

# Graphical Passwords in the Context of Contra Harmonic Mean Labeling

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## Abstract

Graphical Passwords have proved to be efficient and user-friendly authentication method compared to usual text-based passwords. Topsnut-GPWs are new type of graphical passwords made using topological structures and number theory. In this article, we construct Topsnut-GPWs using a certain type of caterpillar tree and Alphabet graphs by means of Contra Harmonic Mean labeling. In addition, we explore the possibility of Alphabet graphs and caterpillar tree being the key-model and lock-model respectively.

## 1. Preliminaries

The increasing complexity of cyber threats has exposed the vulnerabilities of traditional text-based passwords (TB-paws), which are often susceptible to brute force attacks, phishing and shoulder surfing. This has led to significant researches in alternative authentication mechanisms. Among these, graphical passwords stands as a promising method due to human brain's ability to recognize and recall visual patterns when compared to abstract string of characters.

A novel class of graphical passwords known as Topsnut-GPWs was introduced by Wang et. al which uses the concept of creating graphical passwords using 'topological structure + number theory', later called as 'Topsnut-GPW' in [5] and [10]. Subsequent research has expanded on this concept, leading to variants like Hanzi-GPWs, which utilizes Chinese characters.

The mathematical foundation of many Topsnut-GPWs lie in graph labelings. The security and uniqueness of a Topsnut-GPW depend on the type of labeling employed. In this paper, we utilize the Contra Harmonic Mean labeling, a concept introduced by

Sandhya et al. In this article i) we establish that a certain class of caterpillar trees admits a Contra Harmonic Mean labeling, thereby qualifying them for use as key models ii) we introduce Alphabet graphs and obtain an algorithm to Super Contra Harmonic mean label the Alphabet graphs iii) we demonstrate the construction of Topsnut-GPWs using Alphabet graphs and a special class of caterpillar tree to create efficient lock and key models.

We provide a brief summary of definitions which are necessary for the present investigation.

**Definition 1.1** [3]: A graph  $G(V, E)$  with  $p$  vertices and  $q$  edges is said to be Contra Harmonic mean graph if it is possible to label the vertices  $x \in V$  with distinct labels  $f(x)$  from  $0, 1, 2, \dots, q$  in such a way that when each edge  $e = uv$  is labeled with  $f(e = uv) = \left\lfloor \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rfloor$  or  $\left\lceil \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rceil$  with distinct edge labels. Then  $f$  is called Contra Harmonic mean labeling of  $G$ .

**Definition 1.2** [3]: An injective function  $V(G) \rightarrow \{1, 2, \dots, p + q\}$  is called a Super Contra Harmonic mean labeling if the induced edge labeling  $f(e = uv)$  is defined as  $f(e) = \left\lfloor \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rfloor$  or  $\left\lceil \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rceil$ , so that  $\{f(V(G))\} \cup \{f(e) : e \in E(G)\} = \{1, 2, \dots, p + q\}$ . A graph which allows Super Contra Harmonic Mean labeling is called Super Contra Harmonic Mean graph.

**Definition 1.3** [3]: A graph  $G(V, E)$  with  $p$  vertices and  $q$  edges is said to be Odd Contra Harmonic Mean graph if it is possible to label the vertices  $x \in V$  with distinct labels  $f(x)$  from  $0, 1, 2, \dots, 2q$  in such a way that each edge  $e = uv$  labeled with  $f(e = uv) = \left\lfloor \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rfloor$  or  $\left\lceil \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rceil$  gets distinct edge labels  $1, 3, 5, \dots, 2q - 1$ . Then  $f$  is called Odd Contra Harmonic Mean labeling of  $G$ .

**Definition 1.4** : Let  $G_1(a, q_1)$  and  $G_2(a, q_2)$  be two graphs such that the vertex identified graph  $G(p, q) = G_1 \circ G_2$  has a mapping  $f: V(G) \rightarrow [0, q]$  such that i)  $f(x) \neq f(y)$  for any pair of vertices of  $x, y \in G$ , ii)  $f$  is an Odd Contra Harmonic Mean labeling of  $G_1$  iii) the set  $f(E(G_i)) = [1, 2q_i - 1]^o$  with  $i = 1, 2$ . Then  $G_1$  is said to be a source graph (key-model) and  $G_2$  an associated graph (lock-model) of  $G_1$ .

## 2. Main Results

### 2.1 Contra Harmonic Mean Labeling on Caterpillar Trees

**Theorem 2.1.1:** A caterpillar tree such that no vertices have degree  $> 4$  is Contra Harmonic Mean iff it admits Super Contra Harmonic Mean labeling.

**Proof**

Let  $T(p, q)$  be a caterpillar tree such that no vertices have degree  $> 4$ .

Suppose that the tree  $T$  has a Contra Harmonic Mean labeling defined as

$$f(x_i) = i - 1, x_i \in V(T), i \in [1, V(T)] \text{ such that}$$

$$f(x_i x_j) = \left\lfloor \frac{f(x_i)^2 + f(x_j)^2}{f(x_i) + f(x_j)} \right\rfloor \text{ or } \left\lceil \frac{f(x_i)^2 + f(x_j)^2}{f(x_i) + f(x_j)} \right\rceil \in [1, E(T)]$$

We construct a new labeling  $g: V(T) \rightarrow \{1, 2, \dots, p + q\}$  of  $T$  by

$$g(x_i) = 2f(x_i) + 1, g(x_i x_j) = f(x_i x_j)$$

Hence  $g(x_i x_j)$  and  $g(x_i)$  are distinct and  $g[V(T) \cup E(T)] = [1, V(T) + E(T)]$ .

