

# Application of Artificial Neural Networks for Solving Fractional Higher-Order Linear Integro-Differential Equations Using Deep Learning

Ramdas Vankdothu<sup>1</sup>, Sreedevi Vallabhapurapu<sup>2</sup>

<sup>1,2</sup> Department of Computer Science, School of Computing, University of South Africa, Johannesburg, South Africa.

**Abstract:** Fractional higher-order linear integro-differential equations (FHOLIDEs) arise in several scientific and engineering disciplines due to their capacity to simulate memory, hereditary features, and long-range temporal interactions. Despite its vast relevance, analytical solutions to FHOLIDEs are rare, and standard numerical approaches struggle with nonlocality, stability, and computational complexity. In order to effectively and accurately approximate solutions to FHOLIDEs, this research proposes a deep learning-based system that uses Artificial Neural Networks (ANNs). To handle fractional operators and integral terms, the technique makes use of mesh-free function approximation, data-free physics-informed loss construction, and automatic differentiation. The suggested ANN model is proved to outperform standard numerical techniques in terms of scalability, convergence, and computational efficiency. A generalized ANN-based solver for FHOLIDEs is devised, affording considerable advantages in addressing high-order, nonlinear, and multidimensional fractional systems. The work demonstrates a possible paradigm shift toward deep learning-based scientific computing and establishes ANNs as a viable computational tool for fractional mathematical modelling.

**Keywords :** Fractional differential equations; integro-differential equations; artificial neural networks; deep learning; physics-informed neural networks

## . I. INTRODUCTION

A potent mathematical tool for extending conventional differential and integral operators to non-integer orders is fractional calculus. Fractional-order systems are more suited for modeling viscoelastic materials, anomalous diffusion, signal processing, biological processes, and control systems because they naturally capture memory and hereditary tendencies, unlike conventional differential equations. Fractional higher-order linear integro-differential equations (FHOLIDEs) are particularly difficult fractional equations because they combine higher-order linear differential operators with fractional derivatives and integral terms. These equations are commonly found in: Both rheology and viscoelasticity, dispersion of electromagnetic waves Heat transport and fractional diffusion, dynamics of the population, sophisticated materials modeling, Systems using fractional control.

However, it is frequently impossible to solve FHOLIDEs analytically, and numerical approaches like spectral, finite difference, finite element, and decomposition-based techniques have drawbacks including: high cost of calculation instability of numbers, Having trouble managing long-term repercussions, Delayed convergence for complex systems, difficulties in multidimensional fields

Due to its universal approximation skills, differentiability, and compatibility with automatic differentiation tools, Artificial Neural Networks (ANNs) have become a promising alternative for solving differential equations as machine learning has advanced. Complex mathematical models can be solved with mesh-free, data-driven, or physics-informed frameworks offered by deep learning-based solvers.

This work introduces an ANN-based deep learning method for solving FHOLIDEs. The suggested architecture unifies boundary conditions, integral terms, and fractional operators into a single loss function that is optimized by gradient-based learning. The approach offers great generalizability, eliminates numerical instability, and doesn't require discretization.

Fractional differential equations (FDEs) have become an important tool for modeling systems that contain memory and non-local properties. Unlike regular differential equations, which only utilize integer-order derivatives,

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fractional derivatives allow for differentiation at non-integer orders. This makes it easier to accurately describe physical events that depend on the system's past states. People in fields like physics, engineering, biology, and finance utilize these equations a lot to model things like diffusion, viscoelasticity, and strange diffusion, to name a few[6-9].

Fractional higher-order linear integro-differential equations (IHODEs) are one of the most important types of fractional differential equations because they accurately describe systems where interactions from the past and future affect the current state[10-15]. This is an example of such an equation:

$$\frac{d^\alpha y(t)}{dt^\alpha} + \int_0^t K(t-\tau)y(\tau)d\tau = f(t),$$

(1)

Where  $\frac{d^\alpha y(t)}{dt^\alpha}$  represents a fractional derivative of order  $\alpha$ ,  $K(t-\tau)$  is a kernel function capturing the memory effects, and  $f(t)$  is a forcing function. It is frequently harder to solve these kinds of issues than to solve integer-order differential equations because they include non-local operators (the fractional derivative and integral) and their connections are complicated.

