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Some results of weak 2-vertex duplication self-switching graphs

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Abstract

Let G be a graph and let $\sigma \subseteq V$ be a non-empty subset of $V - \sigma$. σ is said to be a self switching if $G \cong G^{\sigma}$ where G^{σ} is obtained from G by removing all edges between σ and $V - \sigma$ and adding edges between all non-adjacent vertices of σ and $V - \sigma$. We also call it as $|\sigma|$ −vertex self switching. When $|\sigma| = 2$, we call it as 2 −vertex self switching. Duplication of a vertex v of a graph G produces a new graph G by adding a new vertex v such that all vertices which are adjacent to v in G are also adjacent to v in G. In this paper, we characterize path, minimum and maximum degree vertices of weak 2-vertex duplication self switching.

Keywords: Switching, 2-vertex self switching, duplication 2-vertex self switching , $dss_2(G), \delta(G), \Delta(G)$.

1. Introduction

For a finite undirected simple graph $G(V, E)$ with $|V(G)| = p$ and a non- empty set $\sigma \subseteq V$, the switching of G by σ is defined as the graph $G^{\sigma}(V, E')$ which is obtained from G by removing all edges between σ and its complement, $V - \sigma$ and adding as edges all non-edges between σ and $V - \sigma$. Switching has been defined by Seidel |8| and is also referred to as Seidel switching. When $\sigma = \{v\} \subset V$, the corresponding switching $G^{\{v\}}$ is called a *vertex switching* and is denoted by G^{v}. A non- empty set $\sigma \subseteq V$ is said to be self switching if $G \cong G^{\sigma}$ .We also call it as $|\sigma|$ −vertex self switching. The set of all k −vertex self switchings of G each with cardinality k is denoted by $SS_k(G)$ and its cardinality by $ss_k(G)$. If $k = 1$, then we call the corresponding self switching as self vertex switching . The survey on self vertex switchings of graphs by Jayasekaran |1| motivated us to study on graphs with $|\sigma| = 2$. Jayasekaran and Prabavathy [3, 4] introduced the concept of duplication self vertex switching. A vertex v' is said to be the duplication of v if all the vertices which are adjacent to v in G are also adjacent to v' in G'. The new graph obtained after duplication of a vertex v is denoted as $D(vG)$.

For any vertex $v \in V(G)$, the open neighbourhood $N(v)$ of v is the set of all vertices adjacent to v. That is $N(v) = \{u \in V(G) / uv \in E(G)\}$. The minimum degree among

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the vertices of G is denoted by $\delta(G)$ and is defined as min $\{d(v)/v \in V(G)\}$and the maximum degree represented by $\Delta(G)$ and is defined as max $\{d(v)/v \in V(G)\}$.

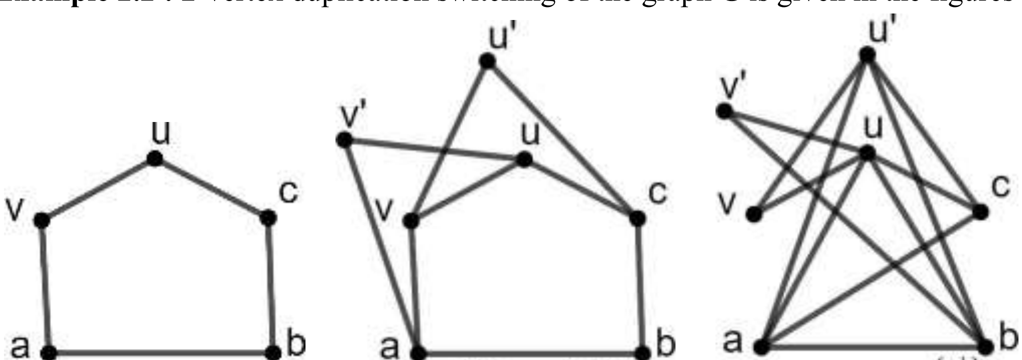
G.Sumathy and K.S Shruthi [10,12] introduced the concept of 2-vertex duplication self switching. $2-$vertex duplication of a graph G is the duplication of any two vertices $u, v \in V(G)$is u', v'such that $x', y'$are adjacent to all the vertices to u & v.It is denoted as $D((u, v)G)$.

