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# A Truth Logic of Modalities Consistent and Incompatible: A Computational Program and Experimental Studies among College Students

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## ABSTRACT

College students give true judgment routinely, logicians can infer modal inferences, i.e., inferences that concern three alethic modalities; necessary (⊃), possible (◊), and impossible (✧) from the premises that are logically consistent and incompatible. To achieve the desired results, logicians have intermingled alethic modalities with logical consistency and incompatibility, i.e., necessary consistent, impossible incompatible, possible consistent, or possible incompatible, and write them in classical modal logic as ⊃C (necessary consistent), ◊C (possible consistent) or ◊I (possibly incompatible) and ✧I (impossible incompatible). M paradigm [Modality, Mental logic theory (MLT), and Mental model theory (MMT)] have been built to enlighten this truth logic. The basic idea of this study is to examine the theory; how do students judge whether two or more different propositions are possible? and, whether their judgment is true. Truth logic is used to construct some principles that help to justify the above theory. First, inferences have either ◊C/ ◊I or ⊃C but assertions are consistent. Second, each ◊C/ ◊I inference and premise is evaluated for both single and double model assertions, and they have consistency (i.e., ⊃C/◊C) and incompatibility (i.e., ◊I or ✧I). As logicians have predicted, students mostly endorse inferences as ◊C/ ◊I rather than ⊃C and the ✧I rate is higher in multi-model assertions. Syllogistic logical reasoning with conditionals (if, then...), conjunctions (and), disjunctions (or), and quantifiers "all" and "some" have been used in the M paradigm to evaluate predictions. Moreover, a computational program and experimental studies have strongly supported the all given principles and predictions.

## INTRODUCTION

Logic is often personified as a principle of true reasoning. It is widely used in "mathematics" as a toolbox (for example, probability, propositions, and proof calculations) or as a rule of interdisciplinary research (that is, what logic does). This study used two or more propositions explaining truth logics have consistency to incompatibility. However, there are many different ways of truth logic using propositions, and many logicians have different agendas; these multiple claims are controversial and further need to investigate. Most often, logicians study a single phenomenon of propositional truth logic as a traditional logic proposition that is divided into descriptive and normative dimensions. In this study, logicians investigated two or more proportional truth logics. In terms of norms, logical processing is applied to responsible belief maintenance strategies for real cognitive subjects, while logical description materials deal with Alethic relationships defined according to the purpose of the true belief with consistency, validity, and effectiveness. This is an option on how to determine the characteristics of Alethic logic because we have to summarize not only descriptive and normative topics of a single proposition but also define whether two or more propositional truth logics are consistent or incompatible. The truth modality "Alethic" translates as necessary, possible, impossible, and contingency (Sambin & Valentini, 1982). Alethic as truth logic in the form of modality is most frequently studied in modal logic (Blackburn *et al.*, 2001; Chellas, 1980; Fitting & Mendelsohn, 2012; Gabbay, 2012; Garson, 2013; Girle, 2014; Kracht & Kracht, 1999; Rönnedal, 2015; Wright, 2007). Modal logic affects probability which can be closest to the possibility (P. N. Johnson-Laird & M. Ragni, 2019). Using the available data, students use every day reasoning to determine if certain consistent hypotheses are possible or necessary (Johnson-Laird *et al.*, 1999; St BT Evans *et al.*, 1999). Logicians consider here every day conditional example;

It's necessary that if I fall into the river, I will get wet.

It's possible that if I fall into the river, I will get wet.

Alethic logic works with truth-based possibilities and necessities, and when taking into account the available data of college students, deductive implications are legitimate since its inferences are taken into consideration together with the validity of its premises. Standard logic makes inferences about what the beliefs are. The mental logic theory (MLT), however, asserts what is a necessary belief or what is a possible belief. Most logicians investigate the reasoning regarding consistency in their experimental research among people using simple inferences about what is belief. This study's experiments have taken data from college students' beliefs.

It is important to look at whether views that are founded on more than two propositions are coherent. According to the consistency mental model theory (MMT), beliefs are said to be consistent if the entity of inferences and provided propositions are true.(Legrenzi *et al.*, 2003). The

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proposition draws an inference that later retracts beliefs and converts them into ones.

Assume the following students' beliefs: It's possible that if they fall into the river, they will get wet. And When they fall into the river.

They believe, "they'll get wet." The deductive inference has valid logic; that is, the inference of this belief (they'll get wet) must be true if its premise (if they fall into the river) is true. Suppose, "If they try to fall into the river they may be dove, swim or/and, all/some die." So, it has been perceived other propositions that justify the incompatibility of consistent invalid inferences. Logicians have necessity and possibility in their inference that "they will get wet". But, to hold facts with inferences incompatibility with beliefs is a property of irrationality. Students will possibly try to logically rationale their way to consistency and may have incompatibility in their beliefs about either they fall into the river or about possible inference, "they will get wet and/or, all/some dove/ die." This assertion is an example of human reasoning. Students try to describe the incompatibility between the consistency of true propositions and inferences. Students are either asked to measure the validity of inferences or to generate the inferences on their own. Similarly major conflict occurs in research of physics "relativity theory" and "quantum theory" which has shown incompatibility in their consistent beliefs (Granger & Greene, 2000). Time is constant on the earth under the gravitation force, but if the person moves out of the gravitation, force-time will dilate. Thus, students try to describe the incompatibility between the consistency of true premises and inferences. Logic is monotonous. Future information cannot invalidate previous valid conclusions. Caret (2019) defined that logical theory has occupied a true bond with Alethic in terms of consistency in its belief, which is valid and rational. Alethic logic continues with each additional true premise the inferences drawn from the propositions are valid and consistent. But, if the premise denies the earlier inferences, there is incompatibility or contradiction, but logically speaking, contradiction implies any inference. Therefore, logic never asks to withdraw the inference. Some logical formulas and some theories of psychological reasoning include MLT or formal reduction rules (L. Rips, 1994). This rule defines if Logicians are right for beliefs, may prove incompatibility, and then they have the right to deny this assumption. When Logicians are being inferred to validate the contradiction caused by only one assertion is false, however, the rule cannot determine which premise has reprobated Morality in logic that it allows Logicians to find incompatibility and practice it to produce additional inferences, but certainly, not appeals to withdraw an inference.

A particular nature of logical reasoning which retracts inference is called non-monotonic (Genin, 2019). The MMT in the logic would consider non-monotonic (Todorovikj *et al.*, 2021). In this belief, students can withdraw a new alethic interpretation of inference based on succeeding facts. On the other hand, they can get the inference in the light of a hypothesis that they consider to be true by faulty, such as in the above example students believe that they will get wet when they fall into the river but when they think to fall, they learned more possibility (they can die, dove, swim, etc.). So, morality is that we can withdraw or retract our inferences based on true logical reasoning. As (Johnson-Laird *et al.*, 2004) remarked that some logical inferences are non-monotonic you force to retract them, having no option, and tend to revise your belief (Khemlani *et al.*, 2014). Logic could also be not specified with propositions which one can retract or which premise is true—this dichotomy of propositions and their inferences present incompatibility between true logical beliefs. Researchers have also allowed retracting their beliefs in light of incompatibility (Constantin, 2018; Craig, 2000; Shtulman & Young, 2019).

Truth logic defines three alethic modalities; necessary (logically consistent), impossible (logically incompatible), and possible (logically consistent but also incompatible). Logicians measure the inferences regarding what is a necessity and what is a possibility within deductions of syllogistic reasoning. Sandu and Tanninen (2017) defined the concept of Wright (1951) regarding Alethic logic who developed his method of truth-table that either expressed model assertions of truth logic or not. According to Von Wright, those truth logics have been dependent on the specific nature of logical model connectives [such as $\lozenge R \wedge \supset (R \rightarrow S) \rightarrow \lozenge S$ or *necessary ($\supset$) and possible ($\lozenge$)] (Wright, 1951).

Suppose; Possible R and Necessary (R → S) → Possible S or It is possible that fast car driving causes accidents and it is necessary that if I drive fast then, I cause accidents. So, it is a possibility of an accident.

