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APPLICATION OF MULTI-VALUED ROUGH NEUTROSOPHIC SET AND MATRIX IN MULTI-CRITERIA DECISION-MAKING

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Abstract. The rough set concept is a methodology of information processing for relational databases. It is a unique uncertainty mathematics topic closely connected to fuzzy set theory. When the rough set is combined with neutrosophic set theory, an effective tool for working with indeterminacy arises. In this study, we defined a multi-valued rough neutrosophic set and a multi-valued rough neutrosophic matrix. Using separation measures, we introduced a new approach for a multi-valued neutrosophic with a rough structure. The suggested approach makes it simple to evaluate the alternatives by applying the separation formula for the multi-valued rough neutrosophic set. The flowchart for the procedure is clearly presented. Then we consider the problem of determining the condition of dengue-affected patients in a specific hospital. Using this method, we create a multi-valued, rough neutrosophic decision matrix that clearly displays the relationship between patient conditions and symptoms. We can determine which one has a serious condition by solving this problem and presenting it on the graph. The comparison of the results with the existing approaches demonstrates the reliability of the suggested method. We analyze the advantages and disadvantages of the multi-valued neutrosophic rough set and matrix to highlight the significance of the suggested structure.

1. Introduction

The term rough set was proposed by Pawlak [19] in 1982, and it looks into approximation set operations, approximate set equality, and approximate set inclusion. Rough sets are founded on the idea that sets can be approximated by a pair of sets known as the lower and upper approximations of a set. Those approximations are based on the equivalence relation in this case. Then, in 1985, the rough set concept was compared with the fuzzy set [20]. Rough fuzzy set and fuzzy rough set was developed by Dubois and Prade [11] in 1990. The rough set approach to incomplete information systems was presented by Kryszkiewicz [15]. Then the generalized fuzzy rough set was presented by Wu et al. [33] in 2003. In this research, both constructive and axiomatic approaches are used to study fuzzy rough sets. In 2008, interval-valued rough sets and fuzzy interval-valued rough sets were given by several authors [12, 31]. In order to get a comprehensive structure for the research of rough interval type-2 fuzzy set, the researcher applied both axiomatic and constructive methods in 2012 [36].

Smarandache [28] invented the neutrosophic set first in 1998, and neutrosophy, the neutrosophic set, logic and probability were proposed to address the problem of indeterminacy in particular. He also built other forms of neutrosophic sets, including single and interval-valued, hesitant, and dual hesitant neutrosophic sets. In the paper "n-valued extended neutrosophic logics and their significance to the field of physics" [30], he further developed the multi-valued neutrosophic sets. It's an extension of the neutrosophic sets. Neutrosophic set theory and rough set theory both are going to be effective tools for handling the incomplete, indeterminate, and unclear data. The concept of rough neutrosophic set was

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defined by Broumi et al. [6] in 2014, and its operations were dicussed. Then the rough neutrosophic set in interval-values was introduced by Broumi ans Samarandache [7] in 2015. Mondal and Pramanik [18] developed a rough grey relation analysis approach to neutrosophic MADM and they defined the rough decision matrix in neutrosophic environment in 2015, using this matrix they solve a MCDM problem. In 2017, many authors proposed a different concept of a rough neutrosophic environment [2, 24, 34]. The authors looked at rough neutrosophic multiset, neutrosophic rough set with single-valued, and MCDM using neutrosophic rough set including coefficient correlation. New multi-granulation rough single-valued neutrosophic sets and its applications were discused by Bo et al. [4] in 2018.

In medical diagnosis, the rough neutrosophic set was used by Samuel and Narmadhagnanam [25] in 2018. The application of medical diagnostics for determine the disease affecting the patient is discussed in this paper. Then Zhang et al. [37] authors developed the process of medical evaluation concentrated on single-valued rough neutrosophic unclear multisets of two dimensions in 2018. Neutrosophic rough sets Pi-distance of diagnostic purpose was published by Samuel and Narmadhagnanam [26] in 2019. The aim of the research is to determine the link between the disease and the patient's condition with the symptoms, as well as to use a rough neutrosophic set to look at the patient's condition. Then, in 2021, this concepts will be further expanded on the soft set neutrosophic over rough sets [8] and single-valued neutrosophic rough set and containing topology [13]. As a result, the rough set is significant to each component of the neutrosophic surroundings.

Multi-valued neutrosophic sets were proposed by Peng and Wang [21] in 2015 and its operations, as well as a comparative method developed from earlier study on intuitionistic and hesitant fuzzy set. After that, the power weighted average operator and power weighted geometric operator of multi-valued neutrosophic [22] were built. The same authors used an ELECTRE approach [23] in 2017, the qualitative efficient multiple criteria technique, in which the criteria values are represented by multi-valued neutrosophic information, is used to analyze MCDM difficulties. A paper containing "N-valued straight neutrosophic soft set and its use in issues with decision-making and diagnosis in medicine" was presented by Alkhazaleh Hazaymeh [3] in 2017. By using a multi-valued neutrosophic set for taking decisions [29], the author constructed a 3 way n-valued neutrosophic idea matrix of various granulations. Ye et al. [35] published a paper with the discussion of neutrosophic multi-valued sets and their multi-attribute decision-making strategy and relation to the consistency of neutrosophic set correlation coefficients.

