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ACCURATE CERTIFIED DOMINATION POLYNOMIALS OF GRAPHS

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Abstract: Let $G = (V, E)$ be a non-trivial graph. An accurate dominating set D' is an accurate certified dominating set, if D' has either zero or atleast two neighbours in $V - D'$. $\gamma_{acer}(G)$ is the minimum cardinality of an accurate certified dominating set and it is called an accurate certified domination number of G. The accurate certified domination polynomial of a graph G of order n is the polynomial $D_{acer}(G, x) = \sum_{i=\gamma_{acer}(G)}^{n} d_{acer}(G, i)x^i$, where $d_{acer}(G, i)$ is the number of accurate certified dominating sets of G of size i. In this paper, we determined the accurate certified domination polynomial of path, star, cycle, helm and complete graph.

Keywords: Accurate certified domination polynomial, domination number, accurate domination number, accurate certified domination number.

1. Introduction

By a graph $G = (V, E)$ we mean a finite undirected simple graph. The order and size of G are denoted by n and m respectively. For graph theoretical terms, we refer Harary [3] and for terms related to domination we refer to Haynes [6]. A subset D of V is said to be a dominating set in G if every vertex in $V - D$ is adjacent to atleast one vertex in D. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set in G. Berge and Ore [1,8] formulated the concept of domination in graphs. It was further extended to define many other domination related parameters in graphs. A dominating set D of a graph $G = (V, E)$ is an accurate dominating set, if $V - D$ has no dominating set of cardinality $|D|$ [7]. An accurate dominating set D' is an accurate certified dominating set, if D' has either zero or atleast two neighbours in $V - D'$. $\gamma_{acer}(G)$ is the minimum cardinality of an accurate certified dominating set and it is called an accurate certified domination number of G [2]. Domination polynomial of graph was introduced by Saeid Alikhani and Yee-hock Peng [5]. The accurate certified domination polynomial of a graph G of order n is the polynomial $D_{acer}(G, x) = \sum_{i=\gamma_{acer}(G)}^{n} d_{acer}(G, i)x^i$, where $d_{acer}(G, i)$ is the number of accurate certified dominating sets of G of size i. In this paper, we determined the accurate certified domination polynomial of path, star, cycle, helm and complete graph.

We recall the following results for future study.

Theorem 1.1: [2] For any graph $G, 1 \leq \gamma_{acer}(G) \leq n$ and the bound is sharp.

Result 1.2: [2] (i) For any path graph of order n, $\gamma_{acer}(P_n) = \begin{cases} \frac{n}{3} &, \ n \equiv 0(mod\ 3) \\ \frac{n+2}{3}, & n \equiv 1(mod\ 3) \\ \frac{n+1}{3}, & n \equiv 2(mod\ 3) \\ n & , n = 2,4. \end{cases}$

(ii) For any cycle of order $n \geq 3$, $\gamma_{acer}(C_n) = n$.

(iii) For any star graph of order $n \geq 2, \gamma_{acer}(K_{1,n}) = 1$.

(iv) For any complete graph of order $n \geq 3$, $\gamma_{acer}(K_n) = \begin{cases} n &, \ if\ n < 5 \\ \left\lfloor \frac{n}{2} \right\rfloor + 1, & if\ n \geq 5. \end{cases}$

Theorem 1.3: [2] If $G = H \circ K_1$, where H is any non-trivial connected graph then $\gamma_{acer}(G) = n$.

Theorem 1.4: [4] There does not exist a graph G with $\gamma_{acer}(G) = n - 1$.

2. Main Results

Definition 2.1:

The accurate certified domination polynomial of a graph G of order n is the polynomial $D_{acer}(G, x) = \sum_{i=\gamma_{acer}(G)}^{n} d_{acer}(G, i)x^i$, where $d_{acer}(G, i)$ is the number of accurate certified dominating sets of G of size i and $\gamma_{acer}(G)$ is the accurate certified domination number of G.

