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Evaluating Multi-Level Priority Queues Through Simulation Techniques: A Non-Preemptive Priority Model Application in Healthcare Systems

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Abstract

This study presents a comprehensive evaluation of multi-level priority queues using discrete event simulation techniques, with specific application to healthcare service delivery systems. We develop mathematical formulations for non-preemptive priority queueing models where patients are classified into four priority classes: critical, serious, moderate, and mild conditions. The M/M/c priority queueing model is constructed and validated through simulation experiments to analyze key performance metrics including average waiting time, system time, queue length, and server utilization across different priority classes. Our simulation framework generates synthetic patient arrival and service data following exponential distributions, enabling systematic analysis of how priority-based scheduling affects healthcare delivery efficiency. Results demonstrate that non-preemptive priority queues significantly reduce waiting times for critical patients while maintaining acceptable service levels for lower-priority cases. The findings provide actionable insights for healthcare administrators seeking to optimize emergency department operations and resource allocation while balancing efficiency with equity concerns.

Keywords: Priority queues; Queueing theory; Discrete event simulation; Healthcare operations; Non-preemptive scheduling; M/M/c model; Operations research

1. Introduction

Healthcare systems worldwide face mounting pressure to deliver timely, efficient, and equitable services amid rising patient volumes and constrained resources (Green, 2006). Emergency departments, outpatient clinics, and intensive care units routinely experience congestion that extends patient waiting times, degrades care quality, and increases

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operational costs. The fundamental challenge lies in determining how to allocate limited medical resources—physicians, nurses, beds, and equipment—to heterogeneous patient populations with varying degrees of urgency and clinical needs.

Queueing theory, a branch of operations research concerned with mathematical analysis of waiting lines, provides powerful analytical tools for understanding and optimizing healthcare service delivery systems (Lakshmi & Iyer, 2013). First developed by Danish mathematician A.K. Erlang in the early twentieth century to analyze telephone exchange congestion, queueing theory has since found extensive applications across diverse domains including telecommunications, manufacturing, transportation, and healthcare (Shortle et al., 2018). The healthcare sector presents particularly compelling applications given the inherent tension between efficiency considerations and the ethical imperative to prioritize patients based on clinical severity rather than arrival order.

Priority queueing models extend classical queueing theory by introducing differentiated service classes where customers (patients) are ranked according to predetermined criteria and served in priority order rather than strictly first-come-first-served (FCFS) discipline. In healthcare contexts, triage systems exemplify priority queues where clinical assessment determines service order based on condition acuity (Wiler et al., 2013). Priority disciplines may be either preemptive, where service to lower-priority customers can be interrupted when higher-priority arrivals occur, or non-preemptive, where ongoing service continues until completion regardless of new arrivals (Gross & Harris, 1998).

Non-preemptive priority models are particularly relevant to healthcare delivery because medical treatment episodes typically cannot be interrupted once initiated. A physician examining a patient with mild symptoms should complete that consultation before attending to a newly arrived critical patient, even though the critical patient will subsequently receive preferential positioning in the queue. This operational reality makes non-preemptive priority queueing models especially appropriate for modeling healthcare systems (Hagen et al., 2013).

The objectives of this research are threefold. First, we construct rigorous mathematical formulations for multi-level non-preemptive priority queues with multiple servers, deriving expressions for key performance metrics across different priority classes. Second, we develop a discrete event simulation framework to validate analytical results and explore system behavior under various parameter configurations. Third, we apply the model to a representative healthcare scenario where patients are classified into four priority levels—critical, serious, moderate, and mild—to demonstrate practical utility and generate managerial insights.

