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Equitable Power Domination Number of Certain Silicate Structure

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ABSTRACT

The power dominating set $S \subseteq V$ in G = (V, E) is said to be equitable power dominating set when each vertex $v \in V - S$ has observed nearby vertex $u \in S$ such that difference between degree of u and the degree of v is less than or equal to 1, i.e. $|d\ (u) - d\ (v)| \leq 1$. The equitable power domination number of a graph G is the minimal cardinality of its equitable power dominating set, and it is represented by the notation $\gamma_{epd}\ (G)$. This paper explores its use in mineralogy, a discipline that relies heavily on silicate minerals. By demonstrating significant theorems related to chain and sheet silicate graphs, this study advances our understanding of the structural characteristics of silicates. Theorists and chemists involved in mineral study will find these theorems useful. In this research, we examine equitable power domination number of certain silicate structure.

Keywords: Molecular structure, silicate structures, and equitable power domination number.

1. Introduction

These days, chemical science is vital. Graph theory is a useful tool for chemical chemists. The majority of the usage is silicate, according to this mineralogy. The definitions of domination, power domination, and equitable power domination are reviewed first. [13]A dominating set S of G is a collection of vertices with at least one neighbour for each vertex v in $v \in V - S$. The "domination number of G," denoted by $\gamma_d\ (G)$, is the lowest cardinality of a dominating set S in G.[12] A set is said to be a power dominating set (PDS) of G if every vertex u ∈ V-S is observed by some vertices in S using the following rules, (i) If a vertex v in G is in PDS, then it dominates itself and all the adjacent vertices of v . (ii) If an observed vertex v in G has K>1 adjacent vertices and if K–1 of these vertices are already observed, then the remaining non-observed vertex will also be observed by v in G. The minimum cardinality of an power dominating set of G is called power domination number of G. A power dominating set's lowest cardinality is its power domination number, represented as γ_{pd} (G).[1] A power dominating set $S \subseteq V$ in G = (V, E) is said to be equitable power dominating set. If the difference between the degrees of u and v is less than or equal to 1 and every vertex $v \in V - S$ has an adjacent vertex $u \in S$, i.e. $|d\ (u) - d\ (v)| \leq 1$. A graph's equitable power domination number, denoted by $\gamma_{\ epd}$ (G), is the least cardinality of the equitable power dominating set [1].

Silicate is the biggest, most fascinating, and most complex class of minerals. The (SiO_4) tetrahedron is the fundamental chemical unit of silicates. A silicate sheet is a ring of tetrahedrons connected by common oxygen nodes in a two- and three-dimensional network. Dimension plane that

10.48047/jocaaa.2025.34.10.07

produces a sheet-like shape. Sand is combined with metal carbonates or oxides to create silicate. SiO_4 tetrahedral may be found in almost all silicate materials. In terms of chemistry, the middle vertex of the SiO_4 tetrahedron represents the silicon ion, while the corner vertices stand for oxygen ions. According to graph theory, the corner vertices stand in for oxygen nodes, and the center vertex for silicon. In this paper the equitable power domination number of the Certain Silicate Structure in this research.

2. Equitable power domination in silicate network:

The most prevalent minerals that from rocks are silicates. Their highly hierarchical organizational structure produces a great deal of diversity: Silicate make up more than 25% of all known minerals.

2.1 Nesosilicates (Island Silicates)

Every SiO^4 as a 4 tetrahedron will be isolated if the corner oxygens are not shared with any other tetrahedrons. As a result, this group is commonly called the island silicate group. Thus, SiO^4 – 4 is the basic structural unit. Mg^{+2}, Fe^{+2}, and Ca^{+2} are examples of octahedral groups that share the oxygens in this group. Olivine is the ideal illustration.

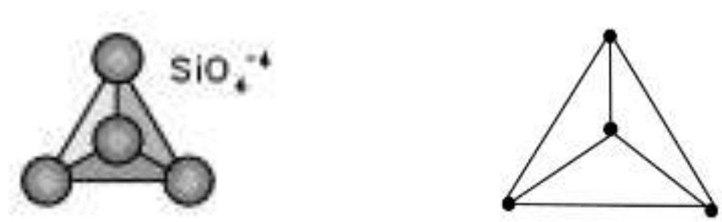

Fig. 2.1: $(SiO_4)^{4-}$

2.2 Sorosilicates (Double Island Silicates)

The sorosilicate group is created when one of the corner oxygens is shared by two tetrahedrons. Because there are two connected tetrahedrons that are separated from every other tetrahedron, this group is frequently referred to as the "double island group." Si^2O^7 - 6 is the fundamental structural unit in this instance. Hemigmorphite is a mineral that is a good example of a sorosilicate. Certain sorosilicates, such as epidote, combine single and double islands.