2. Weak 2 –vertex duplication self switching of graphs

Definition 2.1: The **2-vertex duplication switching** of G byλ is the graph obtained by duplicating any two vertices $u,\ v$ and by removing all edges between λ and its complement $V-\lambda$also by adding all edges between $\lambda$and $V-\lambda$ which are not in G, without affecting the adjacency and non-adjacency of vertices in λ.

It is denoted by $\boldsymbol{D((u,v)G)^{\lambda}}$**or** $\boldsymbol{D((u,v)G)^{\{x,y\}}}$.

Example 2.2 : 2-vertex duplication switching of the graph G is given in the figures 2.1 to 2.3.



G $Fig.2.1$ $D((u,v)G)$ $Fig.2.2$ $D((u,v)G)^{\{a,b\}}$ $Fig.2.3$

Definition 2.3: If $\lambda = \{\{x, y\}/\{x, y\} \subseteq V(G)\}$ is called a **2 –vertex duplication self switching** of a graph Gif the resultant graph obtained after duplication of $\{u, v\}$ has λ as a self vertex switching. $(ie)\ D((u, v)G) \cong D((u, v)G)^{\{x,y\}}$.

If $\lambda = \{\{x, y\}/\{x, y\} \neq \{u, v\} \subseteq V(G)\}$, then λ is called the **weak 2 –vertex duplication self switching** of G.

Example 2.4: Weak 2-vertex duplication self switching of a graph is given in the figures 2.4 to 2.6.

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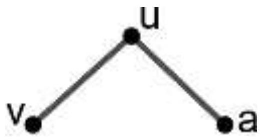
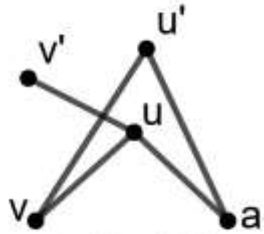
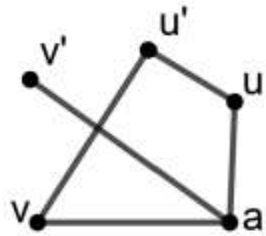


G
$Fig.2.4$



$D((u,v)G)$
$Fig.2.5$



$D((u,v)G)^{\{u,a\}}$
$Fig.2.6$

Theorem 2.5: If $\lambda = \{x, y\} \subseteq V$ is a weak 2 –vertex duplication self switching of a graph G, then $d_G(x) + d_G(y) = p$ if $xy \in V(G)$.

Proof:

Let $\lambda = \{x, y\} \subseteq V$ be a weak 2 –vertex duplication self switching of a graph G. By the definition, $D((u,v)G) \cong D((u,v)G)^{\lambda}$ and hence $|E(D(u,v)G)| = |E(D(u,v)G)^{\lambda}|$.

This implies that,

$$q + d_G(x) + d_G(y) = q + d_G(x) + d_G(y) + [p + 2 - 1 - d_{D((u,v)G)}(x)] - d_{D((u,v)G)}(x) + [p + 2 - 1 - d_{D((u,v)G)}(y)] - d_{D((u,v)G)}(y) + 2.$$

$$= p + 1 - [d_G(x) + 1] - (d_G(x) + 1) + p + 1 - [d_G(y) + 1] - (d_G(y) + 1) + 2.$$

0

$$= 2p + 2 - 2d_G(x) - 2 - 2d_G(y) - 2 + 2.$$

$$= 2p - 2d_G(x) - 2d_G(y).$$

$$p = d_G(x) + d_G(y).$$

0

0

Note 2.6: The converse of the theorem need not be true. For example, let us consider the graph G with 5 vertices given in figure 2.1. In this graph $uv_1 \in E(G)$ and $d_G(u) + d_G(v_1) = 5 =$ but $D((u,v)G) \ncong D((u,v)G)^{\{u,v1\}}$.