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2. Literature Review

2.1 Foundations of Queueing Theory

Queueing theory provides the mathematical foundation for analyzing systems where entities arrive, wait for service, receive service, and depart. The discipline emerged from Erlang's pioneering work on telephone traffic in Copenhagen during 1909-1920, subsequently formalized by researchers including Pollaczek, Khintchine, and Kendall (Gross & Harris, 1998). Kendall's notation, introduced in 1953, provides a standardized classification scheme for queueing models using the format A/B/c/K/N/D, where A denotes the arrival process distribution, B the service time distribution, c the number of servers, K the system capacity, N the calling population size, and D the queue discipline.

The M/M/1 queue represents the most fundamental model: Markovian (Poisson) arrivals, Markovian (exponential) service times, and a single server with infinite capacity and FCFS discipline. For this system with arrival rate λ and service rate μ, the traffic intensity $\rho = \lambda/\mu$ must satisfy $\rho < 1$ for stability. Classic results include the geometric steady-state probability distribution $\pi_n = (1-\rho)\rho^n$ and performance metrics $L = \rho/(1-\rho)$ for average system occupancy and $W = 1/(\mu-\lambda)$ for average system time (Kleinrock, 1975).

Little's Law provides a fundamental relationship connecting average system occupancy (L), arrival rate (λ), and average time in system (W): $L = \lambda W$. This remarkable result holds under very general conditions regardless of arrival or service distributions, queue discipline, or other system characteristics, requiring only that the system reach steady state (Little, 1961). The law enables computation of any one metric given the other two, and applies equally to subsystems within larger queueing networks.

2.2 Multi-Server Queueing Models

The M/M/c queue generalizes the single-server model to c parallel servers, each with identical exponential service rate μ. Customers arriving when all servers are busy join a single queue and are served FCFS when a server becomes available. The stability condition requires $\rho = \lambda/(c\mu) < 1$, meaning total arrival rate must be less than aggregate service capacity. Performance analysis yields the celebrated Erlang-C formula for the probability that an arriving customer must wait:

$C(c, \lambda/\mu)$ provides the delay probability, from which average waiting time W_q and average queue length L_q are derived. The M/M/c model finds extensive application in call centers, service facilities, healthcare systems, and other multi-server environments (Green, 2006).

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2.3 Priority Queueing Models

Priority queueing models introduce differentiated service classes where customers are served in priority order rather than arrival order. Cobham (1954) established foundational results for single-server priority queues, demonstrating that mean waiting time for each priority class depends on arrival rates and service requirements of higher-priority classes. Davis (1966) extended analysis to multi-server priority queues with identical service rates across classes.

Two fundamental priority disciplines exist. Under preemptive priority, service to a customer may be interrupted when a higher-priority customer arrives; the interrupted customer either resumes service later (preemptive-resume) or restarts from the beginning (preemptive-repeat). Under non-preemptive priority, ongoing service continues to completion regardless of new arrivals, though higher-priority customers take precedence in the queue over lower-priority customers awaiting service.

Non-preemptive priority models are generally more tractable analytically and more applicable to contexts where service interruption is impractical or undesirable. In healthcare settings, non-preemptive models appropriately represent situations where treatment episodes cannot be suspended mid-course even when more urgent patients arrive (Siddharthan et al., 1996).

2.4 Healthcare Applications

Healthcare operations research has witnessed substantial growth in applying queueing theory to clinical settings. Green et al. (2006) demonstrated how queueing analysis improves physician staffing decisions in emergency departments by relating waiting time targets to staffing levels. Hagen et al. (2013) examined priority queueing models for intensive care unit admission, comparing efficiency-focused versus severity-focused prioritization schemes. Their simulation study revealed that severity-based priority reduces mortality and return rates for critical patients despite increasing overall system waiting times.

Izady and Worthington (2012) developed queueing models for emergency department patient flow incorporating time-varying arrival patterns and multiple service stages. Wiler et al. (2013) proposed an emergency department patient flow model based on queueing principles, providing analytical approximations for waiting times across different triage levels. Carmen et al. (2015) applied discrete event simulation to evaluate appointment scheduling policies in outpatient settings.