The present study inspired by Wright (1951) connectives (necessary, possible, and impossible) with logical consistency and incompatibility such as necessary consistency (Omnes, 1988) impossible incompatibility (Ratti, 2018), possible consistency (Miori *et al.*, 2010), or possible incompatibility (Thibault, 2014), wrote them in classical modal logic as ⊃C (necessary consistent), ◊C (possible consistent) or ◊I possibly incompatible) and ⟡I (impossible incompatible). M paradigm [Modality, MLT, and MMT] have been built to enlighten this logic. The main idea of this study is to examine the theory; how do students judge whether two or more different propositions are possible? and, whether their judgment is correct. Truth logic is used to construct some principles that help to justify the above theory. First, inferences have either ◊C/ ◊I or ⊃C but assertions are consistent. Second, each ◊C/ ◊I inference and premise is evaluated by both single and double model assertions and they have consistency, i.e., ⊃C/◊C, or incompatibility i.e., ◊I or ⟡I. As logicians have predicted that students mostly endorsed inferences as ◊C/ ◊I rather than ⊃C and the ⟡I rate is higher in multi-model assertions. For theory justification, they present "M" the paradigm (see Figure 1) with the implementation of a computer program and psychological views, and then they assess theory through experimental studies among college students in the light of given principles and predictions. Finally, based on the result, they conclude.

## METHODOLOGY

### Computational implementation of Truth logic

A computational program is implemented to describe the truth logic consistently having if, and/or, and all/some incompatible assertions with the help of three main "M" paradigms; Modality, MLT, and MMT (see Figure 1).

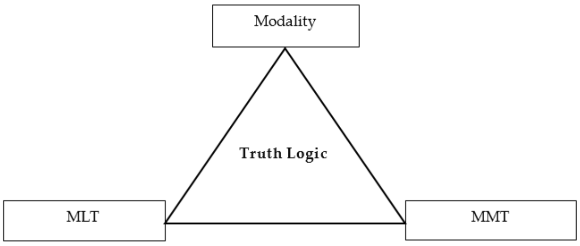


**Figure 1:** Alethic logic M paradigm

### Modality

To compute the principles (as below), students must detect modalities of Truth logic. The conventional Alethic modalities include; possibility ("Possible R", "It is possible that R"), necessity ("Necessary R", "It is necessary that R"), and impossibility ("Impossible R", "It is impossible that R") (Prior, 1962).

**Principal 1: Inferences have either ◊C/ ◊I or ⊃C but assertions are consistent.**

Logical modalities have consistency in modal operators such as;

It will be snowing tomorrow and It's possible that tomorrow may bring snowfall.

"Possibility" considers an operator in modal logic. It has incompatibility to describe necessity and possibility. In general, it knows as the fallacy of modal. The possibility of reasoning believes the fundament of daily life (P. Johnson-Laird & M. Ragni, 2019) and it's a core cognitive ability. Many Western countries describe modal as possible and necessary (Markovits & Barrouillet, 2002) and modal logics formalize these concepts (Chellas, 1980; Fitting & Mendelsohn, 2012). Modal operators are usually written as *necessary (⊃) and possible (◊)] (Wright, 1951).

Suppose; R presents the assertion "it will snow tomorrow" and the possibility of assertion would be presented as ◊ R means "it is possible that it may be snow tomorrow". Same as ⊃ R means "it is necessary that it will be snow tomorrow". Classical modal logic defines negation as;

◊ R ↔ ~⊃~ R or It is possible that it may snow tomorrow if and only if it is not necessary that it will not snow tomorrow.

Both assertions are consistent. Similarly, necessary can also be defined as possible and their symbols can be used as modal operators (Hughes *et al.*, 1996). "~" negation shows incompatibility in assertion which describes the second principle.

**Principal 2: Each possible inference and premise is evaluated as both single and multi-propositions assertions and they have consistency or incompatibility.**

To compute the second principle, firstly students process the truth logic from consistency to detect incompatibility among propositions. Students believe that incompatibility is a conflict between two propositions; one is R and the second is Not-R (considering the contrary and contradiction of R). P. N. Johnson-Laird and M. Ragni (2019) state that Alethic interpreting logic is considered constantly dyadic, such as they have concerned two possibilities' sets and the relationships among them. If the propositions are true their conclusion is true (Jeffrey, 1981). As a result, it appears that the assertion has the same validity as the preceding assertion; logically, it can be considered validated. Hence, human reasoning has concerns about possible inferences, semantic interpretation, antecedent causality, and its effects (Quelhas *et al.*, 2017). Truth logic has true consistency between propositions i.e.

It is possible that if I have a car accident, then I will be injured or It is possible that if I have no car accident, then I will not be injured.

Here, both possibilities are consistent, based on "if…., then……". But the second possibility is incompatible too. Semantically explains as;

R infers S, and so not R infers not-S.

The above assertion deals with the consistency of alethic propositional relationships. If the premise (R) infers the possibility of an inference (S), then the inference of a given premise is valid and true and not both R and S (called NAND connective) means R is incompatible with S. Deductive conclusion has been used in formal rules to explain incompatibility and it will be true when both of them false or when one of the propositions will be false, and the other will true (Rautenberg *et al.*, 2006). Additionally, assertions are incompatible if more than two propositions occur at the same time or condition. Between two propositions (R and S), if an inference S has followed only as possible, then the consequence yields possible S valid. So, a dyadic of alethic logic interpretation is the occurrence of a "sentential adverb," including 'necessary,' describing the modality of an inference. A necessary relationship is that R implies S that has shown consistency in the possible description if R then S. but if the alethic logic interpretation is occurred "impossible" in their consequence that has described incompatibility in the assertion. Its possibility refers to these given possibilities: R S, R ~S, and ~R ~S. Here, '~' signifies the negation of the clause. The relationships of these given possibilities can be explained in other truth logical ways:

Suppose,

If an inference R only follows as a possibility, then the consequence yielding possibly S is correct.

R holds S necessary (R S) (logical consistency)

The statement R holds S impossible (R ~S)(Incompatibility)

This statement has equivalent to R develops not-S necessary. And, the connection:

R holds S possible (~R~S) (Truth logic from consistency to incompatibility)

This statement explains without R, S does not exist, although it can refer to all four possibilities. So here, three possibilities are discussed operators R ~S, and ~R ~S. Truth logic's modal is necessary or impossible as the negations can create equivalence, which is hard to grapple with: if an inference is necessary the with premise, then it does not consider the possibility impossible and if it deems impossible then its possibility must be incompatible. Based on the above description, we can draw table 1 of assertions.

Hence, the truth table justified the main concept of this study. Moreover, the set of propositions is also incompatible when the proper subset has consistency, suppose; the Propositions' set has been three atoms, r, s, and t.

r or s, or both.

**Table 1:** Truth table modality represents Truth logic consistent having if, and/or, and all/some incompatible assertions

| Syllogism | Alethic logic | Consistency | Incompatibility |
|---|---|---|---|
| Possibility based on | Necessary (if then S) | Yes | No |
| "If…... then……" | Impossible (if R then ~S) | No | Yes |
| | Possible (if~R then~S) | Yes | Yes |
| And | Necessary (R and S) | Yes | No |
| | Impossible (R and ~S) | No | Yes |
| | Possible (~R and~S) | Yes | Yes |
| Or | Necessary (R or S) | Yes | No |
| | Impossible (R or~S) | No | Yes |
| | Possible (~R or~S) | Yes | Yes |
| All | Necessary (All R S) | Yes | No |
| | Impossible (All R ~S) | No | Yes |
| | Possible (All~R ~S) | Yes | Yes |
| Some | Necessary (Some R S) | Yes | No |
| | Impossible (Some R ~S) | No | Yes |
| | Possible (Some~R ~S) | Yes | Yes |

*Note: Here, capital letters R and S explain simple assertions, "If…... then……" is conditional "and" operations are conjunction, "or" operations are disjunction, "all/some" quantifiers and symbolized~ indicates negation.*

Not-s or t, or both.

Not-r and Not-t.

All propositional pairs have consistency; however, propositions together have incompatibility. Incompatibility is considered intractable (Cook, 1971) which is a greater demand of memory and time as its atoms' number increases the set of propositions. This demand increases day by day, but there is still no feasible computation system that yields results. The proposition of small scale has been investigated psychologically, computationally, and experimentally. The prior researcher did not address the multi-propositional question as stated in this research; how do students judge whether two or more different propositions are possible? and, whether their judgment is correct. Further, Mental logic theory and mental model theory have justified these research questions.