Example 2.2:

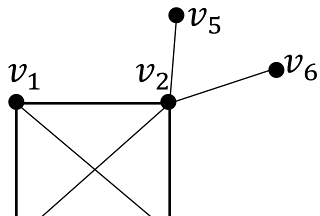

Figure 2.1

In figure 2.1, Clearly, $\gamma_{acer}(G) = 1$ and there are only one accurate certified dominating set of size 1 is $\{v_2\}$, three accurate certified dominating set of size 2, namely $\{v_1, v_2\}, \{v_2, v_3\}$ and $\{v_2, v_4\}$, there is no dominating set of size 3, four accurate certified dominating set of size 4, namely $\{v_1, v_2, v_3, v_4\}, \{v_1, v_2, v_5, v_6\}, \{v_2, v_3, v_5, v_6\}$ and $\{v_2, v_4, v_5, v_6\}$, there is no dominating set of size 5 and one accurate certified dominating set of size 6 is $\{v_1, v_2, v_3, v_4, v_5, v_6\}$. Hence, $D_{acer}(G) = x + 3x^2 + 0x^3 + 4x^4 + 0x^5 + x^6 = x + 3x^2 + 4x^4 + x^6$.

Theorem 2.3: Let G be a graph with $|V(G)| = n$. Then,

(i) If G is connected, then $d_{acer}(G, n) = 1$ and $d_{acer}(G, n-1) = 0$.

(ii) $d_{acer}(G, i) = 0$ if $i < \gamma_{acer}(G)$ or $i > n$.

(iii) $D_{acer}(G, x)$ has no constant term.

(iv) Let G be a graph and H be any induced subgraph of G. Then, deg $(D_{acer}(G, x)) \geq$ deg $(D_{acer}(H, x))$.

(v) Zero is a root of $D_{acer}(G, x)$ with multiplicity n.

Proof:

(i) Since G has n vertices, there is only one possible way to choose n vertices. Therefore, $d_{acer}(G, n) = 1$. By theorem 1.4, there is no dominating set of size $n - 1$ if order of G has n vertices. Hence, $d_{acer}(G, n-1) = 0$.

(ii) Trivially, this is true.

(iii) By theorem 1.1, $1 \leq \gamma_{acer}(G) \leq n$. Hence, there is no constant term.

(iv) Since, the number of vertices of induced subgraph H less than or equal to the number vertices of a graph G, deg $(D_{acer}(G, x)) \geq$ deg $(D_{acer}(H, x))$.

(v) By (iii), $D_{acer}(G, x)$ has no constant term. Also, $D_{acer}(G, x)$ is a polynomial of degree n. Therefore, Zero is a root of $D_{acer}(G, x)$ with multiplicity n.

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Result 2.4: If G and H are isomorphic graphs, then the accurate certified domination polynomial of G and the accurate certified domination polynomial of H are equal.

Example 2.5:

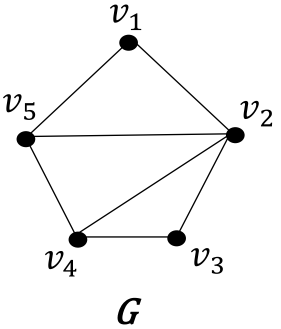

Figure 2.2

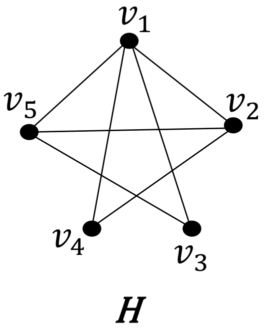

Figure 2.3

In figure 2.2 and 2.3, the graphs G and H are isomorphic. Here, $d_{acer}(G,1) = 1, d_{acer}(G,2) = 0, d_{acer}(G,3) = 0, d_{acer}(G,4) = 0, d_{acer}(G,5) = 1$. Therefore, $D_{acer}(G,x) = x + x^5$.

Also, $d_{acer}(H,1) = 1, d_{acer}(H,2) = 0, d_{acer}(H,3) = 0, d_{acer}(H,4) = 0, d_{acer}(H,5) = 1$. Therefore, $D_{acer}(H,x) = x + x^5$.

Hence, the accurate certified domination polynomial of two isomorphic graphs are equal.

Result 2.6: The converse part of the above result is need not be true. That is, if the accurate certified domination polynomials of two simple graphs G and H are equal, then G and H need not be isomorphic.