Fuzzy and neutrosophic-related networks were proposed in 2004 by Kandasamy and Smarandache [14]. Additionally, they included square neutrosophic matrices in this. In 2014, they created the neutrosophic matrix and associated algebraic operations [9]. The idea of rough matrix theory was first proposed to the decision-making community in 2013 by Vijayabalaji and Balaji [32]. Abobala et al. [1] published the neutrosophic square matrix algebraic creativity in 2021. The definition of intuitionistic fuzzy rough matrices and a study of certain of their theoretical characteristics, including some operations between them, are presented by Shanthi [27] in 2021. This concept helps to develop and make decisions using intuitionistic fuzzy rough matrices. The idea of the multi-valued neutrosophic matrix was expanded by Martina and Deepa [16]. This study presented the importance of multi-valued numbers in a neutrosophic environment. This included a discussion of the operations and characteristics of the suggested matrix, the creation of linguistic terms for that matrix, and its uses for the neutrosophic simplified TOPSIS approach. Then they included the matrix concept in the rough environment. In 2022, they provided the rough neutrosophic matrix's energy [10] and how it applied to handle the MCDM challenge. In this, they discussed the matrix energy in a rough neutrosophic structure and suggested a novel solution to the problem of decision-making. In 2023, the rough neutrosophic matrix [17] was introduced and applied to multi-criteria decision-making. In this paper, the authors discuss the rough neutrosophic matrices operations and their applications. Many researchers have established the concepts of neutrosophic matrices and rough matrices, but there is no research on the multi-valued neutrosophic rough matrix and its operations and applications.

One of the strongest areas of operation research is multi-criteria decision-making. Then it was applied in many different areas. When the concept of decision-making is applied to the neutrosophic set, it also solves the indeterminacy situation. Then the researchers developed this idea in a rough set that included a neutrosophic environment. A rough set is especially used in the medical sector. The neutrosophic idea has many different kinds of rough sets, including single, interval-valued rough neutrosophic sets. A multi-valued neutrosophic set is one of the applicable types of neutrosophic sets in decision-making situations. Because this set can be reduced to single values or interval values. It was connected to the hesitant neutrosophic set. The multi-valued neutrosophic fuzzy set is a subset of the neutrosophic hesitant fuzzy set. Our present research connects a rough set and a multi-valued neutrosophic set in circumstances requiring medical decision-making. Accordingly, we developed the concept of a rough set in a multi-valued neutrosophic set. Then it will be applied to healthcare situations in MCDM.

The multi-valued rough neutrosophic set and matrix and their attributes are defined in this paper. Section 2 contains the basic definitions and concepts. Section 3 presents the definition and properties of a multi-valued rough neutrosophic set. Section 4 provides the multi-valued rough neutrosophic matrix and its determinant, operations, and energy. A novel strategy for addressing the MCDM problem using the separation measures of a multi-valued neutrosophic set is presented, and the flow chart of the given approach is presented in the picture in Section 5. For evaluating alternatives, we use a multi-valued neutrosophic rough set by using linguistic terms for multi-valued neutrosophic numbers. Section 6 provides a numerical illustration of the suggested approach. The comparative results are given in Section 7. Finally, there were results, discussion, and a conclusion.

2. Basic Definitions

Definition 2.1. Neutrosophic Set [28]

Let U be considered a universal set and let each element 'a' be a member of U has degree of truth, indeterminacy, falsity membership functions in set S. Which is called neutrosophic set. Then it is defined as

$$S = \{\langle a, T_S(a), I_S(a), F_S(a)\rangle : a \in U\}$$

where, $0 \leq T_S(a) + I_S(a) + F_S(a) \leq 3$
and T_S is the truth membership function, I_S is the indeterminacy membership function, F_S is the false membership function, every function lies between [0,1] in U.

Definition 2.2. Rough Set [19]

Let R be considered an equivalence relation (indiscernibility relation) on U and U be the universal set. An approximation space is defined as A= U/R, which is a collection of all equivalence classes of U over R.

Let $X \subseteq U$ be a subset of U. The terms $\underline{A}(X)$ and $\overline{A}(X)$, which stand for the lower and higher approximations of X in A, respectively, are defined below.

$$\underline{A}(X) = \{a \in U : [a]_R \subseteq X\},$$
$$\overline{A}(X) = \{a \in U : [a]_R \cap X \neq \emptyset\}$$

where $[a]_R$ represents the equivalence class of R that contains an element a.

The pair $A(X) = (\underline{A}(X), \overline{A}(X))$ is known as rough set of X in A.