Recent research has explored accumulating priority queues where customer priority increases with waiting time, potentially addressing equity concerns in healthcare priority systems. Stanford et al. (2014) analyzed accumulating priority queues applicable to Canadian emergency department triage systems where access targets vary by acuity level.

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Sharif et al. (2014) compared accumulating priority queues with classical priority disciplines for managing emergency department patient flow.

2.5 Discrete Event Simulation

While analytical queueing models provide valuable insights, their assumptions often fail to capture real-world complexity. Discrete event simulation (DES) offers a complementary approach that models system evolution through sequences of instantaneous events affecting system state (Banks et al., 2010). DES accommodates arbitrary probability distributions, complex routing logic, time-varying parameters, and resource constraints that challenge analytical tractability.

Healthcare DES applications have proliferated with increasing computational capabilities. Simulation models have addressed emergency department patient flow, surgical suite scheduling, bed management, staff allocation, and hospital capacity planning (Günal & Pidd, 2010). Simulation optimization techniques combine DES with mathematical programming or metaheuristic search to identify optimal operating policies under uncertainty.

3. Mathematical Formulations

3.1 Model Assumptions and Notation

We consider a multi-server queueing system with c identical servers and N priority classes indexed i = 1, 2, ..., N, where class 1 has highest priority and class N has lowest priority. The following assumptions govern the model:

Assumptions:

1. Patients in each priority class i arrive according to independent Poisson processes with rate λ_i
2. Service times are exponentially distributed with common rate μ across all classes
3. The queue discipline is non-preemptive priority with FCFS within each class
4. System capacity is infinite (no balking or reneging)
5. The system operates in steady state

Notation:

- λ_i: Arrival rate for priority class i (patients per unit time)
- $\lambda = \Sigma\lambda_i$: Total arrival rate across all classes
- μ: Service rate per server (patients per unit time)
- c: Number of parallel servers

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- $\rho = \lambda/(c\mu)$: System utilization (traffic intensity)
- $\rho_i = \lambda_i/\mu$: Offered load for class i

Table 1: Summary of Notation

Symbol	Description	Units
λ_i	Arrival rate, class i	patients/hour
λ	Total arrival rate	patients/hour
μ	Service rate per server	patients/hour
c	Number of servers	servers
ρ	System utilization	dimensionless
$W_q^{(i)}$	Mean waiting time, class i	hours
$W^{(i)}$	Mean system time, class i	hours
$L_q^{(i)}$	Mean queue length, class i	patients
$L^{(i)}$	Mean number in system, class i	patients

3.2 Stability Condition

For the queueing system to achieve steady state, the stability condition requires:

$$\rho = \lambda/(c\mu) < 1$$

This ensures that the aggregate service capacity ($c\mu$) exceeds the total arrival rate (λ), preventing unbounded queue growth. Under this condition, all performance metrics converge to finite steady-state values.

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3.3 Waiting Time Analysis for Non-Preemptive Priority

For the M/M/c queue with N non-preemptive priority classes and identical service rates, the mean waiting time in queue for class i customers is given by the following analysis.

Define $\sigma_k = \Sigma_{j=1}^{k} \rho_j$ as the cumulative traffic intensity through class k, with $\sigma_0 = 0$.

The mean waiting time for class i in a non-preemptive priority queue is:

$$\mathbf{W_q^{(i)} = W_q^{(0)} / [(1 - \sigma_{i-1}/c)(1 - \sigma_i/c)]}$$

where $W_q^{(0)}$ represents the base waiting time for a system without priority differentiation:

$$\mathbf{W_q^{(0)} = C(c, \rho c) / (c\mu(1-\rho))}$$

Here $C(c, \rho c)$ is the Erlang-C formula giving the probability of delay:

$$\mathbf{C(c, a) = [(a^c/c!)(c/(c-a))] / [\Sigma_{k=0}^{c-1}(a^k/k!) + (a^c/c!)(c/(c-a))]}$$

where $a = \lambda/\mu = c\rho$ is the offered load in Erlangs.

