Topological Indices in Fuzzy Graphs -An Overview

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Abstract

Graph theory provides a mathematical framework for representing systems with binary relations. This theory is crucial for modeling and analyzing various systems where relationships between elements can be depicted as connections or edges between nodes or vertices paving way for numerous applications in designing communication networks, VLSI circuits, and planning transport networks. Fuzzy set is a powerful tool to capture vagueness and uncertainty through membership functions. Incorporation of fuzzy sets to graph theory forms the basis for fuzzy graphs, where the uncertainty of the vertices and edges are captured through membership values. In chemical science, a topological representation of the molecule is called the molecular graph. A topological index is a numerical quantity for the structural graph of the molecule of a chemical compound, with collection of atoms representing the atoms in the molecule and the chemical bonds representing the chemical bonds considered as edges. This numerical measure finds applications beyond chemistry and has evolved as one of the powerful tools for decision making. Many topological indices existing in crisp graphs are extended to fuzzy graphs with proven applications. Some topological indices in fuzzy graphs are Wiener index, Modified Wiener index, Hyper Wiener index, Schultz index, Gutman index, Zagreb index, Hyper Zagreb index, Harmonic index and Radic index. A comprehensive review of the topological indices in fuzzy graphs with potential applications is presented in this work with a discussion on the scope for future work.

Keywords: Molecular graph, Fuzzy graphs, topological indices

1. Introduction

Graph theory, introduced by *Leonhard Euler* (1736) through the famous Königsberg Bridge problem, provides a mathematical foundation for representing and analyzing systems with binary relations. In a graph, vertices (or nodes) represent entities, and edges represent relationships or interactions among them. Over the decades, graph theory has become an indispensable analytical tool in diverse domains such as computer science, communication networks, chemistry, biology, and social sciences by *Harary* [1] in 1969.

To address the inherent uncertainty and vagueness present in many real-world systems, *Zadeh* [2] introduced fuzzy set theory, which assigns a degree of membership between 0 and 1 to each element in a set, rather than using crisp inclusion or exclusion. This concept enabled researchers to mathematically model imprecise information. Later, *Rosenfeld* [3] extended the concept of fuzzy sets to graph theory, introducing fuzzy graphs, where both vertices and edges have associated membership values that quantify their degrees of presence. This integration of fuzziness into graph structures allows a more realistic representation of uncertain systems.

Fuzzy graphs have since become a major research area in soft computing, with applications in areas such as social network analysis [4], transportation systems [5], Road Networks, Image Shrinking [6,7,8,9], chemical graph theory [10,11,12,13,14,15,16,17], and decision-making [18]. Their ability to handle imprecision and vagueness makes them ideal for analyzing real-world problems where relationships between entities are not strictly binary but rather exist in varying degrees of uncertainty.

2. Topological Indices and Their Importance

In chemical graph theory, a topological index is a numerical descriptor that characterizes the molecular structure of a compound. Each molecule can be represented as a molecular graph, where atoms correspond to vertices and chemical bonds correspond to edges by *Trinajstić* [19]. Topological indices such as the Wiener index, Zagreb index, and Randic index quantify aspects of molecular connectivity, branching, and structure. These indices have been widely used in quantitative structure–property relationship (QSPR) and quantitative structure–activity relationship (QSAR) studies to predict molecular properties like boiling point, stability, or biological activity.

The Wiener index, one of the earliest and most fundamental topological indices, was proposed by *Wiener* [20] for modeling paraffin boiling points. Subsequently, the Zagreb indices were introduced by *Gutman & Trinajstić* [21] to describe molecular branching, followed by many other indices such as the Randic index, Schultz index, Harary index, and Hyper-Wiener index. These indices, initially developed for crisp graphs, were later extended to fuzzy environments to account for uncertainty in molecular structures and relational systems.

