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On k- Heinz Quarter Mean Labeling of Graph

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Abstract:

A graph G with p vertices and q edges is said to have k- Heinz Quarter Mean Labeling if it is possible to label the vertices $v \in V$ with distinct labels $f(x)$ from $k, k+1, k+2, \ldots.. k+q$` in such a way that in each edge $e = uv$ is labeled with $f(e = uv) = \left\lfloor \frac{\sqrt[4]{f(u)f(v)}\,(\sqrt{f(u)}+\sqrt{f(v)})}{2} \right\rfloor$ or $\left\lceil \frac{\sqrt[4]{f(u)f(v)}\,(\sqrt{f(u)}+\sqrt{f(v)})}{2} \right\rceil$, and the edge labelings are distinct. In this case f is called k- Heinz Quarter Mean labeling of Graph. The graph which admits k- Heinz Quarter mean labeling is called k- Heinz Quarter mean graph . Here we investigate k- Heinz Quarter mean labeling of $D_2(P_n)$, Mirror graph ,Split graph of Comb, $P_n \odot K_{1,2}$.

Mathematics subject classification: 05C78

Keywords :

Graph, k- Graph, $D_2(P_n)$, Mirror graph , Split graph of Path, Split graph of Comb.

1. INTRODUCTION

Most graph labeling methods trace their origin to one introduced by Rosa in 1967,or one given by Graham and Sloane in 1980. Rosa called a function f a β-valuation of a graph G with q edges if f is an injection from the vertices of G to the set $\{0,1,\ldots,q\}$ such that, when each edge xy is assigned the label $|f(x) - f(y)|$, the resultingedge labels are distinct. Golomb subsequently called such labelings graceful and this is now the popular term. Alternatively, Buratti, Rinaldi and Traetta define a graph G with q edges to be graceful if there is an injection f from the vertices of G to the set $\{0,1,\ldots,q\}$ such that every possible difference of the vertex labels of all the edges is the set $\{1,2,\ldots,q\}$.

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. A useful survey on graph labeling by Gallian (2016) can be found in [1]. Somasundaram and Pondraj have introduced the notion of mean labeling of graphs. In [9], Bhatia introduced the Heinz mean $f(u, v;\ y) = \frac{f(u)^y\, f(v)^{1-y} + f(u)^{1-y} f(v)^y}{2}$, for $0 \le y \le \frac{1}{2}$. We

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introduced Heinz Quarter mean labeling in [7]. In this paper we investigate k- Heinz Quarter mean labeling of some special graphs.

2. PRELIMINARIES

Definition 2.1: A graph $G = (V, E)$ having number of vertices as p and number of edges as q can be called as **Heinz Quarter Mean graph,** if the vertices $x \in V$ can be labeled with distinct labels $f(x)$ from $1, 2, 3, \dots \dots, q + 1$ and each edge $e = uv$ is labeled with $f(e = uv) = \left\lfloor \frac{\sqrt[4]{f(u)f(v)}\left(\sqrt{f(u)}+\sqrt{f(v)}\right)}{2} \right\rfloor$ or $\left\lceil \frac{\sqrt[4]{f(u)f(v)}\left(\sqrt{f(u)}+\sqrt{f(v)}\right)}{2} \right\rceil$, then the resulting edge labels are distinct. Here, f is called Heinz Quarter mean labeling of G. This graph is said to Heinz Quarter Mean Graph.

Definition 2.2: $D_2(P_n)$ is a graph obtained from two copies of P_n so that the pair of vertices u, v' and u', v are adjacent in $D_2(P_n)$ whenever uv and $u'v'$ are the respective edges of the copies of P_n.

Definition 2.3: For a graph G, the split graph is obtained by adding to each vertex v, a new vertex v' such that v' is adjacent to every vertex that is adjacent to v in G. The resultant graph is denoted as Spl (P_n).

Definition 2.4: Let G be a graph. Let G' be a copy of G. The Mirror graph $M(G)$ of G is defined as the disjoint union of G and G' with additional edges joining each vertex of G to its corresponding vertex in G'.

