neutrosophic Soft Graph:** The complex neutrosophic soft graph represents a comprehensive mix of complex neutrosophic numbers, sets, and soft sets. It captures truth, indeterminacy, and falsity with complex values while modeling ambiguity and a variety of criteria. As a result, CNSG is well adapted for high-dimensional, dynamic decision-making systems, such as medical diagnostics, big data analysis, and intelligent transportation systems.

2. PRELIMINARIES

This section provides a basic description and example to support the key findings.

Definition 1. [4] A fuzzy soft graph $G = (G^*, \mathfrak{S}, \mathfrak{R}, L)$ is defined as an ordered quadruple that satisfies the following conditions:

- $G^* = (V, E)$ is a simple graph,
- L is a non-empty set of parameters,
- (\mathfrak{S}, L) is a fuzzy set defined over the vertex set V ,
- (\mathfrak{R}, L) is a fuzzy set defined over the edge set E ,
- For each $a \in L$, the pair $(\mathfrak{S}(a), \mathfrak{R}(a))$ forms a fuzzy graph associated with G^* , such that:

$$\mathfrak{R}(a)(xy) \leq \min\{\mathfrak{S}(a)(x), \mathfrak{S}(a)(y)\}$$

holds for all $x, y \in V$.

For ease of use, $H(a)$ is used to represent the fuzzy graph $(\mathfrak{S}(a), \mathfrak{R}(a))$. A fuzzy soft graph may therefore be thought of as a family of fuzzy graphs that are parameterized by L . $S(G^*)$ represents the class of all fuzzy soft graphs over G^* .

Definition 2. [6] An intuitionistic fuzzy soft graph $G = (G^*, \mathfrak{S}, \mathfrak{R}, L)$ is defined as an ordered quadruple that satisfies the following conditions:

- $G^* = (V, E)$ is a simple graph,
- L is a non-empty set of parameters,
- (\mathfrak{S}, L) constitutes an intuitionistic fuzzy set defined over the vertex set,
- (\mathfrak{R}, L) represents an intuitionistic fuzzy set defined over the edge set E ,
- For each $a \in L$, the pair $(\mathfrak{S}(a), \mathfrak{R}(a))$ forms an intuitionistic fuzzy graph based on G^* . Specifically,

$$\mathfrak{U}_{\mathfrak{R}(a)}(xy) \leq \min\{\mathfrak{U}_{\mathfrak{S}(a)}(x), \mathfrak{U}_{\mathfrak{S}(a)}(y)\}$$

$$\mathfrak{C}_{\mathfrak{R}(a)}(xy) \geq \max\{\mathfrak{C}_{\mathfrak{S}(a)}(x), \mathfrak{C}_{\mathfrak{S}(a)}(y)\}$$

For ease of use, $H(a)$ is used to represent the intuitionistic fuzzy graph $(\mathfrak{S}(a), \mathfrak{R}(a))$. An intuitionistic fuzzy soft graph may therefore be thought of as a family of intuitionistic fuzzy graphs that are parameterized by L . $\mathfrak{S}(G^*)$ represents the class of all intuitionistic fuzzy soft graphs over G^* . for all $a \in L$ and $x, y \in V$.

Definition 3. [7] A neutrosophic soft graph $G = (G^*, \mathfrak{S}, \mathfrak{R}, L)$ is defined as a quadruple satisfying the following conditions:

- $G^* = (V, E)$ represents a simple (i.e., undirected and without loops or multiple edges) graph,
- L is a non-empty collection of parameters,
- (\mathfrak{S}, L) denotes a neutrosophic set defined over the vertex set V ,
- (\mathfrak{R}, L) denotes a neutrosophic set defined over the edge set E ,
- For every $a \in L$, the pair $(\mathfrak{S}(a), \mathfrak{R}(a))$ forms a neutrosophic subgraph of G^* . Specifically, the following conditions hold:

$$\mathfrak{U}_{\mathfrak{R}(a)}(xy) \leq \min\{\mathfrak{U}_{\mathfrak{S}(a)}(x), \mathfrak{U}_{\mathfrak{S}(a)}(y)\}$$

$$\mathfrak{B}_{\mathfrak{R}(a)}(xy) \leq \min\{\mathfrak{B}_{\mathfrak{S}(a)}(x), \mathfrak{B}_{\mathfrak{S}(a)}(y)\},$$

$$\mathfrak{C}_{\mathfrak{R}(a)}(xy) \geq \max\{\mathfrak{C}_{\mathfrak{S}(a)}(x), \mathfrak{C}_{\mathfrak{S}(a)}(y)\}$$

For ease of use, $H(a)$ is used to represent the neutrosophic fuzzy graph ($\mathfrak{S}(a)$, $\mathfrak{K}(a)$). A neutrosophic fuzzy soft graph may therefore be thought of as a family of neutrosophic fuzzy graphs that are parameterized by L . $\mathfrak{S}(G^*)$ represents the class

of all neutrosophic fuzzy soft graphs over G^* . for all $a \in Q$ and $x, y \in V$.

Definition 4. [10] Assume $U \neq \phi$. A complex fuzzy set A is an entity with the following structure:

$$A = \{(x, \rho_A(x)): x \in U\} = \{x, \mathfrak{A}_A(x)e^{i w_A(x)}\}$$

$$\text{where } i = \sqrt{-1}, \mathfrak{A}_A(x) \in [0, 1] \text{ and } 0 \leq w_A(x) \leq 2\pi.$$

Definition 5. [1] Assume $U \neq \phi$. An item of the type complex intuitionistic fuzzy set A

$$A = \{(x, \mathfrak{A}_A(x), \mathfrak{C}_A(x)): x \in U\}$$

$$A = \{(x, \mathfrak{A}_A(x)e^{i\alpha_A(x)}, \mathfrak{C}_A(x)e^{i\gamma_A(x)}): x \in U\}$$

$$\text{where } i = \sqrt{-1}, \mathfrak{A}_A(x), \mathfrak{C}_A(x) \in [0, 1], \alpha_A(x), \gamma_A(x) \in [0, 2\pi] \text{ and } 0^- \leq \mathfrak{A}_A(x) + \mathfrak{C}_A(x) \leq 1^+.$$

Definition 6. [1] Assume $U \neq \phi$. An object with the shape of a complex neutrosophic set A is

$$A = \{(x, \mathfrak{A}_A(x), \mathfrak{B}_A(x), \mathfrak{C}_A(x)): x \in U\}$$

$$A = \{(x, \mathfrak{A}_A(x)e^{i\alpha_A(x)}, \mathfrak{B}_A(x)e^{i\beta_A(x)}, \mathfrak{C}_A(x)e^{i\gamma_A(x)}): x \in U\}$$

$$\text{where } i = \sqrt{-1}, \mathfrak{A}_A(x), \mathfrak{B}_A(x), \mathfrak{C}_A(x) \in [0, 1], \alpha_A(x), \beta_A(x), \gamma_A(x) \in [0, 2\pi] \text{ and } 0^- \leq \mathfrak{A}_A(x) + \mathfrak{B}_A(x) + \mathfrak{C}_A(x) \leq 3^+.$$

Definition 7. [9] When A is a complex neutrosophic set on X and B is a complex neutrosophic relation in X , then $G = (A, B)$ is a complex neutrosophic graph on $X = \phi$.

$$\mathfrak{A}_B(xy)e^{i\alpha_B(xy)} \leq \min\{\mathfrak{A}_A(x), \mathfrak{A}_A(y)\}e^{i\min\{\alpha_A(x), \alpha_A(y)\}}$$

$$\mathfrak{B}_B(xy)e^{i\beta_B(xy)} \leq \min\{\mathfrak{B}_A(x), \mathfrak{B}_A(y)\}e^{i\min\{\beta_A(x), \beta_A(y)\}}$$

$$\mathfrak{C}_B(xy)e^{i\gamma_B(xy)} \leq \max\{\mathfrak{C}_A(x), \mathfrak{C}_A(y)\}e^{i\max\{\gamma_A(x), \gamma_A(y)\}}$$

3. COMPLEX NEUTROSOPHIC SOFT GRAPHS.

Definition 8. A Complex Neutrosophic Soft Graph (CNSG) is defined as an ordered quadruple $G = (G^*, \mathfrak{S}, \mathfrak{K}, P)$ that satisfies the following conditions:

- $G^* = (V, E)$ is a simple graph.
 - L is a non-empty set of parameters.
 - (\mathfrak{S}, L) constitutes a complex neutrosophic soft (CNS) set over the vertex set V .
 - (\mathfrak{K}, L) represents a CNS set over the edge set E .
 - For each parameter $l \in L$, the pair $(\mathfrak{S}(l), \mathfrak{K}(l))$ forms a complex neutrosophic graph (CNVG) on G^* .
- Let $V = \{l_1, l_2, \dots, l_n\}$, and consider the functions:

$$\mathfrak{A}_{\mathfrak{S}(l)}: V \rightarrow [0, 1], \quad \mathfrak{B}_{\mathfrak{S}(l)}: V \rightarrow [0, 1], \quad \mathfrak{C}_{\mathfrak{S}(l)}: V \rightarrow [0, 1]$$

The degrees of truth-membership, indeterminacy-membership and falsity-membership for each vertex $p_i \in V$ under the parameter $l \in L$ are represented by these functions, respectively.

Furthermore, in $[0, 2\pi]$, let $X_{\mathfrak{S}(l)}, Y_{\mathfrak{S}(l)}$ and $Z_{\mathfrak{S}(l)}$ represent the corresponding argument functions for every membership function. Afterward, for every vertex p_i :

$$0 \leq \mathfrak{A}_{\mathfrak{S}(l)}(p_i)e^{iX_{\mathfrak{S}(l)}(p_i)} + \mathfrak{B}_{\mathfrak{S}(l)}(p_i)e^{iY_{\mathfrak{S}(l)}(p_i)} + \mathfrak{C}_{\mathfrak{S}(l)}(p_i)e^{iZ_{\mathfrak{S}(l)}(p_i)} \leq 2$$

For the edge set $E \subseteq V \times V$, the corresponding functions are defined as:

$$\mathfrak{A}_{\mathfrak{K}(l)}: E \rightarrow [0,1], \quad \mathfrak{B}_{\mathfrak{K}(l)}: E \rightarrow [0,1], \quad \mathfrak{C}_{\mathfrak{K}(l)}: E \rightarrow [0,1]$$

With regard to parameter l , these functions indicate the degrees of truth-membership, indeterminacy membership and falsity-membership for every edge $p_i p_j \in E$. The functions $X_{\mathfrak{K}(l)}, Y_{\mathfrak{K}(l)}$ and $Z_{\mathfrak{K}(l)} \in [0, 2\pi]$ are argument functions. Also correspond to the corresponding membership functions.