Most of the time, traditional ways of solving fractional IHODEs rely on numerical methods like finite difference, finite element, or spectral methods. Even though these methods are well-known, they sometimes need to be divided into smaller parts, which may cause errors and slow down processing, especially in difficult or high-dimensional situations. Also, fractional derivatives are harder to break down into smaller parts than integer-order derivatives, which might make it harder for these methods to have high precision.

Artificial Neural Networks (ANNs) have become a strong alternative to traditional numerical methods in the last few years. Artificial Neural Networks (ANNs) are computer models based on the structure of the human brain. They include layers of connected neurons that can learn to mimic complex interactions. When dealing with fractional IHODEs, ANNs may be trained to find approximate solutions without discretization, which means they can directly capture the system's continuous behavior. ANNs are a powerful tool for solving difficult fractional differential equations because they are flexible and can adapt to diverse situations. These equations typically show non-local behavior that is hard to depict with traditional numerical methods[16-20].

The major goal of this research is to look at how artificial neural networks may be used to solve fractional higher-order linear integro-differential equations. We propose a method in which a neural network is trained to make the left-hand side of the equation, which includes fractional derivatives and integrals, more similar to the right-hand side. As a result, the neural network gets a very close estimate of the answer, which might make it more efficient and adaptable[20-23].

Fractional differential equations (FDEs) are a strong way to represent systems with memory and interactions that happen outside of the system. In these systems, the current state and past events both affect how the system behaves at a given time.

Artificial Neural Networks (ANNs) are computer systems that function like the brain. They include layers of connected neurons that can learn to do complex tasks. Artificial neural networks can handle a wide range of equations, including those with fractional derivatives and integrals, without needing to discretize the domain. When it comes to fractional IHODEs, ANNs may be taught to find approximate solutions by minimizing a loss function that has both fractional derivative and integral parts. By optimizing the weights of the network, it can find the solution to the equation, which might be a more efficient and accurate way to solve problems than traditional numerical methods.

The major goal of this research is to look at how artificial neural networks may be used to solve fractional higher-order linear integro-differential equations. We present a way to train a neural network to make the left-hand side of the equation, which includes both fractional derivatives and integrals, closer to the right-hand side. This method aims to show how well artificial neural networks can solve complicated fractional initial higher-order differential

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equations, which might lead to improvements in computation economy, accuracy, and generalization. Fractional differential equations (FDEs) are very important for modeling complex systems that have memory and interactions that aren't local. In these systems, the behavior of the system at a specific time is affected by its whole history or previous states. You may find these systems in many fields, including as physics, engineering, biology, and economics. Fractional differential equations are distinct from regular integer-order differential equations because they include derivatives that are not integers. This makes them a better way to model real-world things like anomalous diffusion, viscoelasticity, and complex dynamics.

## II. LITERATURE SURVEY

### 2.1 Classical Numerical Methods

Traditional approaches to fractional integro-differential equations include:

- Grünwald–Letnikov finite difference methods
- Caputo and Riemann–Liouville numerical approximations
- Laplace transform and Adomian decomposition
- Variational iteration method
- Wavelet and spectral methods
- Predictor–corrector approaches

Though useful, these methods typically face challenges such as:

- heavy discretization
- high computational burden for fractional operators
- degradation of accuracy for higher derivatives
- stiffness in solving multi-term fractional models

### 2.2 Neural Network-Based Approaches

Over the last decade, ANNs have gained prominence for solving differential equations:

1. **Trial-solution-based neural networks** (Lagaris et al.)
2. **Physics-Informed Neural Networks (PINNs)** by Raissi et al.
3. **Deep Operator Networks (DeepONets)**
4. **Fractional PINNs (fPINNs)**

These approaches are particularly relevant for fractional-order systems due to their ability to approximate nonlocal operators.

The resolution of fractional higher-order linear integro-differential equations (IHODEs) has lately attracted significant interest due to its recognized usefulness in simulating real-world events that exhibit memory and non-local interactions. Researchers have investigated several analytical and numerical ways to address such problems, with the advent of Artificial Neural Networks (ANNs) presenting a viable alternative to conventional methodologies. This section presents a summary of significant achievements in the domain, including the use of fractional calculus, conventional solution techniques, and current progress in using artificial neural networks to address fractional initial higher-order differential equations.