Definition 2.5 : $P_n \odot K_{1,2}$ is a graph obtained by attaching each vertex of P_n to the central vertex of $K_{1,2}$

Definition 2.6: $P_n \odot K_{1,3}$ is a graph obtained by attaching each vertex of P_n to the central vertex of $K_{1,3}$

3. Main Results

Definition 3.1: A graph G with p vertices and q edges is said to have k- Heinz Quarter Mean Labeling if it is possible to label the vertices $v \in V$ with distinct labels $f(x)$ from $k, k+1, k+2, \dots.. k+q$` in such a way that in each edge $e = uv$ is labeled with $f(e = uv) = \left\lfloor \frac{\sqrt[4]{f(u)f(v)}\left(\sqrt{f(u)}+\sqrt{f(v)}\right)}{2} \right\rfloor$ or $\left\lceil \frac{\sqrt[4]{f(u)f(v)}\left(\sqrt{f(u)}+\sqrt{f(v)}\right)}{2} \right\rceil$, and the edge labelings are distinct. In this case f is called k- Heinz Quarter Mean labeling of Graph. The graph which admits k- Heinz Quarter mean labeling is called k- Heinz Quarter mean graph.

Theorem 3.2: $D_2(P_n)$ is k –Heinz Quarter Mean graph for all $n \geq 2$ & $k \geq 3$

Proof : Let $V(D_2(P_n)) = \{u_i, v_i\,; 1 \leq i \leq n\}$

$$E(D_2(P_n)) = \{u_i u_{i+1}, v_i v_{i+1}\,; 1 \leq i \leq n-1\} \cup \{u_i v_{i+1}, v_i u_{i+1}; 1 \leq i \leq n-1\}$$

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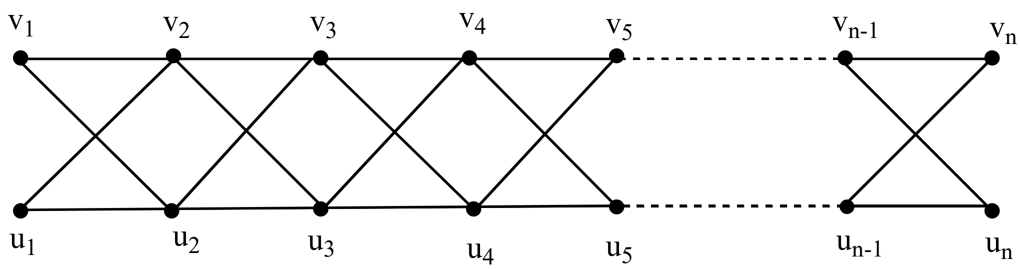
The ordinary labeling



Define a function $f: V(D_2(P_n)) \to \{k, k+1, k+2, \dots \dots k+4n-4\}$

$$f(u_i) = k + 4i - 3 \quad if\ 1 \le i \le n-1$$

$$f(u_n) = k + 4n - 4$$

$$f(v_1) = k$$

$$f(v_i) = k + 4i - 6 \quad if\ \ 2 \le i \le n$$

Then the induced edge labels are

$$f(u_i\, u_{i+1}) = k + 4i - 1 \qquad if \quad 1 \le i \le n-1$$

$$f(v_i\, v_{i+1}) = k + 4i - 4 \qquad if \quad 1 \le i \le n-1$$

$$f(u_i\, v_{i+1}) = k + 4i - 3 \qquad if \quad 1 \le i \le n-1$$

$$f(u_{n-1}\, v_n) = k + 4n - 2$$

$$f(v_i\, u_{i+1}) = k + 4i - 2 \qquad if \quad 1 \le i \le n-1$$

$$f(v_{n-1}\, u_n) = k + 4n - 7$$

The above defined function f provides k- Heinz Quarter mean labeling of the graph.

Hence $D_2(P_n)$ is k-Heinz Quarter Mean graph for all $n \geq 2$ & $k \geq 3$.

