

MULTIFACTORIZATION OF COMPLETE GRAPHS INTO EQUAL COPIES OF CYCLES AND PATHS.

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ABSTRACT

Let $K_n(\lambda)$ denote a complete graph with n vertices with edge multiplicity λ . Factorization of a graph is a partition of the given graph into isomorphic spanning subgraphs. A (G,H) – Multifactorization of a graph is a factorization of given graph into G and H with at least one copy of G and H . In this paper, we studied about the (G,H) – Multifactorization of K_n with equal copies of G and H , where G is a Cycle factor and H is a Path factor.

Keyword: Complete Graphs, Multifactorization, Path and Cycle.

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1. INTRODUCTION

Let G be a simple and finite graph. Let K_n denote a complete graph on n vertices. $G(\lambda)$ denote a graph G with edge multiplicity λ . Let C_n denote a cycle and P_n denote a path on n vertices respectively. Let $K_n(\lambda)$ denote a complete graph on n vertices with edge multiplicity λ . Decomposition of a graph is a partition of the given graph into isomorphic subgraphs. Factorization of a graph is a partition of the given graph into isomorphic spanning subgraphs. A (G,H) – Multidecomposition of a graph is a decomposition of given graph into G and H with at least one copy of G and H . A (G,H) – Multifactorization of a graph is a factorization of given graph into G - factors and H - factors with at least one copy of G and H . The study of multidecomposition was introduced by Atif Abeiuda and Mike Daven in 2003 [1]. Priyadharsini and Muthusamy extended the study of Multidecomposition and Multifactorization for many cases [3]. More studies on multidecomposition can be found in book [5].

Hurd and Sarvate [4] studied a decomposition of a $K_n(\lambda)$, ($\lambda \geq 2$) into H_3 graphs with equal copies. They gave a necessary and sufficient condition for the decomposition of $K_n(\lambda)$ into equal copies of C_3 and P_3 . Now we rewrite the above problem as (G,H) – Multidecomposition of $K_n(\lambda)$ with equal copies of G and H . In this paper we extend the above problem as (G,H) – Multifactorization of $K_n(\lambda)$ with equal copies of (C_n, P_n) .

To prove our results, we need the following results.

Walecki Construction [2]:

- (i) The graph K_{2n+1} has a Hamilton cycle decomposition.
- (ii) The graph K_{2n} has a Hamilton Path decomposition.

Lemma [1.1]:

- (i) For even $m > 2$, the graph $2K_{2m}$ has a C_{2m} decomposition.

- (ii) The graph $2K_{2m+1}$ has a Hamilton path decomposition.

Corollary [1.2]:

The graph K_{2n} has a Hamilton path decomposition.

2. (P_n, C_n) – Multifactorization of $K_n(\lambda)$ with equal copies of P_n and C_n .**Lemma 2.1:[3]**

For $n \geq 3$, $K_n(\lambda)$ has a (P_n, C_n) – multifactorization if and only if

- (i) $|E(K_n(\lambda))| = r(n-1) + sn$, $r, s > 0$ and
(ii) $\lambda > 2$.

The above theorem gave a necessary and sufficient conditions for multifactorization of P_n and C_n .

Theorem 2.2:

For $n \geq 3$, $K_n(\lambda)$ has a $(P_n, C_n)_r$ – multifactorization into equal copies (r copies) if

- (i) $|E(K_n(\lambda))| = r(2n-1)$, $r > 0$ and
(ii) $\lambda \equiv 0 \pmod{2n-1}$.

Proof:

Assume that $K_n(\lambda)$ has a $(P_n, C_n)_r$ – multifactorization. Then there exists an integer 'r' such that

$$K_n(\lambda) = rP_n \oplus rC_n. \text{ Hence } |E(K_n(\lambda))| = r(n-1) + sn = r(2n-1), r > 0$$

(i.e), $\lambda n(n-1)/2 = r(2n-1)$. This is possible only $\lambda/2(2n-2)$, as $(n, n-1) = 1$, $(n, 2n-1) = 1$ and $(n-1, 2n-1) = 1$.

Hence $\lambda \equiv 0 \pmod{2n-1}$.

Conversely, Assume conditions (i) and(ii) are satisfied.

We give the construction for $\lambda = (2n-1)$. Consider $(2n-1) K_n$

Case(a): n is odd

$$\begin{aligned} (2n-1) K_n &= (2n-3+2) K_n = (n+n-3+2) K_n \\ &= (n-3)/2 nP_n \oplus n(n-1)/2 C_n \oplus nP_n \\ &= n(n-1)/2 P_n \oplus n(n-1)/2 C_n. \text{ [By Waleckis Construction and Lemma1.1]} \end{aligned}$$

Case(b): n is even

$$\begin{aligned} (2n-1) K_n &= (2n-3+2) K_n = (n+n-3+2) K_n \\ &= (n-3) K_n \oplus n K_n \oplus 2 K_n \\ &= (n-3)n/2 P_n \oplus n/2 (n-1) C_n \oplus nP_n \\ &= n(n-1)/2 P_n \oplus n(n-1)/2 C_n. \text{ [By Waleckis Construction and Corollary 1.2]} \end{aligned}$$

Hence the Proof for all $\lambda \equiv 0 \pmod{2n-1}$.

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