Example 2.7:

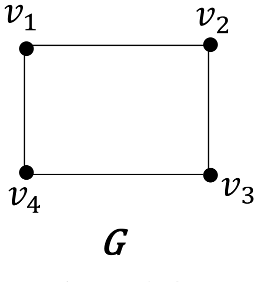

Figure 2.4

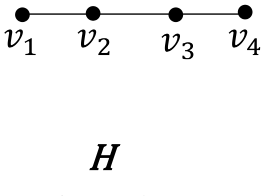

Figure 2.5

In figure 2.4 and 2.5, $D_{acer}(G) = x^4$ and in figure 5, $D_{acer}(H) = x^4$. Hence, $D_{acer}(G) = D_{acer}(H)$ but the graph G and H are not isomorphic.

Lemma 2.8: If $G = H \circ K_1$, where H is any disconnected graph then $\gamma_{acer}(G) = n$.

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Proof: Let n be the number of vertices in $G = H \circ K_1$. Let l be the set of all pendant vertices in G. Then $|l| = \frac{n}{2}$. By theorem, the minimum accurate dominating set of G containing all the support vertices and one pendant vertex. That is, $\gamma_a(G) = |V(H)| + 1$. Then, $\gamma_a(G) - 2$ vertices have one neighbour in $V - \gamma_a(G)$. Hence, $\gamma_{acer}(G) = |V(H)| + 1 + |l| - 1 = \frac{n}{2} + 1 + \frac{n}{2} - 1 = n$. Thus, $\gamma_{acer}(G) = n$.

Theorem 2.9: If $G = H \circ K_1$, where H is any graph then $D_{acer}(G) = x^n$.

Proof: By theorem 1.3 and lemma 2.8, $\gamma_{acer}(G) = n$ and $d_{acer}(G, n) = 1$. Therefore, $D_{acer}(G) = x^n$.

Theorem 2.10: For any path $P_n, n \geq 2, D_{acer}(P_n, x) =$

$$\begin{cases} x^n & if\ n = 2,4 \\ x^n+\binom{k-1}{0}x^k+\binom{k}{3}x^{k+1}+\cdots+\binom{\frac{3k-4}{2}}{\frac{3k-6}{2}}x^{\frac{3k-2}{2}},\ k\ is\ even & \multirow{2}{*}{$if\ n = 3k$} \\ x^n+\binom{k-1}{0}x^k+\binom{k}{3}x^{k+1}+\cdots+\binom{\left\lfloor\frac{3k-2}{2}\right\rfloor}{\left\lfloor\frac{3k-2}{2}\right\rfloor}x^{\left\lfloor\frac{3k-2}{2}\right\rfloor+1},\ k\ is\ odd & \\ x^n+\binom{k}{2}x^{k+1}+\binom{k+1}{5}x^{k+2}+\cdots+\binom{\frac{3k-2}{2}}{\frac{3k-2}{2}}x^{\frac{3k}{2}},\ k\ is\ even & if\ n = 3k+1 \\ x^n+\binom{k}{2}x^{k+1}+\binom{k+1}{5}x^{k+2}+\cdots+\binom{\left\lfloor\frac{3k-2}{2}\right\rfloor}{\left\lfloor\frac{3k-2}{2}\right\rfloor-1}x^{\left\lfloor\frac{3k-2}{2}\right\rfloor+1},\ k\ is\ odd\ \&\ k\neq 1 & \\ x^n+\binom{k}{1}x^{k+1}+\binom{k+1}{4}x^{k+2}+\cdots+\binom{\frac{3k-2}{2}}{\frac{3k-4}{2}}x^{\frac{3k}{2}},\ k\ is\ even\ \&\ k\neq 0 & if\ n = 3k+2 \\ x^n+\binom{k}{1}x^{k+1}+\binom{k+1}{4}x^{k+2}+\cdots+\binom{\left\lceil\frac{3k-2}{2}\right\rceil}{\left\lceil\frac{3k-2}{2}\right\rceil}x^{\left\lceil\frac{3k-2}{2}\right\rceil+1},\ k\ is\ odd & \end{cases}$$

Proof: Let P_n be a path graph with n vertices. We know that, by theorem 2.3 (i), $d_{acer}(P_n, n) = 1$.