Example 3.3

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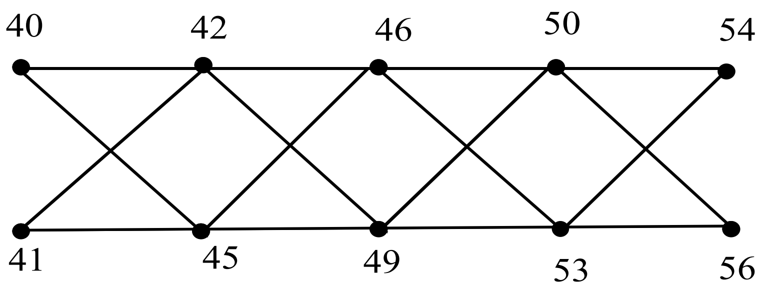


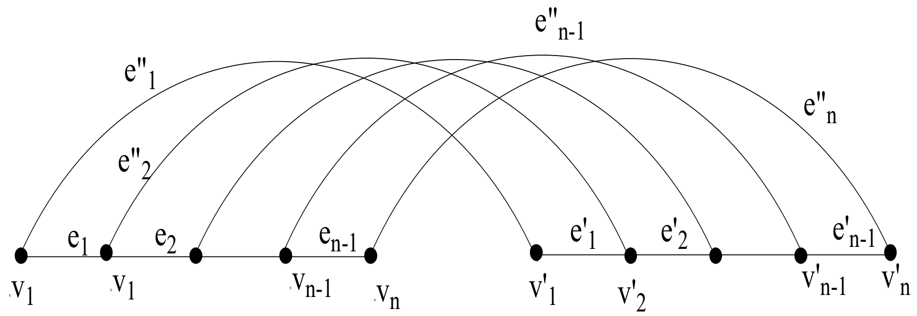
40 - Heinz Quarter Mean Labeling of $D_2(P_n)$

Theorem 3.4: The Mirror graph $M(P_n)$ is k-Heinz Quarter Mean graph for all n & k

Proof : Let $V\big(M(P_n)\big) = \{v_i\ , v_i'; 1 \le i \le n\}$

$$E\big(M(P_n)\big) = \{\ v_i v_{i+1}\,; 1 \le i \le n-1\} \cup \{e_i' = (v_i' v_{i+1}'): 1 \le i \le n-1\} \\ \cup \big\{\{e_i' = (v_i v_i'): 1 \le i \le n-1\}\big\}$$

The ordinary labeling is



Define a function $V\big(M(P_n)\big) \to \{k, k+1, \ldots .\, k+3n-2\}$

$$\text{By} \quad f(v_i) = k + 3i - 3 \qquad if\ 1 \le i \le n$$

$$f(v'_{2i-1}) = k + 6i - 5 \quad if\ 1 \le i \le \tfrac{n}{2}$$

$$f(v_{2i}') = k + 6i - 1 \qquad if\ 1 \le i \le \tfrac{n}{2}$$

Then the induced edge labels are

$$f(e_i) = k + 6i - 5 \qquad if\ 1 \le i \le n-1$$

$$f(e_i') = k + 3i - 1 \qquad if\ 1 \le i \le n-1$$

$$f\big(e_i^{"}\big) = k + 3i - 3 \quad if\ 1 \le i \le n$$

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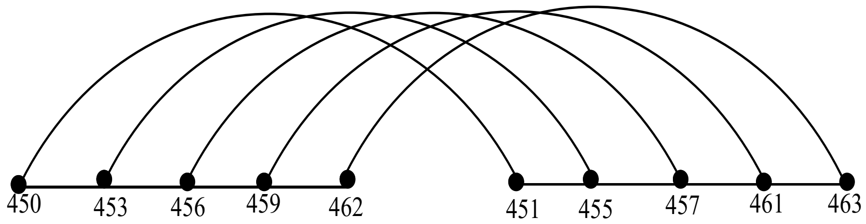
The above defined function f provides k- Heinz Quarter mean labeling of the graph.

Hence $M(P_n)$ is k-Heinz Quarter Mean graph for all n & k

Example 3.5



450 - Heinz Quarter Mean Labeling of M(P_7)

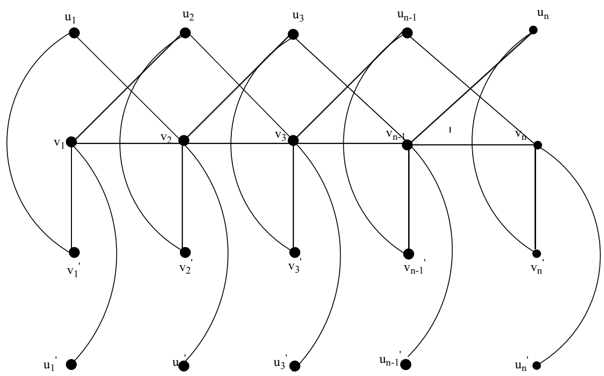
Theorem 3.6: The Splitting graph of comb Spl $(P_n \odot K_1)$ is k – Heinz Quarter mean graph for all n &$k \geq 4$.