**Mental Logic Theory (MLT)**

To justify research questions, logicians further compute mental logic Theory (MLT). In earlier works of reasoning, psychologists have relied on orthodox logic (i.e., Feigl, H.,1970), and they have searched to understand that the mind formulizes logic. They had no awareness of system axioms that's why they joined assumptions with some rules of inferences akin to theoretical proof for propositional calculus (Bloom & Suszko, 1972; Sandqvist, 2015; Zhan, 2022). The founder of mental logic L. J. Rips (1994) defined MLT as close to orthodox logic. We implemented this theory into a computer program that's called Psychology of Proof (PSYOP). Truth logic propositions are inputs of PSYOP. Its natural language relies on single and multi-propositions and rules of inference. Truth logic exists in the assertions of inferences to evaluate the rule of whether the assertions are; necessarily consistent, impossible incompatible, possibly consistent, or possibly incompatible. Truth logic as alethic logic defined such logic as if in the rule of belief something is possible (consistent/ incompatible), it means it is true in one possible world at least, but if something in the belief has necessary (consistent), it's mean that it is considered to be true in all possible worlds. This study investigated the possibility of whether two or more different propositions are consistent.

Truth logic produces new rules of MLT with additional consequences. The basis of MLT is the iteration of the modal operator; there is countless evidence of infinite numbers because, in ⊃C ⊃C possibility, it confirms the essential probability, but maybe any necessity is ⊃C. The modality in everyday life rarely presents propositions: Heap a bunch of modal operators just for emphasis formal rule, for example:

It is possible that it may flood.

The MLT designs the rules of 3 modal operators (⊃C, ◊C/◊I, and ✧I) eager for 2 (⊃C, ◊C/◊I) but not legal to fold in 2 modal operators (⊃C, ◊C/◊I) into 1 (⊃C). This principle can also extend at random by simplifying an operator to n-1 is legal, but not in addition, wherever n has a natural number. The inference has counted infinite sets of diverse MLT because previous researchers focused on MLT as necessary not possible.

The MLT explains PSYOP approaches to the rule of expression exist in a possible world. Mainly if a logic proposition is possibly consistent (◊C), it means that it may be a rule to happen in a possible world (PW) as seen in example 18. Possible incompatible (◊I) means that there is at least one PW who has the rule of assertion true, i.e. If it is possible that it may flood in Myanmar, then there is at least one possible world in which it may flood there. As a result, Alethic logicians took a series of PW, usually including the real world, so that the possibility is true, the possibility is true in at least one world PW, and PW corresponds to the possible members of the world.

One sort of the rule as above if R then S; R; therefore S, allows students to judge from single and multi-propositions. A formal rule that inclusive deduction R; therefore, R or S, or both. Where S can be applied premise to its inference. This belief yields as; therefore, (R or S, or both) or T, or both. It can be applied to this inference too, so that it's an exclusive deduction. PSYOP approaches the rules. It's demoted to the 2nd set's rules, which can only be performed to infer premises from an assumed inference. Although the idea forbade students from drawing their inferences, it was the pinnacle of descriptions of everyday reasoning based on MLT. One sign of impending issues was the following rules, which are valid in Alethic logic for the condition;

It's not the belief that if R then S. Therefore, R and not S. PSYOP excludes these rules as it includes only those who accept this rule. Students mostly do not endorse this denial condition. If R then not S. Since PSYCOP, it has been obvious that the notion that common sense inferences depend on alethic logic has multiple possible incompatibilities. The first is that any set of premises can lead to infinite valid inferences. The second incompatibility is that alethic logic never suggests that a valid inference should be withdrawn given any premises, not even those that contradict one another. Consider the following propositions, for instance:

The elected Prime minister has no permission to own a home.

Trudeau is an elected Prime Minster

So, the truth logic with common sense suggests the inference: He has no own home.

So, let's say that didn't occur. Now, truth logic and common sense disagree. Nothing is logical. The inference is in contradiction with reality, yet logically, a self-contradiction consistently necessitates any inferences at all. Because more premises lead to more inferences, here truth logic is monotonic. However, it never consistently necessitates the retraction of inference, not even when evidence contradicts it. In contrast, common sense dictates that one should abandon the inference, reconsider the premises, and seek a description that resolves the conflict. So, human reasoning is non-monotonic: additional assertions might result in the withdrawal of prior inferences and the modification of assertions. Defeasibility is embedded into the model theory, according to certain theorists (Stenning & Van Lambalgen, 2012).

The third issue is the ⊃C of a group of beliefs, i.e., whether their judgment may all be true at a similar time. Students tend to reject conflicting claims once they become aware of the incompatibility: at least one of them must be falsity (◊I). There are rules for proving inferences in logic, but it is not immediately evident how to use them to evaluate the ◊C of a set of beliefs. The set is ◊I if the denial of one statement in the set derives from the other statements. Otherwise, the collection is ◊C after an extended but futile search for proof. The technique appears ✧I in the real world. Yet investigations demonstrate that, according to the hypothesis, students mostly endorse inferences as ◊C/ ◊I rather than ⊃C and the ✧I rate is higher in multi-model assertions. A straightforward approach exists for determining how students judge whether two or more different propositions are possible and, whether their judgment is correct. However, the implausibility of truth logic in everyday reasoning has not produced deductions' simulation from ordinary claims as opposite to their truth logical forms. Logicians can rely on the implied mental logic, but another theory holds that it is a spiritual simulation of possibilities (Khemlani *et al.*, 2014) in the world isomorphic model (Johnson-Laird & Khemlani, 2017) as defined as MMT has discussed below in detail. Additionally, syllogistic logic has been originally developed by the model theory, such as

Some artists are rich people,

All the rich people are artists,

Therefore, some baggers are artists.

Here, each sentence has a quantifier "all" or "some". This shows that students adopt diverse approaches for deal with syllogistic logic (Johnson-Laird & Bara, 1984). So, the MMT has been extended to include more than one quantifier (Johnson-Laird *et al.*, 1999) and propositions. Let's say:

There are three thieves: Jill, Joe, Jack,

All Loves Gold,

Someone loves gold more than all, Jill stole gold,

What follows? Students can parse such a model. Here, arrows show the love and desire in the figure 2.

The inference is that everyone loves gold. But one person loves gold more than others. Jill stole the gold. So, we can follow whether Joe loves gold or Jack loves gold but just because Jill stole the gold, he loves the gold more than others. Although we can infer that Jill has more power

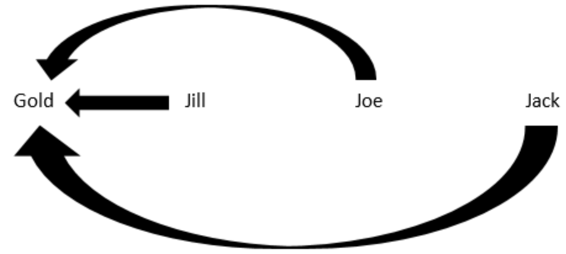


**Figure 2:** Multi-Model Quantifiers

or technique to steal the Gold or Joe loves gold but has no power or Jack loves gold more than others but he has no technique to steal gold. There are other possibilities to justify inferences. Thus, no completed model theory exists for results that are quantifiers and propositions (Bara *et al.*, 2001) but the theme of the theory can apply. Mental models of possibilities represent what is true only.

### Mental Models Theory (MMT)

Hinterecker *et al.* (2016) stated that the MMT, often known as the "model theory" because of its abbreviated version, accounts for what is computed in reasoning: the authors explained that for deduction and maintenance of semantic information, to shorten, and to reach a novel inference. It's also necessary for how the computational method is conducted, predicting that inferences are supported on models of possibilities' sets (Johnson-Laird, 1983; Johnson-Laird, 1994), uses deductive reasoning for maintaining semantic connectives, simplifies them, and makes new inferences.

Mental models (MMs) are as iconic as possible; that is, their stages accord to the stages they characterize, and their configurations accord to the configuration they signify (Peirce, 1974). Visual images (VI) are also iconic, and MMs can occur as VI. Though, logicians can have MMs of abstracts proposition, For example:

The elected Prime minister has no permission to own a home.

As the MMs of an abstract logic among proposition, logicians can think about the Prime minister and a home. But not just the image may indicate negation, consent, or rights of a Prime minister. The iconic mental model has one of the hallmark advantages - The advantages that for logics (Alethic logic), Pierce takes advantage of in his diagrammatic system - logicians can practice some suggestions building the model and after that practice the model to appeal to a

**Table 2:** An Exclusive disjunction's the Truth Table among two propositions: spade (♠) and club (♣).

| **Spade (♠)** | **Club (♣)** | **Spade (♠) or else club (♣), but not both** |
|---|---|---|
| Truth | Truth | False |
| Truth | False | Truth |
| False | Truth | Truth |
| False | False | False |

*Note: "spade (♠) denotes R here and club (♣) denotes S.*

new inference that cannot match any of these statements. Suppose the following statement explains "xor" in which only one clause is true between two:

There is a spade (♠), or else there is a club (♣)

Statements are often used to express theorem. For ease, students practice standard terminology in the logic propositional theorem referring to statements' use. Students practiced another standard word "atomic proposition" referring to a statement that does not contain any negation or connective. For example, the previous statement contains two atoms: one with a spade (♠) and one with a club (♣).