Definition 2.3. Rough Neutrosophic Set [6]

Let U be considered a universal set and each element a $\in$ U. Let S be the neutrosophic set contains truth T_S, indeterminacy I_S and false membership function F_S in U and let R be an equivalence relation on U. The lower and upper approximations of S in U/R is represented by $\underline{N}(S)$ and $\overline{N}(S)$ respectively, and are defined as follows:

$$\underline{N}(S) = \left\{ \left\langle a, T_{\underline{N}(S)}(a), I_{\underline{N}(S)}(a), F_{\underline{N}(S)}(a) \right\rangle \ : b \in [a]_R, a \in U \right\},$$

$$\overline{N}(S) = \left\{ \left\langle a, T_{\overline{N}(S)}(a), I_{\overline{N}(S)}(a), F_{\overline{N}(S)}(a) \right\rangle \ : b \in [a]_R, a \in U \right\}$$

where

$$\begin{array}{r|r} T_{\underline{N}(S)}(a) = \bigwedge_{b \in [a]_R} T_S(b) & T_{\overline{N}(S)}(a) = \bigvee_{b \in [a]_R} T_S(b) \\ I_{\underline{N}(S)}(a) = \bigvee_{b \in [a]_R} I_S(b) & I_{\overline{N}(S)}(a) = \bigwedge_{b \in [a]_R} I_S(b) \\ F_{\underline{N}(S)}(a) = \bigvee_{b \in [a]_R} F_S(b) & F_{\overline{N}(S)}(a) = \bigwedge_{b \in [a]_R} F_S(b) \end{array}$$

where $0 \leq T_{\underline{N}(S)}(a) + I_{\underline{N}(S)}(a) + F_{\underline{N}(S)}(a) \leq 3$ and $0 \leq T_{\overline{N}(S)}(a) + I_{\overline{N}(S)}(a) + F_{\overline{N}(S)}(a) \leq 3$ where, $T_S(a), I_S(a), F_S(a)$ are truth membership, indeterminacy membership and falsity membership function of element a on neutrosophic set S. $\bigvee$ stand for 'max' and $\bigwedge$ stand for 'min'. Therefore $\underline{N}(S)$ and $\overline{N}(S)$ are two neutrosophic sets in U. The pair $(\underline{N}(S), \overline{N}(S))$ is called the rough neutrosophic set in U/R.

If $\underline{N}(S) = \overline{N}(S)$ for any $a \in U$, then S is known as a definable neutrosophic set.

Example 2.1. *Let R taken as an equivalence relation on universal set U.*
where, $U = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$ *and*

$$R = \{(\alpha_1, \alpha_1), (\alpha_2, \alpha_2), (\alpha_3, \alpha_3), (\alpha_1, \alpha_2), (\alpha_2, \alpha_1), (\alpha_3, \alpha_4), (\alpha_4, \alpha_3), (\alpha_4, \alpha_4), (\alpha_4, \alpha_5), (\alpha_5, \alpha_4), (\alpha_3, \alpha_5), (\alpha_5, \alpha_3)(\alpha_5, \alpha_5)\}$$

An equivalence classes of U under R are given by $[\alpha_1] = \{\alpha_1, \alpha_2\}, [\alpha_2] = \{\alpha_1, \alpha_2\}$,
$[\alpha_3] = \{\alpha_3, \alpha_4, \alpha_5\}$, $[\alpha_4] = \{\alpha_3, \alpha_4, \alpha_5\}, [\alpha_5] = \{\alpha_3, \alpha_4, \alpha_5\}$
All of the equivalence classes of U over R are collected together as
$U/R = \{[\alpha_1], [\alpha_2], [\alpha_3], [\alpha_4], [\alpha_5]\} = \{\{\alpha_1, \alpha_3\}, \{\alpha_2, \alpha_4, \alpha_5\}\}$
Let $N(S) = \{(\alpha_1, \langle 0.11, 0.45, 0.52\rangle), (\alpha_3, \langle 0.23, 0.33, 0.64\rangle), (\alpha_4, \langle 0.55, 0.21, 0.83\rangle)\}$ *be a neutrosophic set of U.*
Then lower approximation $\underline{N}(S)$ *and upper approximation* $\overline{N}(S)$ *of the neutrosophic set S,*

$$\underline{N}(S) = \{(\alpha_1, \langle 0.11, 0.45, 0.64\rangle), (\alpha_3, \langle 0.11, 0.45, 0.64\rangle)\}$$

$$\overline{N}(S) = \{(\alpha_1, \langle 0.23, 0.33, 0.52\rangle), (\alpha_3, \langle 0.23, 0.33, 0.52\rangle), (\alpha_2, \langle 0.55, 0.21, 0.83\rangle), (\alpha_4, \langle 0.55, 0.21, 0.83\rangle), (\alpha_5, \langle 0.55, 0.21, 0.83\rangle)\}$$

If the neutrosophic set $N(S) = \{(\alpha_1, \langle 0.25, 0.53, 0.72\rangle), (\alpha_3, \langle 0.25, 0.53, 0.72\rangle)$
Then the values of upper and lower approximation will be equal $\underline{N}(S) = \overline{N}(S)$.

$$\underline{N}(S) = \{(\alpha_1, \langle 0.25, 0.53, 0.72\rangle), (\alpha_3, \langle 0.25, 0.53, 0.72\rangle)\}$$

$$\overline{N}(S) = \{(\alpha_1, \langle 0.25, 0.53, 0.72\rangle), (\alpha_3, \langle 0.25, 0.53, 0.72\rangle)\}$$

Now the neutrosophic set is known as definable set.