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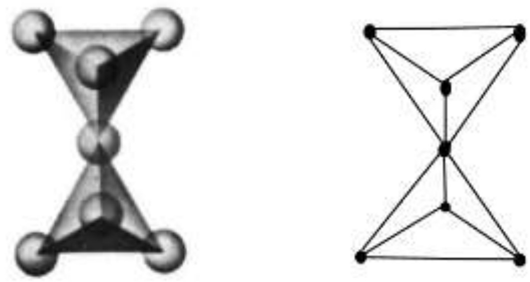

Fig. 2.2: $(Si_2O_7)^{6-}$

2.3 Chain Silicate:

A straight line connecting the tetrahedral is the simplest way to visualize the formation of an inosilicate or chain silicate. Each neighbor of a SiO_4 tetrahedron shares two oxygen atoms in this manner. The tetrahedral apices of these chain silicates all point in the same direction, and the excess charges are balanced by cations such as Ca^{2+} and Mg^{2+}. The chains are held together by these two ions, where calcium has eight oxygen atoms and magnesium has six. This kind of single chain silicate, which includes pyroxenes, has a chemical formula made up of several Si_2O_6 units arranged parallel to the c-axis.

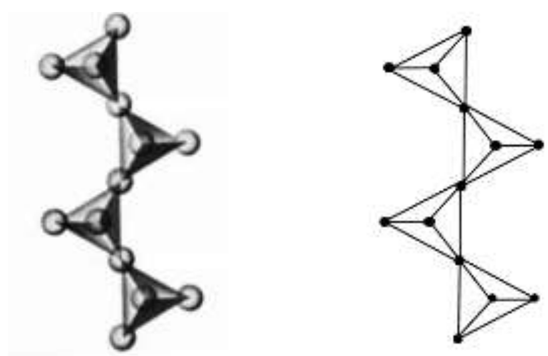

Fig. 2.3: $(Si_2O_6)^{4-}$

2.4 Double chain silicate:

A pair of adjacent chains with the same orientation and a shared oxygen bond make up double chain silicates, which include the amphiboles. Si_4O_{11} is the most basic unit of the structure formed by the periodic sharing of oxygen atoms along the chain. Ions such as Ca^{2+} and Mg^{2+} are responsible for the bonding of adjacent paired chains. The structure also contains hydroxyl groups, which fluorine may substitute for. Aluminum could also be used in place of silicon.

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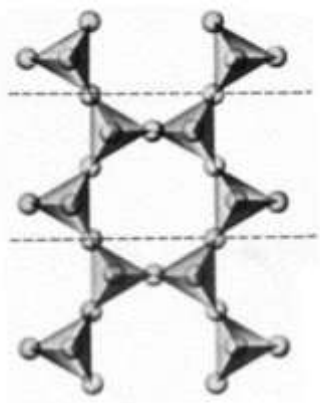
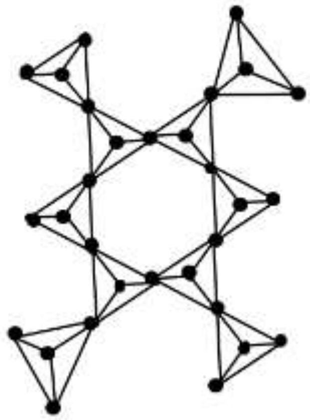

Fig. 2.4: $(Si_4O_{11})^{6-}$

2.5 Cyclic silicate:

Many tetrahedral silicates are joined together and then closed to form a ring to form cyclic silicates, also known as ring silicates. Three SiO_4 tetrahedral links that share two oxygen atoms with each other to form a six-membered ring make up one subgroup. This indicates that the ring structure has an overall charge of 6 and the chemical formula Si_3O_9. On the other hand, rings could be made of four or six interconnected SiO_4 tetrahedral. In the second scenario, a structure with the formula Si_6O_{18} has an overall charge of 12. The Si:O ratio is lowered to 1:3 in each instance due to the neighboring tetrahedral sharing of oxygen atoms. In each of these configurations, aluminum can be used in place of silicon, and the cations that typically balance the charges include Fe^{2+}, Fe^{3+}, Mg^{2+} and Ca^{2+}. The more unusual elements beryllium, lithium, boron and fluorine are also present in some cyclic silicates.