### The Model Theory of Alethic Logic Depends on the Truth Principle Which

"Naive signify propositions with the construction of mental models' sets. Each model stands for true possibilities. The proposition of each clause, either atomic or negation, only when it will be true and ◊C or ◊I, then it represents in MMT". This principle is subtle. Students can grasp it easily by considering descriptive examples. The "xor" mental model (MM), "There is a spade (♠), or else there is a club (♣)" represent two possibilities (such as ◊C or ◊I). Only when they are alethically true within possibility then one of them is true, ◊C or ◊I. We describe these two mental models (MMs) through figures in which respectively a row defines a divided model of possibilities:

~♠
    ♣

Here, the "~" symbolizes negation, "♠" symbolizes a MM presence of spade, and "♣" indicates a MM presence of club. The first MM is not explicitly stated it's falsity in this ◊I. It is a club; the second model suggests that no spade is false. The possibility is that there is a spade. The argument is "mental footnote" to track errors or fallacies and incompatibilities, but they quickly forgot these footnotes. logicians have ensured to keep up footnotes, but be able to use them and enrich them, MMs, to fully explicit models (FEM), so the principle of truth can be overcome. Previous exclusive MMs disjunction "xor" can be supplemented to complete the following operations model:

~♠ ~♣
♠ ♣

These models also correspond to bi-conditional theorems; if and only if there is not a spade, then there is not a club. But when most students feel uncomfortable with disjunction contact, they don't understand ◊C because they trust the main MMT. ◊I/◊C MMs are elementary. For one there, if there is a spade then there is a club. The MMT ◊C represents the preceding ◊C clause (with a spade) is true, and the ◊C/◊I possibility of falsehood is represented by a whole implicit model ⊃C (here called ellipsis):

♠ ♣
. . .

Logicians must keep the above footnote in mind when the ◊C/◊I possibility proposed by the implicit model is

**Table 3:** MMS and FEMs for the Main Sentential Connective

| Connective | MMs | | FEMs | |
|---|---|---|---|---|
| ♠ and ♣ | ♠ | ♣ | ♠ | ♣ |
| ♠ or else ♣ | ♠ | | ♠ | ~♣ |
| | | ♣ | ~♠ | ♣ |
| ♠ or ♣, or both | ♠ | | | ~♣ |
| | | ♣ | ~♠ | ♣ |
| | ♠ | ♣ | ♠ | ♣ |
| If ♠ then ♣ | ♠ | ♣ | ♠ | ♣ |
| | . . . | | ~♠ | ♣ |
| | | | ~♠ | ~♣ |
| If and only if ♠ then ♣ | ♠ | ♣ | ♠ | ♣ |
| | . . . | | ~♠ | ~♣ |

*Note. "~" denotes negation, and ". . ." denotes implicit model wholly. spade (♠) denotes A here and club (♣) denotes B. All rows represent MM of a conditional possibility.*

false, by retaining this footnote, they can fully constitute MMs, impossible incompatible (⟡I) explicit model:

♠ ♣
~♠ ♣
~♠ ~♣

A bi-conditional MM says that if and only if there is a spade, then there is a club, the same as the ◊C club as above. The only difference is that the MM footnote shows this. The propositions and their inferences are a fallacy in the possibilities expressed by an implicit MM. Table 2 briefs the FEMs with the basic model of sentential connectives. However, these explanations are influenced by two factors: semantic connectives and pragmatic factors (Evans & Over, 2013; Garnham & Oakhill, 1996). Logicians explained the procedure by using these factors which can adjust the module's possible explanation (Johnson-Laird *et al.*, 2004). They can form a model with additional steps in sequence, but it can also be built as a model of possibility.

The table detected ⊃C and ⟡I among propositions' set and tempted thinking ⟡I as an encounter among just two propositions, ⟡I can occur in propositions' set in which any proper subset is ⊃C. It should be looked into further. Logicians can psychologically write a computer program; however, it can also be based on MLT inference rules. First of all, three stages of the computational program have been established, as shown below with a flowchart (fig: 3).

**Stage 1**

The first program is used according to model ⊃C. It evaluates whether the set of propositions is consistent or

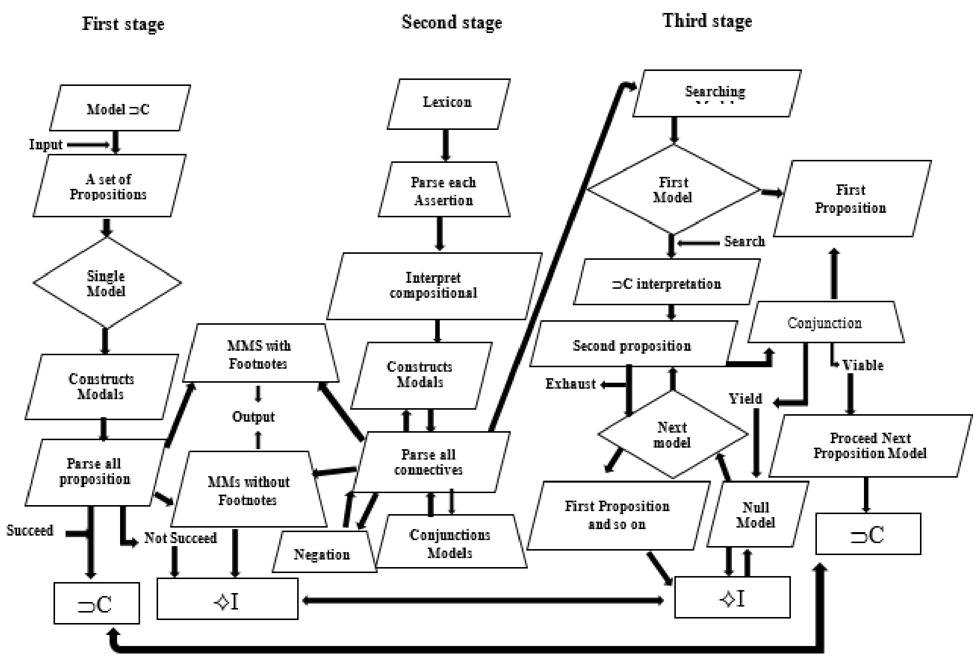


**Figure 3:** A flowchart of computational three stages program.

not. This program input a propositional set and searches for a single MM that satisfy each propositional set. All the propositions are true base propositions. Then, it constructs the model to parse all the propositions in two ways: MMs with footnotes and MMs without footnotes (Holyoak, 2012). If the model is succeeded, it is concluded that the propositions are consistent; but if the results are not succeeded its means there are ⟡I in assertions.

Moreover, MMs without footnotes carry out a process in a simple way which represents the assertions that have main connectives (see table: 3). The MMT of Alethic logic: if there is a spade (♠)), then there is a club (♣), are as followed:

♠ ♣

. . .

Here, ellipses are denoted as an implicit mental model (IMM) for possibilities. On the other side, MMs with footnotes consider expert or advanced level. It is used for a fully explicit mental model (FEMM) that makes no fault (Johnson-Laird, 2012) (for example; see table: 2). These two levels are produced output of given assertions.

Considering examples with more details, firstly this program inputs a set of propositions containing "if," "or," "and," and negation (see table: 2).

There is a ♠ xor, not a comma there is a ♣ and there is a ❤.

There is a ♠ and not there is a ♣.

Here, "❤" presents the Heart and an atomic proposition presents a program of affirmative clauses: xor denotes exclusive disjunctions (see table: 2), "not" denotes negation, which is preceded negate clause, and "comma" denotes the left parenthesis. Thus, two propositions are evaluated as follows:

♠ xor not [♣ and ❤]

♠ and ♣

**Stage 2**

This stage presents the Lexicon program base on assertions that produce models. The Lexicon program parses each assertion and interprets compositional connectives as explained in Table: 3. The compositional connectives construct models containing grammatical assertions. The clauses of connectives interrelate with connectives themselves and recurse the program of constructive models. The compositional process parses for negation and makes conditional models. This process can deal with each sentential conditional; presented in Table 3.