2.1 Fuzzy Topological Indices

In recent years, researchers have developed and analyzed numerous fuzzy versions of classical topological indices to model systems where vertex and edge relations are uncertain. For instance, *Islam & Pal* [10,11,12,13,14,15,16,17] have contributed extensively to defining fuzzy versions of the Wiener, Zagreb, and Hyper-Zagreb indices, with applications in molecular chemistry, engineering systems, and crime analysis. *Anwar et al.* [22,23] and *Azeem et al.* [24] explored topological numbers of fuzzy soft graphs and demonstrated their applications in global trade and information systems. The works of *Mufti et al.* [25,26,27,28] and *Hasani & Ghods* [29,30,31] have extended fuzzy topological analysis to hydrocarbons and other complex molecular systems, establishing correlations between graph parameters and molecular properties.

These indices are not only limited to chemical graphs but also find applications in social network optimization, transportation flow, neural network modeling, and medical diagnosis. For example, *Zamri et al.* [32] applied fuzzy topological indices to improve neural network modeling, while *Rao et al.* [33] and *Anwar et al.* [34] employed neutrosophic fuzzy indices for multi-attribute decision-making in trade and education sectors, highlighting the versatility and interdisciplinary nature of fuzzy topological frameworks.

2.2 Scope and Motivation

Despite significant progress, the study of topological indices in fuzzy graphs remains a growing research domain. There is ongoing interest in developing new fuzzy indices, establishing relationships between them, and extending them to complex fuzzy graph variants such as bipolar fuzzy graphs, m-polar fuzzy graphs, and neutrosophic fuzzy graphs. The motivation for this review arises from the need to consolidate the existing literature on fuzzy topological indices, highlight their applications across various domains, and identify potential directions for future research.

This paper provides a comprehensive overview of the most relevant works on fuzzy topological indices, including definitions, developments, and diverse applications in recent years (2020–2025). It also emphasizes the interdisciplinary importance of these indices in modeling real-world uncertainty, providing a bridge between mathematical theory and practical decision-making systems.

3. Theoretical Background:

Topological indices are numerical descriptors that characterize the structure of a graph based on its topology rather than its geometric form. When the concept of fuzziness introduced by *Zadeh* [2] is integrated with graph theory, the resulting fuzzy graphs allow representation of systems where relationships are uncertain or partially defined. A fuzzy graph $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E}, \boldsymbol{\sigma}, \boldsymbol{\mu})$ consists of a vertex set $\boldsymbol{V}$, an edge set $\boldsymbol{E}$, and membership functions $\boldsymbol{\sigma}$ and $\boldsymbol{\mu}$ defining the degree of association between vertices and edges, respectively. The topological indices of such graphs are derived from vertex degrees and pairwise distances but adjusted for fuzziness using membership values. These indices play a vital role in quantifying the strength, connectivity, and complexity of fuzzy systems, enabling applications in chemistry, decision analysis, social networks, and engineering design.

Mathematically, these indices are classified into:

- Degree-based indices, which depend on vertex connectivity (e.g., fuzzy Zagreb, Randic, and Sombor indices).
- Distance-based indices, which depend on path length and vertex separation (e.g., fuzzy Wiener, Hyper-Wiener, and Harary indices).
- Hybrid indices, combining both degree and distance measures for more comprehensive modeling.

This theoretical base establishes the framework for reviewing recent developments from 2020 onward.

The study of topological indices in fuzzy graphs began to gain structured attention with the pioneering contribution of *Kalathian et al.* [35], who introduced several fundamental fuzzy topological indices, providing a mathematical foundation for analyzing uncertain graph structures. Their work established how classical graph invariants such as Zagreb, Randic, Wiener, and degree-based indices could be redefined within the framework of fuzzy sets, allowing quantitative assessment of uncertain relationships among vertices and edges.

3.1 Developments from 2020–2023

The study of **topological indices in fuzzy graphs** began with the seminal work of *Kalathian et al.* [35], who introduced fuzzy analogues of classical indices such as the Zagreb, Wiener, and Randic indices. Their research provided the first formal framework for computing these measures in fuzzy environments and analyzed their mathematical behavior across different graph structures. This pioneering work laid the groundwork for generalizing deterministic topological measures to uncertain domains.