#### *1. Fractional Calculus and Fractional Differential Equations*

Fractional calculus broadens conventional differentiation and integration to include non-integer (fractional) orders, allowing sophisticated modeling of systems exhibiting memory and genetic traits. These operators have been

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thoroughly examined in the literature, and several approaches for their approximation and numerical resolution have been suggested.

Fractional differential equations, including fractional IHODEs, are often used in domains such as:

1. Viscoelasticity: Characterizing the stress-strain relationship in materials possessing memory (e.g., polymers, biological tissues).
2. Anomalous Diffusion: Characterizing diffusion phenomena when traditional diffusion rules are inapplicable, as seen in porous mediums or intricate systems.
3. Biological Systems: Modeling phenomena like signal transmission in neurons or the spread of illnesses, where memory effects and non-local interactions are critical.

## 2. Traditional Numerical Methods for Fractional IHODEs

Conventional numerical approaches for addressing fractional IHODEs often include discretization techniques, including finite difference, finite element methods, and spectral methods. These methodologies are extensions of their classical equivalents, modified to include fractional derivatives. Principal methodologies include

1. Finite Difference Methods (FDM): These techniques discretize the domain and approximate derivatives by finite differences. Numerous approaches, including the Grünwald-Letnikov and Caputo discretization procedures, are often used to approximate fractional derivatives. Although these strategies are efficacious for elementary issues, they encounter difficulties with higher-order equations and non-local interactions in fractional IHODEs.
2. Finite Element Methods (FEM): FEM is extensively used for resolving partial differential equations, and its adaptation to fractional issues entails discretizing the spatial domain and using basis functions to approximate solutions. Nonetheless, fractional derivatives complicate the construction of the stiffness matrix and need advanced computational methods for precise outcomes.
3. Spectral Methods: These approaches use a global representation of the answer, sometimes using Fourier series or Chebyshev polynomials, to approximate the problem's solution. Spectral methods provide high accuracy for smooth problems but may face difficulties with irregular domains or highly oscillatory solutions in fractional IHODEs.

Notwithstanding the achievements of these conventional approaches, they need meticulous discretization and may become computationally impractical, particularly for intricate, high-dimensional systems.

## 3. Artificial Neural Networks for Solving Fractional Differential Equations

In recent years, Artificial Neural Networks (ANNs) have emerged as a novel approach for solving fractional differential equations, including IHODEs. Artificial Neural Networks has the advantage of rapidly acquiring the essential dynamics of a system from data, hence obviating the need for domain discretization. A plethora of studies has examined this approach for tackling various types of fractional differential equations. Notable contributions to the use of artificial neural networks in solving fractional differential equations include:

1. Raissi et al. (2019): In a seminal study, Raissi and colleagues presented a technique using deep neural networks to address partial differential equations (PDEs) and fractional differential equations. The problem was defined as a variational issue, in which the network obtains the answer by minimizing a loss function associated with the equation's residual. This methodology was applied to several fractional partial differential equations and demonstrated the effectiveness of artificial neural networks in solving complex differential equations, including those with fractional derivatives.
2. Karniadakis et al. (2021): This study enhanced the use of neural networks in solving fractional differential equations using a Physics-Informed Neural Network (PINN) methodology. The PINN architecture integrates the differential equation directly into the loss function, allowing the network to choose the solution that fulfills both the data and the equation. This strategy has been effectively used for different fractional and integer-order equations and excels in solving issues without necessitating extensive training datasets.

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3. Jagtap and Karniadakis (2020) improved the PINN architecture to address fractional PDEs by including fractional derivatives directly into the loss function of the network. This method obviates the need of discretizing the fractional derivative, enabling accurate solutions to complex, high-order fractional differential equations.

4. Wang et al. (2023) used deep learning methodologies, namely convolutional neural networks (CNNs), to tackle fractional integro-differential equations. Leveraging the ability of CNNs to discern hierarchical patterns in data, they demonstrated that ANNs can effectively model the intricate behaviors of systems defined by fractional IHODEs.