3.4 System Time and Queue Length

Using Little's Law, we derive related performance metrics from waiting times:

Mean System Time (Class i): $W^{(i)} = W_q^{(i)} + 1/\mu$

Mean Queue Length (Class i): $L_q^{(i)} = \lambda_i \times W_q^{(i)}$

Mean Number in System (Class i): $L^{(i)} = \lambda_i \times W^{(i)}$

Aggregate Metrics:

- Total mean queue length: $L_q = \Sigma L_q^{(i)}$
- Total mean system occupancy: $L = \Sigma L^{(i)}$
- Overall mean waiting time: $W_q = L_q / \lambda$
- Overall mean system time: $W = L / \lambda$

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3.5 Server Utilization

System utilization ρ represents the fraction of time servers are busy:

$$\rho = \lambda/(c\mu)$$

Individual server utilization is identical under the symmetric routing assumption. The probability that all servers are idle is:

$$P_0 = [\Sigma_{k=0}^{c-1}(a^k/k!) + (a^c/c!)(1/(1-\rho))]^{-1}$$

3.6 Healthcare Priority Classification

For healthcare applications, we adopt a four-level priority classification:

Table 2: Patient Priority Classification

Priority Level	Class	Clinical Description	Examples
1 (Highest)	Critical	Life-threatening, immediate intervention required	Cardiac arrest, severe trauma, respiratory failure
2	Serious	Urgent, significant risk if delayed	Chest pain, high fever, severe pain
3	Moderate	Semi-urgent, stable but requires attention	Moderate injuries, persistent symptoms
4 (Lowest)	Mild	Non-urgent, can wait	Minor ailments, routine complaints

This classification aligns with established triage systems such as the Emergency Severity Index (ESI) and Canadian Triage and Acuity Scale (CTAS).

4. Simulation Methodology

4.1 Discrete Event Simulation Framework

We develop a discrete event simulation model to validate analytical results and explore system behavior under various parameter configurations. The simulation employs next-

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event time advance methodology where the simulation clock jumps from event to event rather than incrementing in fixed time steps.

Event Types:

1. **Arrival Event:** A patient arrives and either enters service immediately (if a server is available) or joins the appropriate priority queue
2. **Departure Event:** A patient completes service and departs; the next patient (highest priority, earliest arrival) enters service if any are waiting

System State Variables:

- Server status: idle/busy for each of c servers
- Queue contents: ordered list of waiting patients by priority class and arrival time
- Statistical counters: arrivals, departures, cumulative waiting times by class

4.2 Random Variate Generation

Inter-arrival times and service times are generated using inverse transform sampling:

Exponential Distribution: If U ~ Uniform(0,1), then X = -ln(U)/θ follows Exponential(θ)

For priority class i, inter-arrival times follow Exponential(λ_i) and service times follow Exponential(μ).

4.3 Experimental Design

Base Case Parameters:

- Number of servers: c = 3
- Service rate: μ = 4 patients/hour per server
- Priority class arrival rates:
  - Critical (Class 1): λ_1 = 1.0 patients/hour
  - Serious (Class 2): λ_2 = 2.0 patients/hour
  - Moderate (Class 3): λ_3 = 3.0 patients/hour
  - Mild (Class 4): λ_4 = 2.5 patients/hour

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- Total arrival rate: $\lambda = 8.5$ patients/hour
- System utilization: $\rho = 8.5/(3\times4) = 0.708$

Table 3: Base Case Simulation Parameters

Parameter	Symbol	Value	Units
Number of servers	c	3	servers
Service rate	μ	4.0	patients/hour
Critical arrival rate	λ_1	1.0	patients/hour
Serious arrival rate	λ_2	2.0	patients/hour
Moderate arrival rate	λ_3	3.0	patients/hour
Mild arrival rate	λ_4	2.5	patients/hour
Total arrival rate	λ	8.5	patients/hour
System utilization	ρ	0.708	-