For each edge $p_i p_j \in E$, the following conditions must be satisfied:

$$\mathfrak{A}_{\mathfrak{K}(l)}(p_i p_j)e^{iX_{\mathfrak{K}(l)}(p_i p_j)} \leq \min\{\mathfrak{A}_{\mathfrak{S}(l)}(p_i), \mathfrak{A}_{\mathfrak{S}(l)}(p_j)\}e^{i \min\{X_{\mathfrak{S}(l)}(p_i), X_{\mathfrak{S}(l)}(p_j)\}}$$

$$\mathfrak{B}_{\mathfrak{K}(l)}(p_i p_j)e^{iY_{\mathfrak{K}(l)}(p_i p_j)} \leq \min\{\mathfrak{B}_{\mathfrak{S}(l)}(p_i), \mathfrak{B}_{\mathfrak{S}(l)}(p_j)\}e^{i \min\{Y_{\mathfrak{S}(l)}(p_i), Y_{\mathfrak{S}(l)}(p_j)\}}$$

$$\mathfrak{C}_{\mathfrak{K}(l)}(p_i p_j)e^{iZ_{\mathfrak{K}(l)}(p_i p_j)} \geq \max\{\mathfrak{C}_{\mathfrak{S}(l)}(p_i), \mathfrak{C}_{\mathfrak{S}(l)}(p_j)\}e^{i \max\{Z_{\mathfrak{S}(l)}(p_i), Z_{\mathfrak{S}(l)}(p_j)\}}$$

Furthermore, each edge satisfies the bounded magnitude condition:

$$0 \leq \mathfrak{A}_{\mathfrak{K}(l)}(p_i p_j)e^{iX_{\mathfrak{K}(l)}(p_i p_j)} + \mathfrak{B}_{\mathfrak{K}(l)}(p_i p_j)e^{iY_{\mathfrak{K}(l)}(p_i p_j)} + \mathfrak{C}_{\mathfrak{K}(l)}(p_i p_j)e^{iZ_{\mathfrak{K}(l)}(p_i p_j)} \leq 2$$

For simplicity, the pair $(\mathfrak{S}(l), \mathfrak{K}(l))$ is known as the complex neutrosophic graph G . Thus, a CNSG is a parameterized structure that expands on the idea of a complex neutrosophic graph.

Example 1. Assume the set of vertices be $V = \{p_1, p_2, p_3, p_4\}$ and the set of edges be $E = \{p_1 p_2, p_2 p_3, p_3 p_4, p_1 p_4\}$, which together form a graph denoted as G^* . Consider (\mathfrak{S}, L) to be a complex neutrosophic soft set (CNSS) over V , and let $L = (l_1, l_2)$ be a parameterized set. $\mathfrak{S} : L \rightarrow L(V)$ is the definition of the complex neutrosophic approximation function.

$$\mathfrak{S}(l_1) = \begin{cases} \{p_1, (.60e^{i\pi(1.7)}, .40e^{i\pi(1.8)}, .30e^{i\pi(1.7)})\} \\ \{p_2, (.50e^{i\pi(1.5)}, .30e^{i\pi(1.6)}, .50e^{i\pi(1.8)})\} \\ \{p_3, (.70e^{i\pi(1.3)}, .50e^{i\pi(1.7)}, .20e^{i\pi(1.6)})\} \\ \{p_4, (.50e^{i\pi(1.4)}, .40e^{i\pi(1.5)}, .40e^{i\pi(1.6)})\} \end{cases}$$

$$\mathfrak{S}(l_2) = \begin{cases} \{p_1, (.40e^{i\pi(1.2)}, .80e^{i\pi(1.4)}, .50e^{i\pi(1.8)})\} \\ \{p_2, (.30e^{i\pi(1.7)}, .50e^{i\pi(1.3)}, .60e^{i\pi(1.6)})\} \\ \{p_3, (.20e^{i\pi(1.5)}, .70e^{i\pi(1.5)}, .70e^{i\pi(1.5)})\} \\ \{p_4, (.40e^{i\pi(1.4)}, .60e^{i\pi(1.6)}, .50e^{i\pi(1.7)})\} \end{cases}$$

Let (\mathfrak{K}, L) be a CNSS over E and let $L = (l_1, l_2)$ be a set of parameters. The complex neutrosophic approximation function $\mathfrak{K} : L \rightarrow L(E)$ is defined by

$$\mathfrak{K}(l_1) = \begin{cases} \{p_1 p_2, (.40e^{i\pi(1.5)}, .20e^{i\pi(1.4)}, .60e^{i\pi(1.8)})\} \\ \{p_2 p_3, (.30e^{i\pi(1.2)}, .20e^{i\pi(1.5)}, .70e^{i\pi(1.9)})\} \\ \{p_3 p_4, (.40e^{i\pi(1.3)}, .40e^{i\pi(1.4)}, .40e^{i\pi(1.7)})\} \\ \{p_1 p_4, (.50e^{i\pi(1.4)}, .30e^{i\pi(1.3)}, .50e^{i\pi(1.8)})\} \end{cases}$$

$$\mathfrak{K}(l_2) = \begin{cases} \{p_1p_2, (.20e^{i\pi(1.2)}, .40e^{i\pi(1.3)}, .60e^{i\pi(1.9)})\} \\ \{p_2p_3, (.10e^{i\pi(1.4)}, .30e^{i\pi(1.2)}, .80e^{i\pi(1.7)})\} \\ \{p_3p_4, (.20e^{i\pi(1.3)}, .50e^{i\pi(1.4)}, .80e^{i\pi(1.8)})\} \\ \{p_1p_4, (.30e^{i\pi(1.2)}, .50e^{i\pi(1.3)}, .70e^{i\pi(1.8)})\} \end{cases}$$

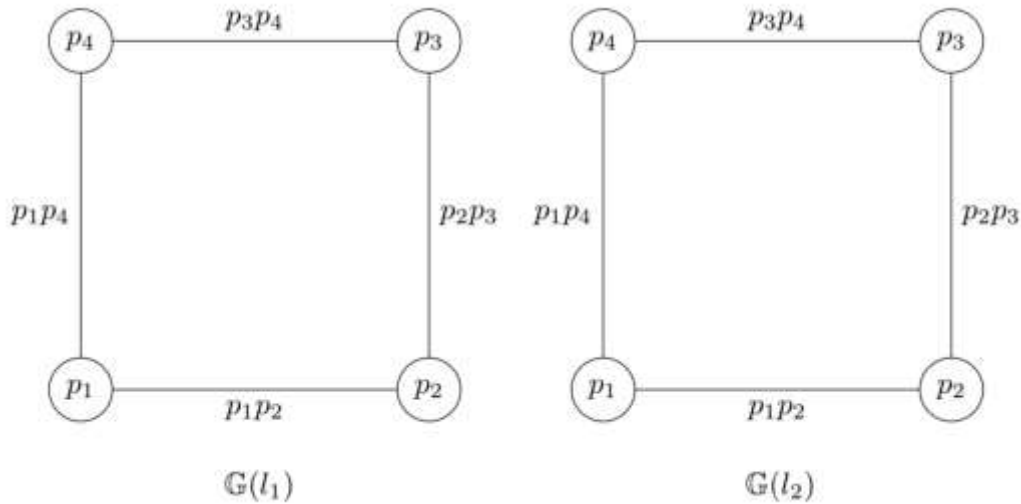


Figure 1: CNSG $\{G(l_1), G(l_2)\}$

Definition 9. Strong complex neutrosophic soft graph (SCNSG) are defined as CNSG $G = (G^*, \mathfrak{S}, \mathfrak{K}, L)$ of $G^* = (V, E)$ if

$$\mathfrak{A}_{\mathfrak{K}(l)}(p_i p_j) e^{i\mathfrak{X}_{\mathfrak{K}(l)}(p_i p_j)} = \min\{\mathfrak{A}_{\mathfrak{S}(l)}(p_i), \mathfrak{A}_{\mathfrak{S}(l)}(p_j)\} e^{i \min\{\mathfrak{X}_{\mathfrak{S}(l)}(p_i), \mathfrak{X}_{\mathfrak{S}(l)}(p_j)\}}$$

$$\mathfrak{B}_{\mathfrak{K}(l)}(p_i p_j) e^{i\mathfrak{Y}_{\mathfrak{K}(l)}(p_i p_j)} = \min\{\mathfrak{B}_{\mathfrak{S}(l)}(p_i), \mathfrak{B}_{\mathfrak{S}(l)}(p_j)\} e^{i \min\{\mathfrak{Y}_{\mathfrak{S}(l)}(p_i), \mathfrak{Y}_{\mathfrak{S}(l)}(p_j)\}}$$

$$\mathfrak{C}_{\mathfrak{K}(l)}(p_i p_j) e^{i\mathfrak{Z}_{\mathfrak{K}(l)}(p_i p_j)} = \max\{\mathfrak{C}_{\mathfrak{S}(l)}(p_i), \mathfrak{C}_{\mathfrak{S}(l)}(p_j)\} e^{i \max\{\mathfrak{Z}_{\mathfrak{S}(l)}(p_i), \mathfrak{Z}_{\mathfrak{S}(l)}(p_j)\}}$$

Example 2. Assume the set of vertices be $V = \{p_1, p_2, p_3, p_4\}$ and the set of edges be $E = \{p_1p_2, p_2p_3, p_3p_4, p_1p_4\}$, which together form a graph denoted as G^* . Consider (\mathfrak{S}, L) to be a strong complex neutrosophic soft set (SCNSS) over V and let $L = (l_1, l_2)$ be a parameterized set. $\mathfrak{S} : L \rightarrow L(V)$ is the definition of the strong complex neutrosophic approximation function.

$$\mathfrak{S}(l_1) = \begin{cases} \{p_1, (.60e^{i\pi(1.7)}, .40e^{i\pi(1.8)}, .30e^{i\pi(1.7)})\} \\ \{p_2, (.50e^{i\pi(1.5)}, .30e^{i\pi(1.6)}, .50e^{i\pi(1.8)})\} \\ \{p_3, (.70e^{i\pi(1.3)}, .50e^{i\pi(1.7)}, .20e^{i\pi(1.6)})\} \\ \{p_4, (.50e^{i\pi(1.4)}, .40e^{i\pi(1.5)}, .40e^{i\pi(1.6)})\} \end{cases}$$

$$\mathfrak{S}(l_2) = \begin{cases} \{p_1, (.40e^{i\pi(1.2)}, .80e^{i\pi(1.4)}, .50e^{i\pi(1.8)})\} \\ \{p_2, (.30e^{i\pi(1.7)}, .50e^{i\pi(1.3)}, .60e^{i\pi(1.6)})\} \\ \{p_3, (.20e^{i\pi(1.5)}, .70e^{i\pi(1.5)}, .70e^{i\pi(1.5)})\} \\ \{p_4, (.40e^{i\pi(1.4)}, .60e^{i\pi(1.6)}, .50e^{i\pi(1.7)})\} \end{cases}$$