The operator of~ of MM set is its adjunctions from all possible models (◊I/◊C) established with the same atoms, such as considering exclusive disjunctions of the form (see MM)

Not ♠ xor ♣

Its FEMs are as charts:

~♠ ~♣

♠ ♣

This program recovers each model of the propositional set that is present in the set (♠ ♣) to negate this set. So, it constructs all possibilities (⊃C, ◊I or ◊C and ⟡I) of a given set.

♠ ♣

♠ ~♣

~♠ ♣

~♠ ~♣

For disjunction each atom of the propositional set returns that is not in the model.

♠ ~♣

~♠ ♣

The process relies on FEM, so to define the MMs set, students lack to extend their MMs in FEM. As the task is difficult, students can't imagine the possibility that all but the simplest statements are falsity. Table 4 lists the process for forming MM conjunction and FEM pairs.

Call for the first two processes, only MMs can be implicit. Each process has accompanied by examples.

The MMs of the proposition,

**Table 4:** The process for forming conjunctive Models pair

| |
|---|
| 1. The conjunctions of Models pair, one proposition ♣ in one MM does not represent in another MM, it depends on the model's sets whose other MM is a follower. If ♣ appears in only one of these MMs, then the absence of ♣ in the present MM is considered negative such as ~ ♣ and it is equivalent with its affirmation. The next procedures then apply (♠ ♣ and ♠ yields ♠ ♣). |
| 2. A model represents propositions with the conjunctions of implicit MM yielded the null MM with default: such as:<br>. . . and ♣ yield nil.<br>The conjunction yields the propositional models if none of the propositional connectives (♣ ❤) represent in MMs sets contained the implicit MM: such as<br>. . . and ♣ ❤ yield ♣ ❤.<br>Same as<br>. . . and . . . yield . . . |
| 3. The conjunctions of Models pair contain: "atomic propositions" and "negations" yielded the null MM, such as<br>♠ ♣ and ~ ♠ ♣ yield nil |
| 4. The null MM in conjunction with another MM yielded the null MM, such as:<br>♠ ♣ and nil yield nil |
| 5. The conjunctive pair of FEMs free from inconsistency updating another MM with all the new propositions from the first MM: such as:<br>♠ ♣ and ♠ ❤ yielding ♠ ♣ ❤. |

*Note: Spade (♠) denotes A here, Club (♣) denotes B, and Heart (❤) denotes C. Source by (Johnson-Laird, P. N. 2012)*

♠ xor not comma ♣ and ❤ are

♠

~♣ ❤

♣ ~❤

~♣ ~❤

The MMs of the proposition, Spade and not Club, is

♠ ~♣

**Stage 3**

This program holds all propositional connectives searching for a model. This program starts searching the first MM of the first propositional connective investigate the ⊃C interpretation with the second propositional connective. Specifically, it navigates each MM of the second propositional connective and uses Table 4 to form a conjunction with the MM of the first propositional connective. If the conjunction of a MM is viable it shows that it can proceed to the next propositional MM and so on but it can't return towards null MM. Although if the conjunction of a MM tends to yield the null MM then it will try the next propositional MM of another propositional connective. However, when it exhausts the entire second propositional connective, it will try the next propositional MM of the 1st propositional, and so on. If the 2nd MMs are -◇I, then their conjunctional yields the null MM (Table 4 Process 3rd above). In MM, the absence of a propositional connective is considered as corresponding to its denial (Table 4 Process 1 above). Thus, the conjunctional 1st propositional MM of the 1st propositional connective with the 1st propositional MM of the second propositional connective yields a ⊃ C interpretation:

♠ ~♣

In addition, this program yields the inference in which the propositions are ⊃C considered IMM. At its expert level, this program uses a fully explicit mental model (see stage 1 in detail). The fully explicit mental model represents the possibility of what is true or false. The FEMs case of the first propositional connectives is as under:

♠ ♣ ❤

~♠ ♣ ~❤

~♠ ~♣ ❤

~♠ ~♣ ~❤

Here, this computational program describes all conjunctions pair's models of second propositional connectives yield the null model. Thus, the program parses the contrary assertions which makes the MM when it uses the two propositions are -◇I. Why because students working memory (WM) has a limited capacity to process and MMT suggests that if the task is harder, it should construct the greater model's number. It must be difficult to consider backtracking and must be used as an alternative model of a prior propositional connective. But the output has been revealed by the computational program.

The model theory has also predicted surprises, such as illusions of ⊃C (considering ◊C) can occur when the expert level is opposed to performing MM for students who rely. For example, in simple beliefs, students judge propositions' set as ⊃C but in fact, propositions' set is -◇I. This computational program has also predicted the -◇I rate is higher in multi-model assertions and students mostly endorsed inferences as ◊I or ◊C rather than ⊃C. Hence, a computational program and psychological views strongly justified the theory, principle, and predictions. Furthermore, logicians collected experiments studies among college students to validate all given principles and predictions. A large experimental research program was conducted by MMs theorists demanding aid for their theory predictions (Johnson-Laird *et al.*, 1992). Such as, Johnson-Laird and Bara (1984) conducted a study of large-scale syllogism reasoning, and explained the methodology of production tasks. All possible propositions of the classical syllogism's forms (see as under) were offered, and students were enquired to draw inferences.

Logicians have analyzed these pairs of propositions based on whether they have ⊃C among one, two, or three models. As predicted, multi-model propositions have a higher -◇I rate. The psychological principle here is that considering multi-models puts pressure on working memory ability. Especially, students frequently have illusions of ⊃C (considering ◊C) because they cannot support a counter-example MM in stage 3, although such a MM exists. Few findings support the MMT, although, rule theorists dispute the theory (Evans *et al.*, 1993). Theorists argued that It has considered difficult to choose between more than two propositions and these propositions are not fully specified in theory (Evans & Over, 2013). For this reason, logicians have tried to test predictions that follow the principles. Computational results are qualitative data needed to justify quantitative studies' results. As the student's judgment regarding necessary consistent (⊃C), impossible incompatible (-◇I), possible consistent (◊C), or possible incompatible (◊I) inferences are driven below.

**Assessments of the Theory**

The modal logic made a distinction between truth logic with consistency and incompatibility such as necessary consistent (⊃C), possible consistent (◊C), or possible incompatible (◊I) and possible incompatible (◊I). If the assertion is Alethic, it is true factual; if the assertion is ⊃C it must be true, and if the assertion is ◊C/ ◊I it may be true. To study these concepts following alethic propositions are as under;

1. If Abdul Jabber is a cager then, he is tall. (⊃C)
2. If Abdul Jabber is a cager then, he is not tall. (◊C)
3. If Abdul Jabber is not a cager then, he is tall. (◊I)
4. If Abdul Jabber is not a cager then, he is not tall. (-◇I)
5. Abdul Jabber is a cager and he is tall. (⊃C)
6. Abdul Jabber is a cager and he is not tall. (◊C)
7. Abdul Jabber is not a cager and he is tall. (◊I)
8. Abdul Jabber is not a cager or he is not tall. (-◇I)
9. Abdul Jabber is a cager or he is tall. (⊃C)
10. Abdul Jabber is a cager or he is not tall. (◊C)
11. Abdul Jabber is not a cager or he is tall. (◊I)

12. Abdul Jabber is not a cager or he is not tall. (-◊I)
13. All the cagers are tall,
Abdul Jabber is a cager,
Therefore, it's necessary that Abdul Jabber is tall. (⊃C)
14. All the cagers are tall,
Abdul Jabber is a cager,
Therefore, it's possible that Abdul Jabber is tall. (◊C)
15. All the cagers are tall,
Abdul Jabber is a cager,
Therefore, it's possible that Abdul Jabber is not tall. (◊I)
16. All the cagers are tall,
Abdul Jabber is a cager,
Therefore, it's impossible that Abdul Jabber is not tall. (-◊I)
17. Some cagers are tall,
Abdul Jabber is a cager,
Therefore, it's necessary that Abdul Jabber is tall. (⊃C)
18. Some cagers are tall,
Abdul Jabber is a cager,
Therefore, it's possible that Abdul Jabber is tall. (◊C)
19. Some cagers are tall,
Abdul Jabber is a cager,
Therefore, it's possible that Abdul Jabber is not tall. (◊I)
20. Some cagers are tall,
Abdul Jabber is a cager,
Therefore, it's impossible that Abdul Jabber is not tall.

