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SOME COMMON FIXED-POINT THEOREMS FOR OCCASIONALLY WEAKLY COMPATIBLE MAPPINGS IN FUZZY MENGER SPACES

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ABSTRACT

In this paper, we prove some common fixed point theorems for occasionally weakly compatible mappings in complete Fuzzy Menger spaces.

Key words: Common fixed point, Menger spaces, Fuzzy Menger spaces, Complete, Occasionally weakly compatible

1 INTRODUCTION

Menger [15,16] introduced the notion of Probabilistic Metric spaces(PM spaces) which is in fact, a generalization of metric spaces. Sehgal [25] initiated the study of contraction mapping theorems in Probabilistic Metric spaces. Many mathematicians weakened the notion of commutativity by introducing the notions of weak commutativity, compatibility by weak compatibility in metric spaces and proved a number of fixed point theorems using these notions. Jungck and Rhoades [12] studied fixed point results for occasionally weakly compatible mappings. Chenand Chang [4] proved a common fixed point theorem in a complete Menger spaces by using the notion of compatibility. In this paper we prove some common fixed point theorems for occasionally weakly compatible mappings in Fuzzy Menger spaces.

2.PRELIMINARIES

Definition 2.1 A fuzzy probabilistic metric space is an ordered pair (X, F_α) consisting of a nonempty set X and a mapping F_α from $X \times X$ into the collections of all fuzzy distribution functions $F_\alpha \in R$ for all $\alpha \in [0,1]$. For $x, y \in X$ we denote the fuzzy distribution function $F_\alpha(x, y)$ by $F_{\alpha(x,y)}$ and $F_{\alpha(x,y)}(u)$ is the value of $F_{\alpha(x,y)}$ at *u* in *R*.

The functions $F_\alpha(x, y)$ for all $\alpha \in [0,1]$ assumed to satisfy the following conditions:

1. $F_{\alpha(x,y)}(u) = 1$ for all $u > 0$ if and only if $x = y$

10.48047/jocaaa.2024.33.08.357

2. $F_{\alpha(x,y)}(0)=0$ for all x,y in X

3. $F_{\alpha(x,y)}=F_{\alpha(y,x)}$ for all x,y in X

4.If $F_{\alpha(x,y)}(u)=1$ and $F_{\alpha(y,z)}(v)=1$ implies $F_{\alpha(x,z)}(u+v)=1$ for all x,y,z in $X$and $u,v>0$.

Definition 2.2 A commutative, associative and non decreasing mapping $*:[0,1]\times[0,1]\to[0,1]$ is a *t*-norm if and only if

1. $a*1=a$ for all $a\in[0,1]$

2. $0*0=0$

3. $c*d\geq a*b$ for $c\geq a, d\geq b$

Definition 2.3 A Fuzzy Menger space is a triplet $(X,F_{\alpha},*)$, where (X,F_{α}) is a fuzzy probabilistic metric space, $*$ is a *t*-norm and the generalized triangle inequality $F_{\alpha(x,z)}(u+v)\geq F_{\alpha(x,y)}(u)*F_{\alpha(y,z)}(v)$ holds for all x,y,z in $X, u,v>0$ and $\alpha\in[0,1]$

Definition 2.4 Let $(X,F_{\alpha},*)$ be a Fuzzy Menger space. If $x\in X$, $\varepsilon>0$ and $\lambda\in$ (0, 1), then (ε,λ) neighbourhood of x, called $U_x(\varepsilon,\lambda)$, is defined by

$$U_x(\varepsilon,\lambda) = \{y\in X:\ F_{\alpha(x_n,x)}(\varepsilon)>1-\lambda\ \}$$

An (ε,λ) - topology in X is the topology induced by the family $\{U_x(\varepsilon,\lambda): x\in X, \varepsilon>0, \alpha\in[0,1]$ and $\lambda\in$ (0, 1)$\}$ of neighbourhood.

Definition 2.5 A sequence $\{x_n\}$ in $(X,F_{\alpha},*)$ is said to be convergent to a point x in X if for every $\varepsilon>0$ and $\lambda>0$, there exists an integer $N=N(\varepsilon,\lambda)$ such that $x_n\in U_x(\varepsilon,\lambda)$ for all $n\geq N$ or equival ently $F_{\alpha(x_n,x)}(\varepsilon)>1-\lambda$ for all $n\geq N$ and $\alpha\in[0,1]$

Definition 2.6 A sequence $\{x_n\}$in $(X,F_{\alpha},*)$ is said to be Cauchy sequence if for every $\varepsilon>0$ and $\lambda>0$, there exists an integer $N=N(\varepsilon,\lambda)$ such that $F_{\alpha(x_n,x_m)}(\varepsilon)>1-\lambda$ for all $n,m\geq N$ and $\alpha\in[0,1]$

10.48047/jocaaa.2024.33.08.357

Definition 2.7 A Fuzzy Menger space $(X, F_\alpha, *)$ with the continuous *t*-norm is said to be complete if every Cauchy sequence in *X* converges to a point in *X* for all $\alpha \in [0,1]$

Definiion2.8 Let $(X, F_\alpha, *)$ be a Fuzzy Menger space. Two mappings $f, g : X \to X$ are said to be weakly compatible if they commute at coincidence point for all $\alpha \in [0,1]$. That is $fx = gx$ for some $x \in X$ then $fgx = gfx$.

Definition2.9 Let $(X, F_\alpha, *)$ be a Fuzzy Menger space. Two mappings $f, g : X \to X$ are said to be compatible if and only if $F_{\alpha(fgx_n, gfx_n)}(t) \to 1$ for all $t > 0$ whenever $\{x_n\}$ in X such that $fx_n, gx_n \to z$ for some $z \in X$.

