Numerical Solutions of Oscillatory Dynamics via Haar Wavelets

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Abstract

An efficient numerical approach has been developed for solving second-order differential equations using the Haar Wavelet Collocation Method (HWCM) in dynamical systems that change over time, including applications in physics and engineering. From a physics perspective, we systematically investigate three sets of differential equations representing simple and damped harmonic motion using the Haar Wavelet Collocation Method (HWCM) and compare the results with the Taylor series method. Numerical results indicate that HWCM provides more accurate approximations than the Taylor series approach. Further, the Haar solutions and exact solutions with absolute errors are calculated. Interestingly, the analysis indicates that error drops exponentially as the resolution level increases. These results provides the accurate tools for predicting displacement, velocity, and acceleration in oscillatory systems. The main key features of the HWCM are that it is simple and effective in solving a wide variety of Initial Value Problems (IVPs).

Keywords: Haar Wavelet; HWCM; Taylor series method; ODE; SHM; DHM

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1. Introduction

The mass-spring system of the mathematical model is used for physical sound synthesis, which can be expressed as differential equations [1]. Differential equations are used in many scientific and technological applications [2] and are applied to various fields, including numerical computation, free vibrations, data compression, applied chemistry, diagnostic image processing, de-noising data, etc. [3-5]. In recent years, many mathematicians and physicists have become interested in the study of IVPs in the second-order Ordinary Differential Equation (ODE) [6, 7]. IVPs are specially used in dynamic systems that change over time, such as the mass-spring system. There have been several sound synthesis systems built using mass-spring systems, and in the literature, they are based on numerical methods [1, 8]. I. Singh and S. Kumar presented the Taylor series method for a damped forced oscillator problem using Haar wavelets [8]. However, their approximate solutions have not addressed the issue of numerical technique stability or accuracy. This has been an important part of criticism for the spring-mass system in physical sound synthesis.

Currently, wavelets represent an interesting and efficient method for addressing differential equations [9]. The Wavelet theory is a novel field of mathematics with extensive applications in image processing, signal analysis, numerical analysis, etc. [4, 10–12]. Alfred Haar [9] presented the collection of piecewise constant functions referred to as Haar wavelets, which play a crucial role in the numerical solution of differential and integral equations. Over the last 2 decades, the Haar wavelet-based collocation technique for numerical approximations has gained significant popularity [13–15]. The Haar wavelet approach has a few advantages, including higher precision with a smaller number of grid points, fast convergence, mathematical simplicity, and the ability to implement conventional algorithms. Therefore, these characteristics of Haar functions have provided a strong basis for numerically approximating ordinary differential equations (ODEs) [15].

In recent research, the analysis of systems using the Haar wavelet has been conducted by Chen and Hsiao [16, 17]. They were pioneers in creating the Haar operational matrix for integrating the Haar function vector, employing Haar analysis for dynamic systems, which included the characterization of Brownian motion and oscillatory motions [18]. Lepik applied the HWCM to address differential and integral equations [9, 19]. To solve the ODEs, Phang Chang and Phang Piau [21] created operational matrices for Haar wavelets in 2008. All calculations were carried out using the wavelet matrix form and its integrals, which reduced the procedure's complexity. Novel scale-3 Haar wavelets were recently introduced and used in fractional dynamical systems by Mittal and Pandit [22] and other technological applications [11]. This highlights their advantages over typical Haar wavelets.

In the present article, we intend to create a novel numerical approach utilizing Haar wavelets to address second-order ODEs representing the problems of SHM and DHM of the mass-spring system. For this purpose, we proposed HWCM, which provides simple solution procedures and involves minimal calculations; precise outcomes are achieved even at lower resolution levels. We mainly focus on the IVPs and consider three sets of problems of mass-spring systems and calculated exact solution, Haar solution and Taylor’s solution with their absolute errors with the maximal resolution J, which are discussed in the below section. The results of this investigation are very beneficial in the study of the motion of an object to design the character of animation in further research work.

The article is organized as follows: Section 2 presents the Taylor-series approach. Section 3 outlines the notations for Haar wavelets and their integrals and the theory behind mass-spring model systems. Section 4 discuss the numerical analysis. Section 5 compares the findings of three specific problems. The last section contains the findings from our investigation.

