

IMPACT OF THERMAL RADIATION ON MHD BOUNDARY LAYER FLOW OF A NANOFUID OVER AN EXPONENTIALLY STRETCHING SHEET

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ABSTRACT

This investigation explores the complex interplay of boundary layer dynamics by combining the impacts of thermal radiation, an external magnetic field, and fluid thermal conductivity boosted by nanoparticles. An exponentially stretching sheet is used as a model in situations where the stretching rate is not constant, which is prevalent in several real-world industrial operations. The flow of the MHD boundary layer was investigated using a radiation-influenced exponentially stretched sheet. Using the MATLAB solver bvp5c, the governing equations are numerically solved. We verified that skin friction, flow area, and heat transfer rate are all affected by non-dimensional factors.

Keywords: Nanofluid, Magneto hydrodynamics, Radiation, Heat Transfer, Stretching

I. INTRODUCTION

Magnetohydrodynamics is a branch of fluid dynamics that studies the effects of magnetic fields on conducting fluids. Applying a magnetic field changes the flow characteristics of a conducting fluid due to the Lorentz force, which acts in a direction perpendicular to both the flow and the field. Interactions like this alter the heat and mass transfer rates by changing the fluid's velocity and temperature. Investigating the effects of MHD in nanofluids has taught scientists more about the altered flow behavior and enhanced thermal conductivity brought about by nanoparticles.

Nanofluids were first suggested by Choi in the early 1990s as a means of improving fluids' thermal conductivity and other thermal properties by the addition of nanoparticles. The addition of nanoparticles to a base fluid (often water or oil) enhances its heat transfer properties, making nanofluids a viable option for several heat transfer applications. The interaction between nanofluids and magnetic fields further complicates the study. This is because, when exposed

to a magnetic field, nanoparticles in a fluid may induce changes to its heat transfer and flow characteristics.

The concept of stretching sheets has been the center of much study about boundary layer fluxes. When a stretching sheet is extended over the boundary layer, it alters the distribution of fluid velocities and temperatures. Extra complexity is introduced by exponential stretching, which is based on the fact that the rate of stretching increases exponentially as the sheet length increases. This form of stretching becomes important in real-world scenarios when the rate of stretching is not constant, such as in manufacturing operations.

Thermal radiation considerably affects boundary layer fluxes as well. Fluid and stretched sheet undergo heat exchange with one another and external environment as part of the process. When it comes to high-temperature industrial operations, radiation heat transfer may have a significant impact on how the fluid and the stretched sheet's surface are heated. To optimize process parameters and make accurate predictions about thermal performance, it is essential to understand how heat radiation affects the MHD boundary layer flow of nanofluids.

Advanced analytical and numerical methods are required to fully comprehend the complex system formed by thermal radiation, MHD interactions, and nanofluid properties. In order to study these effects, academics have developed a plethora of models, including analytical solutions and numerical simulations. With these models, we can anticipate the fluid's behavior under a wide range of conditions, such as those involving different amounts of radiation, nanoparticles, and magnetic fields.

II. LITERATURE REVIEW

Kishan, Naikoti et al., (2016) In this particular scenario, the steady-state magnetohydrodynamic boundary layer flow of an electrically conducting nanofluid is investigated via the use of computer simulations. An exponentially porous stretched sheet has the potential to act as either a heat source or a sink. To ensure that all pertinent aspects are taken into account, heat conduction, thermophoresis, and nanoparticle volume fraction are all taken into consideration. Through the use of an appropriate similarity transformation, the partial differential equations that regulate the mass, momentum, energy, and nanoparticle volume fraction are converted into ordinary differential equations.

Yanala, Dharmendar. (2016) In this study, the results of a numerical analysis on multi-scale boundary layer flow across a non-linear stretching sheet are provided. The analysis included thermal radiation, magnetic field, suction/injection, and nanofluid heat transfer. The control partial differential equations are transformed into ordinary differential equations by the use of similarity transformations. In order to solve the modified non-linear equations, a finite difference technique that is often referred to as the Keller Box method is used. It is remarkable that these findings and those from earlier papers are in complete accord with one another.

Rao, Jakkula et al., (2015) The purpose of this work is to give the results of a numerical research that investigates the flow behavior of a nanofluid across an exponentially stretched sheet in a

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porous medium. The study also includes the features of heat and mass transfer. In the event that the sheet is permeable, we will proceed with our plan. Through the application of appropriate similarity transformations, the connected nonlinear ordinary differential equations may be derived from the partial differential equations (PDEs) that regulate the system. For the purpose of numerically solving the updated equations, the Keller Box methodology, which is a well-known explicit finite difference method, is used. For the purpose of determining the impact that physical factors have on the profiles of longitudinal velocity, temperature, nanoparticle volume fraction, local skin-friction coefficient, local Nusselt number, and local Sherwood number, a comprehensive parametric analysis was carried out. Tables and graphs are used to show the findings of the study.