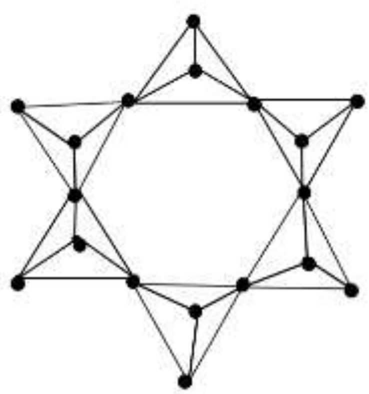

Fig. 2.5: $(Si_6O_{18})^{12-}$

2.6 Sheet silicate (Phyllosilicate):

Sheet silicates are imagined that several of the amphiboles' double chain structures are laid down next to one another and are subsequently joined by the sharing of oxygen atoms. This is how a sheet is formed. In this manner, tetrahedral rings unite to form a single plane with a tessellated pattern when viewed from above. This results in the Si:O ratio dropping to l:2.5 because one oxygen atom in each tetrahedron is free and pointed away from the sheet, while the other three oxygens are shared with neighboring tetrahedral. Following that, layers of cations and other related ions, elements, and molecules bind adjacent sheets to one another. The comparatively weak ionic chemical bonds utilized in this situation make the sheets prone to separation. As a result, the final mineral frequently has a plate appearance with perfect basal cleavage and is relatively soft (hardness I to 3). Phyllosilicates are incredibly robust

10.48047/jocaaa.2025.34.10.07

despite this, withstanding both chemical weathering and metamorphic changes. They are therefore widespread in igneous, metamorphic, and sedimentary settings. Phyllosilicates exhibit remarkable diversity and comprise micas, minerals belonging to the serpentine group, and the most prevalent silicate minerals, clay minerals.

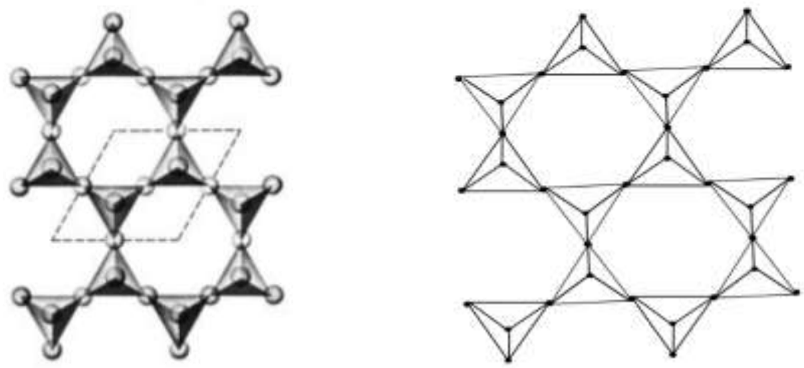

Fig. 2.6: $(Si_2O_5)^{2-}$

2.7 Framework Silicates (Tectosilicates):

A structure known as a framework forms if every corner oxygen is shared by another SiO4 tetrahedron. Next, SiO2 becomes the fundamental structural group. This structural basis is shared by the minerals tridymite, cristobalite, and quartz. More ions coordinated in various configurations can be discovered inside the framework structure if some of the Si+4 ions are swapped out for Al+3. Charges become unbalanced as a result. That being said, the tectosilicate structure also serves as the basis for the feldspar and feldspathoid minerals.

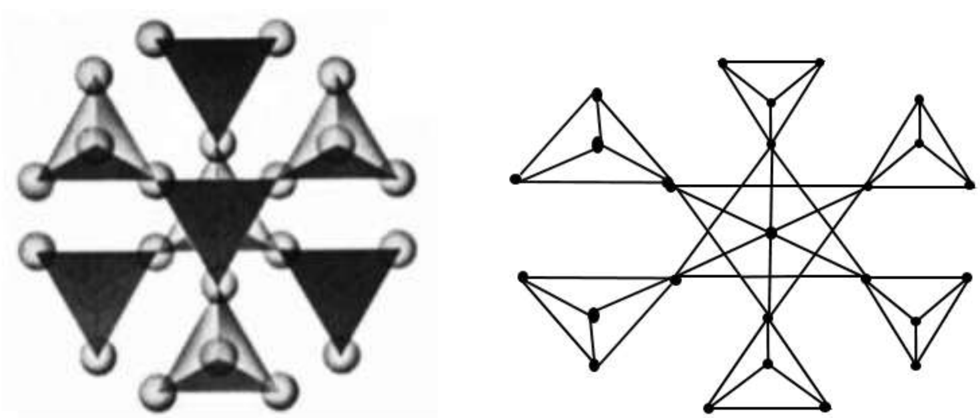

Fig. 2.7: (SiO_2)