2. Taylor-series method

The well-known Taylor-series approach allows for the numerical solution of ODEs. Let's examine the second-order ODE [8].

$$\frac{d^2x}{dt^2} = \phi(t, x, \frac{dx}{dt}) \tag{1}$$

By replacing, $\frac{dx}{dt} = y$ It can be simplified into a pair of 1st-order simultaneous differential equations.

$$\frac{dx}{dt} = y = f(t, x, y) \tag{1.1}$$

and

$$\frac{dy}{dt} = \phi(t, x, y) \tag{1.2}$$

Let $x(t_0) = x_0$ and $y(t_0) = y_0$ be the initial conditions. Initially at (t_0, x_0, y_0) and h be the increment, for $x_1 = x(t_0 + h)$ and $y_1 = y(t_0 + h)$. By using, Taylor's series method for Eq. (1) and Eq. (1.1) gives

$$x_1 = x_0 + hx_0' + \frac{h^2}{2!}x_0'' + \frac{h^3}{3!}x_0''' + \ldots \tag{1.3}$$

$$y_1 = y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \ldots \tag{1.4}$$

Differentiating Eq. (1) and Eq. (1.1) consecutively, we obtain x'', y'' etc. Thus $x_0', x_0'', x_0''', \ldots$ and $y_0', y_0'', y_0''', \ldots$ are noted. Inserting these values into the equations stated above, we obtain x_1, y_1. Similarly, we have the Taylor's series method.

$$x_2 = x_1 + hx_1' + \frac{h^2}{2!}x_1'' + \frac{h^3}{3!}x_1''' + \dots \qquad (1.5)$$

$$y_2 = y_1 + hy_1' + \frac{h^2}{2!}y_1'' + \frac{h^3}{3!}y_1''' + \dots \qquad (1.6)$$

Since x_1, y_1 are known. We can calculate $x_1', x_1'', x_1''', \ldots$ and $y_1', y_1'', y_1''', \ldots$ Inserting these values into the equations stated above, we obtain x_2, y_2. By following this method, we can evaluate the other values of x and y stepwise.

2.1. Haar Wavelet Technique

In the present paper, we construct an orthogonal set for the system of $L^2[a,b]$ constitute a family of the Haar wavelet and symbols represented by Lepik [9, 23] were used. The equidistance $\left(\Delta t = \frac{(b-a)}{2^{J+1}}\right)$ of subintervals are generated from the range $[a,b]$ is 2^{J+1}, here J indicates the dilation parameters or maximal level of a wavelet. The two parameters as the dilation parameter and translation parameter values respectively $j = 0, 1, 2, \ldots, J$ $k = 0, 1, 2, \ldots, 2^j - 1$

The, i^{th} Haar wavelet is as follows.

$$h_i(t) = \begin{cases} 1, & \text{for } t \in [\xi_1(i), \xi_2(i)), \\ -1, & \text{for } t \in [\xi_2(i), \xi_3(i)), \\ 0, & \text{elsewhere}, \end{cases} \qquad (1.7)$$

where $\mu = 2^{J-j}$ and $i = m + k + 1$, $\xi_1(i) = a + 2k\mu\Delta t$, $\xi_2(i) = a + (2k+1)\mu\Delta t$, $\xi_3(i) = a + 2(k+1)\mu\Delta t$, for $i > 2$ *and* $h_1(t)$ *and* $h_2(t)$ are called scaling function, and mother wavelets for the family of Haar wavelets are defined as

$$h_1(t) = \begin{cases} 1, & \text{for } t \in [a,b), \\ 0, & \text{elsewhere}, \end{cases} \qquad (1.8)$$

$$h_2(t)=\begin{cases}1, \text{ for t } \in\left[\text{a},\dfrac{\text{a+b}}{2}\right), \\ \text{-1, for t } \in\left[\dfrac{\text{a+b}}{2},\text{b}\right), \\ 0, \text{ elsewhere,}\end{cases} \tag{1.9}$$