Let (\mathfrak{K}, L) be an SCNSS over E and let $L = (l_1, l_2)$ be a set of parameters. The strong complex neutrosophic approximation function $\mathfrak{K} : L \rightarrow L(E)$ is defined by

$$K(l_1) = \begin{cases} \{p_1p_2, (.50e^{i\pi(1.4)}, .30e^{i\pi(1.6)}, .50e^{i\pi(1.8)})\} \\ \{p_2p_3, (.50e^{i\pi(1.3)}, .30e^{i\pi(1.6)}, .50e^{i\pi(1.8)})\} \\ \{p_3p_4, (.50e^{i\pi(1.5)}, .40e^{i\pi(1.5)}, .40e^{i\pi(1.6)})\} \\ \{p_1p_4, (.50e^{i\pi(1.4)}, .40e^{i\pi(1.5)}, .40e^{i\pi(1.7)})\} \end{cases}$$

$$K(l_2) = \begin{cases} \{p_1p_2, (.30e^{i\pi(1.2)}, .50e^{i\pi(1.3)}, .60e^{i\pi(1.8)})\} \\ \{p_2p_3, (.20e^{i\pi(1.5)}, .50e^{i\pi(1.3)}, .70e^{i\pi(1.6)})\} \\ \{p_3p_4, (.20e^{i\pi(1.4)}, .60e^{i\pi(1.5)}, .70e^{i\pi(1.7)})\} \\ \{p_1p_4, (.40e^{i\pi(1.2)}, .60e^{i\pi(1.4)}, .50e^{i\pi(1.8)})\} \end{cases}$$

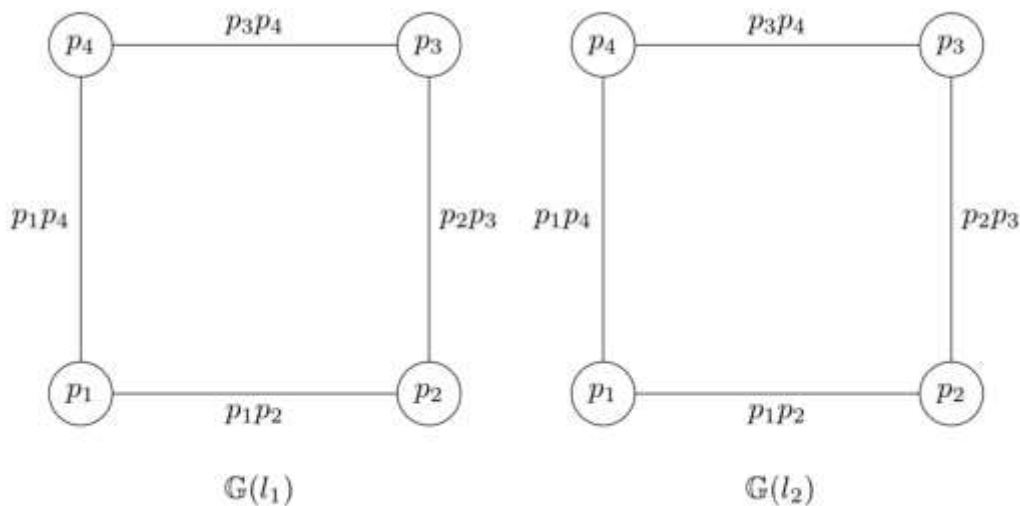


Figure 2: SCNSG $\{G(l_1), G(l_2)\}$

Definition 10. A CNSG $G = (G^*, \mathfrak{S}, \mathfrak{A}, L)$ is considered complete if

$$\mathfrak{A}_{\mathfrak{A}(l)}(p_i p_j) e^{iX_{\mathfrak{A}(l)}(p_i p_j)} = \min\{\mathfrak{A}_{\mathfrak{S}(l)}(p_i), \mathfrak{A}_{\mathfrak{S}(l)}(p_j)\} e^{i \min\{X_{\mathfrak{S}(l)}(p_i), X_{\mathfrak{S}(l)}(p_j)\}}$$

$$\mathfrak{B}_{\mathfrak{A}(l)}(p_i p_j) e^{iY_{\mathfrak{A}(l)}(p_i p_j)} = \min\{\mathfrak{B}_{\mathfrak{S}(l)}(p_i), \mathfrak{B}_{\mathfrak{S}(l)}(p_j)\} e^{i \min\{Y_{\mathfrak{S}(l)}(p_i), Y_{\mathfrak{S}(l)}(p_j)\}}$$

$$\mathfrak{C}_{\mathfrak{A}(l)}(p_i p_j) e^{iZ_{\mathfrak{A}(l)}(p_i p_j)} = \max\{\mathfrak{C}_{\mathfrak{S}(l)}(p_i), \mathfrak{C}_{\mathfrak{S}(l)}(p_j)\} e^{i \max\{Z_{\mathfrak{S}(l)}(p_i), Z_{\mathfrak{S}(l)}(p_j)\}}$$

for all $p_i, p_j \in V$ and $p_i p_j \in E$.

Example 3. Assume the set of vertices be $V = \{p_1, p_2, p_3, p_4\}$ and the set of edges be $E = \{p_1p_2, p_2p_3, p_3p_4, p_1p_4, p_1p_3, p_2p_4\}$, which together form a graph denoted as G^* . Consider (\mathfrak{S}, L) to be a complete complex neutrosophic soft set over V and let $L = (l_1, l_2)$ be a parameterized set. $\mathfrak{S} : L \rightarrow L(V)$ is the definition of the complete complex neutrosophic approximation function.

$$\mathfrak{S}(l_1) = \begin{cases} \{p_1, (.60e^{i\pi(1.7)}, .40e^{i\pi(1.8)}, .30e^{i\pi(1.7)})\} \\ \{p_2, (.50e^{i\pi(1.5)}, .30e^{i\pi(1.6)}, .50e^{i\pi(1.8)})\} \\ \{p_3, (.70e^{i\pi(1.3)}, .50e^{i\pi(1.7)}, .20e^{i\pi(1.6)})\} \\ \{p_4, (.50e^{i\pi(1.4)}, .40e^{i\pi(1.5)}, .40e^{i\pi(1.6)})\} \end{cases}$$

$$\mathfrak{S}(l_2) = \begin{cases} \{p_1, (.40e^{i\pi(1.2)}, .80e^{i\pi(1.4)}, .50e^{i\pi(1.8)})\} \\ \{p_2, (.30e^{i\pi(1.7)}, .50e^{i\pi(1.3)}, .60e^{i\pi(1.6)})\} \\ \{p_3, (.20e^{i\pi(1.5)}, .70e^{i\pi(1.5)}, .70e^{i\pi(1.5)})\} \\ \{p_4, (.40e^{i\pi(1.4)}, .60e^{i\pi(1.6)}, .50e^{i\pi(1.7)})\} \end{cases}$$

Let (\mathfrak{K}, L) be a complete CNSS over E and let $L = (l_1, l_2)$ be a set of parameters. The complete complex neutrosophic approximation function $\mathfrak{K} : L \rightarrow L(E)$ is defined by

$$K(l_1) = \begin{cases} \{p_1p_2, (.50e^{i\pi(1.4)}, .30e^{i\pi(1.6)}, .50e^{i\pi(1.8)})\} \\ \{p_2p_3, (.50e^{i\pi(1.3)}, .30e^{i\pi(1.6)}, .50e^{i\pi(1.8)})\} \\ \{p_3p_4, (.50e^{i\pi(1.5)}, .40e^{i\pi(1.5)}, .40e^{i\pi(1.6)})\} \\ \{p_1p_4, (.50e^{i\pi(1.4)}, .40e^{i\pi(1.5)}, .40e^{i\pi(1.7)})\} \\ \{p_1p_3, (.60e^{i\pi(1.3)}, .40e^{i\pi(1.7)}, .30e^{i\pi(1.7)})\} \\ \{p_2p_4, (.50e^{i\pi(1.4)}, .30e^{i\pi(1.5)}, .50e^{i\pi(1.8)})\} \end{cases}$$

$$K(l_2) = \begin{cases} \{p_1p_2, (.30e^{i\pi(1.2)}, .50e^{i\pi(1.3)}, .60e^{i\pi(1.8)})\} \\ \{p_2p_3, (.20e^{i\pi(1.5)}, .50e^{i\pi(1.3)}, .70e^{i\pi(1.6)})\} \\ \{p_3p_4, (.20e^{i\pi(1.4)}, .60e^{i\pi(1.5)}, .70e^{i\pi(1.7)})\} \\ \{p_1p_4, (.40e^{i\pi(1.2)}, .60e^{i\pi(1.4)}, .50e^{i\pi(1.8)})\} \\ \{p_1p_3, (.20e^{i\pi(1.2)}, .70e^{i\pi(1.4)}, .70e^{i\pi(1.8)})\} \\ \{p_2p_4, (.30e^{i\pi(1.4)}, .50e^{i\pi(1.3)}, .60e^{i\pi(1.7)})\} \end{cases}$$

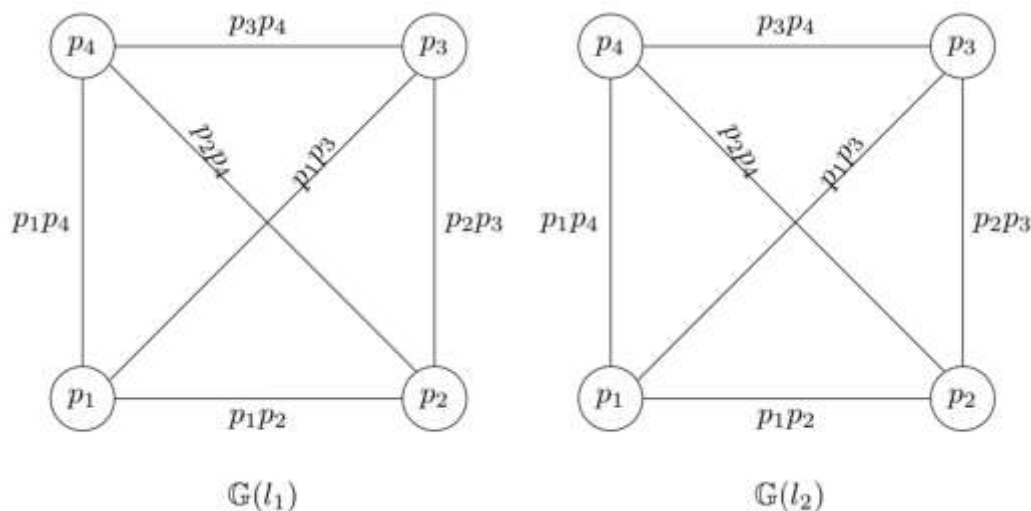


Figure 3: Complete CNSG $\{G(l_1), G(l_2)\}$

Definition 11. The complement of an CNSG $G = (G^*, \mathfrak{S}, \mathfrak{K}, L)$ on G^* is an CNSG G^* where