Hence  $g$  is Super Contra Harmonic Mean labeling.

Conversely, assume  $g'$  be the Super Contra Harmonic Mean of  $T$  such that

$$g'(x_i) = 2i - 1, i \in [1, V(T)] \text{ and}$$

$$g'(x_i x_j) = \left\lfloor \frac{g'(x_i)^2 + g'(x_j)^2}{g'(x_i) + g'(x_j)} \right\rfloor \text{ or } \left\lceil \frac{g'(x_i)^2 + g'(x_j)^2}{g'(x_i) + g'(x_j)} \right\rceil \in [1, V(T) + E(T)]$$

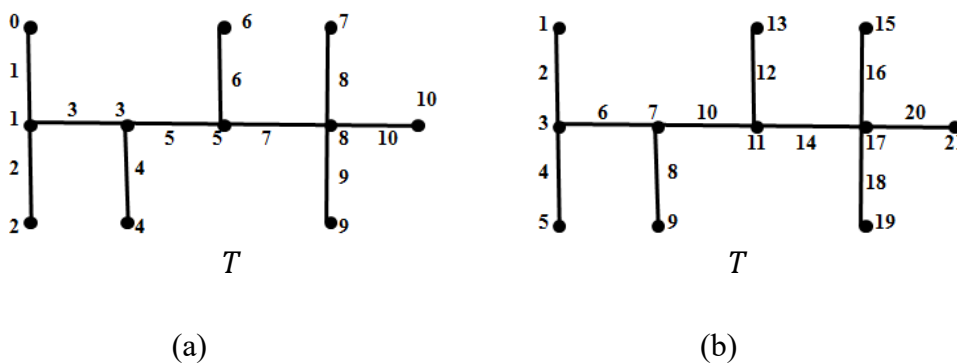
Now, construct a labeling  $f': V(T) \rightarrow \{1, 2, \dots, p\}$  defined as

$$f'(x_i) = g'(x_i) - 1 \text{ and } f'(x_i x_j) = g'(x_i x_j)$$

Clearly,  $f'(V(T)) = [0, |E(T)|]$  and  $f'(E(T)) = [1, |E(T)|]$

Hence  $f'$  is Contra Harmonic Mean labeling.

Fig. 1 shows the illustration of theorem 2.1.1



**Fig.1:** (a) Contra Harmonic Mean labeling of  $T$  and Fig.1(b) Super Contra Harmonic Mean labeling of  $T$ .

Next, we construct large scale caterpillar trees by using vertex identification operation as follows

**Theorem 2.1.2:** If the vertex-identification operation on two disjoint trees  $T_1(p_1, q_1)$  and  $T_2(p_2, q_2)$  is performed by combining  $u_i$  of  $T_1$  and  $v_j$  of  $T_2$  where  $f(u_i) = q_1$  and  $f(v_j) = 0$  or  $f(u_i) = 0$  and  $f(v_j) = q_2$  to form a new graph  $T$ , then  $T$  is a caterpillar tree.

**Proof**

Let  $T_1$  be a tree such that  $f(u_i) = q_1$  and  $f(v_j) = 0$

We consider two cases,

Case (i): Let  $u_i$  be the vertex on the central path of  $T_1$ .

Let  $u_1, u_2, \dots, u_i$  be the vertices on the central path and let  $v_1, v_2, \dots, v_h$  be the vertices on the central path of  $T_2$ .

Then by adjoining  $u_i$  to  $v_j$ , we get a tree  $T$  with central path  $u_1 u_2 \dots u_i v_j v_1 v_2 \dots v_h$ .

Hence  $T$  is a caterpillar tree.

Case (ii): Let  $u_i$  be a leaf of  $T_1$

Let  $u_1, u_2, \dots, u_g$  be the vertices on the central path and let  $v_1, v_2, \dots, v_h$  be the vertices on the central path of  $T_2$ .

Then by adjoining  $u_i$  to  $v_j$ , we get a tree  $T$  with central path  $u_1 u_2 \dots u_g u_i v_1 v_2 \dots v_h$ .

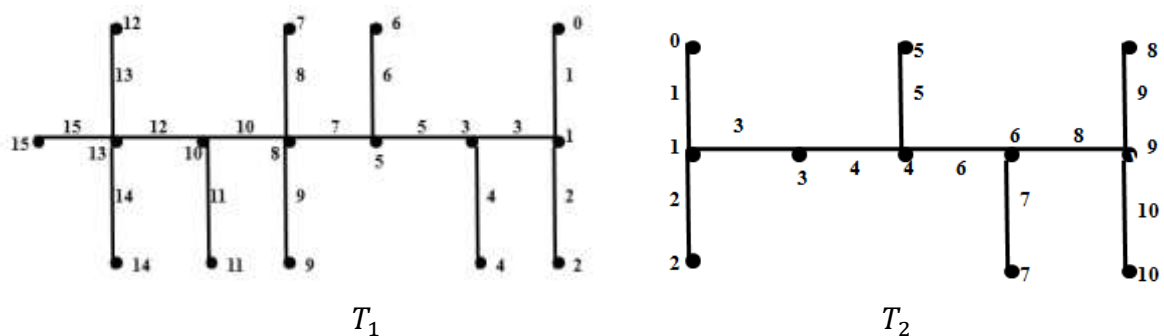
Hence  $T$  is a caterpillar tree.