Proof :

Let $V\big(Spl(P_n \odot K_1)\big) = \{v_i, v_{i\,,}' u_i' \,; 1 \leq i \leq n\}$

$E\left(Spl\big((P_n \odot K_1)\big)\right) = \{v_i, v_i', v_i u_i', v_i v_i'; 1 \leq i \leq n-1\} \cup \{v_i v_{i+1}, v_i u_{i+1}, u v_{i+1}; 1 \leq i \leq n-1\}$

The ordinary labeling of Split graph of comb is



Define a function $V\big(Spl(P_n \odot K_1)\big) \rightarrow \{k, k+1, \dots . k+6n-3\}$ by

$f(v_i) = k + 6i - 4 \quad if\ 1 \leq i \leq n\,; \quad f(u_i') = k + 6i + 2 \quad if\ 1 \leq i \leq n\,;$

$f(u_i) = k + 6i - 5 \quad if\ 1 \leq i \leq n\,; \quad f(v_i') = k + 6i - 6 \quad if\ 1 \leq i \leq n$

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And the induced edge labels are

$$f(v'_i\, u_i) = k + 6i - 6 \qquad if \quad 1 \le i \le n$$

$$f(v_i\, u'_i) = k + 6i - 4 \qquad if \quad 1 \le i \le n$$

$$f(v_i\, v'_i) = k + 6i - 5 \qquad if \quad 1 \le i \le n$$

$$f(v_i\, v_{i+1}) = k + 6i - 1 \qquad if \quad 1 \le i \le n-1$$

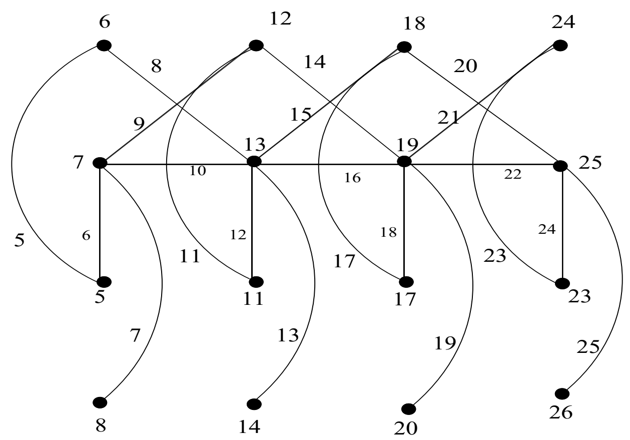
$$f(u_i\, v_{i+1}) = k + 6i - 3 \qquad if \quad 1 \le i \le n-1$$

$$f(v_i\, u_{i+1}) = k + 6i - 2 \qquad if \quad 1 \le i \le n-1$$

Thus the above defined function f provides k- Heinz Quarter mean labeling of the graph.

Hence $Spl(P_n \odot K_1)$ is k-Heinz Quarter Mean graph for all n & $k \ge 4$.

Example 3.7



5-Heinz QuarterMean Labeling of Spl($P_4 \odot K_1$)

Theorem 3.8: $P_n \odot K_{1,2}$ admits k- Heinz Quarter mean labelling, for all positive integer k.

Proof : Let $G = P_n \odot K_{1,2}$ where P_n is a path with vertices $u_1, u_2, \dots, u_n$ and v_i, w_i be the vertices of $K_{1,2}$ which are attached to the vertex $u_1$of P_n, for $1 \le i \le n$, respectively.