Case 1: $n = 2{,}4$.

By result 1.2 (i), $\gamma_{acer}(P_n) = n$. Therefore, $D_{acer}(P_n, x) = x^n$.

Case 2: $n = 3k$.

Subcase 2.1: k is even.

The number of accurate certified dominating set of size k can be done in $\binom{k-1}{0}$ ways. An accurate certified dominating set of size $k + 1$ is obtained by $\binom{k}{3}$ ways. By a similar way, we can obtain the value $d_{acer}(P_n, k+2) = \binom{k+1}{6}$ ways, $d_{acer}(P_n, k+3) = \binom{k+2}{9}$ ways, …, $d_{acer}\left(P_n, \frac{3k-2}{2}\right) = \binom{\frac{3k-4}{2}}{\frac{3k-6}{2}}$ ways.

Hence,

$$D_{acer}(P_n, x) = d_{acer}(P_n, n)\, x^n + d_{acer}(P_n, k)\, x^k + d_{acer}(P_n, k+1)\, x^{k+1} + \cdots + d_{acer}\left(P_n, \frac{3k-2}{2}\right) x^{\frac{3k-2}{2}}$$

$$= x^n + \binom{k-1}{0} x^k + \binom{k}{3} x^{k+1} + \cdots + \binom{\frac{3k-4}{2}}{\frac{3k-6}{2}} x^{\frac{3k-2}{2}}.$$

Subcase 2.2: k is odd.

To find the number of accurate certified dominating set of size k is obtained by $\binom{k-1}{0}$ ways. An accurate certified dominating set of size $k+1$ can be done in $\binom{k}{3}$ ways. By a similar way, we can obtain the value $d_{acer}(P_n, k+2) = \binom{k+1}{6}$ ways, $d_{acer}(P_n, k+3) = \binom{k+2}{9}$ ways, …, $d_{acer}\left(P_n, \left\lfloor\frac{3k-2}{2}\right\rfloor + 1\right) = \binom{\left\lfloor\frac{3k-2}{2}\right\rfloor}{\left\lfloor\frac{3k-2}{2}\right\rfloor}$ ways.

Hence,

$$D_{acer}(P_n, x) = d_{acer}(P_n, n)\, x^n + d_{acer}(P_n, k)\, x^k + d_{acer}(P_n, k+1)\, x^{k+1} + \cdots + d_{acer}\left(P_n, \left\lfloor\frac{3k-2}{2}\right\rfloor + 1\right) x^{\left\lfloor\frac{3k-2}{2}\right\rfloor+1}$$

$$= x^n + \binom{k-1}{0} x^k + \binom{k}{3} x^{k+1} + \cdots + \binom{\left\lfloor\frac{3k-2}{2}\right\rfloor}{\left\lfloor\frac{3k-2}{2}\right\rfloor} x^{\left\lfloor\frac{3k-2}{2}\right\rfloor+1}.$$

Case 3: $n = 3k+1$.

Subcase 3.1: k is even.

To find the number of accurate certified dominating set of size $k+1$ is obtained by $\binom{k}{2}$ ways. An accurate certified dominating set of size $k+2$ is obtained by $\binom{k+1}{5}$ ways. By a similar way, we can obtain the value $d_{acer}(P_n, k+3) = \binom{k+2}{8}$ ways, $d_{acer}(P_n, k+4) = \binom{k+3}{11}$ ways, …, $d_{acer}\left(P_n, \frac{3k}{2}\right) = \binom{\frac{3k-2}{2}}{\frac{3k-2}{2}}$.

Hence,

$$D_{acer}(P_n, x) = d_{acer}(P_n, n)\, x^n + d_{acer}(P_n, k+1)\, x^{k+1} + d_{acer}(P_n, k+2)\, x^{k+2} + \dots + d_{acer}\left(P_n, \frac{3k}{2}\right) x^{\frac{3k}{2}}$$

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$$= x^n + \binom{k}{2} x^{k+1} + \binom{k+1}{5} x^{k+2} + \cdots + \binom{\frac{3k-2}{2}}{\frac{3k-2}{2}} x^{\frac{3k}{2}}.$$

Subcase 3.2: k is odd and $k \neq 1$.