⊃C is a modus ponens inference that concludes necessity state, their propositions are true, indeed it's necessary that Abdul Jabber is tall, and this inference is valid. ◊C inferences are also valid, however as compared with strong inference ⊃C (it's necessary that Abdul Jabber is tall.), ◊C (it's possible that Abdul Jabber is tall.) would be invalid, why because their propositions have consistency with the situation where there are players who are not tall, so Abdul Jabber may be a cager who is not tall. ◊I am invalid as ⊃C shows, Abdul Jabber is necessarily tall given the premise and it's impossible that Abdul Jabber is not tall is completely invalid. Logicians refer to these states as necessary consistent (⊃C), possible consistent (◊C), or possible incompatible (◊I), and impossible incompatible (-◊I). Deductive reasoning standard guidelines require students to mark decisions about the validity of Alethic inferences.

**The Theory First**
How do students judge whether two or more different propositions are consistent?

**The Theory S**econd
Is whether their judgment is correct.
Logicians asked students to decide whether multi-propositions are ⊃C, where multi-propositions are assertions, and they must endorse the ⊃C inferences and reject the ◊C/ ◊I and -◊I. Conversely, if students are requested to choose whether multi-propositions is ◊C/ ◊I, then they would be endorsed ⊃C inferences as well as ◊C/ ◊I inferences and only reject -◊I inferences. The ability to distinguish ◊C/ ◊I from -◊I inferences is negligible in formal psychological investigations on reasoning.

Considering the three common stages of MMT as explained initially. Among the first two stages, logicians construct a single model of the propositions that supports a tentative inference. This inference can be accounted as ◊C/ ◊I except for any further logical reasoning. The inference validity stage (looking for counter-examples) works when students are requested to demonstrate that the supposed inference is ⊃C. Thus, according to the MMT, ◊C/ ◊I inference must be easiest than ⊃C.

**Prediction First**
Students have mostly endorsed inferences as possible consistent (◊C) or possible incompatible (◊I) rather than necessarily consistent (⊃C).

**Prediction Second**
Impossible incompatible (-◊I) inferences are higher than necessary consistent (⊃C) in single and multi-model assertions.
Logicians report three studies to design the test, the first two address theory and predictions, and the third one defines the issues that arise from the first two studies.
⊃C is a modus ponens inference that conclude necessity state, their propositions are true, indeed it's necessary that Abdul Jabber is tall, and this inference is valid. ◊C inferences are also valid, however as compared with strong inference ⊃C (it's necessary that Abdul Jabber is tall.), ◊C (it's possible that Abdul Jabber is tall.) would be invalid, why because their propositions have consistency with the situation where there are cagers who are not tall, so Abdul Jabber may be a cager who is not tall. ◊I am invalid as ⊃C shows, Abdul Jabber is necessarily tall given the premise and it's impossible that Abdul Jabber is not tall is completely invalid. Logicians refer to these states as necessary consistent (⊃C), possible consistent (◊C), or possible incompatible (◊I) and impossible incompatible (-◊I). Deductive reasoning standard guidelines require people to mark decisions about the validity of Alethic inferences.

**The Theory First**
How do people judge whether two or more different propositions are consistent?

**The Theory S**econd
Is whether their judgment is correct.
Logicians asked people to decide whether multi-propositions are ⊃C, where multi-propositions are assertions, and they must endorse the ⊃C inferences and reject the ◊C/ ◊I and -◊I. Conversely, if people are requested to choose whether multi-propositions is ◊C/ ◊I, then they would be endorsed ⊃C inferences as well as ◊C/ ◊I inferences and only reject -◊I inferences. The ability to distinguish ◊C/ ◊I from -◊I inferences is negligible in formal psychological investigations on reasoning.
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**Prediction First**

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Impossible incompatible (⟡I) is higher than necessary consistent (⊃C) in single and multi-model assertions.

Logicians report three studies to design the test, the first two addresses theory and predictions, and the third one defines the issues that arise from the first two studies.

In study one, Logicians tested the first theory, how do people judge whether two or more different propositions are consistent? and the prediction one by using the simple task of syllogistic reasoning. In this task, logicians give people a single premise of syllogistic reasoning and suggest that they make two or more different propositional inferences; this task is called the "immediate inference task" (see Newstead & Griggs, 1983). Logicians have used more than two different propositions to justify the task. Such as a possible syllogistic logic is:

1. All ♠ are ❤,
   Some ♣ are not ❤,
   Therefore, some ♠ are not ❤

Here, Spade (♠) denotes A, Club (♣) denotes B, and Heart (❤) denotes C. Above assertions show all three modal operators ⊃ C, ◊ C/ ◊ I and ⟡I. Instructions were presented to the people with yes or no responses which were either ⊃ C or ◊ C/ ◊ I with immediate inference propositions. That helps logicians to test the prediction one identified initially. Note that in study one, there is an inevitable confusion among the type of guidance ⊃ C or ◊ C/ ◊ I and the type of response that constitutes the correct answer. For example, since more than two different inferences are ◊C/ ◊I than ⊃ C, any general bias in accepting inferences will be led to more right responses in the possibility (◊) group than in the necessity

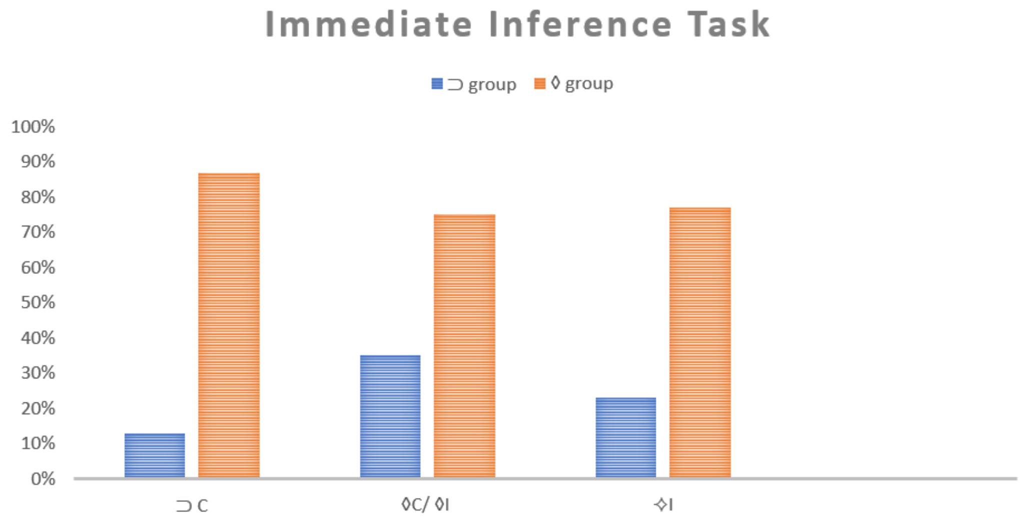


**Figure 4:** Percentage of Immediate Inference Task accepted in study one between ⊃ and ◊ groups.

(⊃) group. Logicians tested two groups ⊃ vs ◊ and three groups ⊃ C vs ◊ C/ ◊ I vs ⟡I. In groups ⊃ vs ◊, people responded 75% yes to ◊ C/ ◊ I inference and just 13% said yes to ⊃ C inferences. So, the results highly accepted the prediction one that people endorsed ◊ C/ ◊I more than ⊃ C. People responded to 23 % of ⟡I inference which is even higher than ⊃ C inferences. So, the results show that prediction two also accepted significantly as impossible incompatible (⟡I) is higher than necessary consistent (⊃C) in single propositions (see figure 4).