Definition 2.4. Properties [6]

If $N(S_1)$ and $N(S_2)$ are two rough neutrosophic sets of the neutrosophic sets S_1 and S_2 respectively in U, then we define the following

(i) $N(S_1) = N(S_2)$ Iff $\underline{N}(S_1) = \underline{N}(S_2)$ and $\overline{N}(S_1) = \overline{N}(S_2)$;

(ii) $N(S_1) \subseteq N(S_2)$ Iff $\underline{N}(S_1) \subseteq \underline{N}(S_2)$ and $\overline{N}(S_1) \subseteq \overline{N}(S_2)$;

(iii) $N(S_1) \cup N(S_2) = \left\langle \underline{N}(S_1) \cup \underline{N}(S_2), \overline{N}(S_1) \cup \overline{N}(S_2) \right\rangle$;

(iv) $N(S_1) \cap N(S_2) = \left\langle \underline{N}(S_1) \cap \underline{N}(S_2), \overline{N}(S_1) \cap \overline{N}(S_2) \right\rangle$;

(v) $N(S_1) + N(S_2) = \left\langle \underline{N}(S_1) + \underline{N}(S_2), \overline{N}(S_1) + \overline{N}(S_2) \right\rangle$;

(vi) $N(S_1) \cdot N(S_2) = \left\langle \underline{N}(S_1) \cdot \underline{N}(S_2), \overline{N}(S_1) \cdot \overline{N}(S_2) \right\rangle$;

(vii) $\sim N(S) = (\underline{N}(S)^c, \overline{N}(S)^c)$ where

$$\underline{N}(S)^c = \left\{ \left\langle a, T_{\underline{N}(S)}(a), 1 - I_{\underline{N}(S)}(a), F_{\underline{N}(S)}(a) \right\rangle \ ; a \in U \right\},$$

$$\overline{N}(S)^c = \left\{ \left\langle a, T_{\overline{N}(S)}(a), 1 - I_{\overline{N}(S)}(a), F_{\overline{N}(S)}(a) \right\rangle \ ; a \in U \right\}.$$

Definition 2.5. Multi Valued Neutrosophic Set [30]

Let U be considered a universal set and let each element 'a' be a member of U. A multi-valued neutrosophic set $\tilde{S}$ in U is denoted by the membership functions $\tilde{T}_{\tilde{S}}(a), \tilde{I}_{\tilde{S}}(a)$ *and* $\tilde{F}_{\tilde{S}}(a)$, which lies between [0, 1], The definition is given below

$$\tilde{S} = \left\{ \left\langle a, \tilde{T}_{\tilde{S}}(a), \tilde{I}_{\tilde{S}}(a), \tilde{F}_{\tilde{S}}(a) \right\rangle : a \in U \right\}$$

where, $\tilde{T}_{\tilde{S}}(a), \tilde{I}_{\tilde{S}}(a)$ *and* $\tilde{F}_{\tilde{S}}(a)$ are representing the truth, indeterminacy and false membership function respectively and $0 \le \tilde{T}_{\tilde{S}}(a) + \tilde{I}_{\tilde{S}}(a) + \tilde{F}_{\tilde{S}}(a) \le 3$. Which hold the following conditions:

$$0 \le \alpha, \beta, \gamma \le 1 \ and \ 0 \le \alpha^+ + \beta^+ + \gamma^+ \le 3$$

where, $\alpha \in \tilde{T}_{\tilde{S}}(a)$, $\beta \in \tilde{I}_{\tilde{S}}(a)$, $\gamma \in \tilde{F}_{\tilde{S}}(a)$ and $\alpha^+ = sup\tilde{T}_{\tilde{S}}(a)$, $\beta^+ = sup\tilde{I}_{\tilde{S}}(a)$, $\gamma^+ = sup\tilde{F}_{\tilde{S}}(a)$. We write the the multi-valued neutrosophic number in the form of, $\tilde{S} = \{\left\langle \tilde{T}_{\tilde{S}}, \tilde{I}_{\tilde{S}}, \tilde{F}_{\tilde{S}} \right\rangle\}$.

Apparently, Multi-valued neutrosophic sets are a specialized form of neutrosophic sets. Especially,

when $\tilde{T}_{\tilde{S}}, \tilde{I}_{\tilde{S}}$ and $\tilde{F}_{\tilde{S}}$ have single value in each α, β and γ respectively and $0 \le \alpha + \beta + \gamma \le 3$, then the multi-valued neutrosophic set is reduced to single-valued neutrosophic set.

when $\tilde{I}_{\tilde{S}} = \emptyset$, then the multi-valued neutrosophic set is transformed into the dual hesitant fuzzy set.
when $\tilde{I}_{\tilde{S}} = \tilde{F}_{\tilde{S}} = \emptyset$, then the multi-valued neutrosophic set is reduced to hesitant fuzzy set.
The MVNS is a generalization of the sets mentioned above.