Loganathan, P. & Vimala, C.. (2014) A sheet that has been exponentially stretched is utilized as a model in order to theoretically study the influence that heat radiation has on the flow of a nanofluid that is contained inside a laminar magnetohydrodynamic boundary layer environment. Through the use of a similarity transformation, the equations that regulate the boundary layer of the issue are converted into ordinary differential equations. A numerical solution is obtained by using the shooting method to solve the ordinary differential equations that have been produced.

III. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider the steady two-dimensional mixed-mode hydrodynamic flow of a viscous incompressible fluid through a porous medium, as shown in Figure 1, using electrical currents generated by a stretching plate. The fluid around the plate is calm and has a constant temperature T_∞ .

We shall assume that the induced magnetic is ignored and that a variable magnetic field $B(x)$ is provided perpendicular to the sheet in order to get an understanding of MHD flow at lower Reynolds numbers. This will allow us to better understand the flow. Using these continuity, momentum, and temperature equations, regulate the flow and heat exchange while taking radiation implications into consideration. This should be done in line with the estimated boundary layer conditions that are often used.

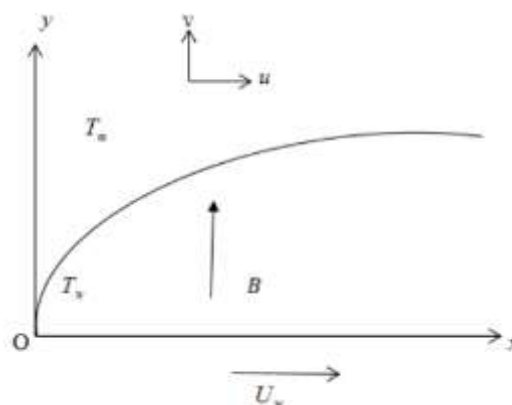


Figure 1. The fundamental nature of both the model and the coordinate system

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} \quad (3)$$

Here is the definition of the necessary constraint:

$$v = 0, u = U_w(x); T = T_w(x), \text{ at } y \rightarrow 0$$

$$u \rightarrow 0, T \rightarrow T_\infty, \text{ as } y \rightarrow \infty.$$

First, let's talk about the following parameters:

$$U_w(x) = ae^{x/L}$$

$$T_w(x) = T_\infty + T_0 e^{x/2L}$$

Where q_r is the Rosseland approximation such that:

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}. \quad (4)$$

By stimulating T^4 in the Taylor series expansion around T_∞ , we may prevent higher-order terms by using the formula if the temperature change within the flow is small enough.

$$T^4 \cong T_\infty^3 (4T - 3T_\infty) \quad (5)$$

Equations (4) and (5) take into account equation (3).

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} \left(\alpha + \frac{16\sigma^* T_\infty^3}{\rho C_p} \right) \quad (6)$$

The stream functions are defined by and satisfy the equation (1).

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \dots$$

The next step is to implement the dimension-less similarity variables that have been introduced.

$$\psi = \sqrt{2uL} f(\eta) e^{x/2L}, \quad u = ae^{x/2L} f'(\eta), \quad v = -\sqrt{\frac{\nu a}{2L}} e^{x/2L} (f(\eta) + \eta f'(\eta))$$

$$T = T_\infty + T_0 e^{x/2L} \theta(\eta), \quad \eta = y \sqrt{(a/2\nu L)} e^{x/2L}.$$

There was a modification in the form of Equations (2) and (6) to

$$\begin{aligned} f''(2f' + M) &= f''' + f''', \\ \left(1 + \frac{4}{3}K\right) \theta'' &= \text{Pr}(f'\theta - f\theta') \end{aligned} \quad (7)$$

For nondimensional boundary conditions, the correct formula shifts to

$$\begin{aligned} f(\eta) &= 1, f'(\eta) = 0, \theta(\eta) = 1, \text{ at } \eta \rightarrow 0 \\ f'(\eta) &\rightarrow 0, \theta(\eta) \rightarrow 0, \text{ as } \eta \rightarrow \infty. \end{aligned} \quad (8)$$

Wall shear stress, heat transfer, and mass transfer are the three phenomena that take place when two surfaces attempt to come into contact with a fluid whose density does not change.

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} \quad q_w(x) = -k \left(\frac{\partial T}{\partial y} \right)_{y=0} \quad (9)$$

In terms of dimensions, the local skin friction, the Nusselt number, and the Sherwood numbers are the physical components that make up this issue.

$$\begin{aligned} C_f = \tau_w / \rho_f e^{2x/L} u_w^2 &\Rightarrow C_f = f''(0) / \sqrt{2 \text{Re}_x}, \\ Nu_x = x q_x / k (T_w(x) - T_\infty) &\Rightarrow Nu_x = - \left(\sqrt{x \text{Re}_x} / L \right) \theta'(0) \end{aligned}$$

IV. RESULTS AND DISCUSSION

The ODE system (7)-(9) is solved using numerical approaches and the integrated MATLAB solver bvp4c. Visualized in Figures 2 are the impacts of the Prandtl number $\text{Pr} = 1$, magnetic parameter $M = 1$, and radiation number $K = 1$ on the velocity profile $f(\eta)$ and temperature profile $\theta(\eta)$.