Study two tested the second theory: whether their judgment is correct. And given predictions. This study involves full syllogism reasoning and asks people to define each pair of ◊C/ ◊I inference regarding each ◊C/ ◊I premises through valid ⊃C judgments. These studies include groups of people required to develop judgments of ⊃C as well as ◊C/ ◊I inference. Both studies have been designed together by running on the same people in similar sessions. The subsequent procedure is as follows. people have divided into ⊃ and ◊ groups and completed the short tasks of study one with an appropriate instructional form. Each people then did the large task required in study two using the same form of instruction assigned to them in study one. These groups have also sub-divided in study two permitting instruction terms in the inferences they have assessed (see the methods select for study two for details). This evaluation task follows the repetition of Johnson-Laird and Bara (1984) by defining the four figures as under:

Figure 1 ♠ - ♣
♣ - ❤

Figure 2 ♣ - ♠
❤ - ♣

Figure 3 ♠ - ♣

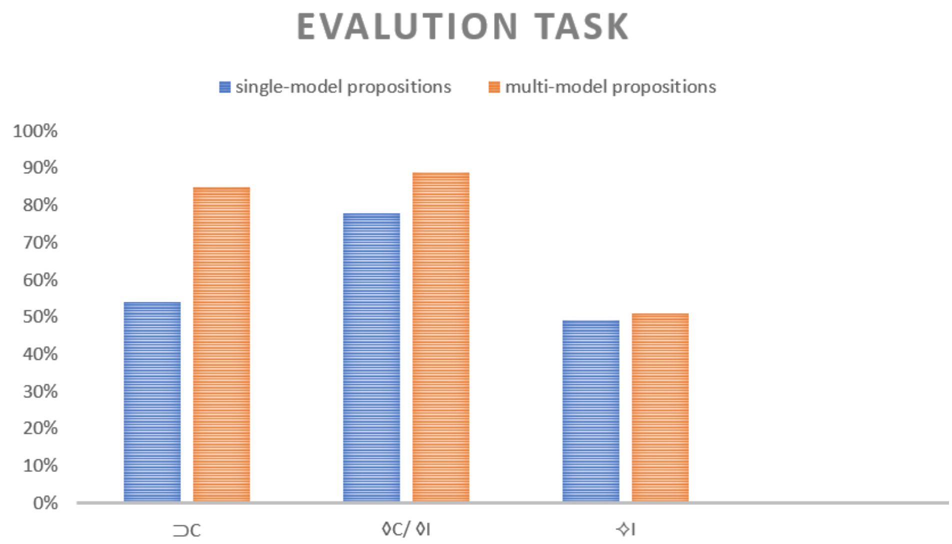


**Figure 5:** Percentage of Evaluation Task accepted in study two between single-model propositions and multi-model propositions.

❤ - ♣
Figure 4 ♣ - ♠
♣ - ❤

Here, Spade (♠) denotes R, Club (♣) denotes S and Heart (❤) denotes T. In an evaluation task, inferences are based on multi-modal propositions as the above figures have four possibilities of mood. People evaluate ♠ ❤ or ❤ ♠ order but not both. ♠ ❤ present ⊃ group and ❤ ♠ present ◊ group. In ⊃ group, people 85% responded yes in single-model assertions and 54% responded yes in multi-model assertions. As well accepted that ◊C/ ◊I rate is higher than ⊃ C as people 89% responded yes in single-model assertions and 78% responded yes in multi-model assertions. Moreover, logicians have computed the significant rate among both models and declared that multi-model propositions have highly significant rather than single-model propositions and multi-model propositions have strongly significant with ◊C/ ◊I rather than ⊃ C (see Figure 5).

Study third was designed to evaluate further validity of the finding and some fallacies created effects and need to design. Logicians have also reported study three that replicated some imported findings of study two by using quite smaller subsets of syllogistic reasoning with a simple method of propositions presentation. Study three is also provided a check of study two's results that are not influenced by study one's results. Moreover, both studies strongly supported the predictions. Study three separated possible consistency (◊C) and possible incompatibility (◊I) into two different subgroups that define as:

Possible consistent (◊C): Assertions whose inferences are ◊C but not ⊃ C gave their propositions, but that frequently endorse as having ⊃ C inferences.

Possible incompatible (◊I): Assertions whose inferences are ◊I then not ⊃ C assumed their propositions, but that rarely endorse as concerning ⊃ C inferences.

Moreover, ◊C syllogistic logic is the fallacy people are tended to commit (under ⊃ C instruction), and likely ◊I syllogistic logic is the fallacy they are tended to avoid. The two studies reported so far that the identity of these two sub-groups was necessarily retrospective. Therefore, Logicians have decided to take possible propositions

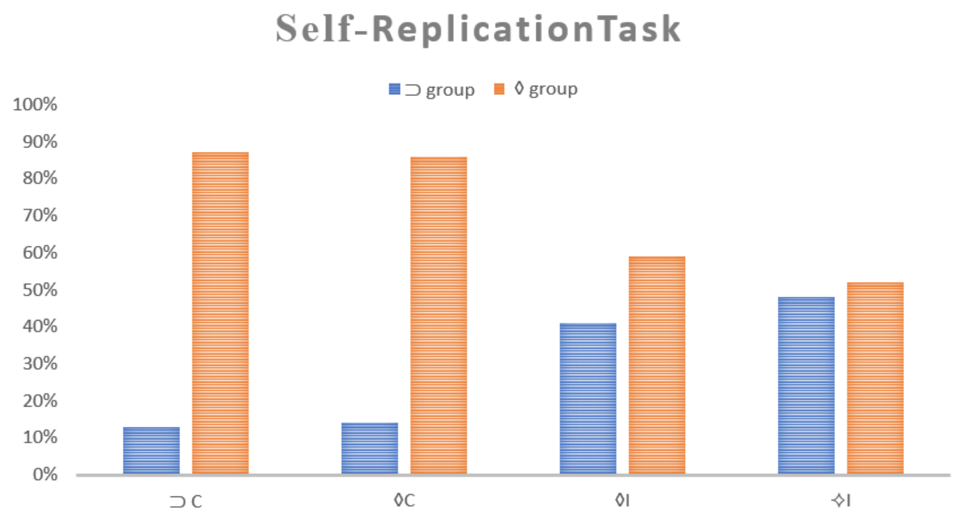


**Figure 6:** Percentage of Self-Replication Task accepted in study three between ⊃ and ◊ groups.

as great and less endorsing assertions in study two and yield replications of autonomous study three. Figure c shows the acceptance frequency of different logic groups under the necessity directive and the corresponding data from study two. As can be seen, study three provided an adjacent replication, eliminating in the least possibility that differences among ◊C and ◊I assertions were amplified by a retrospective selection of this sub-group. The situation in study three is again that the acceptance rates for ⊃ C (87%) and ◊C (86 %) assertions are very similar. ◊I (59%) assertions are likewise only accepted with a slightly higher frequency than -◊I (48%) assertions (see Figure 6).

Figure 6 compares the frequency of acceptance for diverse question groups under the Possibility directive and again compares the data from study two and study three. Again, good replication is seen, but there is a tendency for ◊I assertions to be more acceptable than ◊I assertions to replicate. Hence, it is noted that significant replication presents among both studies two and three (see Figure 7). The comparison with study two contradicts two possible criticisms of the earlier experimental design. Firstly, it can be argued that earlier knowledge contributing to study one has some influence on performance in study two; study


**Figure 7:** Percentage of Self-Evaluation Task accepted in study three between study two and study three.

three has no such prior task. Second, people arguably became tired or uninterested even though handling the big number of syllogistic tasks investigated in study two, which may have added noise to their statistics. Logicians consider this improbable to be true anyway because the self-evaluations provided as well as the examination show very strong and systematic tendencies. The fact that the statistics matching syllogisms were very similar in study three, while only a quarter of the syllogisms were used in study three, supports this idea.

**DISCUSSION**

Sometimes true beliefs create conflict in thinking and these beliefs generate inferences that are consistent or incompatible. truth logic with the help of its modalities not only accepts this challenge but also accepts the contrary facts of inferences. It also tries to come up with a reasonable diagnosis of what went wrong. Truth logical inferences, by contrast, are monotonic and cumulate: as logicians learn more propositions, they may be able to derive additional inferences. Can some inferences in the possible world be irrefutable and can they not be corrected? Inferences created conditional propositions such as opposed to necessary truth or axiomatic premise that may give guarantee constantly. Any inference derived from valid reasoning can be inverted by overwhelming suggestion to the contradiction. Skeptics may have used this information to downplay the status of deductive logic in everyday life expectancy. The opposite morality must be derived. Solitary the conflict between the truth and the valid inferences of logicians' beliefs forces them to reason about necessary consistent (⊃C). However, human reasoning is usually conducted in a protected mental atmosphere where it is a kind of logical lab - in which inferences can be evaluated in terms of true beliefs or facts. This evaluation process can prime to numerous consequences, counting acceptance or withdrawal of inferences. Artificial intelligence students established many computer programs for non-monotonic logical reasoning, among them earlier inferences are discarded in the phase of advanced evidence, and beliefs are revised in the phase of incompatibility. Merely few cognitive psychologists have studied these courses (BYRNE & JOSEPHINE, 2002; Byrne & Walsh, 2005; Khemlani & Johnson-Laird, 2011). Oaksford and Chater (1991) claimed logical reasoning as ⊃C must be a computationally manageable procedure. Table 5 showed a program to check ⊃C, ◊C/ ◊I and -◊I among a set of propositions by using MMS and FEMs, a computer program established stages output.