Definition 2.6. Multi Valued Neutrosophic Matrix [16]

Let P be a multi-valued neutrosophic matrix with $m \times n$ order. It's described by

$$P = [P(\tilde{T}_{ij}), P(\tilde{I}_{ij}), P(\tilde{F}_{ij})]_{m \times n}$$

where, $P(\tilde{T}_{ij}), P(\tilde{I}_{ij}), P(\tilde{F}_{ij})$ are lies between 0 and 1, i=1,2,... m and j=1,2,... n, with the conditions $0 \le \alpha_{ij}, \beta_{ij}, \gamma_{ij} \le 1$, $0 \le \alpha^+_{ij} + \beta^+_{ij} + \gamma^+_{ij} \le 3$ where, $\alpha_{ij} \in P(\tilde{T}_{ij}), \beta_{ij} \in P(\tilde{I}_{ij}), \gamma_{ij} \in P(\tilde{F}_{ij})$, $\alpha^+_{ij} = sup\ P(\tilde{T}_{ij})$, $\beta^+_{ij} = sup\ P(\tilde{I}_{ij})$, $\gamma^+_{ij} = sup\ P(\tilde{F}_{ij})$ and crisp values $\in$ [0,1].
If each element of $P(\tilde{T}_{ij}), P(\tilde{I}_{ij}), P(\tilde{F}_{ij})$ has one values, then the multi-valued neutrosophic matrix is transformed to single-valued neutrosophic matrix. If each element of $P(\tilde{T}_{ij}), P(\tilde{I}_{ij}), P(\tilde{F}_{ij})$ has interval values, then the multi-valued neutrosophic matrix is transformed to interval-valued neutrosophic matrix.

Definition 2.7. Rough matrix. [32]

We can define a rough matrix $R_M = [r_{ij}]$ of order $m \times n$ as follows

$$R_M = [r_{ij}] = \begin{pmatrix} r_{11} & r_{12} & \dots & r_{1n} \\ r_{21} & r_{22} & \dots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \dots & r_{mn} \end{pmatrix}$$

where, each $r_{ij} \in \mu_x^R$. μ_x^R is rough membership function.

Definition 2.8. Rough Neurosophic Matrix [18]

$$D_N = \left\langle \underline{N}_{ij}(S), \overline{N}_{ij}(S) \right\rangle_{m\times n} = \begin{matrix} & C_1 & C_2 & \dots & C_n \\ A_1 & \left\langle \underline{N}_{11}, \overline{N}_{11} \right\rangle & \left\langle \underline{N}_{12}, \overline{N}_{12} \right\rangle & \dots & \left\langle \underline{N}_{1n}, \overline{N}_{1n} \right\rangle \\ A_2 & \left\langle \underline{N}_{21}, \overline{N}_{21} \right\rangle & \left\langle \underline{N}_{22}, \overline{N}_{22} \right\rangle & \dots & \left\langle \underline{N}_{2n}, \overline{N}_{2n} \right\rangle \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & \left\langle \underline{N}_{m1}, \overline{N}_{m1} \right\rangle & \left\langle \underline{N}_{m2}, \overline{N}_{m2} \right\rangle & \dots & \left\langle \underline{N}_{mn}, \overline{N}_{mn} \right\rangle \end{matrix}$$

The element of the D_N matrix is a rough neutrosophic matrix of m alternatives over n criteria with the order $m \times n$. where, $\underline{N}_{ij}$ is lower approximation and $\overline{N}_{ij}$ is upper approximation of S (neutrosophic set). The order of the matrix depends on the number of alternatives and criteria, if both are equal, the matrix is a square matrix with $n \times n$ elements.

Example 2.2. *Let D_N be the rough neutrosophic matrix with order 3×2.*

$$\begin{pmatrix} \left\langle (0.12, 0.72, 0.94), (0.66, 0.57, 0.27) \right\rangle & \left\langle (0.59, 0.82, 0.65), (0.2, 0.29, 0.11) \right\rangle \\ \left\langle (0.85, 0.42, 0.13), (0.66, 0.55, 0.32) \right\rangle & \left\langle (0.23, 0.89, 0.99), (0.75, 0.68, 0.38) \right\rangle \\ \left\langle (0.48, 0.71, 0.54), (0.3, 0.57, 0.68) \right\rangle & \left\langle (0.94, 0.51, 0.22), (0.75, 0.43, 0.11) \right\rangle \end{pmatrix}$$

This kind of matrix is used in multi-criteria problems with various alternatives and situations.