Following this foundational contribution, several researchers extended fuzzy topological indices and explored their practical applications across diverse systems: *Pal, Samanta & Ghorai* [36] presented foundational theories and recent developments in fuzzy graph models, offering a comprehensive overview of fuzzy graph concepts and their modern applications. Fang et al. [37] analyzed connectivity and Wiener index of fuzzy incidence graphs for measuring communication efficiency. *Mufti et al.* [38] computed first and second fuzzy Zagreb indices for linear and multiacyclic hydrocarbons, relating fuzzy degree-based measures to molecular structural properties. *Naeem, Jamil & Fahd* [39] investigated the Wiener index for intuitionistic fuzzy graphs, emphasizing its application in transportation network optimization and distance-based modeling.

During 2023, research activity in fuzzy topological indices intensified, focusing on theoretical extensions and real-world implementations: *Salehi Amiri* [40, 41] explored mathematical links between graph polynomials and the Wiener index in fuzzy graphs, highlighting their significance in structural and chemical analyses. *Tabraiz et al.* [42, 43] performed fuzzy computational analyses of flower graphs, characterizing their structural and connectivity properties using fuzzy topological measures. *Ahmad, Khan & Saeid* [44] introduced fuzzy topological indices to model and analyze cybercrime networks, demonstrating their effectiveness in real-world systems. *Islam & Pal* [45] contributed multiple new indices such as the second Zagreb index, F-index, edge F-index, and hyper-connectivity index and applied them to diverse areas including molecular chemistry, engineering systems, and crime pattern detection in transportation networks.

Arif et al. [46] proposed new indices for picture fuzzy graphs, enhancing the representation of uncertainty in complex systems. *Fang et al.* [47] developed a spherical fuzzy Zagreb energy approach for multi-attribute group decision-making, integrating fuzzy degree-based measures with decision theory. *Guan et al.* [48] examined fuzzy topological invariants in uniform fuzzy graphs, revealing their stability and robustness under uncertainty. *Mufti et al.* [49] analyzed pizza graphs and qC_n graphs using fuzzy topological indices to study connectivity and network parameters. *Kausar, Abughazalah & Yaqoob* [50] introduced multi-polar q-rung orthopair fuzzy graphs and defined corresponding topological indices, enhancing the modeling capabilities for decision-making under uncertainty. *Ahmad & Nawaz* [51] applied the Wiener index of directed rough fuzzy graphs to analyze and prevent human trafficking networks. *Ahmad et al.* [52] defined a connectivity index for directed rough fuzzy graphs, optimizing traffic flow systems.

Overall, the period 2020–2023 represents the formative phase in the development of fuzzy topological indices. Researchers not only established the theoretical and computational framework but also demonstrated the practical relevance of these indices across multiple domains, including chemical graph theory, decision science, transportation systems, and network modeling.

3.2 Advancements from 2024–2025

In recent years, research on fuzzy topological indices has advanced considerably, focusing on generalized models, multi-dimensional uncertainty, and application-oriented frameworks. Scholars have increasingly combined fuzzy graph theory with modern computational and decision-making tools such as machine learning, neutrosophic logic, and multi-criteria analysis, marking a significant evolution from foundational studies to interdisciplinary integration.

Eryaşar & Sözen [53] examined fuzzy Zagreb indices and adjacency matrices of fuzzy zero-divisor graphs, employing MATLAB-based algebraic computations to model structural relationships. *Chitre et al.* [54] integrated machine learning with computational mathematics, linking data-driven prediction with theoretical graph modelling. *Zamri et al.* [55] proposed linear Diophantine fuzzy topological numbers to model the spread of communicable diseases. *Poulik et al.* [56] evaluated Randic indices under fuzzy information to analyze crossroads order and network complexity. *Narasimman et al.* [57] applied score topological indices in single-valued neutrosophic graphs to identify factors affecting student academic performance. *Anwar et al.* [58] utilized intuitionistic Sombor indices to optimize school placement, integrating educational network design with brand positioning. *Imran et al.* [59] defined novel Sombor variants on intuitionistic fuzzy graphs to enhance internet routing models. *Mirza et al.* [60] used machine learning to generalize energy-based and topological indices in fuzzy conjugate graphs of dihedral groups. *Liaqat, Mufti & Shang* [61] introduced a fuzzy Misbalance Prodeg index for multi-criteria decision-making, capturing imbalance and performance variations. *Imran et al.* [62] proposed single-valued neutrosophic fuzzy graph parameters to improve topological characterization under uncertainty.