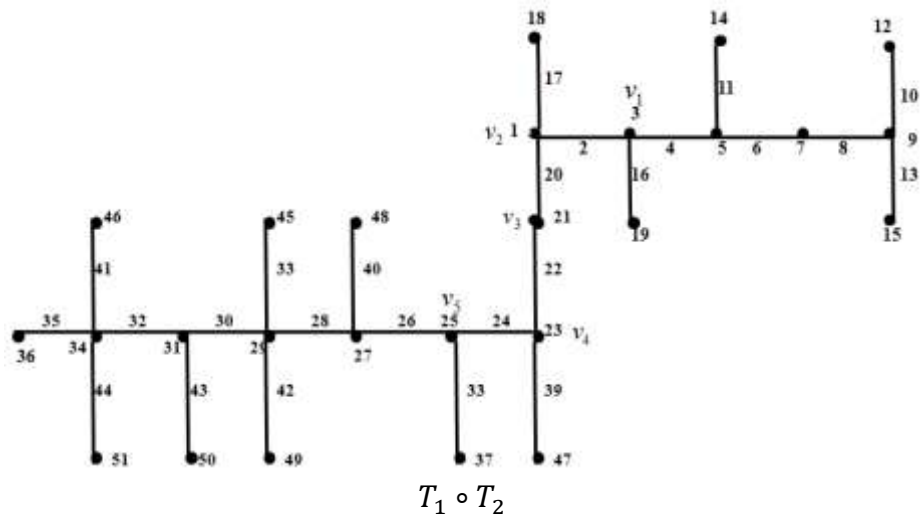
Similarly, we can prove the result for  $f(u_i) = 0$  and  $f(v_j) = q_2$ .

Next, we have the following remark

**Remark 2.1.3:** The theorem 2.1.1 and theorem 2.1.2 can be used to create large scale caterpillar trees. If  $T_1, T_2, \dots, T_n$  are Contra Harmonic Meangraphs and applying the vertex-identification operation form a new tree having Super Contra Harmonic Mean labeling.

Fig. 2 shows the illustration of vertex-identification operation.





**Fig.2:**  $T_1$  and  $T_2$  are Contra Harmonic Mean labeled and  $T_1 \circ T_2$  is Super Contra Harmonic Mean labeled

Using path-neighbour method in Fig. 2, we get vv-type TB-paw along path  $p_1 = v_1v_2v_3v_4v_5$  as

$$P_{vv}(p_1) = 3151931318211211232123212547232523273725$$

### 2.2 Super Contra Harmonic Mean Labeling on Alphabet Graphs

The risk with Topsnut-GPW is that there is a risk of forgetting when not used for a longer time, whereas TB-paws have a greater chance of recalling later in time since users tend to use alphanumeric passwords that are associated with them in some way. Hence, we investigate the idea of converting text-based passwords to graphical structures. Alphabet graphs are English alphabets converted to graphs. Since, English is the most spoken global language, wide number of users benefit from Alphabet graphs.

Here we create an algorithm to construct Super Contra Harmonic Mean labeled Alphabet graphs.

#### Algorithm 2.2.1: Algorithm to Construct Super Contra Harmonic Mean labeled Alphabet Graphs

Let  $A$  denote Alphabet graphs

The vertices and edges of  $A$  are labeled as  $v_1e_2v_3e_4v_5 \dots$  such that  $e_2$  is the edge connecting  $v_1$  and  $v_3$ .

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*Step 1:* First, we consider Tree like alphabets C, E, F, G, H, I, J, K, L, M, N, S, T, U, V, W, X, Y, Z. This is further subdivided into Alphabets that are Paths and Trees with at least one branch.

*Subcase (i):* First, we consider path like alphabets C, G, I, L, M, N, S, U, V, W, Z. The vertices and edges are labeled in the same way as labeling paths using consecutive numbers, i.e.,  $f(v_1) = 1, f(v_3) = 3, \dots, f(v_n) = n$ , where  $n = p + q$

*Subcase (ii):* Next we consider the alphabets E, F, H, J, K, T, X, Y. The labeling is defined as follows:

$$f(v_i) = i, \text{ if } d(v_i) \leq 3, \forall v_i \in A$$

$$f(v_i) = i, f(v_n) = n, f(v_{n-2}) = n - 1, \text{ where } n = p + q, p = |V(A)| \text{ and } q = |E(A)|$$

*Step 2:* Next we consider alphabets that are either cycles or contain cycles which include A, B, D, O, P, Q, R.