Definition 2.9. Energy of Rough Neutrosophic Matrix [10]

Let S be a neutrosophic set and let D be the rough neutrosophic sqaure matrix with lower and upper approximation of set S.

$$D = \left\langle \left(\underline{T}_{ij}(S), \underline{I}_{ij}(S), \underline{F}_{ij}(S) \right), \left(\overline{T}_{ij}(S), \overline{I}_{ij}(S), \overline{F}_{ij}(S) \right) \right\rangle$$

The rough neutrosophic matrix is separated by six matrices, that include truth, indeterminacy, and falsity membership values of lower and upper approximation matrices. Therefore the energy of the rough neutrosophic matrix is described as

$$E[D] = \left(E[\underline{T}_{ij}(S)], E[\underline{I}_{ij}(S)], E[\underline{F}_{ij}(S)] \right), \left(E[\overline{T}_{ij}(S)], E[\overline{I}_{ij}(S)], E[\overline{F}_{ij}(S)] \right)$$

$$E[D] = \left(\sum_{i=1}^{n} |\underline{\lambda}_i - \mu_{\underline{\lambda}}|, \sum_{i=1}^{n} |\underline{\zeta}_i - \mu_{\underline{\zeta}}|, \sum_{i=1}^{n} |\underline{\eta}_i - \mu_{\underline{\eta}}| \right), \left(\sum_{i=1}^{n} |\overline{\lambda}_i - \mu_{\overline{\lambda}}|, \sum_{i=1}^{n} |\overline{\zeta}_i - \mu_{\overline{\zeta}}|, \sum_{i=1}^{n} |\overline{\eta}_i - \mu_{\overline{\eta}}| \right)$$

where, $\underline{\lambda}_i$, $\overline{\lambda}_i$ are the eigenvalues of truth membership lower and upper approximation matrices, $\underline{\zeta}_i$, $\overline{\zeta}_i$ are the eigenvalues of indeterminacy membership lower and upper approximation matrices, and $\underline{\eta}_i$, $\overline{\eta}_i$ are the eigenvalues of false membership lower and upper approximation matrices. $\mu_{\underline{\lambda}}$, $\mu_{\overline{\lambda}}$, $\mu_{\underline{\zeta}}$, $\mu_{\overline{\zeta}}$, $\mu_{\underline{\eta}}$, and $\mu_{\overline{\eta}}$ are the mean values of respective eigenvalues.

3. Multi-Valued Rough Neutrosophic Set (MVRNS)

We defined the multi-valued rough neutrosophic set, example and its attributes in this section.

Definition 3.1. Multi-Valued Rough Neutrosophic Set

Let U be considered a universal set and each element a $\in$ U. Let R be an equivalence relation on U and $\tilde{S}$ be the neutrosophic set in U with truth membership degree $\tilde{T}_{\tilde{S}}$, indeterminacy degree $\tilde{I}_{\tilde{S}}$ and false membership degree $\tilde{F}_{\tilde{S}}$. The lower and upper approximations of neutrosophic set S in U/R is stated by $\underline{N}(\tilde{S})$ and $\overline{N}(\tilde{S})$ and they are described as follows

$$\underline{N}(\tilde{S}) = \left\{ \left\langle a, \tilde{T}_{\underline{N}(\tilde{S})}(a), \tilde{I}_{\underline{N}(\tilde{S})}(a), \tilde{F}_{\underline{N}(\tilde{S})}(a) \right\rangle \ : b \in [a]_R \subseteq \tilde{S}, a \in U \right\},$$

$$\overline{N}(\tilde{S}) = \left\{ \left\langle a, \tilde{T}_{\overline{N}(\tilde{S})}(a), \tilde{I}_{\overline{N}(\tilde{S})}(a), \tilde{F}_{\overline{N}(\tilde{S})}(a) \right\rangle \ : b \in [a]_R \cap \tilde{S} \neq \emptyset, a \in U \right\}$$

where

$$\begin{array}{l|l} \tilde{T}_{\underline{N}(\tilde{S})}(a) = \bigwedge_{b\in[a]_R} \tilde{T}_{\tilde{S}}(b) & \tilde{T}_{\overline{N}(\tilde{S})}(a) = \bigvee_{b\in[a]_R} \tilde{T}_{\tilde{S}}(b) \\ \tilde{I}_{\underline{N}(\tilde{S})}(a) = \bigvee_{b\in[a]_R} \tilde{I}_{\tilde{S}}(b) & \tilde{I}_{\overline{N}(\tilde{S})}(a) = \bigwedge_{b\in[a]_R} \tilde{I}_{\tilde{S}}(b) \\ \tilde{F}_{\underline{N}(\tilde{S})}(a) = \bigvee_{b\in[a]_R} \tilde{F}_{\tilde{S}}(b) & \tilde{F}_{\overline{N}(\tilde{S})}(a) = \bigwedge_{b\in[a]_R} \tilde{F}_{\tilde{S}}(b) \end{array}$$

where $0 \leq \tilde{T}_{\underline{N}(\tilde{S})}(a) + \tilde{I}_{\underline{N}(\tilde{S})}(a) + \tilde{F}_{\underline{N}(\tilde{S})}(a) \leq 3$ and $0 \leq \tilde{T}_{\overline{N}(\tilde{S})}(a) + \tilde{I}_{\overline{N}(\tilde{S})}(a) + \tilde{F}_{\overline{N}(\tilde{S})}(a) \leq 3$ satisfies the following conditions:

$$0 \leq \alpha, \beta, \gamma \leq 1 \ and \ 0 \leq \alpha^+ + \beta^+ + \gamma^+ \leq 3$$