1. $(\bar{\mathfrak{S}} = \mathfrak{S})$

2.

$$\bar{\mathfrak{U}}_{\mathfrak{S}(l)}(p_i) e^{iX_{\mathfrak{S}(l)}(p_i)} = \mathfrak{U}_{\mathfrak{S}(l)}(p_i) e^{iX_{\mathfrak{S}(l)}(p_i)}$$

$$\bar{\mathfrak{B}}_{\mathfrak{S}(l)}(p_i) e^{iY_{\mathfrak{S}(l)}(p_i)} = \mathfrak{B}_{\mathfrak{S}(l)}(p_i) e^{iY_{\mathfrak{S}(l)}(p_i)}$$

$$\bar{\mathfrak{C}}_{\mathfrak{S}(l)}(p_i) e^{iZ_{\mathfrak{S}(l)}(p_i)} = \mathfrak{C}_{\mathfrak{S}(l)}(p_i) e^{iZ_{\mathfrak{S}(l)}(p_i)}$$

3.

$$\bar{\mathfrak{U}}_{\mathfrak{K}(l)}(p_i p_j) e^{iX_{\mathfrak{K}(l)}(p_i p_j)} = \min[\mathfrak{U}_{\mathfrak{S}(l)}(p_i), \mathfrak{U}_{\mathfrak{S}(l)}(p_j)] e^{i \min\{X_{\mathfrak{S}(l)}(p_i), X_{\mathfrak{S}(l)}(p_j)\}} - \mathfrak{U}_{\mathfrak{K}(l)}(p_i p_j) e^{iX_{\mathfrak{K}(l)}(p_i p_j)}$$

$$\bar{\mathfrak{B}}_{\mathfrak{K}(l)}(p_i p_j) e^{iY_{\mathfrak{K}(l)}(p_i p_j)} = \min[\mathfrak{B}_{\mathfrak{S}(l)}(p_i), \mathfrak{B}_{\mathfrak{S}(l)}(p_j)] e^{i \min\{Y_{\mathfrak{S}(l)}(p_i), Y_{\mathfrak{S}(l)}(p_j)\}} - \mathfrak{B}_{\mathfrak{K}(l)}(p_i p_j) e^{iY_{\mathfrak{K}(l)}(p_i p_j)}$$

$$\begin{aligned} \bar{\mathfrak{C}}_{\mathfrak{K}(l)}(p_i p_j) e^{iX_{\mathfrak{K}(l)}(p_i p_j)} \\ = \max[\mathfrak{C}_{\mathfrak{S}(l)}(p_i), \mathfrak{C}_{\mathfrak{S}(l)}(p_j)] e^{i \max\{Z_{\mathfrak{S}(l)}(p_i), Z_{\mathfrak{S}(l)}(p_j)\}} - \mathfrak{C}_{\mathfrak{K}(l)}(p_i p_j) e^{iZ_{\mathfrak{K}(l)}(p_i p_j)} \end{aligned}$$

for all $p_i p_j \in E$ and $l \in L$.

Definition 12. Let $G = (G^*, \mathfrak{S}, K, L)$ be a CNSG on G^* . The complement of the complement of the complex neutrosophic soft graph $\bar{G} = (\bar{V}, \bar{E})$ on G^* then gives $\bar{\bar{G}}$, according to the description above, where $\bar{V} = V$ and

$$\begin{aligned} \bar{\bar{\mathfrak{U}}}_{\mathfrak{K}(l)}(p_i p_j) e^{iX_{\mathfrak{K}(l)}(p_i p_j)} \\ = \min[\mathfrak{U}_{\mathfrak{S}(l)}(p_i), \mathfrak{U}_{\mathfrak{S}(l)}(p_j)] e^{i \min\{X_{\mathfrak{S}(l)}(p_i), X_{\mathfrak{S}(l)}(p_j)\}} - \mathfrak{U}_{\mathfrak{K}(l)}(p_i p_j) e^{iX_{\mathfrak{K}(l)}(p_i p_j)} \end{aligned}$$

$$\begin{aligned} \bar{\bar{\mathfrak{B}}}_{\mathfrak{K}(l)}(p_i p_j) e^{iY_{\mathfrak{K}(l)}(p_i p_j)} \\ = \min[\mathfrak{B}_{\mathfrak{S}(l)}(p_i), \mathfrak{B}_{\mathfrak{S}(l)}(p_j)] e^{i \min\{Y_{\mathfrak{S}(l)}(p_i), Y_{\mathfrak{S}(l)}(p_j)\}} - \mathfrak{B}_{\mathfrak{K}(l)}(p_i p_j) e^{iY_{\mathfrak{K}(l)}(p_i p_j)} \end{aligned}$$

$$\bar{\bar{\mathfrak{C}}}_{\mathfrak{K}(l)}(p_i p_j) e^{iX_{\mathfrak{K}(l)}(p_i p_j)} = \max[\mathfrak{C}_{\mathfrak{S}(l)}(p_i), \mathfrak{C}_{\mathfrak{S}(l)}(p_j)] e^{i \max\{Z_{\mathfrak{S}(l)}(p_i), Z_{\mathfrak{S}(l)}(p_j)\}} - \mathfrak{C}_{\mathfrak{K}(l)}(p_i p_j) e^{iZ_{\mathfrak{K}(l)}(p_i p_j)}$$

and

$$\bar{\bar{\mathfrak{U}}}_{\mathfrak{K}(l)}(p_i p_j) e^{iX_{\mathfrak{K}(l)}(p_i p_j)} = \mathfrak{U}_{\mathfrak{K}(l)}(p_i p_j) e^{iX_{\mathfrak{K}(l)}(p_i p_j)}$$

$$\bar{\bar{\mathfrak{B}}}_{\mathfrak{K}(l)}(p_i p_j) e^{iY_{\mathfrak{K}(l)}(p_i p_j)} = \mathfrak{B}_{\mathfrak{K}(l)}(p_i p_j) e^{iY_{\mathfrak{K}(l)}(p_i p_j)}$$

$$\bar{\bar{\mathfrak{C}}}_{\mathfrak{K}(l)}(p_i p_j) e^{iZ_{\mathfrak{K}(l)}(p_i p_j)} = \mathfrak{C}_{\mathfrak{K}(l)}(p_i p_j) e^{iZ_{\mathfrak{K}(l)}(p_i p_j)}$$

where the complex neutrosophic relation on V is denoted by E . Since $\bar{\bar{G}}$ is an SCNSG, $G \subseteq \bar{\bar{G}}$ is also an SCNSG.

Proposition 1. If G is the complement of the complement of SCNSG G , then G is an SCNSG

$(G = \bar{\bar{G}})$.

Proof: Let SCNSG G .

Let (\mathfrak{S}, L) be a SCNSS over V and let $L = 1$ be a set of parameters. The strong complex neutrosophic approximation function $\mathfrak{S} : L \rightarrow L(V)$ is defined by

$$\mathfrak{S}(l) = \begin{cases} \{p_1, (.40e^{i\pi(1.7)}, .70e^{i\pi(1.4)}, .50e^{i\pi(1.5)})\} \\ \{p_2, (.30e^{i\pi(1.4)}, .50e^{i\pi(1.6)}, .60e^{i\pi(1.7)})\} \\ \{p_3, (.20e^{i\pi(1.6)}, .80e^{i\pi(1.3)}, .80e^{i\pi(1.4)})\} \\ \{p_4, (.50e^{i\pi(1.3)}, .70e^{i\pi(1.5)}, .70e^{i\pi(1.6)})\} \end{cases}$$

Let (K, L) be a SCNSS over E and let $L = 1$ be a set of parameters. The strong complex neutrosophic approximation function $K : L \rightarrow L(E)$ is defined by

$$\mathfrak{K}(l) = \begin{cases} \{p_1p_4, (.40e^{i\pi(1.3)}, .70e^{i\pi(1.4)}, .70e^{i\pi(1.6)})\} \\ \{p_1p_3, (.20e^{i\pi(1.6)}, .70e^{i\pi(1.3)}, .80e^{i\pi(1.5)})\} \\ \{p_2p_3, (.20e^{i\pi(1.4)}, .50e^{i\pi(1.3)}, .80e^{i\pi(1.7)})\} \end{cases}$$

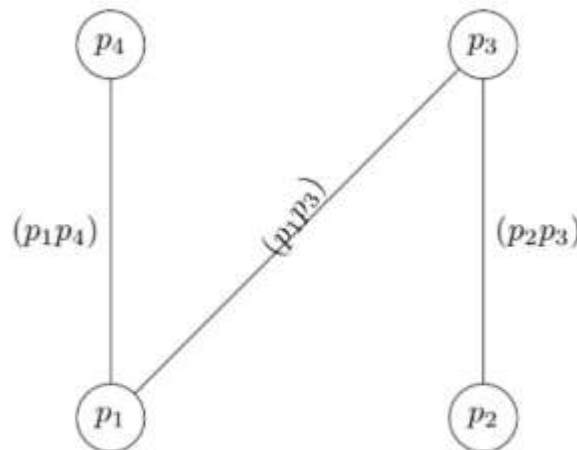


Figure 4: G - SCNSG.

Take the complement for a strong complex neutrosophic soft graph.

Let (\mathfrak{S}, L) be a complement of SCNSS over V and let $L = 1$ be a set of parameters. The complement of the strong complex neutrosophic approximation function $\mathfrak{S} : L \rightarrow L(V)$ is defined by

$$\bar{\mathfrak{S}}(l) = \begin{cases} \{p_1, (.40e^{i\pi(1.7)}, .70e^{i\pi(1.4)}, .50e^{i\pi(1.5)})\} \\ \{p_2, (.30e^{i\pi(1.4)}, .50e^{i\pi(1.6)}, .60e^{i\pi(1.7)})\} \\ \{p_3, (.20e^{i\pi(1.6)}, .80e^{i\pi(1.3)}, .80e^{i\pi(1.4)})\} \\ \{p_4, (.50e^{i\pi(1.3)}, .70e^{i\pi(1.5)}, .70e^{i\pi(1.6)})\} \end{cases}$$

Let (\mathfrak{K}, L) be a complement of SCNSS over E and let $L = 1$ be a set of parameters. The complement of the strong complex neutrosophic approximation function $\mathfrak{K} : L \rightarrow L(E)$ is defined by

$$\bar{\mathfrak{K}}(l) = \begin{cases} \{p_1p_2, (.30e^{i\pi(1.4)}, .50e^{i\pi(1.4)}, .60e^{i\pi(1.7)})\} \\ \{p_2p_4, (.30e^{i\pi(1.3)}, .50e^{i\pi(1.5)}, .70e^{i\pi(1.7)})\} \\ \{p_3p_4, (.20e^{i\pi(1.3)}, .70e^{i\pi(1.3)}, .80e^{i\pi(1.6)})\} \end{cases}$$

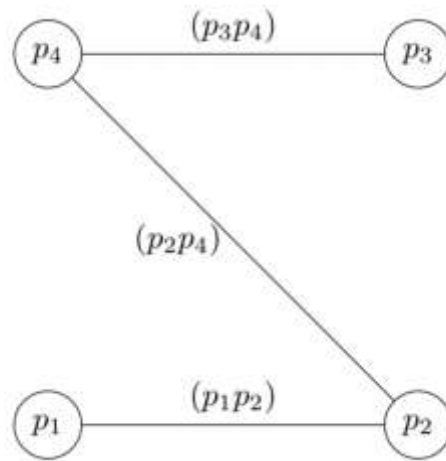


Figure 5: \bar{G} - Complement of SCNSG.