3. Main Results

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Theorem 3.1:

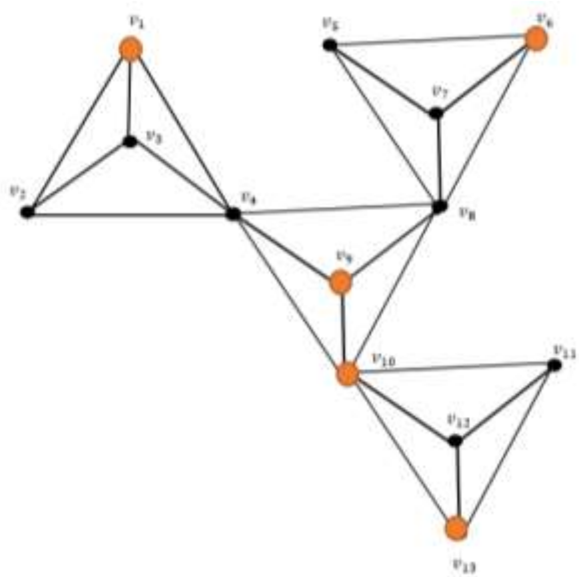
For n ≥ 1, Let, 'N' be the Nesosilicate graph, then $\gamma_{epd}(N_n) = \begin{cases} 1, & \text{if } n=1 \\ n+1, & \text{if } n \geq 2 \end{cases}$

(If 'n' represent number of copies has k4)

Proof:

Let V= {v_1, v_2, …….} indicates vertex set E = {e_1, e_2, e_3, …..} denote the edge set.

Let, S represents the set of equitable power domination.



$\gamma epd(N_4) = 5$

From the above figure, Let us choose v_{10} in s. then v_{10} dominates v_4, v_8. Then choose v_1 in s it dominates v_3, v_2. Choose v_6 in s it dominates v_5, v_7. Choose v_{13} in s it dominates v_{12}, v_{11}. Choose v_9 in s it dominates itself. Likewise one can prove that

$$\gamma epd(N_n) = n+1 \quad , \text{for every } n \geq 2$$

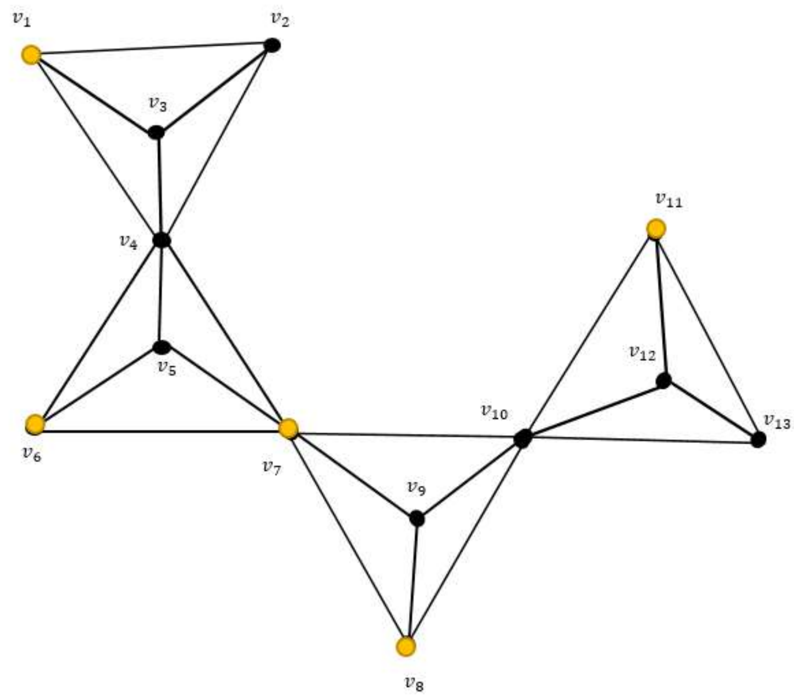
Result 3.2:

For n ≥ 2, Let, S be the sorosilicate graph, then $\gamma_{epd}(S_n)$ =n+1

(if n represent number of copies has k_4)

Proof:

10.48047/jocaaa.2025.34.10.07



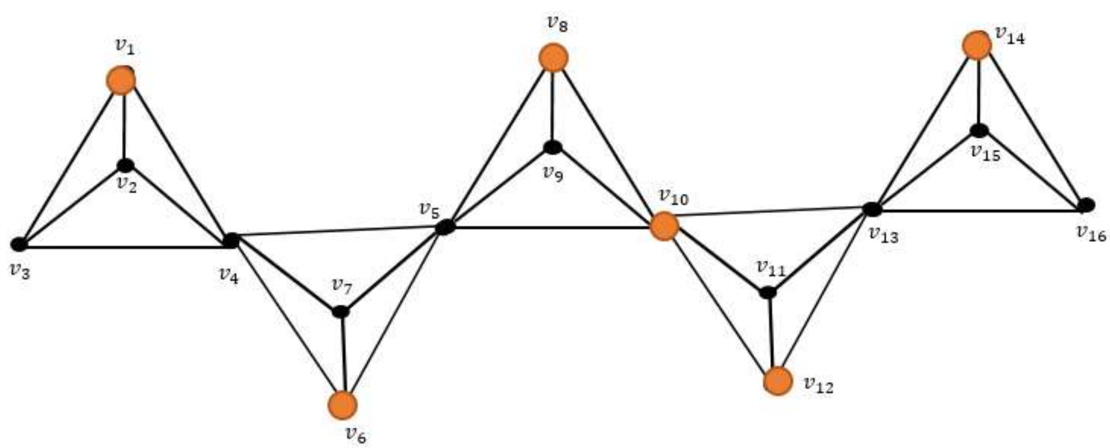
γepd (S_4) = 5

Result 3.3:

For n ≥ 2, Let C be the chain silicate graph, then γ_{epd} (C_n) =n+1.

(If ‘n’ represent number of copies has k_4)

Proof:



γepd (C_5) = 6

Theorem 3.4:

10.48047/jocaaa.2025.34.10.07

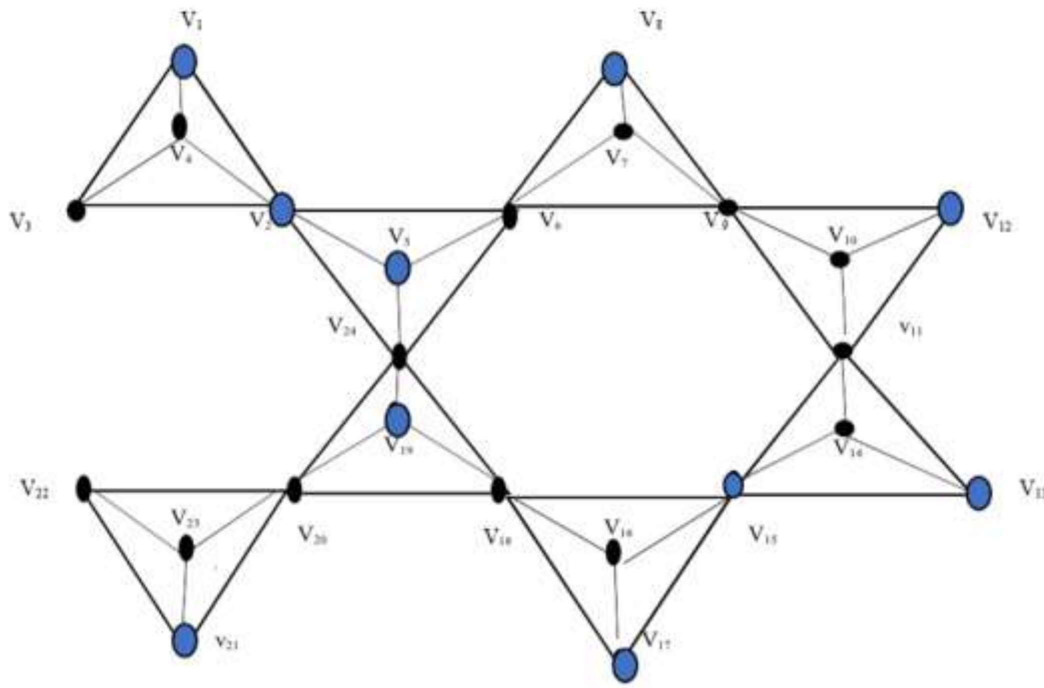
For n ≥ 2, Let DC be the double chain silicate graph, then γ_{epd} (DC $_n$) = 2n+ ⌊n/2⌋

(If 'n' represents double the number of copies has twin k_4)

Proof:

Let V= {v_1, v_2, …….} denote the vertex set E = {e_1, e_2, e_3, …..} denote the edge set.

Let, S be the equitable power domination set.