where, $\alpha \in \tilde{T}_{\tilde{S}}(a), \beta \in \tilde{I}_{\tilde{S}}(a), \gamma \in \tilde{F}_{\tilde{S}}(a)$ and $\alpha^+ = sup\tilde{T}_{\tilde{S}}(a), \beta^+ = sup\tilde{I}_{\tilde{S}}(a), \gamma^+ = sup\tilde{F}_{\tilde{S}}(a)$ and $\bigvee$ stand for 'max', $\bigwedge$ stand for 'min' and $\tilde{T}_{\tilde{S}}(a), \tilde{I}_{\tilde{S}}(a), \tilde{F}_{\tilde{S}}(a)$ are truth, indeterminacy, false membership degree of a on $\tilde{S}$. Hence $\underline{N}(\tilde{S})$ and $\overline{N}(\tilde{S})$ are two multi-valued neutrosophic sets in U.
The pair $(\underline{N}(\tilde{S}), \overline{N}(\tilde{S}))$ is called the multi-valued rough neutrosophic set in U/R.

If $\underline{N}(\tilde{S}) = \overline{N}(\tilde{S})$ for any $a \in U$, then $\tilde{S}$ is known as a definable multi-valued neutrosophic set.

Example 3.1. *Let U be the candidates applying for the job for a particular company,* $U = \{A_1, A_2, A_3, A_4, A_5\}$ *Let R be the equivalence relation between the Age and Qualification of every person in U. Table 1 presents the information of candidates.*

Table 1. Information data

Candidates	*Qualification*	*Age*	*Communication Skill*
A_1	*UG degree*	*18-20*	*Yes*
A_2	*PG degree*	*25-30*	*Yes*
A_3	*UG degree*	*20-25*	*Yes*
A_4	*UG degree*	*20-25*	*No*
A_5	*PG degree*	*25-30*	*Yes*
A_6	*UG degree*	*20-25*	*No*

U/R (Age, Qualification) $= \{\{A_1\}, \{A_2, A_5\}, \{A_3, A_4, A_6\}\}$
Let $N = \{A \in U$ *: Communication skill - yes*$\}$

here we take N be a multi-valued neutrosophic set $\tilde{S}$, its is described as $N(\tilde{S})$.

$$\text{Let } N(\tilde{S}) = \{(A_1, \langle\{0.4, 0.6\}, \{0.3\}, \{0.5\}\rangle), (A_2, \langle\{0.5\}, \{0.6, 0.7\}, \{0.2\}\rangle), (A_3, \langle\{0.7\}, \{0.4, 0.5\}, \{0.6, 0.7\}\rangle), (A_5, \langle\{0.6, 0.7\}, \{0.5\}, \{0.7, 0.8\}\rangle)\}$$

where, the three functions of truth, indeterminacy, and false for MVNS are speaking skill, reading skill, and writing skill respectively. Then lower approximation and upper approximation of the multi-valued neutrosophic set $\tilde{S}$ are given below,

$$\underline{N}(\tilde{S}) = \{(A_1, \langle\{0.4, 0.6\}, \{0.3\}, \{0.5\}\rangle), (A_2, \langle\{0.5\}, \{0.6, 0.7\}, \{0.7, 0.8\}\rangle), (A_5, \langle\{0.5\}, \{0.6, 0.7\}, \{0.7, 0.8\}\rangle), \}$$

$$\overline{N}(\tilde{S}) = \{(A_1, \langle\{0.4, 0.6\}, \{0.3\}, \{0.5\}\rangle), (A_2, \langle\{0.6, 0.7\}, \{0.5\}, \{0.2\}\rangle), (A_5, \langle\{0.6, 0.7\}, \{0.5\}, \{0.2\}\rangle), (A_3, \langle\{0.7\}, \{0.4, 0.5\}, \{0.6, 0.7\}\rangle), (A_4, \langle\{0.7\}, \{0.4, 0.5\}, \{0.6, 0.7\}\rangle), (A_6, \langle\{0.7\}, \{0.4, 0.5\}, \{0.6, 0.7\}\rangle)\}$$

Therefore, the pair $(\underline{N}(\tilde{S}), \overline{N}(\tilde{S}))$ is the multi-valued rough neutrosophic set.
If $N(\tilde{S}) = \{(A_1, \langle\{0.5\}, \{0.3, 0.4\}, \{0.7\}\rangle), (A_2, \langle\{0.2\}, \{0.5\}, \{0.8, 0.9\}\rangle), (A_5, \langle\{0.2\}, \{0.5\}, \{0.8, 0.9\}\rangle)\}$ then, $\underline{N}(\tilde{S})$ is equal to $\overline{N}(\tilde{S})$.

$$\underline{N}(\tilde{S}) = \{(A_1, \langle\{0.5\}, \{0.3, 0.4\}, \{0.7\}\rangle), (A_2, \langle\{0.2\}, \{0.5\}, \{0.8, 0.9\}\rangle), (A_5, \langle\{0.2\}, \{0.5\}, \{0.8, 0.9\}\rangle)\}$$
$$\overline{N}(\tilde{S}) = \{(A_1, \langle\{0.5\}, \{0.3, 0.4\}, \{0.7\}\rangle), (A_2, \langle\{0.2\}, \{0.5\}, \{0.8, 0.9\}\rangle), (A_5, \langle\{0.2\}, \{0.5\}, \{0.8, 0.9\}\rangle)\}$$

Now the set $\tilde{S}$ is known as difinable multi-valued neutrosophic set.