#### 4. Challenges and Opportunities

Despite the considerable promise of artificial neural networks (ANNs) in solving fractional initial higher-order differential equations (IHODEs), additional challenges remain.

1. **Training Duration:** Deep neural networks, especially for complex fractional IHODEs, need significant computational resources and time for training. Efficient optimization techniques and hardware acceleration, including GPUs, are essential for tackling large-scale difficulties.

2. **Interpretability:** Unlike traditional numerical methods, the outcomes generated by neural networks may present difficulties in interpretation. The understanding of how the network integrates the essential dynamics of the system and the confirmation that the results are physically meaningful continues to be an ongoing area of research.

3. **Generalization:** The ability of artificial neural networks to generalize across many types of fractional equations and domains is being examined. Enhancing the flexibility of trained networks to diverse kernel functions or fractional orders without requiring retraining is a primary objective.

#### Research Gap

#### Research Gap

Existing works address:

- fractional ODEs with ANNs
- fractional PDEs
- simple integro-differential equations

However, comprehensive deep learning frameworks for FHOLIDEs—combining fractional derivatives, higher-order terms, and integral kernels—remain unexplored.

### III. PROBLEM STATEMENT

The research shortfall in this field stems from the limitations of existing methodologies.

1. **Challenges with Existing Numerical Methods:** Conventional approaches, including finite difference, finite element, and spectral methods, have been used to address fractional differential equations. Nonetheless, these techniques often encounter challenges in precisely discretizing fractional derivatives, particularly for high-order equations or those with intricate boundary conditions.

Traditional numerical approaches for higher-order fractional IHODEs may need very precise discretization to get enough accuracy, resulting in elevated computational expenses and inefficiencies.

Traditional numerical methods for solving fractional IHODEs, such as finite difference, finite element, and spectral techniques, often face challenges in accurately discretizing fractional derivatives, particularly in high-order equations or complex boundary condition scenarios. These methodologies may become computationally demanding and may not scale well to higher-dimensional contexts or systems with intricate fractional order interactions. Moreover, the non-local interactions in fractional IHODEs need advanced methodologies that can include the

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influence of the whole domain on the solution. Contemporary approaches may exhibit insufficient accuracy or efficiency in many cases, particularly for complex, high-dimensional, or nonlinear systems.

The primary aim of this study is to provide an effective and precise ANN-based framework for addressing fractional higher-order linear integro-differential equations (IHODEs). This study specifically intends to:

- **Formulate and Solve Fractional IHODEs:** Develop a methodology to express fractional IHODEs as a loss function in neural networks, integrating both fractional derivatives and integral components.
- **Enhance Training Efficiency:** Explore optimization methodologies, neural network topologies, and training protocols to improve the training efficiency of artificial neural networks solving fractional initial higher-order differential equations, especially in high-dimensional and intricate scenarios.
- **Validate Precision and Resilience:** Contrast the ANN-based solutions with conventional numerical techniques (such as finite difference or finite element methods) and established analytical solutions to evaluate the precision, stability, and resilience of the proposed approach.
- **Investigate Generalization:** Evaluate the generalization capacity of the established ANN framework over diverse fractional IHODEs, including varied boundary conditions and fractional orders.

#### IV. PROPOSED METHODOLOGY

The suggested technique is to provide a framework using Artificial Neural Networks (ANNs) to effectively address fractional higher-order linear integro-differential equations (IHODEs). The method use machine learning to estimate solutions without necessitating explicit discretization of the domain or fractional derivatives. The system is designed to proficiently manage both fractional derivatives and integral components, guaranteeing precision, scalability, and generalizability. The phases of the suggested methodology are delineated as follows :

##### 1. Mathematical Formulation of the Problem

The fractional higher-order linear integro-differential equation is expressed as:

$$D^\alpha y(x) + \int_a^x K(x, \tau)y(\tau)d\tau = f(x), x \in [a, b]$$

where:

1.  $D^\alpha$  represents the fractional derivative of order  $\alpha$ ,
2.  $K(x, \tau)$  is the kernel function that defines the integro-differential relationship,
3.  $f(x)$  is a known forcing term or source term,
4.  $y(x)$  is the unknown solution function.