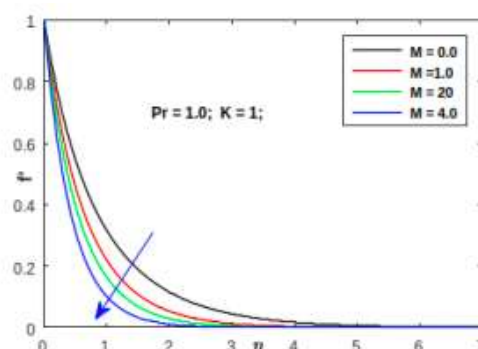
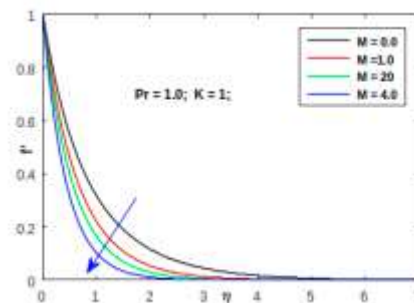
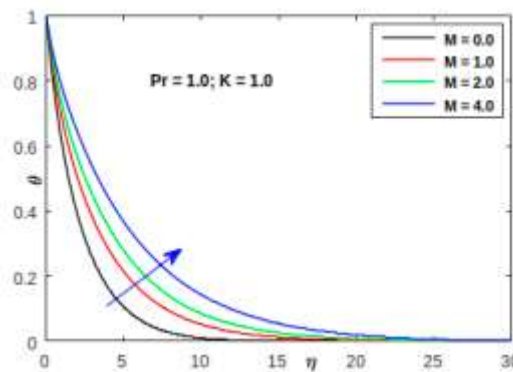


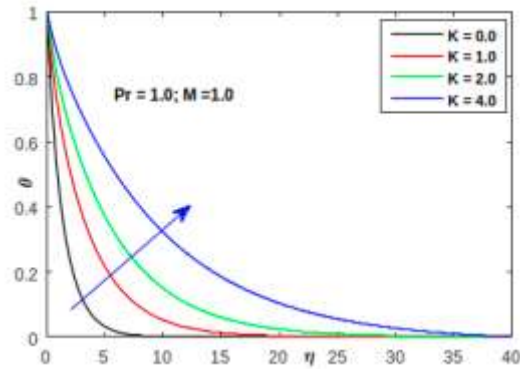
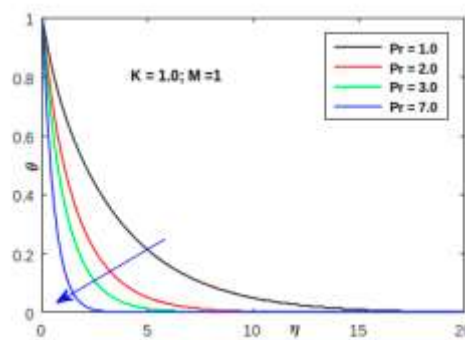
Figure 2. $f(\eta)$, $f'(\eta)$, $\theta(\eta)$ variation with η

The fact that the velocity profiles $f(\eta)$ and $f'(\eta)$ are directly proportional to each other is shown by this. Figure 3 displays the velocity curves for several magnetic parameter M estimations. It

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is evident that the transfer rate drops significantly with increasing M . Everyone knows that the Lorentz force makes transportation phenomena more resistant. What's more, the Lorentz force changes with M and P , although R and r don't affect the flow area in the equation (7). M has an inverse relationship with the gradient speed of surface shear stress. Therefore, M is a magnetic-related variable that regulates the shear stress at the surface. Changing M , K , and P causes all of the other parameters' temperature curves to become equal. Verifying the numerical results, we look at Figures 3-6, which depict the asymptotically complete far-field limit conditions. As can be seen in Figures 4-6, the thickness of the thermal boundary layer increases as M and K do, whereas the reverse is true for higher Pr values. When M and K grow, the neighboring Nusselt number ($-\theta'(0)$) decreases, while the opposite trend is seen when P estimates improve. A larger temperature gradient will form at the heated surface as a result of the gradual absorption and dissipation of more and more heat due to the fact that the warm diffusivity drops with increasing P .

Figure 3. $f'(\eta)$ v/s M Figure 4. $\theta(\eta)$ vs M

Figure 5. $\theta(\eta)$ vs KFigure 6. $\theta(\eta)$ vs P r

V. CONCLUSION

Thermal radiation improves nanofluid heat transfer efficiency by changing the boundary layer properties and temperature distribution, as shown in the research. An example of the difficulty of improving thermal management in advanced applications is the interplay of stretched surfaces, radiative effects, and magnetic fields. This research shows how important it is to include many physical phenomena into thermal analysis, which in turn helps with the creation of better cooling systems and enhanced engineering process performance.

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