γ_{epd} (DC $_4$) = 10

From the above figure, Let us choose v_2 in s. then v_2 dominates v_{24}, v_6. Then choose v_{15} in s it dominates v_{11}, v_{18}. Choose v_1 in s it dominates v_4, v_3. Choose v_8 in s it dominates v_7. Choose v_{12} in s it dominates v_{10}. Choose v_{13} in s it dominates v_{14}. Choose v_{17} in s it dominates v_{16}. Choose v_{21} in s it dominates v_{23}, v_{22}. Choose v_5, v_{19} in s it dominates itself. Now for v_{20}, v_9, is the only one non – observed vertex then v_{24} observed v_{20}, v_6 observed v_9. Likewise one prove that

γ_{epd} (DC $_n$) = 2n + ⌊n/2⌋, for every n.

Theorem 3.5:

For n ≥ 3, Let CS be the cyclic silicate graph, then

10.48047/jocaaa.2025.34.10.07

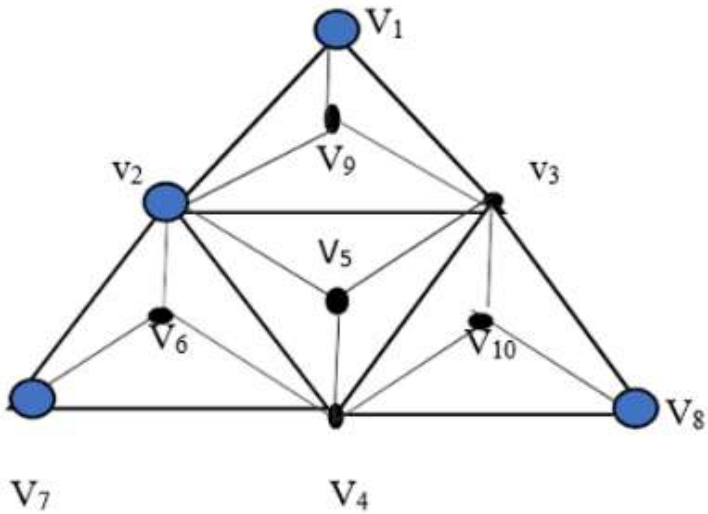
γ_{epd} (CS $_n$) = n+⌊n/4⌋.

(If 'n' represent number of copies has twin k_4)

Proof:

Let V= {v_1, v_2, …….} denote the vertex set E = {e_1, e_2, e_3, …..} denote the edge set.

Let, S be the equitable power domination set.



γ_{epd} (CS $_3$) = 4

From the above figure, Let us choose v_2 in s. then v_2 dominates v_3, v_4. Then choose v_1 in s it dominates v_9. Choose v_7 in s it dominates v_6. Choose v_8 in s it dominates v_{10}. Likewise one prove that

γ_{epd} (CS $_n$) = n+⌊n/4⌋, for every n

Theorem 3.6:

For n ≥ 4, Let SS be the sheet silicate graph, then γ_{epd} (SS $_n$) =2n+3.

(If 'n' represent four number of copies has twin k_4)

Proof:

Let V= {v_1, v_2, …….} denote the vertex set E = {e_1, e_2,e_3,…..} denote the edge set.

Let, S be the equitable power domination set.

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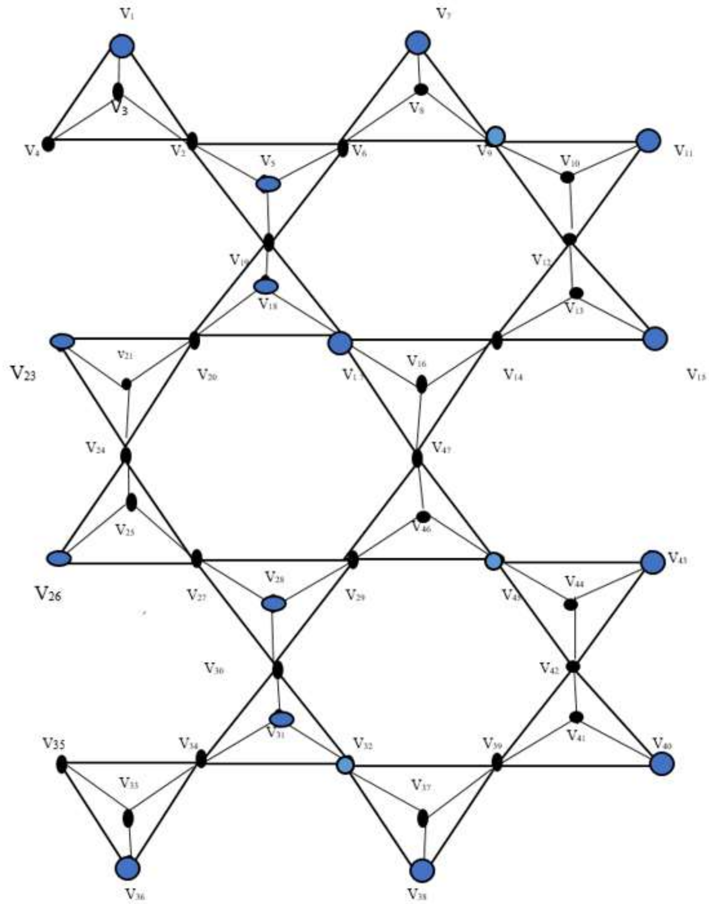