To find the number of accurate certified dominating set of size $k+1$ is obtained by $\binom{k}{2}$ ways. An accurate certified dominating set of size $k+2$ can be done in $\binom{k+1}{5}$ ways. By a similar way, we can obtain the value $d_{acer}(P_n, k+3) = \binom{k+2}{8}$ ways, $d_{acer}(P_n, k+4) = \binom{k+3}{11}$ ways, …, $d_{acer}\left(P_n, \left\lfloor\frac{3k-2}{2}\right\rfloor + 1\right) = \binom{\left\lfloor\frac{3k-2}{2}\right\rfloor}{\left\lfloor\frac{3k-2}{2}\right\rfloor - 1}$ ways.

Hence, $D_{acer}(P_n, x) = d_{acer}(P_n, n)\, x^n + d_{acer}(P_n, k+1)\, x^{k+1} + d_{acer}(P_n, k+2)\, x^{k+2} +$

$$\ldots + d_{acer}\left(P_n, \left\lfloor\frac{3k-2}{2}\right\rfloor + 1\right) x^{\left\lfloor\frac{3k-2}{2}\right\rfloor + 1}$$

$$= x^n + \binom{k}{2} x^{k+1} + \binom{k+1}{5} x^{k+2} + \cdots + \binom{\left\lfloor\frac{3k-2}{2}\right\rfloor}{\left\lfloor\frac{3k-2}{2}\right\rfloor - 1} x^{\left\lfloor\frac{3k-2}{2}\right\rfloor + 1}.$$

Case 4: $n = 3k + 2$.

Subcase 4.1: k is even and $k \neq 0$.

To find the number of accurate certified dominating set of size $k+1$ is obtained by $\binom{k}{1}$ ways. An accurate certified dominating set of size $k+2$ is obtained by $\binom{k+1}{4}$ ways. By a similar way, we can obtain the value $d_{acer}(P_n, k+3) = \binom{k+2}{7}$ ways, $d_{acer}(P_n, k+4) = \binom{k+3}{10}$ ways, …, $d_{acer}\left(P_n, \frac{3k}{2}\right) = \binom{\frac{3k-2}{2}}{\frac{3k-4}{2}}$ ways.

Hence, $D_{acer}(P_n, x) = d_{acer}(P_n, n)\, x^n + d_{acer}(P_n, k+1)\, x^{k+1} + d_{acer}(P_n, k+2)\, x^{k+2} +$

$$\ldots + d_{acer}\left(P_n, \frac{3k}{2}\right) x^{\frac{3k}{2}}$$

$$= x^n + \binom{k}{1} x^{k+1} + \binom{k+1}{4} x^{k+2} + \cdots + \binom{\frac{3k-2}{2}}{\frac{3k-4}{2}} x^{\frac{3k}{2}}.$$

Subcase 4.2: k is odd.

To find the number of accurate certified dominating set of size $k+1$ is obtained by $\binom{k}{1}$ ways. An accurate certified dominating set of size $k+2$ can be done in $\binom{k+1}{4}$ ways. By a similar way, we can obtain the value $d_{acer}(P_n, k+3) = \binom{k+2}{7}$ ways, $d_{acer}(P_n, k+4) = \binom{k+3}{10}$ ways, …, $d_{acer}\left(P_n, \left\lceil\frac{3k-2}{2}\right\rceil + 1\right) = \binom{\left\lceil\frac{3k-2}{2}\right\rceil}{\left\lceil\frac{3k-2}{2}\right\rceil}$.

Hence, $D_{acer}(P_n, x) = d_{acer}(P_n, n)\, x^n + d_{acer}(P_n, k+1)\, x^{k+1} + d_{acer}(P_n, k+2)\, x^{k+2} +$

$$\ldots + d_{acer}\left(P_n,\ \left\lceil\frac{3k-2}{2}\right\rceil + 1\right)\, x^{\left\lceil\frac{3k-2}{2}\right\rceil+1}$$

$$= x^n + \binom{k}{1} x^{k+1} + \binom{k+1}{4} x^{k+2} + \cdots + \binom{\left\lceil\frac{3k-2}{2}\right\rceil}{\left\lceil\frac{3k-2}{2}\right\rceil} x^{\left\lfloor\frac{3k-2}{2}\right\rfloor+1}.$$

Table 1: $d(P_n, i)$ The number of dominating set of P_n with cardinality i.