*Subcase (i):* First we define labeling in cyclic alphabets D and O

$$f(v_j) = j, \text{ where } v_j \text{ denotes the initial vertex in } A,$$

$$f(v_{n-1}) = n \text{ where } n = p' + q', p' \text{ and } q' \text{ are number of vertices and edges in the cycle.}$$

Now the remaining vertices are labeled in such a way that,

$$f(v_{i+2}) = \begin{cases} f(v_i) + 3, & \text{if either } f(v_{i+2}) = f(e_n) \text{ or } f(v_i v_{i+2}) = f(e_n) \\ f(v_i) + 2, & \text{otherwise} \end{cases}$$

*Subcase (ii):* Here we consider the alphabets that contain cycle which include A, B, P, R, Q. These alphabets have at most 6 vertices and there exist at least one vertex  $v_i$  in A, such that  $d(v_i) \geq 3$ , then we define

$$f(v_i) = i, f(v_{i-2}) = i - 2, f(v_{i-2}) = i - 2,$$

$$f(v_{i+4}) = p + q, f(v_{i+6}) = f(v_{i+4}) - 2,$$

The remaining vertex is labeled as  $f(v_{i+8}) = f(v_{i+2}) + 2$

Next, we have the edge-series operation process

**Remark 2.2.2:** The Alphabet graphs are converted to Word graphs by adding an edge between  $u_i$  of  $A_1$  and  $v_j$  of  $A_2$  where  $f(u_i) = p_1 + q_1$ , where  $p_1 = |V(A_1)|$  and  $q_1 = |E(A_1)|$  and  $v_j$  is the initial vertex in  $A_2$ .

Obtaining a Word graph from Alphabet graphs is illustrated in Fig. 3

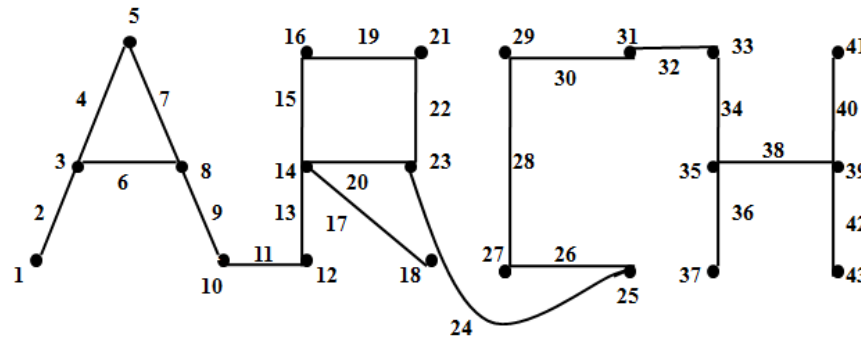


Fig. 3: Word graph obtained from the Alphabets A, R, C and H

### 2.3 Key and Lock Models using Caterpillar trees and Alphabet graphs

To create efficient key models, we use Odd Contra harmonic Mean labelling to label Alphabet graphs.

**Theorem 2.3.1:** Alphabet graphs are the source graphs of caterpillar trees with no vertices having degree  $>4$ .

**Proof:**

Let  $A$  denote Alphabet graph and let  $T$  be a Caterpillar tree with no vertices of degree  $> 4$

Let  $v_i$  be the vertices of  $A$  and  $u_i$  be the vertices of  $T$

Let  $q_1$  and  $q_2$  denote the size of  $A$  and  $T$  respectively

The source graphs are defined in the following three cases

*Case 1:* Alphabets that are cycles or contain cycles

Label the vertices in the cycle in such a way that all vertices in the cycle contain the labels 1, 3, 7, where vertex labeled 1 is attached to the vertex labeled 3 and the vertex labeled 3 is adjacent to the vertex labeled 7 and then label the vertex adjacent to 1 as 0.

Thus, we label the vertices as

$$f(v_1) = 0, f(v_2) = 1, f(v_3) = 3, f(v_4) = 7, f(v_5) = 9$$

If the graph has 6 vertices, then  $f(v_6) = 2e - 1, e = |E(A)|$

Then we get distinct edge labels  $[1, 2q_1 - 1]^o$

*Case 2:* Tree like alphabets C, E, F, G, H, I, J, K, L, M, N, S, T, U, V, W, X, Y, Z. This is further subdivided into Alphabets that are Paths and Trees with at least one branch.

*Subcase (i):* Trees with atleast one branch

Let  $v_1$  be the vertex with the highest degree and let  $v_2$  be the vertex adjacent to  $v_1$

The labeling is defined as

$$f(v_1) = 0, f(v_2) = 1, f(v_i) = 2i - 3, i \geq 3$$

Then we get distinct edge labels  $[1, 2q_1 - 1]^o$

*Subcase (ii): Path like Alphabets*

The labeling is defined as

$$f(v_1) = 0, f(v_2) = 1, f(v_i) = 2i - 3, i \geq 3$$

Then we get distinct edge labels  $[1, 2q_1 - 1]^o$

Now the associated graph  $T$  is defined as a function  $f: V_2 \rightarrow [0, 2q_2]$  such that