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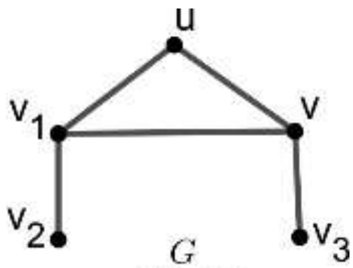
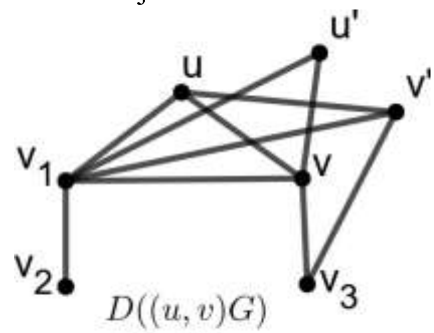
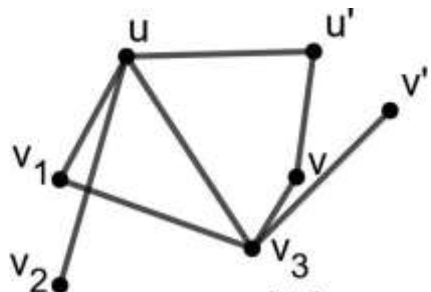


G
$Fig.2.7$



$D((u,v)G)$
$Fig.2.8$



$D((u,v)G)^{\{u,v1\}}$
$Fig.2.9$

Theorem 2.7: For $p \geq 2$, $dss2(Pp\ ) = \begin{cases} 2, & if\ p = 3 \\ 2, & if\ p = 4 \\ 0, & otherwise. \end{cases}$

Proof:

Case (i): If $p = 3$

Let $\lambda = \{uiui{+}1, uiui{+}2/1 \leq i \leq 2\}$.

Then $d(ui) + dG(ui{+}1) = 1 + 2 = 3 = p$ and $d(ui) + dG(ui{+}2) = 1 + 2 = 3 = p$.

Hence by Theorem 2.5, λ is a weak 2 $-$vertex duplication self switching of G.

Therefore, $dss2(P3) = 2$.

Case (ii): If $p = 4$

Let $\lambda = \{u2, u3\}$.Then, $dG(u2) + dG(u3) = 2 + 2 = 4 = p$.

Hence by Theorem 2.5, λ is a weak 2 $-$vertex duplication self switching of G

Therefore, $dss2(P4) = 1$.

Case (iii): If $p > 4$.

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Let u, v be any two adjacent vertices in $V(Pp)$.
Then, $dG(u) + dG\ (v) \neq p$. Hence by the Theorem 2.5, λ is a not a weak 2 −vertex duplication self switching of Cp.

Therefore, $dss2(Pp\ ) = 0$.

Hence the theorem.

Theorem 2.8: In a non-regular graph G, $\lambda = \{\{x, y\}/$ either x *or* y is adjacent to any minimum degree vertex in $D((u, v)G)\}$ is a weak 2 −vertex duplication self switching .

Proof:

Let w be the minimum degree vertex in $D((u, v)G)$. Let $\lambda = \{\{x, y\}/$ either x *or* y is adjacent to any minimum degree vertex in $D((u, v)G)\}$ and x be the vertex adjacent to w in $D((u, v)G)^{\lambda}$. Then $d(D(u,v)G)\lambda(v_i) = d_{(D(u,v)G)}(v_i)$ for all $v_i \in N(x)$ and $v_i \neq y$ and $d(D(u,v)G)\lambda(w) = \delta(D(u, v)G) = d_{(D(u,v)G)}(w)$. This implies that $D((u, v)G) \cong D((u, v)G)^{\lambda}$. Hence λ is a weak 2 −vertex duplication self switching.

Theorem 2.9: In a non-regular graph G, $\lambda = \{\{x, y\}/$ either x *or* y is adjacent to any maximum degree vertex in $D((u, v)G)\}$ is a weak 2 −vertex duplication self switching .

Proof:

Let w be the maximum degree vertex in G. Let $\lambda = \{\{x, y\}/$ either x *or* y is adjacent to any maximum degree vertex in $D((u, v)G)\}$ and x be the vertex adjacent to w in $D((u, v)G)^{\lambda}$. Then $d(D(u,v)G)\lambda(v_i) = d_{(D(u,v)G)}(v_i)$ for all $v_i \in N(x)$ and $v_i \neq y$ and $d(D(u,v)G)\lambda(w) = \Delta(D(u, v)G) = d_{(D(u,v)G)}(w)$. This implies that $D((u, v)G) \cong D((u, v)G)^{\lambda}$. Hence λ is a weak 2 −vertex duplication self switching.

3. Conclusion

In this paper, we have discussed about path, minimum and maximum of 2-vertex duplication self switching graphs. Further we are analyzing more graphs.

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