γ_{epd} (SS $_8$) = 19

From the above figure, Let us choose v_9 in s. then v_9 dominates v_{12}, v_6. Then choose v_{17} in s it dominates v_{19}, v_{20}, v_{14}, v_{47}, v_{20}. Choose v_{32} in s it dominates v_{39}, v_{30}, v_{34}. Choose v_1 in s it dominates v_3, v_4. Choose v_7 in s it dominates v_8. Choose v_{11} in s it dominates v_{10}. Choose v_{15} in s it dominates v_{13}. Choose v_{43} in s it dominates v_{44}. Choose v_{49} in s it dominates v_{41}. Choose v_{38} in s it dominates v_{37}. Choose v_{36} in s it dominates v_{33}, v_{35}. Choose v_{26} in s it dominates v_{25}. Choose v_{23} in s it dominates v_{28}. Choose v_5, v_{18}, v_{16}, v_{46}, v_6, v_{31} in s it dominates itself. Now for v_2, v_{24}, v_{29}, v_{27}, v_{45}, v_{42} is the only one non – observed vertex then v_6 observed v_2, v_{20} observed v_{24}, v_{47} observed v_{29}, v_{30} observed v_{27}, v_{47} observed v_{45}, v_{39} observed v_{42}. Likewise one prove that

γ_{epd} (SS $_n$) = 2n+3, for every n.

Theorem 3.7:

10.48047/jocaaa.2025.34.10.07

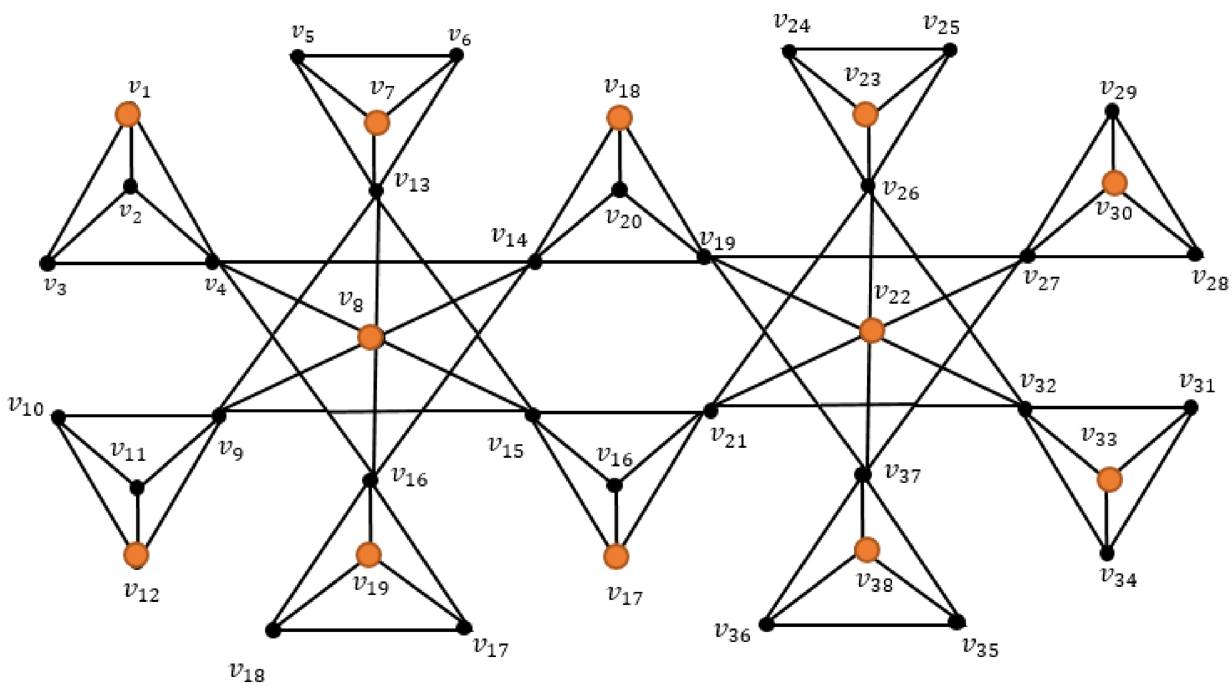
For n ≥ 1, Let TS be the tectosilicate graph, then γ_{epd} (TS $_n$) =5n+2

(If 'n' represent four number of copies has k_4)

Proof:

Let V= {v_1, v_2, …….} indicates vertex set E = {e1, e2, e3, …..} denote the edge set.