$$f(u_2) = 1, f(u_1) = 2, f(u_i) = 2i - 2, \text{ for all } i \geq 3$$

Then we get distinct edge labels  $[1, 2q_2 - 1]^o$

Hence,  $A$  and  $T$  can be Odd Contra Harmonic Mean labeled

Clearly,  $f(u_i) \neq f(v_i)$ , for all  $i \neq 2$

Thus  $A$  is a source graph of associated graph  $T$

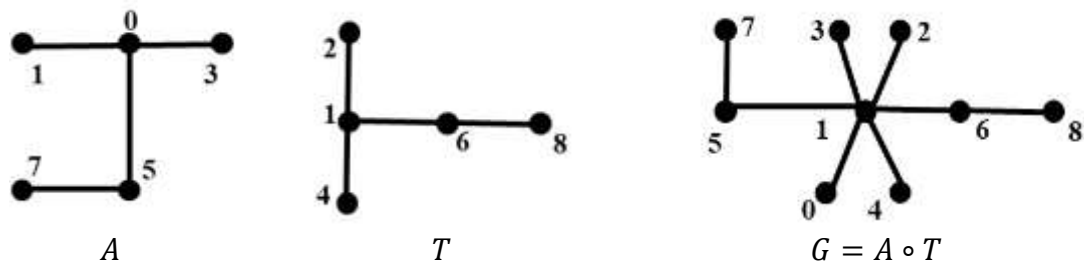


Fig 4: An identifying graph  $G = A \circ T$  with source graph  $A$  and associated graph  $T$

**Conclusion and scope**

In this paper we have investigated the role of Contra Harmonic Mean labeling for information security. While the Super Contra Harmonic Mean labeled graphs can be used for simple user logins, Odd Contra Harmonic Mean labeled graphs can be used to develop lock and key-models. Further research can be done on applying labeling on more user friendly graphs like Eulerian graphs.

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## Abstract

Graphical Passwords have proved to be efficient and user-friendly authentication method compared to usual text-based passwords. Topsnut-GPWs are new type of graphical passwords made using topological structures and number theory. In this article, we construct Topsnut-GPWs using a certain type of caterpillar tree and Alphabet graphs by means of Contra Harmonic Mean labeling. In addition, we explore the possibility of Alphabet graphs and caterpillar tree being the key-model and lock-model respectively.

## 3. Preliminaries

All graphs in this paper are finite, simple and undirected graphs. Topsnut-gpw stands for ‘topological structure + number theory graphical password’ and TB-paw stands for text-based password.  $[1,2q]^o$  denotes the set of all odd integers in the set  $[1,2q]$ .

For a detailed survey of graph labeling we refer to Gallian[1]. For all other standard terminology and notation, we follow Harary [2]. S. S. Sandhya, S. Somasundaram and J. Rajeshni Golda introduced Contra Harmonic Mean labeling of graphs [3]. Wang et al.

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introduced the concept of creating graphical passwords using ‘topological structure + number theory’, later called as ‘Topsnut-GPW’ in [5] and [10]. Topsnut-gpws made from Hanzi-graphs called Hanzi-gpws were introduced by Mu et al [9].

We provide a brief summary of definitions which are necessary for the present investigation.

**Definition 1.1** [3]: A graph  $G(V, E)$  with  $p$  vertices and  $q$  edges is said to be Contra Harmonic mean graph if it is possible to label the vertices  $x \in V$  with distinct labels  $f(x)$  from  $0, 1, 2, \dots, q$  in such a way that when each edge  $e = uv$  is labeled with  $f(e = uv) = \left\lfloor \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rfloor$  or  $\left\lceil \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rceil$  with distinct edge labels. Then  $f$  is called Contra Harmonic mean labeling of  $G$ .

**Definition 1.2** [3]: An injective function  $V(G) \rightarrow \{1, 2, \dots, p + q\}$  is called a Super Contra Harmonic mean labeling if the induced edge labeling  $f(e = uv)$  is defined as  $f(e) = \left\lfloor \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rfloor$  or  $\left\lceil \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rceil$ , so that  $\{f(V(G))\} \cup \{f(e) : e \in E(G)\} = \{1, 2, \dots, p + q\}$ . A graph which allows Super Contra Harmonic Mean labeling is called Super Contra Harmonic Mean graph.

**Definition 1.3:** A graph  $G(V, E)$  with  $p$  vertices and  $q$  edges is said to be Odd Contra Harmonic Mean graph if it is possible to label the vertices  $x \in V$  with distinct labels  $f(x)$  from  $0, 1, 2, \dots, 2q$  in such a way that each edge  $e = uv$  labeled with  $f(e = uv) = \left\lfloor \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rfloor$  or  $\left\lceil \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rceil$  gets distinct edge labels  $1, 3, 5, \dots, 2q - 1$ . Then  $f$  is called Odd Contra Harmonic Mean labeling of  $G$ .

A caterpillar tree is a tree in which all the vertices are within distance one of a central path.

**Definition 1.4:** A caterpillar tree is a tree in which all the vertices are within distance one of a central path.

**Definition 1.5:** Alphabet graphs are obtained by converting English characters into graphs.

**Definition 1.6** [4]: Consider two disjoint trees  $T_1$  and  $T_2$ , *vertex-identification operation* is performed by combining two vertices  $u_i$  of  $T_1$  and  $v_j$  of  $T_2$  to form a new vertex  $w$  called identified vertex.  $T = T_1 \circ T_2$  is called identifying graph with  $p_1 + p_2 - 1$  vertices and  $q_1 + q_2$  edges.