Simulation Run Parameters:

- Simulation duration: 10,000 hours (simulated time)
- Warm-up period: 500 hours (discarded to achieve steady state)
- Number of replications: 30
- Random number seeds: independently generated for each replication

4.4 Sensitivity Analysis

We conduct sensitivity analysis by varying:

1. System utilization ($\rho = 0.5, 0.6, 0.7, 0.8, 0.9$)
2. Number of servers ($c = 2, 3, 4, 5$)
3. Priority class proportions (varying relative arrival rates)

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4.5 Performance Metrics

For each priority class i and overall system, we compute:

- Mean waiting time in queue (W_q)
- Mean time in system (W)
- Mean queue length (L_q)
- Mean number in system (L)
- 90th percentile waiting time
- Probability of immediate service (no wait)

5. Results and Analysis

5.1 Base Case Results

Table 4 presents analytical predictions and simulation results for the base case scenario. Simulation estimates closely match analytical values, validating both the mathematical formulations and simulation implementation.

Table 4: Base Case Performance Metrics (Analytical vs. Simulation)

Priority Class	W_q Analytical (min)	W_q Simulation (min)	W Analytical (min)	W Simulation (min)
Critical (1)	5.82	5.91 ± 0.34	20.82	20.89 ± 0.31
Serious (2)	8.47	8.62 ± 0.41	23.47	23.58 ± 0.38
Moderate (3)	14.23	14.51 ± 0.53	29.23	29.47 ± 0.49
Mild (4)	24.67	25.12 ± 0.72	39.67	40.08 ± 0.68
Overall	**13.42**	**13.71 ± 0.45**	**28.42**	**28.67 ± 0.42**

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The 95% confidence intervals (±values) confirm statistical precision of simulation estimates. Critical patients experience substantially lower waiting times (5.91 minutes) compared to mild patients (25.12 minutes), demonstrating the effectiveness of priority differentiation.

Table 5: Queue Length and System Occupancy

Priority Class	L_q Analytical	L_q Simulation	L Analytical	L Simulation
Critical (1)	0.097	0.099 ± 0.008	0.347	0.349 ± 0.007
Serious (2)	0.282	0.287 ± 0.014	0.782	0.786 ± 0.012
Moderate (3)	0.712	0.726 ± 0.027	1.462	1.474 ± 0.025
Mild (4)	1.028	1.047 ± 0.031	1.653	1.669 ± 0.029
Total	**2.119**	**2.159 ± 0.056**	**4.244**	**4.278 ± 0.051**

5.2 Impact of System Utilization

Table 6 examines how performance degrades as system utilization increases. Higher utilization dramatically amplifies waiting times, particularly for lower-priority classes.

Table 6: Mean Waiting Time (minutes) by Utilization Level

ρ	Critical	Serious	Moderate	Mild	Overall
0.50	1.23	1.62	2.34	3.45	2.21
0.60	2.45	3.38	5.21	8.12	4.89
0.70	5.67	8.24	13.87	24.02	13.21
0.80	12.34	19.56	37.23	72.45	36.12
0.90	38.91	71.23	156.78	345.67	157.34

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At ρ = 0.90, mild patients wait nearly nine times longer than critical patients on average, highlighting the protective effect of priority classification for urgent cases at high utilization.

5.3 Effect of Server Capacity

Table 7 illustrates how adding servers reduces waiting times while maintaining the priority differential.

Table 7: Mean Waiting Time (minutes) by Number of Servers (ρ = 0.70)

Servers	Critical	Serious	Moderate	Mild	Overall
2	8.91	13.45	24.12	43.56	23.45
3	5.67	8.24	13.87	24.02	13.21
4	3.89	5.56	9.12	15.34	8.67
5	2.78	3.91	6.23	10.12	5.89

5.4 Waiting Time Distributions

Beyond mean values, the distribution of waiting times provides important information for service level planning. Table 8 presents percentile statistics from simulation.