Since we have a precise representation for each component of the Haar family [15], we can carry out integration repeatedly according to the specific requirements. The subsequent notations are utilized for γ the number of integrations of the members within the family as defined on (a,b):

$$\begin{aligned} P_{\gamma,i}(t)&=\int_a^t\int_a^t.....\int_a^t h_i(x)\ dx^{\gamma}, \\ E_{\gamma,i}&=\int_a^b P_{\gamma,i}(t)\ dt \end{aligned} \tag{2}$$

$For\ i=1$, Eq. (2) becomes

$$P_{\gamma,i}(t)=\frac{1}{\gamma!}(t-a)^{\gamma}, \tag{2.1}$$

$For\ i\geq 2$, we have

$$P_{\gamma,i}(t)=\begin{cases}0, if \text{ t } \in[\text{a},\xi_1(\text{i})), \\ \dfrac{1}{\gamma!}(t-\xi_1(\text{i}))^{\gamma}, if \text{ t } \in[\xi_1(\text{i}),\xi_2(\text{i})), \\ \dfrac{1}{\gamma!}\{(t-\xi_1(\text{i}))^{\gamma}-2(t-\xi_2(\text{i}))^{\gamma}\}, if \text{ t } \in[\xi_2(\text{i}),\xi_3(\text{i})), \\ \dfrac{1}{\gamma!}\{(t-\xi_1(\text{i}))^{\gamma}-2(t-\xi_2(\text{i}))^{\gamma}+(t-\xi_3(\text{i}))^{\gamma}\}, if \text{ t } \in[\xi_3(\text{i}),\text{b}).\end{cases} \tag{2.2}$$

For $J=2$, The Haar matrix H and the integrated Haar matrices p_1 are as follows

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$$p_1 = \begin{bmatrix} 0.0625 & 0.1875 & 0.3125 & 0.4375 & 0.5625 & 0.6875 & 0.8125 & 0.9375 \\ 0.0625 & 0.1875 & 0.3125 & 0.4375 & 0.4375 & 0.3125 & 0.1875 & 0.0625 \\ 0.0625 & 0.1875 & 0.1875 & 0.0625 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0625 & 0.1875 & 0.1875 & 0.0625 \\ 0.0625 & 0.0625 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0625 & 0.0625 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0625 & 0.0625 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0625 & 0.0625 \end{bmatrix}$$

The infinite series of Haar wavelets for the function that possesses finite energy over the interval $[a,b]$, i.e. $f \in L^2[a,b]$ is

$$f(x) = \sum_{i=1}^{\infty} a_i h_i(x) \tag{3}$$

Here a_i's are referred to as the coefficients of the Haar wavelet.

If f is a piecewise constant on each subinterval in which the series terminates to finite.

2.2 Haar Wavelet Collocation Method:

The suggested method is outlined below [9, 15].

(i) The highest derivative of the Haar wavelets is Approximated as

$$y''(x) = \sum_{i=1}^{2^{J+1}} a_i h_i(x). \tag{4}$$

(ii) The integrated Haar functions can be obtained by decomposing $y^{'}(x)$ *and* $y(x)$ and values are replaced in the linear differential equation.

(iii) The above equation can be discretizing at each Collocation point

$$x_1 = \frac{(\tilde{x}_{l-1} + \tilde{x}_1)}{2}, l = 1, 2, \ldots 2^{J+1},$$

where $\tilde{x}_n = a + n\Delta t, n = 0, 1, 2, \ldots, 2^{J+1}$.

Leading to $2^{J+1} \times 2^{J+1}$ the system of linear algebra.

(iv) Determine the approximate solution for the unknown y by computing the wavelet coefficients a_i's.

2.3 Mass Spring Model

The above model uses three basic parts—mass, spring, and dampers—to construct complex musical instruments [1]. The finite differential techniques are used to separate each element. In this section, we have applied HWCM to study the variation of velocity and acceleration of the dynamical system connected with the mass-spring system.

2.3.1 Simple Harmonic Oscillator

Consider a spring, which is attached to the unit mass (m), and has a spring of stiffness constant and its initial length (l). This position is represented as the mean position say the initial condition., $y\ (0)\ = 0$.