Define $f : V(G) \to \{k, k+1, \dots, k+3n-1\}$

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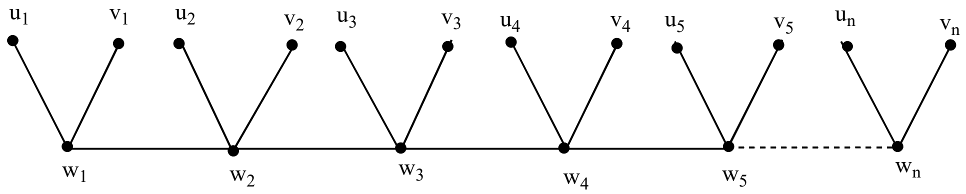


The ordinary labeling is

$$f(u_i) = k + 3i - 3, \qquad 1 \le i \le n$$

$$f(v_i) = k + 3i - 1, \qquad 1 \le i \le n$$

$$f(u_i) = k + 3i - 2, \qquad 1 \le i \le n$$

Then the induced edge labels are

$$f(u_i w_i) = k + 3i - 2$$

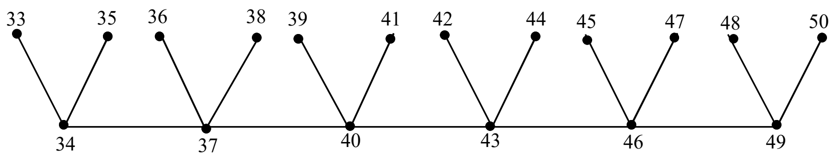
$$f(v_i w_i) = k + 3i - 1$$

$$f(u_i w_i) = k + 3i - 3$$

Thus the above defined function f provides k-Heinz Quarter labeling of the graph.

Hence $P_n \odot K_{1,2}$ admits k- Heinz Quarter mean labeling, for all positive integer k.

Example 3.9



33-Heinz QuarterMean Labeling of $P_6 \odot K_{1,2}$

Theorem 3.10: $P_n \odot K_{1,3}$ admits k- Heinz Quartermean labelling, for all k.

Proof : Let $G = P_n \odot K_{1,3}$ where P_n is a path with vertices $u_1, u_2, \dots, u_n$ and v_i, w_i, z_i be the vertices of $K_{1,3}$ which are attached to the vertex $u_1$of P_n, for $1 \le i \le n$, respectively.

Define $f : V(G) \to \{k, k+1, \dots, k+3n-1\}$

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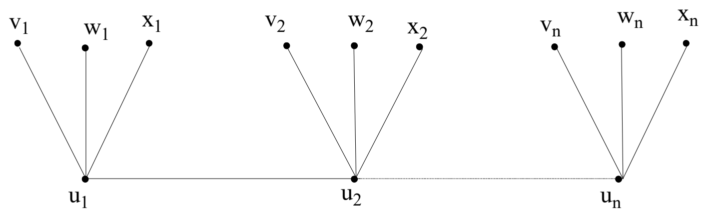


$$f(u_i) = k + 4i - 3, \quad 1 \le i \le n$$

$$f(v_i) = k + 4i - 4, \quad 1 \le i \le n$$

$$f(w_i) = k + 4i - 2, \quad 1 \le i \le n$$

$$f(x_i) = k + 4i - 1, \quad 1 \le i \le n$$

Then, the distint edge labels are :

$$f(u_i u_{i+1}) = k + 4i - 3 \quad 1 \le i \le n - 1$$

$$f(u_i v_i) = k + 4i - 2 \quad 1 \le i \le n$$

$$f(u_i w_i) = k + 4i - 1 \quad 1 \le i \le n$$

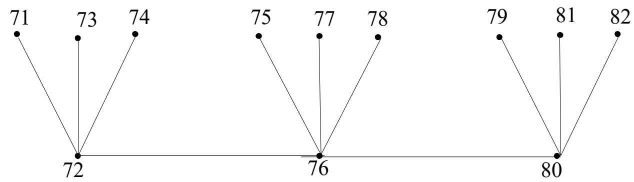
$$f(u_i x_i) = k + 4i - 4 \quad 2 \le i \le n.$$

The graph G gets distinct edge labels.

Thus the above defined function f provides k-Heinz Quarter labeling of the graph.

Hence $P_n \odot K_{1,3}$ admits k- Heinz Quarter mean labeling, for all positive integer k.

Example 3.11



71-Heinz Quarter Mean Labeling of $P_3 \odot K_{1,3}$

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