Let, S represents the set of equitable power domination.



γ_{epd} (TS $_2$) = 12

From the above figure, Let us choose v_8 in s. then v_8 dominates v_4, v_{13}, v_{14}, v_{15}, v_{16}, v_9. Then choose v_{22} in s it dominates v_{26}, v_{27}, v_{32}, v_{37}, v_{21}, v_{19}. Choose v_1 in s it dominates v_2, v_3. Choose v_7 in s it dominates v_5, v_6. Choose v_{18} in s it dominates v_{20}. Choose v_{23} in s it dominates v_{24}, v_{25}. Choose v_{30} in s it dominates v_{29}, v_{28}. Choose v_{33} in s it dominates v_{31}, v_{34}. Choose v_{38} in s it dominates v_{35}, v_{36}. Choose v_{17} in s it dominates v_{16}. Choose v_{19} in s it dominates v_{18}, v_{17}. Choose v_{12} in s it dominates v_{11}, v_{10}. Likewise one prove that

$$\gamma_{epd} (TS_n) = 5n+2, \text{ for every } n$$

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Conclusion:

The study offers a distinctive description of equitable power domination tailored to silicate structures, proving that conventional domination theories may be successfully applied to mineralogical settings. By successfully determining the equitable power dominance numbers for a variety of silicate structure types, including cyclic silicates, chain silicates, double chain silicates, and sheet silicates, the study expands the theoretical framework of domination in graph theory. By demonstrating how mathematical ideas may be used to address practical issues in chemical science, this study successfully closes the gap between graph theory and mineralogy and promotes interdisciplinary cooperation and creativity. Future research might investigate fair power dominance in mineral classes other than silicates, including oxides or carbonates, in order to get a more comprehensive knowledge of mineral structures.

References:

1. Banupriya S and Srinivasan N, Equitable Power Domination Number of Certain Graphs, International Journal Engineering and Technology, 2018.
2. Swaminathan V and Dharmalingam K M, Degree Equitable Domination in Graphs Journal of Mathematics, 2011.
3. B. Rajan, A.William, C. Grigorious, S. Stephen, On certain topological indices of silicate, honeycombs and hexagonal networks. J. Computer. Math. Sci, 2012.
4. S. Stephen, B. Rajan, J. Ryan, C. Grigorious, A. William, Power domination in certain chemical structures, J. Disc. Algorithms 33 (2015) 10–18.
5. C.S. Liao, D.T. Lee, Power domination in circular-arc graphs, Algorithmic 65(2) (2011) 443–466.
6. P. Manuel, I. Rajasingh, Topological properties of silicate networks, in: 5th IEEE GCC Conference, Kuwait, March, 16–19, 2009.
7. K.J. Pai, J.M. Chang, Y.L. Wang, A simple algorithm for solving the power domination problem on grid graphs, in: Proc. the 24th Workshop Combine. Maths and Computer. Theory, 2007, pp. 256–260.
8. S. Varghese, A. Vijayakumar, Power domination in some classes of graphs, in: EuroComb'11, 2011.
9. Y.L. Wang, G.S. Huang, An algorithm for solving the power domination problem on honeycomb meshes, Master Thesis, Department of Computer Science and Information Engineering, National Chi Nan University, 2009.
10. G.J. Xu, L.Y. Kang, E.F. Shan, M. Zhao, Power domination in block graphs, Theory. Computer. Sci. 359 (2006) 299–305. [25] M. Zhao, L.Y. Kang, G.J. Chang, Power domination in graphs, Discrete Math. 306 (2006) 1812–1816.
11. B. Thenmozhi, Dr. R. Prabha , Power Domination of Middle Graph of Path Cycle and Star, Mathematics, Engineering published 2017.
12. Slashi Leel,Sweta Srivastav ,Sangeeta Gupta and Geetha Ganesan ,Domination Number in the Context of Some New Graphs , at the 2nd Computing Congress 2023, Chennai, India, 28–29 December 2023.