Logicians have sympathized with this suggestion, on the other side, there is reasoning for thoughtfulness. If the

**Table 5:** To check ⊃C, ◊C/ ◊I and ⟡I among a set of propositions by using MMS and FEMs, a computer program established stages output. Spade (♠) xor not comma club (♣) and heart (❤); ♠ and not ♣.

| |
|---|
| **1. MMs:** |
| The proposition, ♠ xor not comma ♣ and , satisfies MM |
| ♠ |
| And the propositions such as ♠ and not ♣ satisfy MM |
| ♠ ~ ♣ |
| Hence, students can be judged as the propositions are ⊃C. |
| MMs of proposition; ♠ xor not comma ♣ and ❤: |
| ♠ |
| ♠ ~ ❤ |
| ~♠ ❤ |
| ~♠ ~ ❤ |
| MMs of the proposition, ♠ and not ♣: |
| ♠ ~ ♣ |
| **2. FEMs:** |
| The propositions, ♠ xor not comma ♣ and ❤, satisfy FEMs |
| ♠ ♣ ❤ |
| The propositions such as ♠ and not ♣ have no FEM |
| Hence, FEMs indicate that the propositions are ⟡I |
| FEMs of proposition, ♠ xor not comma ♣ and ❤: |
| ♠ ♣ ❤ |
| ~♠ ♣ ~❤ |
| ~♠ ~♣ ❤ |
| ~♠ ~♣ ~❤ |
| FEMs of proposition, ♠ and not ♣, have MMs |
| ♠ ~ ♣ |

*Note: The computational programs predict that for students who decide to use MMs, their set of propositions are either ⊃C or ◊C/ ◊I whoever, ⟡I is higher in FEMs. "xor" defines exclusive disjunctions, and "comma" defines that the denial is applied as a whole to the conditional.*

correct results can compute for all probable input and the computation's time and WM needs are just a small polynomial in the size of the input, the job is considered manageable. The process of human usual linguistic analyzers appears to be manageable, as students did not continue to lag with considerate longer and longer assertions. Logics as using negation and assertion conditional such as if and or is unwieldy by comparison, and such human logical reasoning quickly breaks down with reasoning based on more and more propositions.

The current work adds significant newness to future directions. Logicians innovatively have presented alethic modal operators necessary (⊃) and possible (◊) (Wright, 1951) with consistency and incompatibility in one plate form such as necessary consistent (Omnes, 1988), impossible incompatible (Ratti, 2018), possible consistency (Miori *et al.*, 2010), and possible incompatible (Thibault, 2014). M paradigm with the help of a psychological view and computational program justified the main theory and predictions. Moreover, all three studies show that logically untrained students are capable of modal reasoning on MLT, MMT, and syllogism reasoning evidence. The consequences also confirmed their two main predictions constructed on MMT. First, Students mostly endorsed inferences as possible consistent (◊C) or possible incompatible (◊I) rather than necessary consistent (⊃C) (prediction one) and impossible incompatible (⟡I) is higher than necessary consistent (⊃C) in single and multi-model assertions (Prediction Second). ⟡I inference holds in any model of the propositions, whereas inferences not required at all hold in only one MM of the premises, so if the logician focuses on a model that holds, then in the latter belief the logician tends to wrong inferences. In other words, they must find counterexamples to the content of their inferences to draw unnecessary inferences. The examination is informal in the belief wherever all models of the propositions are counter-examples then in the situation where at most one is not.

In Study One, students assessed direct inferences from a single quantified proposition to quantify inference; in Study Two, they assessed Prediction Two and all possible modal inferences of syllogistic propositions; and in Study Three, they evaluated a subset of modal inferences drawn from syllogistic propositions. An initially unexpected effects were those questions supported ◊C/ ◊I inferences demolish into two subgroups ◊C and ◊I. Some of these are often considered ⊃C inferences, while others are rarely considered ⊃C inferences, and sometimes not even considered ◊C/ ◊I inferences. This phenomenon was first observed in study one. This also occurs in study two and is confirmed by study three, in which logicians designed to compare two types of states, which we call possible consistent (◊C) and possible incompatible (◊I). Of particular note in Figure C is that ◊C syllogisms are often recognized as ⊃C, whereas ◊I syllogisms are rarely recognized as impossible incompatible (-◇I). This proposes that pursuing counter-example MMs in the current study is weak, and most students based their inferences on the first model they encountered (logicians will return to this advanced). On this statement, falsities are separated into their two sub-groups, since some pairs of propositions consistently present an initial model that supports the inferences (likely ◊C), while others consistently present a MM that denies the inference (likely ◊I). Clearly, these findings cannot be explained at the very general level of their other predictions. However, it has been reported that computational program implementations of the MMT by Johnson-Laird *et al.* (2004) tend to generate MMs of explicit sequences, logicians were furthermore capable to study the program's output to identify their two sub-groups of syllogisms in an attempt to confirm their intuitional work. The outcomes provide strong confirmation. Moreover, studies and predictions strongly supported the given concepts.

## LIMITATIONS AND RECOMMENDATIONS

This study work targeted a spacific sample of college students in Pakistan. It can be studied further with various samples of experimental data. Moreover, the data of college students were not specified that was another limitation of this study. We just focused on theoratical work of this study. Future studies can be elaborated experiments more sufficiently.

Alethic modalities are a hot topic in this era. This study well-defined the interaction between alethic modalities with logical consistency and incompatibility as necessary consistency (⊃C), possible consistency (◊C) or possible incompatibility (◊I) and impossible incompatibility (-◇I). Future studies can extend this work in one plate form and can explain this interaction with another multiple theories. We only used two theories (Mental logic theory (MLT), and Mental model theory (MMT) to justify our concept further studies can use belief-biased theory, hypothetical inferential theory, or possible world theory, etc.

## CONCLUSION

Logicians asked about the judgments of the students regarding different propositions and their responses. They evaluated their results by using MMT. Moreover, the results of the given study have a clear conclusion. Psychological views, computational programs, the "M" paradigm, and experimental studies have justified theory, its principles and predictions'. Assessment of studies have supported the MMT about truth logic consistently having if, and/or, and all/some incompatible assertions. Logicians have detected incompatibility among their MM's true evidences and beliefs. They retracted faulty facts, single or double propositions, and direct conflicts, and attempted to resolve them. This finding also lends support to the mental model (MM) in explaining the bias belief effect (Khemlani & Johnson-Laird, 2022; Oakhill *et al.*, 1989). There is an incentive to find counterexamples to disprove it when a putative inference is implausible. The effect of instruction has also been consistent with the idea that students need some incentive to perform model searches. Evans and Over (2013) presented a durable pedagogical importance on alethic logic necessity considerably reduce biased beliefs, as well as the tendency to endorse ◊C/ ◊I but not ⊃C inferences. If the natural mode of thinking is inductive rather than deductive, then deductive abilities are most plausible in situations where students are motivated to try to reason. Finally, are there any alternatives to MLT or formal rule-based theories for explaining our main findings? The fact that our main predictions and principles are independent of the theory of mental models cannot be explained by the current account of formal rule theory, which describes only the ⊃C inference mechanisms. Nersessian's model inference rules involve only a limited set of inferences based on propositional connections and do not make any predictions about quantified modal inferences. Our findings pose a strong challenge to logicians, reasoners, and psychologists who believe that syllogistic inferences can be enlightened without reference to the deductive of logical reasoning efforts.

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**Authors' Contributions**
Conceptualization: SW, MW, AW, SH. Design and methodology: SW, MW, AW. Conduction of the study: SW, SH. Statistical analysis and interpretation: SW, AW. Writing—original draft preparation: SW. Writing—review, and editing: SW, MW, AW, SH. Resources: SW. Supervision: MW, SW. All authors contributed to the article and approved the submitted version. All authors have read and approved the manuscript.

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**Data Availability**
The dataset used in the current study is available from the corresponding author upon reasonable request.

**Declarations**

**Competing Interests**

The authors declare that they have no competing interests.

**Ethics Approval and Consent to Participate**

The questionnaire and methodology of the current study were approved by the concerned authorities. Ethics approval was obtained from the ethics committee of the Department of Psychology, Shaanxi Normal University.

**Consent for Publication**

Not applicable.

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