Let the applied force act on a system is $1N$ vertically downwards, the spring is elongated to $(l + x)$, spring stretched to unit length, and set free, it underwent oscillation. The Oscillation of a simple mass-spring system is shown in the figure 1.

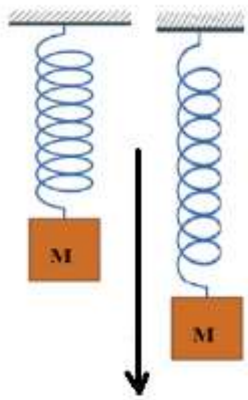

Figure 1 Simple mass-spring systems attached to load

Therefore, the restoring force acting on a body due to the stiffness of the material of spring as provided by

$$F_r = -kx \tag{5}$$

x: Displacement k: Spring Constant

According to Newton's second Law restoring force produces acceleration.

$$\therefore F = ma \tag{5.1}$$

From Eq. (5) & Eq. (5.1), the differential equation is expressed as

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \tag{5.2}$$

where $\omega^2 = \frac{k}{m}$

This is the differential equation for SHM.

2.3.2 Damped Harmonic Oscillations

The loss of energy in an oscillator is referred to as damping. The damping may be due to a) the viscosity of the fluid, b) the frictional force, and c) the structure. At any instant, the system is influenced by the following force:

(i) The restoring force is proportional to displacement but in the opposite direction.

$$F_r = -kx \tag{6}$$

(ii) The frictional force is proportional to velocity but in the opposite direction

$$F^1 = -\gamma v \tag{6.1}$$

γ is the damping constant

Therefore, the resultant force is $F = F_r + F^1$

Write it in a differential form.

$$\frac{d^2x}{dt^2} + (2b)\frac{dx}{dt} + (\omega^2)x = 0 \tag{6.2}$$

Where damping coefficient $\frac{\gamma}{m} = 2b$ and $\frac{k}{m} = \omega^2$

3. Numerical Studies

We considered three sets of examples and their exact solutions are known, arising in SHM. The efficiency of HWCM and Taylor's method were investigated in each example and compared with the exact solution, which is shown in the tables and figures as follows. All the calculations are carried out through MATLAB programming with a maximum level of resolution J and graphs are plotted in Origin Laboratory.

4. Results and Discussion

4.1 Example 1

A frictionless horizontal spring is attached to a mass. Its one end is fixed with a rigid support, and by applying $6N$of force, the spring stretches $1.5m$from its initial length. The resulting second-order SHM equation is

$$\frac{d^2x}{dt^2} + 4x = 0 \tag{7}$$

$0 \le t \le 1$with starting conditions $x(0) = 0$ & $x'(0) = 1$ and the exact solution is $x = 0.5\sin 2t$

Figure 2 indicates the exact solution, HWCM and Taylor series method with a maximum resolution number J designated as 5 for this solution. It's observed that HWCM coincides with the exact solution, and the obtained results are well matched with the periodic motion through Eq. (7). Also, it is evident from Figure 2 that the Taylor series method is slightly deviating from the exact solution.

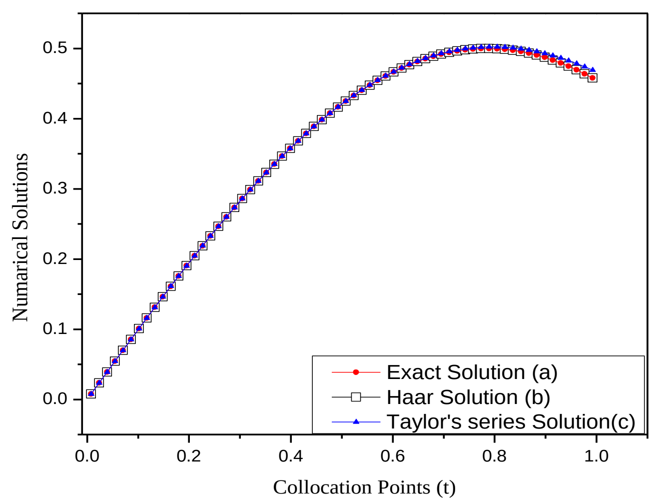

Figure 2 Comparison of Exact solution HWCM and Taylor series method with maximum resolution J=5 of example 1

Table 1 presents the results of the numerical methods using HWCM and the Taylor series, along with the exact solutions and their absolute errors, for comparison. This indicates that the minimum errors are established between HWCM and the exact solution compared to the Taylor series method.