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
n															
1	1														
2	0	1													
3	1	0	1												
4	0	0	0	1											
5	0	1	0	0	1										
6	0	1	0	0	0	1									
7	0	0	1	0	0	0	1								
8	0	0	2	0	0	0	0	1							
9	0	0	1	1	0	0	0	0	1						
10	0	0	0	3	0	0	0	0	0	1					
11	0	0	0	3	1	0	0	0	0	0	1				
12	0	0	0	1	4	0	0	0	0	0	0	1			
13	0	0	0	0	6	1	0	0	0	0	0	0	1		
14	0	0	0	0	4	5	0	0	0	0	0	0	0	1	
15	0	0	0	0	1	10	1	0	0	0	0	0	0	0	1

Theorem 2.11: For any cycle C_n of order n, $D_{acer}(C_n, x) = x^n$.

Proof: Let C_n be a cycle with n vertices. By result 1.2 (ii), $\gamma_{acer}(C_n) = n$. Therefore, by theorem 2.3 (i), $d_{acer}(C_n, n) = 1$. Hence, $D_{acer}(C_n, x) = x^n$.

Theorem 2.12: For any helm H_p, $D_{acer}(H_p, x) = x^p + x^{2p+1}$.

Proof: In H_p, let u be the hub vertex, $v_1, v_2, \dots, v_p$ be the vertices of the outer cycle and $u_1, u_2, \dots, u_p$ be the pendant vertices of H_p. Therefore, the order of H_p is $n = 2p + 1$.

The minimum accurate certified dominating set of size p is obtained by selecting a outer cycle vertices $v_1, v_2, \dots, v_p$. This can be done in only one way. Hence, $d_{acer}(H_p, p) = 1$. Since, any accurate certified dominating set of size $p + 1, p + 2, \dots, 2p$ have one neighbour in $V - D''$, $d_{acer}(H_p, p + 1) = d_{acer}(H_p, p + 2) = \cdots = d_{acer}(H_p, 2p) = 0$. An accurate certified dominating set of size $2p + 1$ is one. That is, $d_{acer}(H_p, 2p + 1) = 1$. Hence, $D_{acer}(H_p, x) = x^p + x^{2p+1}$.

Theorem 2.13: For any star graph $K_{1,n}$, $D_{acer}(K_{1,n}, x) = \sum_{i=1}^{n+1} \binom{n}{i-1} x^i$ $\begin{cases} i \neq n & , \ if\ n\ is\ even \\ i \neq n, \frac{n+1}{2}, & if\ n\ is\ odd. \end{cases}$

Proof: Let v be the centre vertex and let $v_1, v_2, \dots, v_n$ be the end vertices of $K_{1,n}$. Then, $V(G) = \{v, v_i \ / \ 1 \le i \le n\}$ and $E(G) = \{vv_i \ / \ 1 \le i \le n\}$. By result 1.2 (iii), $\gamma_{acer}(K_{1,n}) = 1$. This can be done in only one way. That is, $d_{acer}(K_{1,n}, 1) = \binom{n}{0}$. To find the number of accurate certified dominating set with size 2, take the centre vertex v and any one vertex from $v_i, 1 \le i \le n$. This can be done in $\binom{n}{1}$ ways and hence $d_{acer}(K_{1,n}, 2) = \binom{n}{1}$. In a similar way, we can prove that $d_{acer}(K_{1,n}, 3) = \binom{n}{2}, \dots, d_{acer}(K_{1,n}, n - 1) = \binom{n}{n-2}$. By Theorem 2.3 (i), for any graph G of order n, $d_{acer}(G, n - 1) = 0$. Here, we have $n + 1$ vertices. Therefore, $d_{acer}(K_{1,n}, n) = 0$. An accurate certified dominating set of size $n + 1$ is one. That is, $d_{acer}(K_{1,n}, n + 1) = \binom{n}{n}$.