Take the complement of the complement of a strong complex neutrosophic soft graph.

Let (\mathfrak{S}, L) be a complement of the complement of an SCNSS over V and let $L = I$ be a set of parameters. The complement of the complement of a strong complex neutrosophic approximation function $\mathfrak{S} : L \rightarrow L(V)$ is defined by

$$\bar{\mathfrak{S}}(I) = \begin{cases} \{p_1, (.40e^{i\pi(1.7)}, .70e^{i\pi(1.4)}, .50e^{i\pi(1.5)})\} \\ \{p_2, (.30e^{i\pi(1.4)}, .50e^{i\pi(1.6)}, .60e^{i\pi(1.7)})\} \\ \{p_3, (.20e^{i\pi(1.6)}, .80e^{i\pi(1.3)}, .80e^{i\pi(1.4)})\} \\ \{p_4, (.50e^{i\pi(1.3)}, .70e^{i\pi(1.5)}, .70e^{i\pi(1.6)})\} \end{cases}$$

Let (\mathfrak{K}, L) be a complement of the complement of an SCNSS over E and let $L = I$ be a set of parameters. The complement of the complement of a strong complex neutrosophic approximation function $\mathfrak{K} : L \rightarrow L(E)$ is defined by

$$\bar{\mathfrak{K}}(I) = \begin{cases} \{p_1p_4, (.40e^{i\pi(1.3)}, .70e^{i\pi(1.4)}, .70e^{i\pi(1.6)})\} \\ \{p_1p_3, (.20e^{i\pi(1.6)}, .70e^{i\pi(1.3)}, .80e^{i\pi(1.5)})\} \\ \{p_2p_3, (.20e^{i\pi(1.4)}, .50e^{i\pi(1.3)}, .80e^{i\pi(1.7)})\} \end{cases}$$

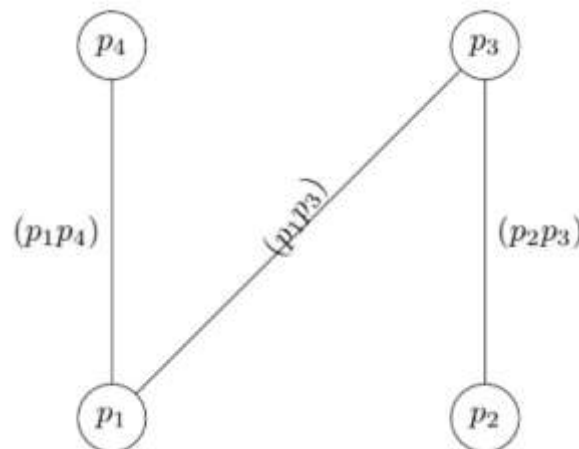


Figure 6: \bar{G} - Complement of the complement of an SCNSG.

Hence, $G = \bar{G}$.

Example 4. Consider a CNSG $G^* = (V, E)$. Take a look at an SCNSG G as shown in figure 7, the complement of SCNSG as shown in figure 8 and the complement of the complement of SCNSG as shown in 9, which shows it clearly.

Let strong complex neutrosophic soft graph G .

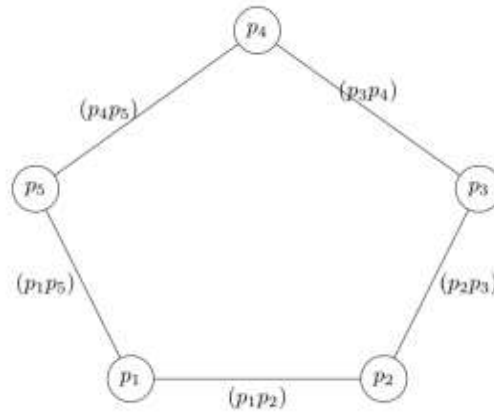


Figure 7: G - SCNSG.

$$\mathfrak{S}(l) = \begin{cases} \{p_1, (.70e^{i\pi(1.5)}, .30e^{i\pi(1.4)}, .20e^{i\pi(1.7)})\} \\ \{p_2, (.50e^{i\pi(1.6)}, .20e^{i\pi(1.6)}, .50e^{i\pi(1.6)})\} \\ \{p_3, (.70e^{i\pi(1.3)}, .40e^{i\pi(1.5)}, .30e^{i\pi(1.8)})\} \\ \{p_4, (.60e^{i\pi(1.8)}, .30e^{i\pi(1.7)}, .40e^{i\pi(1.8)})\} \\ \{p_5, (.80e^{i\pi(1.4)}, .50e^{i\pi(1.4)}, .20e^{i\pi(1.9)})\} \end{cases}$$

$$\mathfrak{K}(l) = \begin{cases} \{p_1p_2, (.50e^{i\pi(1.5)}, .20e^{i\pi(1.4)}, .50e^{i\pi(1.7)})\} \\ \{p_2p_3, (.50e^{i\pi(1.3)}, .20e^{i\pi(1.5)}, .50e^{i\pi(1.8)})\} \\ \{p_3p_4, (.60e^{i\pi(1.3)}, .30e^{i\pi(1.5)}, .40e^{i\pi(1.8)})\} \\ \{p_4p_5, (.60e^{i\pi(1.4)}, .30e^{i\pi(1.4)}, .40e^{i\pi(1.9)})\} \\ \{p_1p_5, (.70e^{i\pi(1.4)}, .30e^{i\pi(1.4)}, .20e^{i\pi(1.9)})\} \end{cases}$$

Take the complement for a strong complex neutrosophic soft graph.

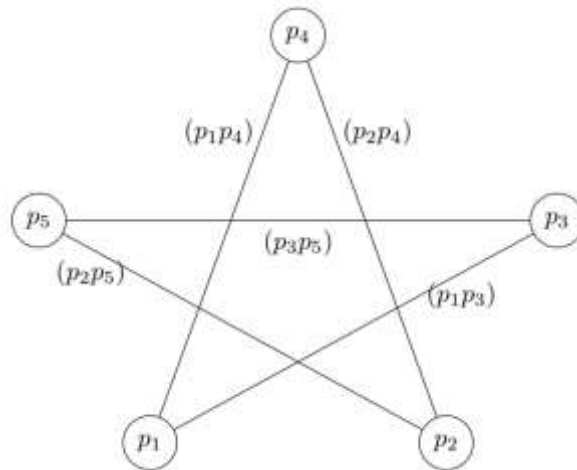


Figure 8: \bar{G} -Complement of SCNSG.

$$\bar{\mathcal{E}}(l) = \left\{ \begin{aligned} &\{p_1, (.70e^{i\pi(1.5)}, .30e^{i\pi(1.4)}, .20e^{i\pi(1.7)})\} \\ &\{p_2, (.50e^{i\pi(1.6)}, .20e^{i\pi(1.6)}, .50e^{i\pi(1.6)})\} \\ &\{p_3, (.70e^{i\pi(1.3)}, .40e^{i\pi(1.5)}, .30e^{i\pi(1.8)})\} \\ &\{p_4, (.60e^{i\pi(1.8)}, .30e^{i\pi(1.7)}, .40e^{i\pi(1.8)})\} \\ &\{p_5, (.80e^{i\pi(1.4)}, .50e^{i\pi(1.4)}, .20e^{i\pi(1.9)})\} \end{aligned} \right\}$$

$$\bar{\mathcal{K}}(l) = \left\{ \begin{aligned} &\{p_1p_3, (.70e^{i\pi(1.3)}, .30e^{i\pi(1.4)}, .30e^{i\pi(1.8)})\} \\ &\{p_1p_4, (.60e^{i\pi(1.5)}, .30e^{i\pi(1.4)}, .40e^{i\pi(1.8)})\} \\ &\{p_2p_4, (.50e^{i\pi(1.6)}, .20e^{i\pi(1.6)}, .50e^{i\pi(1.8)})\} \\ &\{p_2p_5, (.50e^{i\pi(1.4)}, .20e^{i\pi(1.4)}, .50e^{i\pi(1.9)})\} \\ &\{p_3p_5, (.70e^{i\pi(1.3)}, .40e^{i\pi(1.4)}, .30e^{i\pi(1.9)})\} \end{aligned} \right\}$$

Take the complement of the complement of a strong complex neutrosophic soft graph.

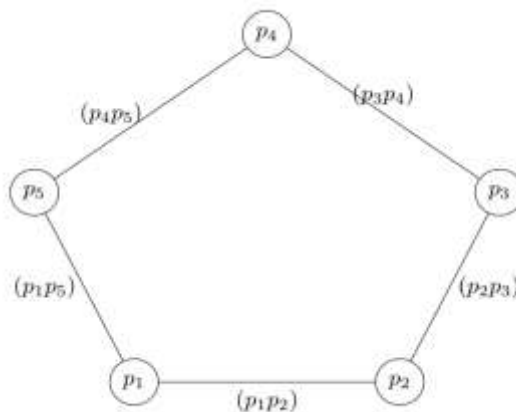


Figure 9: $\bar{\bar{G}}$ - Complement of the complement of an SCNSG.

$$\bar{\bar{\mathcal{E}}}(l) = \left\{ \begin{aligned} &\{p_1, (.70e^{i\pi(1.5)}, .30e^{i\pi(1.4)}, .20e^{i\pi(1.7)})\} \\ &\{p_2, (.50e^{i\pi(1.6)}, .20e^{i\pi(1.6)}, .50e^{i\pi(1.6)})\} \\ &\{p_3, (.70e^{i\pi(1.3)}, .40e^{i\pi(1.5)}, .30e^{i\pi(1.8)})\} \\ &\{p_4, (.60e^{i\pi(1.8)}, .30e^{i\pi(1.7)}, .40e^{i\pi(1.8)})\} \\ &\{p_5, (.80e^{i\pi(1.4)}, .50e^{i\pi(1.4)}, .20e^{i\pi(1.9)})\} \end{aligned} \right\}$$

$$\bar{\mathfrak{K}}(l) = \begin{cases} \{p_1p_2, (.50e^{i\pi(1.5)}, .20e^{i\pi(1.4)}, .50e^{i\pi(1.7)})\} \\ \{p_2p_3, (.50e^{i\pi(1.3)}, .20e^{i\pi(1.5)}, .50e^{i\pi(1.8)})\} \\ \{p_3p_4, (.60e^{i\pi(1.3)}, .30e^{i\pi(1.5)}, .40e^{i\pi(1.8)})\} \\ \{p_4p_5, (.60e^{i\pi(1.4)}, .30e^{i\pi(1.4)}, .40e^{i\pi(1.9)})\} \\ \{p_1p_5, (.70e^{i\pi(1.4)}, .30e^{i\pi(1.4)}, .20e^{i\pi(1.9)})\} \end{cases}$$

Definition 13. A SCNSG G is said to be self-complementary if it is isomorphic to its complement, denoted by \bar{G} . In this context, \bar{G} represents the complement of the CNSG G .