Definition2.10 Let $(X, F_\alpha, *)$ be a Fuzzy Menger space. Two mappings $f, g : X \to X$ are said to be occasionally weakly compatible if there exists a point $x \in X$ such that $fx = gx$ and $fgx = gfx$.

Lemma2.11 Let $\{x_n\}$ be a sequence in a Fuzzy Menger space $(X, F_\alpha, *)$ where $*$ is continuous *t*-norm and $t * t \geq t$ for all $t \in [0,1]$, if there exists a constant $k \in (0,1)$ such that for all $t > 0$ and $n \in N$ $F_{\alpha(x_n, x_{n+1})}(kt) \geq F_{\alpha(x_{n-1}, x_n)}(t)$ for all $\alpha \in [0,1]$ then $\{x_n\}$ is Cauchy sequence.

Lemma 2.12 Let $(X, F_\alpha, *)$ be a Fuzzy Menger space. If there exists a constant $k \in (0,1)$ such that $F_{\alpha(x,y)}(kt) \geq F_{\alpha(x,y)}(t)$ for all $x, y \in X$ and $t > 0$ then $x = y$.

Lemma 2.13 Let X be a set. Let *f,g* be occasionally weakly compatible self maps of *X*. If *f* and *g* have a unique point of coincidence, $w = fx = gx$, then *w* is the unique common fixed point of *f* and *g*.

3 MAIN RESULT

Theorem 3.1 Let *A* and *S* be two self occasionally weakly compatible mappings of a complete Fuzzy Menger space $(X, F_\alpha, *)$ with $t * t \geq t$ such that for each $x \neq y$ in $X, t > 0$ and for $0 < k < 1$. $F_{\alpha(Ax,Ay)}(kt) \geq \min\{F_{\alpha(Sx,Sy)}(t), F_{\alpha(Sx,Ay)}(t) * F_{\alpha(Sy,Ay)}(t), F_{\alpha(Ax,Sx)}(t) * F_{\alpha(Ax,Sy)}(t)\}$. Then *A* and *S* have a unique common fixed point.

10.48047/jocaaa.2024.33.08.357

Proof

Since A and S are occasionally weakly compatible so there exist $a \in X$ such that $Aa = Sa$ implies $ASa = SAa$.

That is there exist $a \in X$ such that $F_{\alpha(Aa,Sa)}(t) = 1$ implies $F_{\alpha(ASa,SAa)}(t) = 1$ for $t > 0$.

Since $Sa = Aa$, we have $SSa = SAa = ASa = AAa$.

Now we show that $Aa = Sa$ is common fixed point of A and S.

Suppose $Aa \neq AAa$. Then we have

$$F_{\alpha(Aa,AAa)}(kt) \geq \min\{F_{\alpha(Sa,SAa)}(t), F_{\alpha(Sa,AAa)}(t) * F_{\alpha(SAa,AAa)}(t), F_{\alpha(Aa,Sa)}(t) * F_{\alpha(Aa,SAa)}(t)\}$$

$$= \min\{F_{\alpha(Aa,AAa)}(t), F_{\alpha(Aa,AAa)}(t) * F_{\alpha(AAa,AAa)}(t), F_{\alpha(Aa,Aa)}(t) * F_{\alpha(Aa,AAa)}(t)\}$$

$$= \min\{F_{\alpha(Aa,AAa)}(t), F_{\alpha(Aa,AAa)}(t) * 1, 1 * F_{\alpha(Aa,AAa)}(t)\}$$

$$= \min\{F_{\alpha(Aa,AAa)}(t), F_{\alpha(Aa,AAa)}(t), F_{\alpha(Aa,AAa)}(t)\}$$

$$= F_{\alpha(Aa,AAa)}(t)$$

$$F_{\alpha(Aa,AAa)}(kt) \geq F_{\alpha(Aa,AAa)}(t)$$

Therefore by Lemma 2.12, $Aa = AAa$.

Thus $AAa = SAa = Aa$.

Hence $Aa = Sa$ is common fixed point of A and S.

Finally we prove that the fixed point is unique.

Let x and y be two common fixed points of A and S.

Then $Ax = Sx = x$ and $Ay = Sy = y$

Now consider

$$F_{\alpha(x,y)}(kt) = F_{\alpha(Ax,Ay)}(kt)$$

$$\geq \min\{F_{\alpha(Sx,Sy)}(t), F_{\alpha(Sx,Ay)}(t) * F_{\alpha(Sy,Ay)}(t), F_{\alpha(Ax,Sx)}(t) * F_{\alpha(Ax,Sy)}(t)\}$$

$$= \min\{F_{\alpha(Ax,Ay)}(t), F_{\alpha(Ax,Ay)}(t) * F_{\alpha(Ay,Ay)}(t), F_{\alpha(Ax,Ax)}(t) * F_{\alpha(Ax,Ay)}(t)\}$$

10.48047/jocaaa.2024.33.08.357

$$= \min\{F_{\alpha(x,y)}(t), F_{\alpha(x,y)}(t) * F_{\alpha(y,y)}(t), F_{\alpha(x,x)}(t) * F_{\alpha(x,y)}(t)\}$$

$$= \min\{F_{\alpha(x,y)}(t), F_{\alpha(x,y)}(t) * 1, 1 * F_{\alpha(x,y)}(t)\}$$

$$= \min\{F_{\alpha(x,y)}(t), F_{\alpha(x,y)}(t), F_{\alpha(x,y)}(t)\}$$

$$= F_{\alpha(x,y)}(t)$$

$$F_{\alpha(x,y)}(kt) \geq F_{\alpha(x,y)}(t)$$

Therefore by Lemma 2.12, $x = y$.

Therefore A and S have a unique common fixed point.

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