If n is odd and $i = \frac{n+1}{2}$, then, $|D| = \frac{n+1}{2}$. Also, $V - D$ has a dominating set of cardinality $|D|$, which is a contradiction to accurate certified dominating set. Therefore, $d_{acer}\left(K_{1,n}, \frac{n+1}{2}\right) = 0$.

Hence,

$$\begin{aligned} D_{acer}(K_{1,n}, x) &= d_{acer}(K_{1,n}, 1)\, x + d_{acer}(K_{1,n}, 2)\, x^2 + \cdots + d_{acer}(K_{1,n}, n - 1)\, x^{n-1} \\ &\quad + d_{acer}(K_{1,n}, n)\, x^n + d_{acer}(K_{1,n}, n + 1)\, x^{n+1} \\ &= \binom{n}{o} x + \binom{n}{1} x^2 + \cdots + \binom{n}{n-2} x^{n-1} + 0 + \binom{n}{n} x^{n+1} \\ &= \sum_{i=1}^{n+1} \binom{n}{i-1} x^i. \end{aligned}$$

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Therefore, $D_{acer}\left(K_{1,n},x\right)=\sum_{i=1}^{n+1}\binom{n}{i-1}x^{i}\begin{cases} i\neq n & , \ if\ n\ is\ even \\ i\neq n,\frac{n+1}{2}, & if\ n\ is\ odd. \end{cases}$

Theorem 2.14: For any complete graph $K_n, D_{acer}(K_n,x)=\begin{cases} x^n & , \quad n<5 \\ \sum_{i=\left\lfloor\frac{n}{2}\right\rfloor+1}^{n}\binom{n}{i}x^i-\binom{n}{n-1}x^{n-1} & , \quad n\geq 5. \end{cases}$

Proof: Let K_n be a complete graph with n vertices. By result 1.2 (iv), $\gamma_{acer}(K_n)=\begin{cases} n & , \quad n<5 \\ \left\lfloor\frac{n}{2}\right\rfloor+1, & n\geq 5. \end{cases}$

Case (i): $n<5$.

By theorem 2.3 (i), $d_{acer}(K_n,n)=1$. Therefore, $D_{acer}(K_n,x)=x^n$.

Case (ii): $n\geq 5$.

The minimum accurate certified dominating set of size $\left\lfloor\frac{n}{2}\right\rfloor+1$ can be done in $\binom{n}{\left\lfloor\frac{n}{2}\right\rfloor+1}$ ways. Hence, $d_{acer}\left(K_n,\left\lfloor\frac{n}{2}\right\rfloor+1\right)=\binom{n}{\left\lfloor\frac{n}{2}\right\rfloor+1}$. By a similar way, $d_{acer}\left(K_n,\left\lfloor\frac{n}{2}\right\rfloor+2\right)=\binom{n}{\left\lfloor\frac{n}{2}\right\rfloor+2},\ldots,d_{acer}(K_n,n-2)=\binom{n}{n-2}$. By Theorem 2.3 (i), $d_{acer}(K_n,n-1)=0$. Clearly, $d_{acer}(K_n,n)=\binom{n}{n}$. Hence, $D_{acer}(K_n,x)=d_{acer}\left(K_n,\left\lfloor\frac{n}{2}\right\rfloor+1\right)x^{\left\lfloor\frac{n}{2}\right\rfloor+1}+d_{acer}\left(K_n,\left\lfloor\frac{n}{2}\right\rfloor+2\right)x^{\left\lfloor\frac{n}{2}\right\rfloor+2}+\cdots+d_{acer}(K_n,n-2)x^{n-2}+d_{acer}(K_n,n-1)x^{n-1}+d_{acer}(K_n,n)x^n=\binom{n}{\left\lfloor\frac{n}{2}\right\rfloor+1}x^{\left\lfloor\frac{n}{2}\right\rfloor+1}+\binom{n}{\left\lfloor\frac{n}{2}\right\rfloor+2}x^{\left\lfloor\frac{n}{2}\right\rfloor+2}+\cdots+\binom{n}{n-2}x^{n-2}+0x^{n-1}+\binom{n}{n}x^n=\sum_{i=\left\lfloor\frac{n}{2}\right\rfloor+1}^{n}\binom{n}{i}x^i-\binom{n}{n-1}x^{n-1}$.

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