Collocation points (t)	Exact solution	Haar solution	Taylor's Solution	Absolute errors of Exact solution with	
				Haar	Taylor's
0.1016	0.1009	0.1009	0.10090	0	0.00
0.1953	0.1904	0.1904	0.19037	0	0.00
0.3047	0.2862	0.2862	0.28619	0	0.00
0.3984	0.3576	0.3576	0.35758	0	0.00
0.5078	0.4249	0.4249	0.42501	0	0.00
0.6016	0.4666	0.4666	0.46695	0	0.00
0.6953	0.4919	0.4919	0.49288	0	0.00
0.8047	0.4996	0.4996	0.50230	0	0.00
0.8984	0.4873	0.4873	0.49302	0	0.01
0.9922	0.4578	0.4578	0.46923	0	0.01

Table 1. Represents the numerical results of Exact, Haar and Taylor's solution with their absolute errors for example 1 with J=5.

4.2 Example 2

Consider the vertical spring it's one end is fixed to a rigid support with a spring constant of $2N/m$ and the oscillation is accelerated from its mean position with a damping coefficient of $3\,rad/s$.

The resulting 2nd-order DHM equation is

$$\frac{d^2x}{dt^2}+3\frac{dx}{dt}+2x=0 \tag{8}$$

$0\le t\le 1$ with starting condition $x(0)=2$ & $x'(0)=-3$ and the exact solution is $x=e^{-t}+e^{-2t}$

Figure 3 shows the decay profile of the amplitude of DHM. It is evident from Eq. (8) that the Haar collocation points are exactly matched with the exact solution by varying time of maximal resolution, $J=5$. And also noticed that the Taylor series method is deviating from the exact solution.

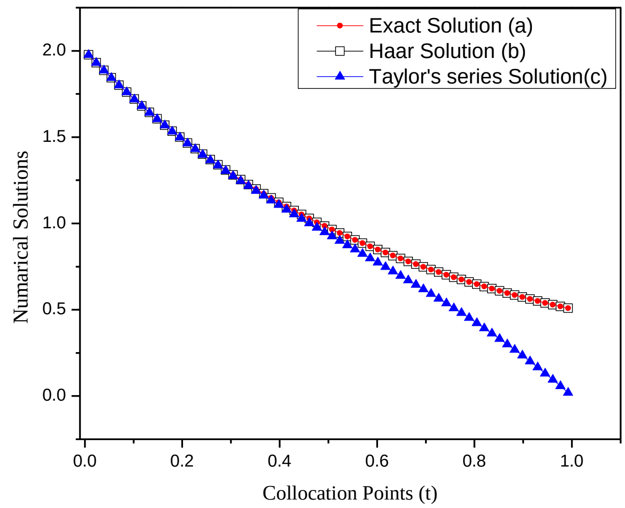

Figure 3 Comparison of Exact solution, HWCM, and Taylor series method of decay profile DHM of example 2 for J=5

Collocation points (t)	Exact solution	Haar solution	Taylor's Solution	Absolute errors of Exact solution with	
				Haar	Taylor's
0.1016	1.7196	1.7196	1.719433	0	0.00
0.1953	1.4992	1.4992	1.498281	0	0.00
0.3047	1.281	1.281	1.275572	0	0.01
0.3984	1.1221	1.1221	1.106754	0	0.02
0.5078	0.985	0.9849	0.924839	0	0.06
0.6016	0.8482	0.8482	0.773407	0	0.07
0.6953	0.7478	0.7478	0.618499	0	0.13
0.8047	0.6472	0.6472	0.42314	0	0.22
0.8984	0.573	0.573	0.234928	0	0.34
0.9922	0.5082	0.5082	0.019379	0	0.49

Table 2. Represents the numerical results of Exact, Haar and Taylor's solution with their absolute errors for example 2 with J=5.