Proposition 2. Let $G = (G^*, \mathfrak{S}, \mathfrak{K}, L)$ be a strong complex neutrosophic soft graph if

$$\mathfrak{A}_{\mathfrak{K}(l)}(p_i p_j) e^{iX_{\mathfrak{K}(l)}(p_i p_j)} = \min[\mathfrak{A}_{\mathfrak{S}(l)}(p_i), \mathfrak{A}_{\mathfrak{S}(l)}(p_j)] e^{i \min\{X_{\mathfrak{S}(l)}(p_i), X_{\mathfrak{S}(l)}(p_j)\}}$$

$$\mathfrak{B}_{\mathfrak{K}(l)}(p_i p_j) e^{iY_{\mathfrak{K}(l)}(p_i p_j)} = \min[\mathfrak{B}_{\mathfrak{S}(l)}(p_i), \mathfrak{B}_{\mathfrak{S}(l)}(p_j)] e^{i \min\{Y_{\mathfrak{S}(l)}(p_i), Y_{\mathfrak{S}(l)}(p_j)\}}$$

$$\mathfrak{C}_{\mathfrak{K}(l)}(p_i p_j) e^{iZ_{\mathfrak{K}(l)}(p_i p_j)} = \max[\mathfrak{C}_{\mathfrak{S}(l)}(p_i), \mathfrak{C}_{\mathfrak{S}(l)}(p_j)] e^{i \max\{Z_{\mathfrak{S}(l)}(p_i), Z_{\mathfrak{S}(l)}(p_j)\}}$$

for all $p_i p_j \in V$ and $l \in L$.

Then G is self-complementary.

Proof: Let $G = (G^*, \mathfrak{S}, K, L)$ be a SCNSG such that

$$\mathfrak{A}_{\mathfrak{K}(l)}(p_i p_j) e^{iX_{\mathfrak{K}(l)}(p_i p_j)} = \min[\mathfrak{A}_{\mathfrak{S}(l)}(p_i), \mathfrak{A}_{\mathfrak{S}(l)}(p_j)] e^{i \min\{X_{\mathfrak{S}(l)}(p_i), X_{\mathfrak{S}(l)}(p_j)\}}$$

$$\mathfrak{B}_{\mathfrak{K}(l)}(p_i p_j) e^{iY_{\mathfrak{K}(l)}(p_i p_j)} = \min[\mathfrak{B}_{\mathfrak{S}(l)}(p_i), \mathfrak{B}_{\mathfrak{S}(l)}(p_j)] e^{i \min\{Y_{\mathfrak{S}(l)}(p_i), Y_{\mathfrak{S}(l)}(p_j)\}}$$

$$\mathfrak{C}_{\mathfrak{K}(l)}(p_i p_j) e^{iZ_{\mathfrak{K}(l)}(p_i p_j)} = \max[\mathfrak{C}_{\mathfrak{S}(l)}(p_i), \mathfrak{C}_{\mathfrak{S}(l)}(p_j)] e^{i \max\{Z_{\mathfrak{S}(l)}(p_i), Z_{\mathfrak{S}(l)}(p_j)\}}$$

for all $p_i p_j \in V$ and $l \in L$. Then $G \approx \bar{G}$ under the identity map $I: V \rightarrow V$.

Hence, G is self-complementary.

Proposition 3. Consider the SCNSG $G = (G^*, \mathfrak{S}, \mathfrak{K}, L)$. Then

$$\sum_{p_i \neq p_j} \mathfrak{A}_{\mathfrak{K}(l)}(p_i p_j) e^{iX_{\mathfrak{K}(l)}(p_i p_j)} = 1/2 \sum_{p_i \neq p_j} \min[\mathfrak{A}_{\mathfrak{S}(l)}(p_i), \mathfrak{A}_{\mathfrak{S}(l)}(p_j)] e^{i \min\{X_{\mathfrak{S}(l)}(p_i), X_{\mathfrak{S}(l)}(p_j)\}}$$

$$\sum_{p_i \neq p_j} \mathfrak{B}_{\mathfrak{K}(l)}(p_i p_j) e^{iY_{\mathfrak{K}(l)}(p_i p_j)} = 1/2 \sum_{p_i \neq p_j} \min[\mathfrak{B}_{\mathfrak{S}(l)}(p_i), \mathfrak{B}_{\mathfrak{S}(l)}(p_j)] e^{i \min\{Y_{\mathfrak{S}(l)}(p_i), Y_{\mathfrak{S}(l)}(p_j)\}}$$

$$\sum_{p_i \neq p_j} \mathfrak{C}_{\mathfrak{K}(l)}(p_i p_j) e^{iZ_{\mathfrak{K}(l)}(p_i p_j)} = 1/2 \sum_{p_i \neq p_j} \max[\mathfrak{C}_{\mathfrak{S}(l)}(p_i), \mathfrak{C}_{\mathfrak{S}(l)}(p_j)] e^{i \max\{Z_{\mathfrak{S}(l)}(p_i), Z_{\mathfrak{S}(l)}(p_j)\}}$$

Proof: If G is a complex neutrosophic soft graph self-complementary. Then, an isomorphism $f: V_1 \rightarrow V_2$ that satisfies

$$\bar{\mathfrak{A}}_{\mathfrak{E}(l)}(f(p_i))e^{iX_{\mathfrak{E}(l)}(f(p_i))} = \mathfrak{A}_{\mathfrak{E}(l)}(f(p_i))e^{iX_{\mathfrak{E}(l)}(f(p_i))} = \mathfrak{A}_{\mathfrak{E}(l)}(p_i)e^{iX_{\mathfrak{E}(l)}(p_i)}$$

$$\bar{\mathfrak{B}}_{\mathfrak{E}(l)}(f(p_i))e^{iY_{\mathfrak{E}(l)}(f(p_i))} = \mathfrak{B}_{\mathfrak{E}(l)}(f(p_i))e^{iY_{\mathfrak{E}(l)}(f(p_i))} = \mathfrak{B}_{\mathfrak{E}(l)}(p_i)e^{iY_{\mathfrak{E}(l)}(p_i)}$$

$$\bar{\mathfrak{C}}_{\mathfrak{E}(l)}(f(p_i))e^{iZ_{\mathfrak{E}(l)}(f(p_i))} = \mathfrak{C}_{\mathfrak{E}(l)}(f(p_i))e^{iZ_{\mathfrak{E}(l)}(f(p_i))} = \mathfrak{C}_{\mathfrak{E}(l)}(p_i)e^{iZ_{\mathfrak{E}(l)}(p_i)}$$

and

$$\begin{aligned} \bar{\mathfrak{A}}_{\mathfrak{R}(l)}(f(p_i)f(p_j))e^{iX_{\mathfrak{R}(l)}(f(p_i)f(p_j))} &= \mathfrak{A}_{\mathfrak{R}(l)}(f(p_i)f(p_j))e^{iX_{\mathfrak{R}(l)}(f(p_i)f(p_j))} \\ &= \mathfrak{A}_{\mathfrak{R}(l)}(p_i p_j)e^{iX_{\mathfrak{R}(l)}(p_i p_j)} \end{aligned}$$

$$\begin{aligned} \bar{\mathfrak{B}}_{\mathfrak{R}(l)}(f(p_i)f(p_j))e^{iY_{\mathfrak{R}(l)}(f(p_i)f(p_j))} &= \mathfrak{B}_{\mathfrak{R}(l)}(f(p_i)f(p_j))e^{iY_{\mathfrak{R}(l)}(f(p_i)f(p_j))} \\ &= \mathfrak{B}_{\mathfrak{R}(l)}(p_i p_j)e^{iY_{\mathfrak{R}(l)}(p_i p_j)} \end{aligned}$$

$$\begin{aligned} \bar{\mathfrak{C}}_{\mathfrak{R}(l)}(f(p_i)f(p_j))e^{iZ_{\mathfrak{R}(l)}(f(p_i)f(p_j))} &= \mathfrak{C}_{\mathfrak{R}(l)}(f(p_i)f(p_j))e^{iZ_{\mathfrak{R}(l)}(f(p_i)f(p_j))} \\ &= \mathfrak{C}_{\mathfrak{R}(l)}(p_i p_j)e^{iZ_{\mathfrak{R}(l)}(p_i p_j)} \end{aligned}$$

for all $(p_i p_j) \in E_1$
we have

$$\begin{aligned} \bar{\mathfrak{A}}_{\mathfrak{R}(l)}(f(p_i)f(p_j))e^{iX_{\mathfrak{R}(l)}(f(p_i)f(p_j))} &= \min [\bar{\mathfrak{A}}_{\mathfrak{E}(l)}(f(p_i)), \bar{\mathfrak{A}}_{\mathfrak{E}(l)}(f(p_j))] e^{i \min \{X_{\mathfrak{E}(l)}(f(p_i)), X_{\mathfrak{E}(l)}(f(p_j))\}} \\ &\quad - \mathfrak{A}_{\mathfrak{R}(l)}(f(p_i)f(p_j))e^{iX_{\mathfrak{R}(l)}(f(p_i)f(p_j))} \end{aligned}$$

i.e.