**Definition 1.7** [4]: Consider two disjoint trees  $T_1$  and  $T_2$ , *edge-series operation* is performed by adding an edge between  $u_i$  of  $T_1$  and  $v_j$  of  $T_2$  so that the newly formed tree  $T$  has  $p_1 + p_2$  vertices and  $q_1 + q_2 + 1$  edges.

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**Definition 1.8** [4]: Let a vertex  $u$  have its neighbour set  $N(u) = \{v_1, v_2, \dots, v_m\}$  with  $f(v_i) < f(v_{i+1})$ , generating TB-paw by *path neighbour method* is defined as  $P_{vv}(u) = f(u), f(v_1), \dots, f(v_m), f(u)$

**Definition 1.9** : Let  $G_1(a, q_1)$  and  $G_2(a, q_2)$  be two graphs such that the vertex identified graph  $G(p, q) = G_1 \circ G_2$  has a mapping  $f: V(g) \rightarrow [0, q]$  such that i)  $f(x) \neq f(y)$  for any pair of vertices of  $x, y \in G$ , ii)  $f$  is an Odd Contra Harmonic Mean labeling of  $G_1$  iii) the set  $f(E(G_i)) = [1, 2q_i - 1]^0$  with  $i = 1, 2$ . Then  $G_1$  is said to be a source graph (key-model) and  $G_2$  an associated graph (lock-model) of  $G_1$ .

The following observations play a vital role for the development of the paper

**Result 1.10:** Paths are Super Contra Harmonic Mean graphs

## 4. Main Results

### 4.1 Contra Harmonic Mean Labeling on Caterpillar Trees

**Theorem 2.1.1:** A caterpillar tree such that no vertices have degree  $> 4$  is Contra Harmonic Mean iff it admits Super Contra Harmonic Mean labeling.

#### Proof

Let  $T(p, q)$  be a caterpillar tree such that no vertices has degree  $> 4$ .

Suppose that the tree  $T$  has a Contra Harmonic Mean labeling defined as

$f(x_i) = i - 1, x_i \in V(T), i \in [1, V(T)]$  such that

$$f(x_i x_j) = \left\lfloor \frac{f(x_i)^2 + f(x_j)^2}{f(x_i) + f(x_j)} \right\rfloor \text{ or } \left\lceil \frac{f(x_i)^2 + f(x_j)^2}{f(x_i) + f(x_j)} \right\rceil \in [1, E(T)]$$

We construct a new labeling  $g: V(T) \rightarrow \{1, 2, \dots, p + q\}$  of  $T$  by

$$g(x_i) = 2f(x_i) + 1, g(x_i x_j) = f(x_i x_j)$$

Hence  $g(x_i x_j)$  and  $g(x_i)$  are distinct and  $g[V(T) \cup E(T)] = [1, V(T) + E(T)]$ .

Hence  $g$  is Super Contra Harmonic Mean labeling.

Conversely, assume  $g'$  be the Super Contra Harmonic Mean of  $T$  such that

$$g'(x_i) = 2i - 1, i \in [1, V(T)] \text{ and}$$

$$g'(x_i x_j) = \left\lfloor \frac{g'(x_i)^2 + g'(x_j)^2}{g'(x_i) + g'(x_j)} \right\rfloor \text{ or } \left\lceil \frac{g'(x_i)^2 + g'(x_j)^2}{g'(x_i) + g'(x_j)} \right\rceil \in [1, V(T) + E(T)]$$

Now, construct a labeling  $f': V(T) \rightarrow \{1, 2, \dots, p\}$  defined as

$$f'(x_i) = g'(x_i) - 1 \text{ and } f'(x_i x_j) = g'(x_i x_j)$$

Clearly,  $f(V(T)) = [0, |E(T)|]$  and  $f(E(T)) = [1, |E(T)|]$

Hence  $f'$  is Contra Harmonic Mean labeling.

Fig. 1 shows the illustration of theorem 2.1.1

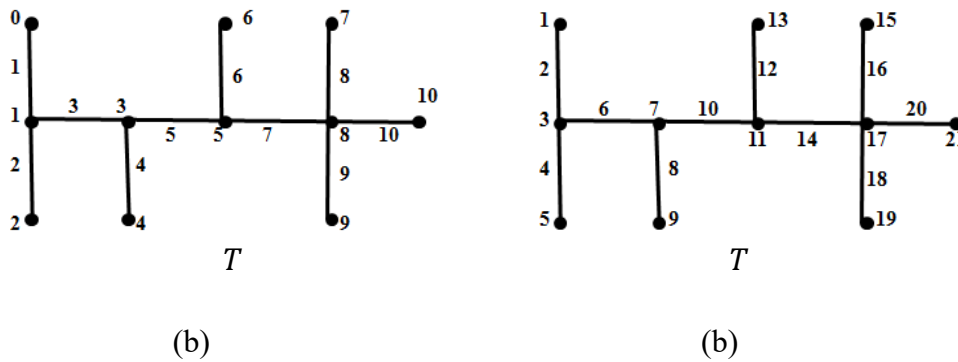


Fig.1

Fig.1(a) shows Contra Harmonic Mean labeling of  $T$  and Fig.1(b) shows Super Contra Harmonic Mean labeling of  $T$ .