Table 2 represents the HWCM and Taylor series method with exact solutions and their absolute errors. We investigated for $J=5$ with 64 collocation points, and the obtained errors are in the order of 10-4, which suggests the above problem is considered for real-world applications

4.3 Example 3

Consider the vertical spring it's one end is fixed to a rigid support with a spring constant of $2N/m$ and the oscillation is being accelerated from its extreme position with a damping coefficient of $3\ rad/s$

The resulting second-order DHM equation is

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 0 \tag{9}$$

$0 \le t \le 1$ with initial condition $x(0) = 2$ & $x'(0) = 3$ with exact solution $x = e^t + e^{2t}$

Figure 4 shows the growth profile of the amplitude of DHM. It is evident from Eq. (9) that the HWCM exactly matches the exact solution of the maximum resolution, $J = 6$. Also observed that the Taylor series method deviated from the exact solution due to frictional force. Additionally, as Table 3 illustrates, the findings produced are more accurate with less error when the number of nodal points is increased.

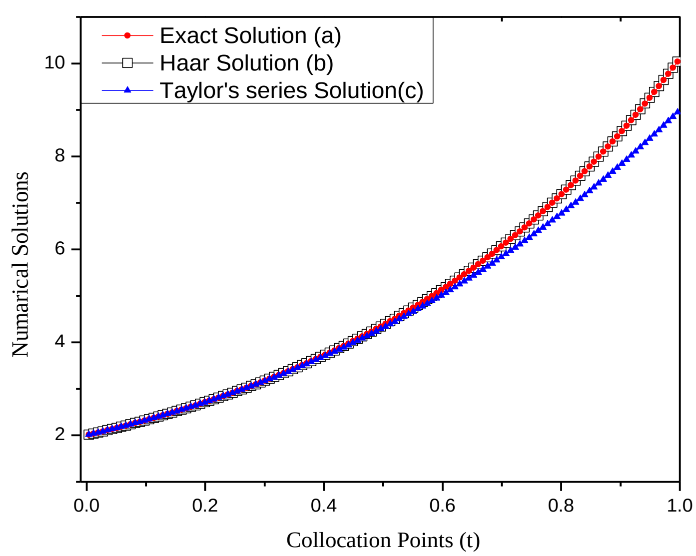

Figure. 4 Comparison of Exact solution, HWCM, and Taylor series method of Growth curve of DHM of example 3 for J=6

Collocation points (t)	Exact solution	Haar solution	Taylor's Solution	Absolute errors of Exact solution with	
				Haar	Taylor's
0.0977	2.3183	2.3183	2.3184	0	0.00
0.1992	2.7099	2.71	2.7087	0	0.00
0.3008	3.1759	3.1759	3.1694	0	0.01
0.4023	3.7313	3.7314	3.7092	0	0.02
0.5039	4.3948	4.3949	4.3384	0	0.06
0.5977	5.1224	5.1226	5.0065	0.0002	0.12
0.6992	6.061	6.0612	5.8325	0.0002	0.23
0.8008	7.1881	7.1883	6.7759	0.0002	0.41
0.9023	8.5434	8.5438	7.8442	0.0004	0.70
0.9961	10.0392	10.0397	8.9514	0.0005	1.09

Table 3. Represents the numerical results of Exact, Haar and Taylor’s solutions with their absolute errors for example 3 with J=6.

Figure 5 represents absolute errors obtained from HWCM for different levels of resolution $\left(J=3,\ 4,\ 5\ and\ 6\right)$ for Eq. (9). As the resolution level increases, the HWCM error curve gets closer to the x-axis, according to the data in Table 4. The data presented in Table 4 indicates that as the resolution level rises, the error curve of HWCM approaches the x-axis. This demonstrates that the outcomes achieved with HWCM are extremely precise. Fig. 6 represents an overlay graph of the normalized amplitude of growth and decay profile of DHM with their collocation points. This will show how the motion of a particle accelerates and decelerates from its extreme and mean positions, respectively. By comparing the values of collocation points, we can calculate the velocity of a particle at any given interval of time, and the obtained solutions are well-matched with the Haar solution. However, these solutions are helpful to design in motion character animation.