$$\begin{aligned} \mathfrak{A}_{\mathfrak{R}(l)}(f(p_i)f(p_j))e^{iX_{\mathfrak{R}(l)}(f(p_i)f(p_j))} &= \min [\mathfrak{A}_{\mathfrak{E}(l)}(f(p_i)), \mathfrak{A}_{\mathfrak{E}(l)}(f(p_j))] e^{i \min \{X_{\mathfrak{E}(l)}(f(p_i)), X_{\mathfrak{E}(l)}(f(p_j))\}} \\ &\quad - \mathfrak{A}_{\mathfrak{R}(l)}(f(p_i)f(p_j))e^{iX_{\mathfrak{R}(l)}(f(p_i)f(p_j))} \end{aligned}$$

$$\begin{aligned} \mathfrak{A}_{\mathfrak{R}(l)}((p_i)(p_j))e^{iX_{\mathfrak{R}(l)}((p_i)(p_j))} &= \min [\mathfrak{A}_{\mathfrak{E}(l)}((p_i)), \mathfrak{A}_{\mathfrak{E}(l)}((p_j))] e^{i \min \{X_{\mathfrak{E}(l)}((p_i)), X_{\mathfrak{E}(l)}((p_j))\}} \\ &\quad - \mathfrak{A}_{\mathfrak{R}(l)}((p_i)(p_j))e^{iX_{\mathfrak{R}(l)}((p_i)(p_j))} \end{aligned}$$

$$\begin{aligned} \sum_{p_i \neq p_j} \mathfrak{A}_{\mathfrak{R}(l)}(p_i p_j)e^{iX_{\mathfrak{R}(l)}(p_i p_j)} + \sum_{p_i \neq p_j} \mathfrak{A}_{\mathfrak{R}(l)}(p_i p_j)e^{iX_{\mathfrak{R}(l)}(p_i p_j)} &= \sum_{p_i \neq p_j} \min [\mathfrak{A}_{\mathfrak{E}(l)}(p_i), \mathfrak{A}_{\mathfrak{E}(l)}(p_j)] e^{i \min \{X_{\mathfrak{E}(l)}(p_i), X_{\mathfrak{E}(l)}(p_j)\}} \end{aligned}$$

$$\begin{aligned} \sum_{p_i \neq p_j} \mathfrak{B}_{\mathfrak{R}(l)}(p_i p_j)e^{iY_{\mathfrak{R}(l)}(p_i p_j)} + \sum_{p_i \neq p_j} \mathfrak{B}_{\mathfrak{R}(l)}(p_i p_j)e^{iY_{\mathfrak{R}(l)}(p_i p_j)} &= \sum_{p_i \neq p_j} \min [\mathfrak{B}_{\mathfrak{E}(l)}(p_i), \mathfrak{B}_{\mathfrak{E}(l)}(p_j)] e^{i \min \{Y_{\mathfrak{E}(l)}(p_i), Y_{\mathfrak{E}(l)}(p_j)\}} \end{aligned}$$

$$\begin{aligned} \sum_{p_i \neq p_j} \mathfrak{C}_{\mathfrak{K}(l)}(p_i p_j) e^{iZ_{\mathfrak{K}(l)}(p_i p_j)} + \sum_{p_i \neq p_j} \mathfrak{C}_{\mathfrak{K}(l)}(p_i p_j) e^{iZ_{\mathfrak{K}(l)}(p_i p_j)} \\ = \sum_{p_i \neq p_j} \max[\mathfrak{C}_{\mathfrak{S}(l)}(p_i), \mathfrak{C}_{\mathfrak{S}(l)}(p_j)] e^{i \max\{Z_{\mathfrak{S}(l)}(p_i), Z_{\mathfrak{S}(l)}(p_j)\}} \end{aligned}$$

Then,

$$\sum_{p_i \neq p_j} \mathfrak{A}_{\mathfrak{K}(l)}(p_i p_j) e^{iX_{\mathfrak{K}(l)}(p_i p_j)} = 1/2 \sum_{p_i \neq p_j} \min[\mathfrak{A}_{\mathfrak{S}(l)}(p_i), \mathfrak{A}_{\mathfrak{S}(l)}(p_j)] e^{i \min\{X_{\mathfrak{S}(l)}(p_i), X_{\mathfrak{S}(l)}(p_j)\}}$$

$$\sum_{p_i \neq p_j} \mathfrak{B}_{\mathfrak{K}(l)}(p_i p_j) e^{iY_{\mathfrak{K}(l)}(p_i p_j)} = 1/2 \sum_{p_i \neq p_j} \min[\mathfrak{B}_{\mathfrak{S}(l)}(p_i), \mathfrak{B}_{\mathfrak{S}(l)}(p_j)] e^{i \min\{Y_{\mathfrak{S}(l)}(p_i), Y_{\mathfrak{S}(l)}(p_j)\}}$$

$$\sum_{p_i \neq p_j} \mathfrak{C}_{\mathfrak{K}(l)}(p_i p_j) e^{iZ_{\mathfrak{K}(l)}(p_i p_j)} = 1/2 \sum_{p_i \neq p_j} \max[\mathfrak{C}_{\mathfrak{S}(l)}(p_i), \mathfrak{C}_{\mathfrak{S}(l)}(p_j)] e^{i \max\{Z_{\mathfrak{S}(l)}(p_i), Z_{\mathfrak{S}(l)}(p_j)\}}$$

Proposition 4. Consider the complement of SCNSG $\overline{G_1} \approx \overline{G_2}$ (isomorphism) and G_1 and G_2 .

Proof: There exists a bijection $f: V_1 \rightarrow V_2$ such that the graphs G_1 and G_2 are isomorphic under this mapping.

$$\mathfrak{A}_{\mathfrak{S}_1(l)}(p_i) e^{iX_{\mathfrak{S}_1(l)}(p_i)} = \mathfrak{A}_{\mathfrak{S}_2(l)}(f(p_i)) e^{iX_{\mathfrak{S}_2(l)}(f(p_i))}$$

$$\mathfrak{B}_{\mathfrak{S}_1(l)}(p_i) e^{iY_{\mathfrak{S}_1(l)}(p_i)} = \mathfrak{B}_{\mathfrak{S}_2(l)}(f(p_i)) e^{iY_{\mathfrak{S}_2(l)}(f(p_i))}$$

$$\mathfrak{C}_{\mathfrak{S}_1(l)}(p_i) e^{iZ_{\mathfrak{S}_1(l)}(p_i)} = \mathfrak{C}_{\mathfrak{S}_2(l)}(f(p_i)) e^{iZ_{\mathfrak{S}_2(l)}(f(p_i))}$$

for all $(p_i \in \mathfrak{S}_1)$,

and

$$\mathfrak{A}_{\mathfrak{K}_1(l)}(p_i p_j) e^{iX_{\mathfrak{K}_1(l)}(p_i p_j)} = \mathfrak{A}_{\mathfrak{K}_2(l)}(f(p_i) f(p_j)) e^{iX_{\mathfrak{K}_2(l)}(f(p_i) f(p_j))},$$

$$\mathfrak{B}_{\mathfrak{K}_1(l)}(p_i p_j) e^{iY_{\mathfrak{K}_1(l)}(p_i p_j)} = \mathfrak{B}_{\mathfrak{K}_2(l)}(f(p_i) f(p_j)) e^{iY_{\mathfrak{K}_2(l)}(f(p_i) f(p_j))},$$

$$\mathfrak{C}_{\mathfrak{K}_1(l)}(p_i p_j) e^{iZ_{\mathfrak{K}_1(l)}(p_i p_j)} = \mathfrak{C}_{\mathfrak{K}_2(l)}(f(p_i) f(p_j)) e^{iZ_{\mathfrak{K}_2(l)}(f(p_i) f(p_j))},$$

For all $p_i p_j \in \mathfrak{K}$ and $l \in L$

We have

$$\begin{aligned} \overline{\mathfrak{A}}_{\mathfrak{K}_1(l)}(p_i p_j) e^{iX_{\mathfrak{K}(l)}(p_i p_j)} \\ = \min[\mathfrak{A}_{\mathfrak{S}_1(l)}(p_i), \mathfrak{A}_{\mathfrak{S}_1(l)}(p_j)] e^{i \min\{X_{\mathfrak{S}_1(l)}(p_i), X_{\mathfrak{S}_1(l)}(p_j)\}} - \mathfrak{A}_{\mathfrak{K}_1(l)}(p_i p_j) e^{iX_{\mathfrak{K}(l)}(p_i p_j)} \end{aligned}$$

$$\begin{aligned} = \min[\mathfrak{A}_{\mathfrak{S}_2(l)}(f(p_i)), \mathfrak{A}_{\mathfrak{S}_2(l)}(f(p_j))] e^{i \min\{X_{\mathfrak{S}_2(l)}(f(p_i)), X_{\mathfrak{S}_2(l)}(f(p_j))\}} \\ - \mathfrak{A}_{\mathfrak{K}_2(l)}(f(p_i) f(p_j)) e^{iX_{\mathfrak{K}(l)}(f(p_i) f(p_j))} \end{aligned}$$

$$= \overline{\mathfrak{A}}_{\mathfrak{K}_2(l)}(f(p_i) f(p_j)) e^{iX_{\mathfrak{K}(l)}(f(p_i) f(p_j))}$$

$$\overline{\mathfrak{B}}_{\mathfrak{K}_1(l)}(p_i p_j) e^{iY_{\mathfrak{K}(l)}(p_i p_j)}$$

$$= \min[\mathfrak{B}_{\mathfrak{S}_1(l)}(p_i), \mathfrak{B}_{\mathfrak{S}_1(l)}(p_j)] e^{i \min\{Y_{\mathfrak{S}_1(l)}(p_i), Y_{\mathfrak{S}_1(l)}(p_j)\}} - \mathfrak{B}_{\mathfrak{K}_1(l)}(p_i p_j) e^{iY_{\mathfrak{K}(l)}(p_i p_j)}$$

$$\begin{aligned}
 &= \min \left[\mathfrak{B}_{\mathfrak{S}_2(l)}(f(p_i)), \mathfrak{B}_{\mathfrak{S}_2(l)}(f(p_j)) \right] e^{i \min\{Y_{\mathfrak{S}_2(l)}(f(p_i)), Y_{\mathfrak{S}_2(l)}(f(p_j))\}} \\
 &\quad - \mathfrak{B}_{\mathfrak{K}_2(l)}(f(p_i)f(p_j)) e^{iY_{\mathfrak{K}(l)}(f(p_i)f(p_j))} \\
 &= \overline{\mathfrak{B}}_{\mathfrak{K}_2(l)}(f(p_i)f(p_j)) e^{iY_{\mathfrak{K}(l)}(f(p_i)f(p_j))} \\
 \overline{\mathfrak{C}}_{\mathfrak{K}_1(l)}(p_i p_j) e^{iZ_{\mathfrak{K}(l)}(p_i p_j)} \\
 &= \min[\mathfrak{C}_{\mathfrak{S}_1(l)}(p_i), \mathfrak{C}_{\mathfrak{S}_1(l)}(p_j)] e^{i \min\{Z_{\mathfrak{S}_1(l)}(p_i), Z_{\mathfrak{S}_1(l)}(p_j)\}} - \mathfrak{C}_{\mathfrak{K}_1(l)}(p_i p_j) e^{iZ_{\mathfrak{K}(l)}(p_i p_j)} \\
 &= \min \left[\mathfrak{C}_{\mathfrak{S}_2(l)}(f(p_i)), \mathfrak{C}_{\mathfrak{S}_2(l)}(f(p_j)) \right] e^{i \min\{Z_{\mathfrak{S}_2(l)}(f(p_i)), Z_{\mathfrak{S}_2(l)}(f(p_j))\}} \\
 &\quad - \mathfrak{C}_{\mathfrak{K}_2(l)}(f(p_i)f(p_j)) e^{iZ_{\mathfrak{K}(l)}(f(p_i)f(p_j))} \\
 &= \overline{\mathfrak{C}}_{\mathfrak{K}_2(l)}(f(p_i)f(p_j)) e^{iZ_{\mathfrak{K}(l)}(f(p_i)f(p_j))}
 \end{aligned}$$

For all $p_i p_j \in \mathfrak{K}$ and $l \in L$

Hence $(\overline{G}_1 \approx \overline{G}_2)$.