Next, we construct large scale caterpillar trees by using vertex identification operation as follows

**Theorem 2.1.2:** If the vertex-identification operation on two disjoint trees  $T_1(p_1, q_1)$  and  $T_2(p_2, q_2)$  is performed by combining  $u_i$  of  $T_1$  and  $v_j$  of  $T_2$  where  $f(u_i) = q_1$  and  $f(v_j) = 0$  or  $f(u_i) = 0$  and  $f(v_j) = q_2$  to form a new graph  $T$ , then  $T$  is a caterpillar tree.

**Proof**

Let  $T_1$  be a tree such that  $f(u_i) = q_1$  and  $f(v_j) = 0$

We consider two cases,

Case (i): Let  $u_i$  be the vertex on the central path of  $T_1$ .

Let  $u_1, u_2, \dots, u_i$  be the vertices on the central path and let  $v_1, v_2, \dots, v_h$  be the vertices on the central path of  $T_2$ .

Then by adjoining  $u_i$  to  $v_j$ , we get a tree  $T$  with central path  $u_1 u_2 \dots u_i v_j v_1 v_2 \dots v_h$ .

Hence  $T$  is a caterpillar tree.

Case (ii): Let  $u_i$  be a leaf of  $T_1$

Let  $u_1, u_2, \dots, u_g$  be the vertices on the central path and let  $v_1, v_2, \dots, v_h$  be the vertices on the central path of  $T_2$ .

Then by adjoining  $u_i$  to  $v_j$ , we get a tree  $T$  with central path  $u_1 u_2 \dots u_g u_i v_1 v_2 \dots v_h$ .

Hence  $T$  is a caterpillar tree.



The risk with Topsnut-GPW is that there is a risk of forgetting when not used for a longer time, whereas TB-paws have a greater chance of recalling later in time since users tend to use alphanumeric passwords that are associated with them in some way. Hence, we investigate the idea of converting text-based passwords to graphical structures. Alphabet graphs are English alphabets converted to graphs. Since, English is the most spoken global language, wide number of users benefit from Alphabet graphs.

Here we create an algorithm to construct Super Contra Harmonic Mean labeled Alphabet graphs.

### **Algorithm 2.2.1: Algorithm to Construct Super Contra Harmonic Mean labeled Alphabet Graphs**

Let  $A$  denote Alphabet graphs

The vertices and edges of  $A$  are labeled as  $v_1e_2v_3e_4v_5 \dots$  such that  $e_2$  is the edge connecting  $v_1$  and  $v_3$ .

*Step 1:* First, we consider Tree like alphabets C, E, F, G, H, I, J, K, L, M, N, S, T, U, V, W, X, Y, Z. This is further subdivided into Alphabets that are Paths and Trees with at least one branch.

*Subcase (i):* First, we consider path like alphabets C, G, I, L, M, N, S, U, V, W, Z. The vertices and edges are labeled in the same way as labeling paths using consecutive numbers, i.e.,  $f(v_1) = 1, f(v_3) = 3, \dots, f(v_n) = n$ , where  $n = p + q$

*Subcase (ii):* Next we consider the alphabets E, F, H, J, K, T, X, Y. The labeling is defined as follows:

$$f(v_i) = i, \text{ if } d(v_i) \leq 3, \forall v_i \in A$$

$$f(v_i) = i, f(v_n) = n, f(v_{n-2}) = n - 1, \text{ where } n = p + q, p = |V(A)| \text{ and } q = |E(A)|$$

*Step 2:* Next we consider alphabets that are either cycles or contain cycles which include A, B, D, O, P, Q, R.

*Subcase (i):* First we define labeling in cyclic alphabets D and O

$$f(v_j) = j, \text{ where } v_j \text{ denotes the initial vertex in } A,$$

$$f(v_{n-1}) = n \text{ where } n = p' + q', p' \text{ and } q' \text{ are number of vertices and edges in the cycle.}$$

Now the remaining vertices are labeled in such a way that,

$$f(v_{i+2}) = \begin{cases} f(v_i) + 3, & \text{if either } f(v_{i+2}) = f(e_n) \text{ or } f(v_i v_{i+2}) = f(e_n) \\ f(v_i) + 2, & \text{otherwise} \end{cases}$$

Subcase (ii): Here we consider the alphabets that contain cycle which include A, B, P, R, Q. These alphabets have at most 6 vertices and there exist at least one vertex  $v_i$  in A, such that  $d(v_i) \geq 3$ , then we define

$$f(v_i) = i, f(v_{i-2}) = i - 2, f(v_{i-2}) = i - 2,$$

$$f(v_{i+4}) = p + q, f(v_{i+6}) = f(v_{i+4}) - 2,$$

$$\text{The remaining vertex is labeled as } f(v_{i+8}) = f(v_{i+2}) + 2$$

Next, we have the edge-series operation process

**Remark 2.2.2:** The Alphabet graphs are converted to Word graphs by adding an edge between  $u_i$  of  $A_1$  and  $v_j$  of  $A_2$  where  $f(u_i) = p_1 + q_1$ , where  $p_1 = |V(A_1)|$  and  $q_1 = |E(A_1)|$  and  $v_j$  is the initial vertex in  $A_2$ .