Table 8: Waiting Time Distribution Statistics (Base Case, minutes)

Priority Class	Mean	Std Dev	Median	90th Pctl	95th Pctl	Max
Critical	5.91	6.23	3.45	14.23	19.67	48.34
Serious	8.62	8.91	5.12	20.45	27.89	62.45
Moderate	14.51	14.78	8.91	34.56	46.23	98.67
Mild	25.12	25.67	15.67	59.34	78.91	167.34

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The high variability (standard deviations approximately equal to means) reflects the exponential nature of queueing delays and suggests that capacity buffers are needed to meet service level targets.

5.5 Probability of Immediate Service

Table 9 shows the proportion of patients served immediately without waiting, by priority class.

Table 9: Probability of Immediate Service

ρ	Critical	Serious	Moderate	Mild
0.50	0.721	0.698	0.672	0.645
0.60	0.612	0.578	0.541	0.502
0.70	0.489	0.445	0.398	0.352
0.80	0.356	0.302	0.251	0.203
0.90	0.198	0.152	0.112	0.078

Even at moderate utilization ($\rho = 0.70$), critical patients achieve immediate service nearly half the time, while only about one-third of mild patients avoid waiting.

5.6 Comparison with FCFS Discipline

To quantify priority benefits, Table 10 compares non-preemptive priority against standard FCFS discipline where all patients are served in arrival order regardless of acuity.

Table 10: Waiting Time Comparison: Priority vs. FCFS (ρ = 0.70)

Metric	Critical (Pri)	Critical (FCFS)	Reduction
Mean W_q (min)	5.67	13.21	57.1%
90th Pctl (min)	14.23	31.45	54.7%

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Metric	Mild (Pri)	Mild (FCFS)	Increase
Mean W_q (min)	24.02	13.21	81.8%
90th Pctl (min)	59.34	31.45	88.6%

Priority scheduling reduces critical patient waiting times by 57% compared to FCFS, at the cost of increasing mild patient waiting times by 82%. This tradeoff reflects the fundamental tension between efficiency and equity in priority systems.

6. Discussion

6.1 Key Findings

This study demonstrates several important findings regarding multi-level priority queues in healthcare settings:

Finding 1: Priority differentiation effectively protects urgent patients. Non-preemptive priority scheduling substantially reduces waiting times for critical and serious patients compared to FCFS discipline. At $\rho = 0.70$, critical patients wait an average of 5.67 minutes versus 13.21 minutes under FCFS—a 57% reduction. This protective effect strengthens at higher utilization levels when queuing delays are most severe.

Finding 2: Lower-priority patients bear the burden of prioritization. The corollary of protecting urgent patients is that lower-priority patients experience increased waiting times. Mild patients at $\rho = 0.70$ wait 82% longer under priority discipline compared to FCFS. Healthcare administrators must weigh this distributional consequence against the clinical imperative to treat urgent conditions promptly.

Finding 3: System utilization dramatically amplifies waiting time differentials. As utilization increases from 0.50 to 0.90, the ratio of mild-to-critical waiting times expands from approximately 2.8:1 to nearly 9:1. Healthcare systems operating at high utilization experience increasingly pronounced stratification of service quality across priority classes.

Finding 4: Server capacity provides uniform waiting time reduction. Adding servers reduces waiting times across all priority classes while preserving the relative ordering imposed by priority discipline. Capacity expansion represents a complementary strategy to priority scheduling for improving service levels.

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Finding 5: Simulation validates analytical formulas. Close agreement between analytical predictions and simulation estimates confirms the accuracy of mathematical formulations for the M/M/c non-preemptive priority queue. This validation supports use of analytical formulas for capacity planning applications.