Definition 14. Let $G = (G^*, \mathfrak{S}, \mathfrak{K}, L)$ be a complete complex neutrosophic soft graph, where $G^* = (V, E)$. The complement of G , denoted by $\overline{G} = (\overline{\mathfrak{S}}, \overline{\mathfrak{K}})$, is defined as another complex neutrosophic soft graph over the same underlying structure $G^* = (\overline{V}, \overline{E})$, where:

- (1) The vertex set remains unchanged, i.e., $\overline{V} = V$.
- (2) The complex neutrosophic soft values on vertices are preserved:

$$\begin{aligned}
 \overline{\mathfrak{A}}_{\mathfrak{S}(l)}(p_i) e^{iX_{\mathfrak{S}(l)}(p_i)} &= \mathfrak{A}_{\mathfrak{S}(l)}(p_i) e^{iX_{\mathfrak{S}(l)}(p_i)}, \\
 \overline{\mathfrak{B}}_{\mathfrak{S}(l)}(p_i) e^{iY_{\mathfrak{S}(l)}(p_i)} &= \mathfrak{B}_{\mathfrak{S}(l)}(p_i) e^{iY_{\mathfrak{S}(l)}(p_i)}, \\
 \overline{\mathfrak{C}}_{\mathfrak{S}(l)}(p_i) e^{iZ_{\mathfrak{S}(l)}(p_i)} &= \mathfrak{C}_{\mathfrak{S}(l)}(p_i) e^{iZ_{\mathfrak{S}(l)}(p_i)},
 \end{aligned}$$

- (2) The complex neutrosophic soft edge functions of the complement $\overline{\mathfrak{K}}$ are defined as:

$$\begin{aligned}
 \overline{\mathfrak{A}}_{\mathfrak{K}(l)}(p_i p_j) e^{iX_{\mathfrak{K}(l)}(p_i p_j)} \\
 &= \min[\mathfrak{A}_{\mathfrak{S}(l)}(p_i), \mathfrak{A}_{\mathfrak{S}(l)}(p_j)] e^{i \min\{X_{\mathfrak{S}(l)}(p_i), X_{\mathfrak{S}(l)}(p_j)\}} - \mathfrak{A}_{\mathfrak{K}(l)}(p_i p_j) e^{iX_{\mathfrak{K}(l)}(p_i p_j)}, \\
 \overline{\mathfrak{B}}_{\mathfrak{K}(l)}(p_i p_j) e^{iY_{\mathfrak{K}(l)}(p_i p_j)} \\
 &= \min[\mathfrak{B}_{\mathfrak{S}(l)}(p_i), \mathfrak{B}_{\mathfrak{S}(l)}(p_j)] e^{i \min\{Y_{\mathfrak{S}(l)}(p_i), Y_{\mathfrak{S}(l)}(p_j)\}} - \mathfrak{B}_{\mathfrak{K}(l)}(p_i p_j) e^{iY_{\mathfrak{K}(l)}(p_i p_j)}, \\
 \overline{\mathfrak{C}}_{\mathfrak{K}(l)}(p_i p_j) e^{iZ_{\mathfrak{K}(l)}(p_i p_j)} \\
 &= \max[\mathfrak{C}_{\mathfrak{S}(l)}(p_i), \mathfrak{C}_{\mathfrak{S}(l)}(p_j)] e^{i \max\{Z_{\mathfrak{S}(l)}(p_i), Z_{\mathfrak{S}(l)}(p_j)\}} - \mathfrak{C}_{\mathfrak{K}(l)}(p_i p_j) e^{iZ_{\mathfrak{K}(l)}(p_i p_j)},
 \end{aligned}$$

for all $p_i p_j \in E$ and for each $l \in L$.

Proposition 5. Consider the complement of a complete complex neutrosophic soft graph that contains no edges. In this case, if G is a complete graph, then its complement \overline{G} contains no edges, i.e., the edge set of \overline{G} is empty.

Proof: Consider the graph $G = (G^*, \mathfrak{S}, \mathfrak{K}, L)$, which represents the complement of a complete complex

neutrosophic soft graph.

$$\bar{\mathfrak{A}}_{\mathfrak{R}(l)}(p_i p_j) e^{iX_{\mathfrak{R}(l)}(p_i p_j)} = \min[\mathfrak{A}_{\mathfrak{E}(l)}(p_i), \mathfrak{A}_{\mathfrak{E}(l)}(p_j)] e^{i \min\{X_{\mathfrak{E}(l)}(p_i), X_{\mathfrak{E}(l)}(p_j)\}}$$

$$\bar{\mathfrak{B}}_{\mathfrak{R}(l)}(p_i p_j) e^{iY_{\mathfrak{R}(l)}(p_i p_j)} = \min[\mathfrak{B}_{\mathfrak{E}(l)}(p_i), \mathfrak{B}_{\mathfrak{E}(l)}(p_j)] e^{i \min\{Y_{\mathfrak{E}(l)}(p_i), Y_{\mathfrak{E}(l)}(p_j)\}}$$

$$\bar{\mathfrak{C}}_{\mathfrak{R}(l)}(p_i p_j) e^{iX_{\mathfrak{R}(l)}(p_i p_j)} = \max[\mathfrak{C}_{\mathfrak{E}(l)}(p_i), \mathfrak{C}_{\mathfrak{E}(l)}(p_j)] e^{i \max\{Z_{\mathfrak{E}(l)}(p_i), Z_{\mathfrak{E}(l)}(p_j)\}}$$

For all $p_i p_j \in V$ and $l \in L$

Hence in \bar{G}

$$\begin{aligned} \bar{\mathfrak{A}}_{\mathfrak{R}(l)}(p_i p_j) e^{iX_{\mathfrak{R}(l)}(p_i p_j)} &= \min[\mathfrak{A}_{\mathfrak{E}(l)}(p_i), \mathfrak{A}_{\mathfrak{E}(l)}(p_j)] e^{i \min\{X_{\mathfrak{E}(l)}(p_i), X_{\mathfrak{E}(l)}(p_j)\}} - \mathfrak{A}_{\mathfrak{R}(l)}(p_i p_j) e^{iX_{\mathfrak{R}(l)}(p_i p_j)} \\ &= \min[\mathfrak{A}_{\mathfrak{E}(l)}(p_i), \mathfrak{A}_{\mathfrak{E}(l)}(p_j)] e^{i \min\{X_{\mathfrak{E}(l)}(p_i), X_{\mathfrak{E}(l)}(p_j)\}} \\ &\quad - \min[\mathfrak{A}_{\mathfrak{E}(l)}(p_i), \mathfrak{A}_{\mathfrak{E}(l)}(p_j)] e^{i \min\{X_{\mathfrak{E}(l)}(p_i), X_{\mathfrak{E}(l)}(p_j)\}} \\ &= 0, \forall i, j, \dots, n \end{aligned}$$

$$\begin{aligned} \bar{\mathfrak{B}}_{\mathfrak{R}(l)}(p_i p_j) e^{iX_{\mathfrak{R}(l)}(p_i p_j)} &= \min[\mathfrak{B}_{\mathfrak{E}(l)}(p_i), \mathfrak{B}_{\mathfrak{E}(l)}(p_j)] e^{i \min\{X_{\mathfrak{E}(l)}(p_i), X_{\mathfrak{E}(l)}(p_j)\}} - \mathfrak{B}_{\mathfrak{R}(l)}(p_i p_j) e^{iX_{\mathfrak{R}(l)}(p_i p_j)} \\ &= \min[\mathfrak{B}_{\mathfrak{E}(l)}(p_i), \mathfrak{B}_{\mathfrak{E}(l)}(p_j)] e^{i \min\{X_{\mathfrak{E}(l)}(p_i), X_{\mathfrak{E}(l)}(p_j)\}} \\ &\quad - \min[\mathfrak{B}_{\mathfrak{E}(l)}(p_i), \mathfrak{B}_{\mathfrak{E}(l)}(p_j)] e^{i \min\{X_{\mathfrak{E}(l)}(p_i), X_{\mathfrak{E}(l)}(p_j)\}} \\ &= 0, \forall i, j, \dots, n \end{aligned}$$

$$\begin{aligned} \bar{\mathfrak{C}}_{\mathfrak{R}(l)}(p_i p_j) e^{iX_{\mathfrak{R}(l)}(p_i p_j)} &= \min[\mathfrak{C}_{\mathfrak{E}(l)}(p_i), \mathfrak{C}_{\mathfrak{E}(l)}(p_j)] e^{i \min\{X_{\mathfrak{E}(l)}(p_i), X_{\mathfrak{E}(l)}(p_j)\}} - \mathfrak{C}_{\mathfrak{R}(l)}(p_i p_j) e^{iX_{\mathfrak{R}(l)}(p_i p_j)} \\ &= \min[\mathfrak{C}_{\mathfrak{E}(l)}(p_i), \mathfrak{C}_{\mathfrak{E}(l)}(p_j)] e^{i \min\{X_{\mathfrak{E}(l)}(p_i), X_{\mathfrak{E}(l)}(p_j)\}} \\ &\quad - \min[\mathfrak{C}_{\mathfrak{E}(l)}(p_i), \mathfrak{C}_{\mathfrak{E}(l)}(p_j)] e^{i \min\{X_{\mathfrak{E}(l)}(p_i), X_{\mathfrak{E}(l)}(p_j)\}} \\ &= 0, \forall i, j, \dots, n \end{aligned}$$

Thus

$$(\bar{\mathfrak{A}}_{\mathfrak{R}(l)}(p_i p_j) e^{iX_{\mathfrak{R}(l)}(p_i p_j)}, \bar{\mathfrak{B}}_{\mathfrak{R}(l)}(p_i p_j) e^{iX_{\mathfrak{R}(l)}(p_i p_j)}, \bar{\mathfrak{C}}_{\mathfrak{R}(l)}(p_i p_j) e^{iX_{\mathfrak{R}(l)}(p_i p_j)}) = (0, 0, 0)$$

Consequently, if G represents a complete complex neutrosophic soft graph, then the edge set of its complement, denoted by \bar{G} , is void.

4. CONCLUSION

This work focuses on the innovative concept of complex neutrosophic soft graphs. Additionally, it explores several fundamental properties of strong, complete, complementary, and self-complementary types of complex neutrosophic soft graphs. The introduced notions are further elucidated through pertinent examples. Future work will emphasize operations related to complex neutrosophic soft graphs, particularly their regular forms and other variants. The framework of complex neutrosophic soft graphs holds potential for diverse applications, such as social network analysis, recommendation systems, traffic modeling, and transportation systems.

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