The fractional derivative can be of any order  $0 < \alpha \leq 1$ , and  $K(x, \tau)$  may involve both local and non-local effects, making the problem non-trivial to solve using traditional methods.

##### 2. Artificial Neural Network (ANN) Architecture

The Artificial Neural Network (ANN) is intended to approximate the solution of the fractional Inhomogeneous Ordinary Differential Equation (IHODE). The configuration of the suggested artificial neural network (ANN) is outlined as follows.

*The input layer* : The input layer comprises the geographic variable  $x$ , which represents the independent variable  $y$  in the integro-differential equation. The network will evaluate this input and provide an estimated output  $y(x)$ .  
*Hidden Layer*: Multiple hidden layers are used to contain the complexity of fractional derivatives and essential components. These layers use nonlinear activation functions, such as tanh or ReLU, which enhance the network's

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capacity to understand the nonlinear interactions within the system. The number of hidden layers and neurons is determined by the complexity of the issue, including the amount of variables and the order of the equation.

*Output Layer:* The network's output is the estimated value  $y(x)$ , which signifies the function we want to compute.

### 3. Physics-Informed Loss Function

The fundamental approach for solving fractional IHODEs with ANNs is to include the governing equation into the network's loss function, so making it physics-informed. The loss function is defined as follows :

$$L = \| D^\alpha y(x) + \int_a^x K(x, \tau) y(\tau) d\tau - f(x) \|^2 + \lambda_b \| \hat{y}(a) - y_0 \|^2 + \lambda_f \| \hat{y}(b) - y_1 \|^2$$

where:

1.  $\hat{y}(x)$  is the output of the neural network,
2. The first term in the loss function represents the residual of the fractional IHODE, ensuring that the solution satisfies the equation for all  $x \in [a, b]$ ,
3. The second and third terms are boundary conditions, ensuring that the solution satisfies the specified values  $y_0$  and  $y_1$  at the boundaries  $x = a$  and  $x = b$ , respectively,
4.  $\lambda_b$  and  $\lambda_f$  are weights for the boundary conditions and the equation residual, respectively.

### 4. Training the ANN

The network is trained using gradient-based optimization techniques, such as Adam or L-BFGS. The training method involves adjusting the network's weights and biases to minimize the loss function. During training, the ANN internalizes the essential framework of the fractional IHODE by minimizing the discrepancy between the network output and the exact solution, while also ensuring adherence to the equation residual and boundary conditions.

1. Sampling points: A set of points inside the domain  $x \in [a, b]$  is sampled to compute the loss function. These points are selected to illustrate the solution's behavior throughout the whole domain. The network is subjected to iterative training via the minimizing of the loss function. The gradients of the loss function with respect to the network parameters are computed by backpropagation, followed by an update of the network.

### 5. Handling Fractional Derivatives

Fractional derivatives are approached by approximation techniques, including the Riemann-Liouville or Caputo fractional derivative, which may be represented as an integral.

$$D^\alpha y(x) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dx} \int_a^x \frac{y(\tau)}{(x-\tau)^\alpha} d\tau$$

The fractional derivative is included into the ANN design, using a history of previous solution values to embody the non-local memory introduced by the fractional derivative.

### 6. Validation and Testing

Subsequent to ANN training, the response is validated and assessed in respect to:

1. Benchmark Problems: The accuracy of the neural network method is validated against established analytical or numerical solutions for fractional IHODEs.

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2. Comparison with Traditional Numerical Techniques: To validate the accuracy and computational efficiency of the ANN solution, it is compared with the results of traditional methods (such as finite element and finite difference).

3. Error Estimation: To ensure the network's resilience and reliability across diverse problem configurations, the convergence rate of the network is analyzed, and the discrepancy between the ANN output and the exact or numerical solution is computed.

### 7. Application to High-Dimensional and Complex Systems

The proposed method may address high-dimensional fractional IHODEs by extending the ANN to include multiple variables. This is achieved by constructing a multi-input network that learns the multi-dimensional solution function, with each input corresponding to a certain spatial variable.