Obtaining a Word graph from Alphabet graphs is illustrated in Fig. 3

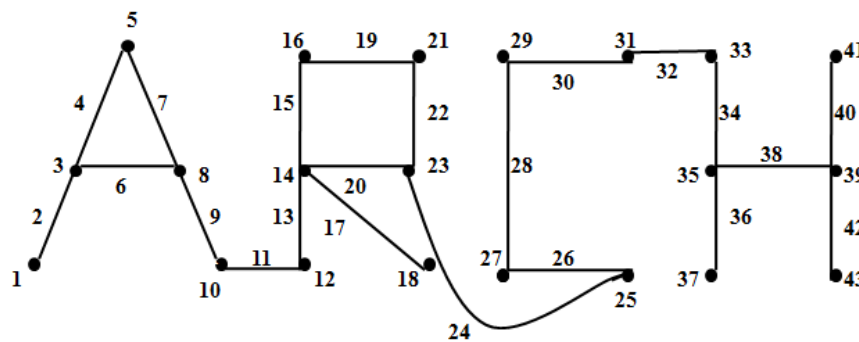


Fig. 3: Word graph obtained from the Alphabets A, R, C and H

### 4.3 Key and Lock Models using Caterpillar trees and Alphabet graphs

To create efficient key models, we use Odd Contra harmonic Mean labelling to label Alphabet graphs.

**Theorem 2.3.1:** Alphabet graphs are the source graphs of caterpillar trees with no vertices having degree  $>4$ .

**Proof:**

Let  $A$  denote Alphabet graph and let  $T$  be a Caterpillar tree with no vertices of degree  $> 4$

Let  $v_i$  be the vertices of  $A$  and  $u_i$  be the vertices of  $T$

Let  $q_1$  and  $q_2$  denote the size of  $A$  and  $T$  respectively

The source graphs are defined in the following three cases

Case 1: Alphabets that are cycles or contain cycles

Label the vertices in the cycle in such a way that all vertices in the cycle contain the labels 1, 3, 7, where vertex labeled 1 is attached to the vertex labeled 3 and the vertex labeled 3 is adjacent to the vertex labeled 7 and then label the vertex adjacent to 1 as 0.

Thus, we label the vertices as

$$f(v_1) = 0, f(v_2) = 1, f(v_3) = 3, f(v_4) = 7, f(v_5) = 9$$

If the graph has 6 vertices, then  $f(v_6) = 2e - 1, e = |E(A)|$

Then we get distinct edge labels  $[1, 2q_1 - 1]^o$

Case 2: Tree like alphabets C, E, F, G, H, I, J, K, L, M, N, S, T, U, V, W, X, Y, Z. This is further subdivided into Alphabets that are Paths and Trees with at least one branch.

Subcase (i): Trees with atleast one branch

Let  $v_1$  be the vertex with the highest degree and let  $v_2$  be the vertex adjacent to  $v_1$

The labeling is defined as

$$f(v_1) = 0, f(v_2) = 1, f(v_i) = 2i - 3, i \geq 3$$

Then we get distinct edge labels  $[1, 2q_1 - 1]^o$

Subcase (ii): Path like Alphabets

The labeling is defined as

$$f(v_1) = 0, f(v_2) = 1, f(v_i) = 2i - 3, i \geq 3$$

Then we get distinct edge labels  $[1, 2q_1 - 1]^o$

Now the associated graph  $T$  is defined as a function  $f: V_2 \rightarrow [0, 2q_2]$  such that

$$f(u_2) = 1, f(u_1) = 2, f(u_i) = 2i - 2, \text{ for all } i \geq 3$$

Then we get distinct edge labels  $[1, 2q_2 - 1]^o$

Hence,  $A$  and  $T$  can be Odd Contra Harmonic Mean labeled

Clearly,  $f(u_i) \neq f(v_i), \text{ for all } i \neq 2$

Thus  $A$  is a source graph of associated graph  $T$

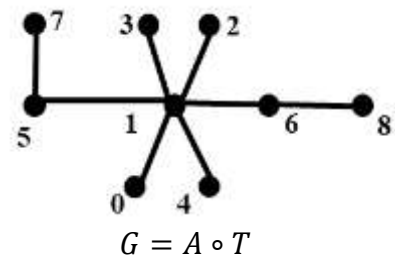
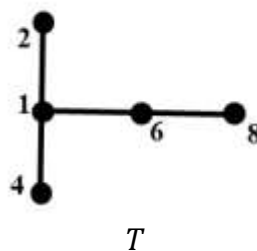
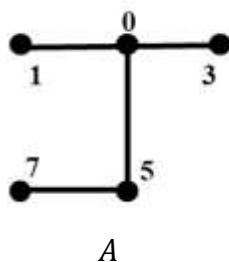


Fig 4: An identifying graph  $G = A \circ T$  with source graph  $A$  and associated graph  $T$

### Conclusion and scope

In this paper we have investigated the role of Contra Harmonic Mean labeling for information security. While the Super Contra Harmonic Mean labeled graphs can be used for simple user logins, Odd Contra Harmonic Mean labeled graphs can be used to develop lock and key-models. Further research can be done on applying labeling on more user friendly graphs like Eulerian graphs.

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