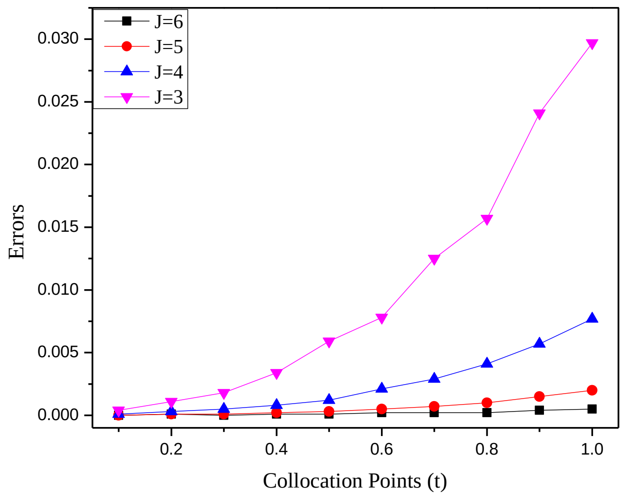

Figure 5 Comparison of absolute errors obtained example 3 for J=3, 4, 5 and 6

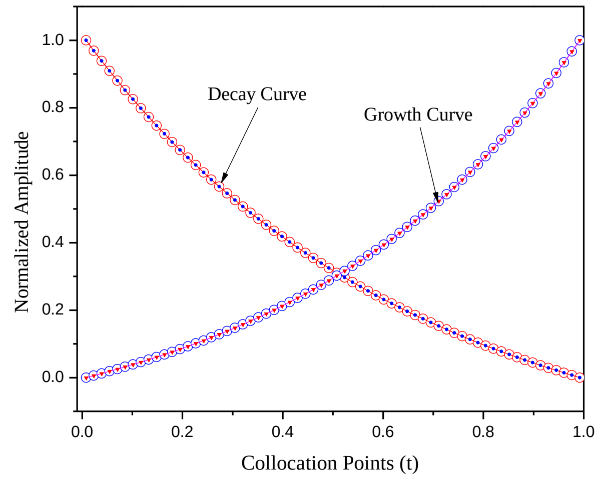

Figure 6. Normalized overlay graph of Growth and Decay curve of DHM

Collocation points(t)	Resolution Levels			
	J=3	J=4	J=5	J=6
0.1	0.0004	0.0001	0	0
0.2	0.0011	0.0003	0.0001	0.0001
0.3	0.0018	0.0005	0.0001	0
0.4	0.0034	0.0008	0.0002	0.0001
0.5	0.0059	0.0012	0.0003	0.0001
0.6	0.0078	0.0021	0.0005	0.0002
0.7	0.0125	0.0029	0.0007	0.0002
0.8	0.0157	0.0041	0.001	0.0002
0.9	0.0241	0.0057	0.0015	0.0004

Table 4. Represents absolute errors of Exact and Haar solutions, for example, 3 with various levels of resolution

5. Conclusion

This work exhibited the applicability of the HWCM to solve second-order differential equations arising in modeling oscillatory motions. We use the Taylor series approach and HWCM to discuss the second-order differential equation for SHM and DHM oscillatory motions. The Haar solutions of the above three problems are well-matched with the exact solution. We noted negligible errors for the simple harmonic motion (SHM) since the oscillatory system is independent of external forces; in these scenarios, we typically resolve the differential equation by employing the Taylor series approach. However, in DHM, the system influences the damping force, so Taylor's solution deviates from the exact solution. From the tables presented above, it can be seen that higher precision is achieved with a smaller number of grid points, highlighting an important feature of this method. We also investigated that the error drops exponentially as the resolution level increases, leading to a more accurate solution than the Taylor series method. The comparison also highlights the importance of selecting appropriate numerical methods for modeling real-world physical systems where analytical solutions are not feasible. These results help to study the motion of the particle to design the character animation and de-noising data. This study opens up new findings, such as the vibration of the musical system by connecting more mass-spring systems to solve higher-order differential equations.

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