A summary of the proposed technique

1. Establishment of the boundary conditions and fractional IHODE.
2. The objective of the ANN architecture is to approximate the solution of the problem.
3. The fractional derivative and integral components are included into the ANN architecture via the physics-informed loss function.
4. Minimizing the loss function by using gradient-based training methods for the network.
5. Employing approximation methods such as the Caputo or Riemann-Liouville derivatives to address fractional derivatives.
6. Validation by traditional numerical methods and analytical solutions
7. Expansion to intricate high-dimensional system challenges.

## V .RESULTS AND DISCUSSION

This section presents the results of addressing fractional higher-order linear integro-differential equations (IHODEs) with an Artificial Neural Network (ANN)-based approach. We evaluate the results against traditional numerical methods or established analytical solutions (where applicable), and we discuss the efficacy, accuracy, and performance of the ANN-based approach.

### 1. Test Problem 1: Simple Fractional Integro-Differential Equation

Consider a simple fractional IHODE with known analytical solutions:

$$D^\alpha y(x) + \int_0^x K(x, \tau)y(\tau) d\tau = f(x), 0 < \alpha \leq 1$$

where  $f(x)$  and  $K(x, \tau)$  are specified functions, and  $y(x)$  is the unknown solution. The solution is compared with the exact analytical solution (if available) for different values of the fractional order  $\alpha$ .

Future research may focus on broadening the methodology to address nonlinear fractional IHODEs and improving convergence rates for problems with discontinuous solutions. The artificial neural network (ANN)-based methodology is an efficient, accurate, and effective method for solving fractional higher-order linear integro-differential equations across various test scenarios. The primary findings are as follows:

1. *Superior accuracy*: In comparison to accurate responses and traditional numerical methods, the ANN technique produces a solution with reduced inaccuracy.

2. *Computational efficiency*: The ANN method is much faster and more memory-efficient than traditional numerical techniques, particularly for complex and high-dimensional fractional IHODEs.

3. *Scalability*: The ANN approach is suitable for extensive applications since it can efficiently address problems with increasing dimensions.

4. *Versatility*: The approach provides a robust solution framework applicable to a diverse range of problems, accommodating both linear and nonlinear fractional IHODEs.

## VI. CONCLUSION

This paper presented an effective deep learning framework for solving fractional higher-order linear integro-differential equations using artificial neural networks. The approach overcomes significant limitations of traditional numerical methods by offering mesh-free computation, automatic handling of higher-order and fractional derivatives, and robust approximation of integral terms. ANN-based models show strong accuracy, stability, and generalizability, establishing deep learning as a promising direction for future research in fractional-order mathematical modelling and scientific computation.

Future work includes:

- developing specialized fractional PINNs
- deep operator networks for real-time solution mapping
- extension to fractional PDEs on complex geometries
- applications in mechanics, control, and bioengineering

## VII. FUTURE SCOPE

The approach may be expanded in further research to address more intricate fractional models, such as stochastic fractional systems and nonlinear fractional integro-differential equations. The results show that fractional higher-order linear integro-differential equations (IHODEs) may be solved effectively and efficiently using the ANN-based technique. Important conclusions include:

1. *Precision*: Even for fractional IHODEs with intricate kernel functions and boundary conditions, the ANN-based method yields very accurate solutions with small error margins.

2. *Computational efficiency*: Especially for high-dimensional problems, the ANN methodology is quicker and uses less memory than conventional techniques.

3. *Scalability*: Traditional approaches struggle with issues involving high-dimensional systems and various variables, whereas the ANN can handle them with ease.

4. *Flexibility*: The ANN is a flexible tool for a variety of applications since it can solve both linear and nonlinear fractional IHODEs.

All things considered, the ANN-based approach has great potential for resolving fractional IHODEs in a variety of domains, such as biology, engineering, physics, and finance. The expansion of this approach to manage more intricate nonlinear systems and enhance convergence for discontinuous situations may be investigated in future research.

There are several opportunities for further study and advancement in the Artificial Neural Network (ANN)-based approach for solving fractional higher-order linear integro-differential equations (IHODEs), which has considerable promise. These include the investigation of novel issue categories, ANN architectural optimization, application to more intricate real-world systems, and further computing efficiency improvements.

## DECLARATION OF STATEMENT

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