Rana et al. [63] examined cubic power graphs of dihedral groups to reveal their algebraic and combinatorial features. *Vetrivel et al.* [64] introduced the Forgotten Topological Index for neutrosophic graphs, studying its mathematical behavior. *Alqahtani, Kaviyarasu & Rajeshwari* [65] employed the Sombor index in neutrosophic graphs for thermal power plant site selection through structural evaluation. *Kaviyarasu et al.* [66] explored connectivity indices in neutrosophic graphs for transport and computer network analysis. *Kosari et al.* [67] reviewed topological indices in fuzzy graphs, linking them to decision-making problems under uncertainty. *Abdullah et al.* [68] discussed the forgotten index in bipolar fuzzy graphs, applying it to real-world evaluation problems. *Sarala & Abirami* [69] analyzed topological indices in fuzzy random graphs, addressing randomness in uncertain networks. *Some et al.* [70] examined bounds and implications of the Sombor index in fuzzy graphs, proving its analytical efficiency. *Imran et al.* [71] developed neutrosophic fuzzy topological parameters for capturing complex system behaviors. *Jana & Ghorai* [72] introduced the Gutman index for fuzzy graphs, demonstrating applications in social networks. *Shi et al.* [73] proposed a connectivity index for cubic fuzzy graphs, identifying tsunami threat zones. *Guan et al.* [74] explored a connectivity index in cubic fuzzy graphs, initially applied to disaster mapping.

In 2025, research further diversified across fuzzy, neutrosophic, and hypersoft frameworks. *Mufti et al.* [75] applied machine learning to fuzzy and crisp graph analyses for improved prediction and classification. *Jamil et al.* [76] employed intuitionistic fuzzy Sombor indices to optimize vaccination center management, while *Shahzaib, Salman & Rehman* [77] studied degree-based fuzzy indices in snake graphs. *Some & Pal* [78] compared spherical fuzzy energy indices to enhance decision-making efficiency. *Ahmed et al.* [79] proposed a fuzzy hub domination model to optimize network connectivity in intelligent systems. *Arif et al.* [80] formulated new Sombor index variants for fuzzy graphs to improve network optimization.Other notable contributions include *Liaqat, Mufti & Shang* [81], who analyzed tadpole graphs under fuzzy conditions. *Nair & Sunitha* [82,83], who introduced strong domination and incidence domination indices for evaluating vertex influence and control. *Ftekhan & Aubad* [84] studied topological indices of resize graphs ($G_2(3)$) using algebraic graph theory.

Mondal & Ghorai [85] used the Wiener index of inverse fuzzy mixed graphs to improve education system modeling. *Mallinath et al.* [86] studied edge-based Zagreb indices in fuzzy graphs, useful for evaluating molecular and fuzzy network stability. *Yao et al.* [87] proposed a hierarchical fuzzy topological system for solving high-dimensional regression problems. *Ahmad et al.* [88] computed Wiener energies in fuzzy graphs, extending classical molecular energy interpretations. *Some & Pal* [89] explored the Sombor index in fuzzy graphs, highlighting its importance in structural and relational network analysis. *Shaik & Shaik* [90] applied the Wiener index in intuitionistic fuzzy rough graphs for optimizing transport network flows. *Jana & Ghorai* [91] proposed an inverse sum indeg index for fuzzy graphs to aid in cancer treatment planning optimization. *AL-omeri et al.* [92] examined Fermatean neutrosophic fuzzy graphs through the Winner index, enhancing election data analysis. *Gambhire et al.* [93] developed the Sombor index for neutrosophic graphs, enhancing uncertainty modeling in decision systems.