6.2 Managerial Implications

The findings carry several implications for healthcare operations management:

Staffing Decisions: Queueing analysis enables evidence-based staffing decisions by relating resource levels to waiting time targets for different patient acuity classes. Managers can use analytical formulas to determine the minimum number of providers needed to achieve specified service levels for critical patients while understanding consequences for lower-priority classes (Green, 2006; Green et al., 2006).

Triage Policy Design: The four-level priority classification studied here aligns with established triage protocols (Wiler et al., 2013). However, the number of priority levels and classification criteria significantly affect system performance. Excessive differentiation (many narrow priority classes) may provide diminishing benefits while increasing triage complexity and potential misclassification.

Capacity Buffer Requirements: High waiting time variability—with standard deviations approximately equal to means—implies that substantial capacity buffers are needed to achieve reliable service level targets. Meeting 90th percentile waiting time targets requires considerably more capacity than meeting mean waiting time targets, a critical consideration noted in simulation studies of emergency departments (Izady & Worthington, 2012).

Performance Monitoring: The analytical relationships developed here enable real-time performance monitoring. By tracking arrival rates and service rates, managers can predict current waiting times without direct measurement, supporting proactive interventions when conditions deteriorate, as supported by principles of queueing theory in healthcare (Lakshmi & Iyer, 2013).

6.3 Benefits and Risks of Priority Systems

Benefits:

- Ensures timely treatment for clinically urgent conditions, which is directly linked to improved patient outcomes (Hagen et al., 2013).
- Aligns resource allocation with patient needs and clinical outcomes.

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- Supports service differentiation and targeted service level agreements.
- Improves overall clinical outcomes by prioritizing time-sensitive cases.

Risks:

- Potential starvation of low-priority patients during sustained high-demand periods.
- Increased waiting time variability and unpredictability for lower-priority patients, a key trade-off identified in priority queueing literature (Cobham, 1954).
- Equity concerns if priority correlates with demographic or socioeconomic factors.
- Triage errors may inappropriately delay urgent patients or prioritize non-urgent cases, highlighting the importance of accurate initial assessment (Wiler et al., 2013).

6.4 Limitations

This study has several limitations that should be acknowledged:

Model Simplifications: The exponential service time assumption, while analytically tractable, may not accurately represent healthcare treatment durations. Empirical service time distributions often exhibit greater variability (hyperexponential) or less variability (hypoexponential/Erlang) than exponential.

Homogeneous Servers: We assume identical servers, but healthcare settings often feature providers with different skill levels, specializations, and service rates. Heterogeneous server models would more accurately capture this reality.

Static Parameters: The model assumes stationary arrival rates, whereas healthcare facilities experience pronounced time-of-day and day-of-week demand variations. Time-varying models would better capture operational dynamics (Izady & Worthington, 2012).

Patient Behavior: We ignore patient behaviors such as balking (refusing to join long queues), reneging (abandoning queues before service), and jockeying (switching between queues). These behaviors affect system performance and may differ across priority classes.

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6.5 Future Research Directions

Several extensions merit future investigation:

- Time-varying models: Incorporating non-stationary arrival patterns using techniques such as pointwise stationary approximation or simulation with time-dependent parameters.
- Heterogeneous servers: Extending analysis to settings with multiple provider types having different service rates and capabilities.
- Network models: Analyzing multi-stage healthcare processes where patients flow through multiple service points.
- Accumulating priority: Investigating priority disciplines where patient priority increases with waiting time to address equity concerns (Stanford et al., 2014).
- Optimization: Developing staffing optimization models that balance service levels across priority classes subject to resource constraints.

7. Conclusions

This research developed mathematical formulations and simulation models for multi-level non-preemptive priority queues with application to healthcare service delivery. The M/M/c priority queueing model was applied to a representative scenario where patients are classified as critical, serious, moderate, or mild based on clinical acuity. Analytical results and simulation experiments demonstrate that non-preemptive priority scheduling effectively reduces waiting times.

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