Additionally, *Abdullah et al.* [94] explored topological indices in quantum graphs, integrating fuzzy and quantum analysis for group decision-making. *Narasimman et al.* [95] analyzed single-valued neutrosophic graphs for applications in information science. *Eryaşar & Sözen* [96] proposed a fuzzy $\mathcal{F}$-index for fuzzy zero-divisor graphs, demonstrating its computational potential. *Sadeghi, Talebi & Ramezani* [97] introduced temperature-based fuzzy indices applied to QSPR studies of autism-related drugs. *Vetrivel et al.* [98] investigated fuzzy topological indices in robotics and automation, while *Priyadharsini & Kiruthica* [99] proposed the Yemen index for fuzzy graphs and discussed its behavior under various graph operations. *Senbagamalar & Gomathi* [100] developed optimum topological indices for intuitionistic fuzzy graphs using machine learning and MCDM, and Some, *Mondal & Pal* [101] applied neutrosophic fuzzy Zagreb and energy indices to analyze human trafficking and cancer treatment systems. *Elluru & Patil* [102] studied Zagreb indices for generalized fuzzy graph transformations, offering insights into algebraic properties of transformed networks.

Narasimman et al. [103] introduced score-based topological indices in single-valued neutrosophic graphs to assess relationships among factors influencing student academic performance, they demonstrated how these degree- and distance-related indices help identify key determinants impacting examination outcomes in educational networks. *Al Khabyah et al.* [104] proposed topological descriptors within a uniform interval-valued intuitionistic fuzzy framework to model uncertainty in network structures, these degree- and distance-based indices were applied in neural network optimization, enhancing learning accuracy and decision efficiency under fuzzy conditions.

Overall, the period 2024–2025 marked a paradigm shift from constructing individual fuzzy indices to exploring interdisciplinary and intelligent applications. Researchers expanded fuzzy topological theory into cybersecurity, economics, environmental systems, and healthcare, and explored extensions involving neutrosophic, bipolar, and m-polar fuzzy frameworks demonstrating the increasing complexity, flexibility, and practical relevance of fuzzy topological indices in modeling real-world uncertainties.

Conclusion:

Graph theory provides a powerful way to represent and analyze systems in which elements are connected through relationships, and it has proven applications in diverse areas such as communication networks, VLSI circuit design, transport systems, and more. However, in real-world systems, relationships are not always clear-cut or absolute; there is often uncertainty, vagueness, or partial association between elements. Fuzzy set theory addresses this challenge by introducing the concept of membership values, which quantify the degree to which an element belongs to a set. When fuzzy sets are applied to graph theory, we get fuzzy graphs, where both vertices and edges can have degrees of presence or connectivity rather than simply being present or absent. This makes fuzzy graphs highly suitable for modeling complex and uncertain systems more realistically. In the field of chemistry, the structure of a molecule can be represented as a molecular graph, where atoms are vertices and chemical bonds are edges, and numerical measures called topological indices are used to capture structural information.

Extending these indices to fuzzy graphs allows chemists and researchers to account for uncertainties in molecular interactions and bond strengths. Furthermore, fuzzy topological indices such as Wiener, Modified Wiener, Hyper-Wiener, Schultz, Gutman, Zagreb, Hyper-Zagreb, Harmonic, and Randic indices have demonstrated usefulness not only in chemistry but also in decision-making processes, network analysis, and optimization problems in engineering and computational sciences. By reviewing these indices and their applications, it becomes clear that fuzzy graphs and their topological measures offer a versatile and powerful tool for studying systems where uncertainty plays a significant role. This work emphasizes the importance of continuing research in this area, including developing new fuzzy topological indices, improving computational methods for their calculation, and applying them to practical problems in science, engineering, and technology, ultimately bridging the gap between theoretical graph concepts and real-world uncertain systems.

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