Definition 3.2. if $N(\tilde{S_1})$ and $N(\tilde{S_2})$ are two multi-valued rough neutrosophic sets of the corresponding multi-valued neutrosophic sets in U. then we define the following (It satisfied all properties of rough neutrosophic set defined in definition (4)

(i) $N(\tilde{S_1}) = N(\tilde{S_2})$ Iff $\underline{N}(\tilde{S_1}) = \underline{N}(\tilde{S_2})$ and $\overline{N}(\tilde{S_1}) = \overline{N}(\tilde{S_2})$;
(ii) $N(\tilde{S_1}) \subseteq N(\tilde{S_2})$ Iff $\underline{N}(\tilde{S_1}) \subseteq \underline{N}(\tilde{S_2})$ and $\overline{N}(\tilde{S_1}) \subseteq \overline{N}(\tilde{S_2})$;
(iii) $N(\tilde{S_1}) + N(\tilde{S_2}) = \left\langle \underline{N}(\tilde{S_1}) + \underline{N}(\tilde{S_2}), \overline{N}(\tilde{S_1}) + \overline{N}(\tilde{S_2}) \right\rangle$;
(iv) $N(\tilde{S_1}) \cdot N(\tilde{S_2}) = \left\langle \underline{N}(\tilde{S_1}) \cdot \underline{N}(\tilde{S_2}), \overline{N}(\tilde{S_1}) \cdot \overline{N}(\tilde{S_2}) \right\rangle$;
(v) $N(\tilde{S_1}) \cup N(\tilde{S_2}) = \left\langle \underline{N}(\tilde{S_1}) \cup \underline{N}(\tilde{S_2}), \overline{N}(\tilde{S_1}) \cup \overline{N}(\tilde{S_2}) \right\rangle$;
(vi) $N(\tilde{S_1}) \cap N(\tilde{S_2}) = \left\langle \underline{N}(\tilde{S_1}) \cap \underline{N}(\tilde{S_2}), \overline{N}(\tilde{S_1}) \cap \overline{N}(\tilde{S_2}) \right\rangle$;
(vii) $\sim N(\tilde{S}) = (\underline{N}(\tilde{S})^c, \overline{N}(\tilde{S})^c)$ where

$$\underline{N}(\tilde{S})^c = \left\{ \left\langle a, T_{\underline{N}(\tilde{S})}(a), 1 - I_{\underline{N}(\tilde{S})}(a), F_{\underline{N}(\tilde{S})}(a) \right\rangle \ ; a \in U \right\},$$

$$\overline{N}(\tilde{S})^c = \left\{ \left\langle a, T_{\overline{N}(\tilde{S})}(a), 1 - I_{\overline{N}(\tilde{S})}(a), F_{\overline{N}(\tilde{S})}(a) \right\rangle \ ; a \in U \right\}.$$

4. Multi-valued rough neutrosophic matrix

Definition 4.1. Multi-valued rough neutrosophic matrix

A multi-valued rough neutrosophic matrix is defined as $D_{ij}(\tilde{S}) = \left\langle \underline{D}_{ij}(\tilde{S}), \overline{D}_{ij}(\tilde{S}) \right\rangle$ with order of

$m \times n$. where $\underline{D}_{ij}(\tilde{S})$ is an upper approximation and $\overline{D}_{ij}(\tilde{S})$ is a lower approximation of the multi-valued rough neutrosophic set $\tilde{S}$. It can be expressed as

$$D_{ij}(\tilde{S}) = \left\langle \underline{D}_{ij}(\tilde{S}), \overline{D}_{ij}(\tilde{S}) \right\rangle = \left\langle \left(\underline{T}_{ij}(\tilde{S}), \underline{I}_{ij}(\tilde{S}), \underline{F}_{ij}(\tilde{S}) \right), \left(\overline{T}_{ij}(\tilde{S}), \overline{I}_{ij}(\tilde{S}), \overline{F}_{ij}(\tilde{S}) \right) \right\rangle$$

where $\underline{T}_{ij}(\tilde{S}), \underline{I}_{ij}(\tilde{S})$, and $\underline{F}_{ij}(\tilde{S})$ are the lower approximation values of truth, indeterminacy, and false membership marices, $\overline{T}_{ij}(\tilde{S}), \overline{I}_{ij}(\tilde{S})$, and $\overline{F}_{ij}(\tilde{S})$ are the upper approximation values of truth, indeterminacy, and false membership matrices of the multi-valued rough neutrosophic matrix $D(\tilde{S})$. With the conditions:

$$0 \leq \underline{T}_{ij}(\tilde{S}) + \underline{I}_{ij}(\tilde{S}) + \underline{F}_{ij}(\tilde{S}) \leq 3, 0 \leq \overline{T}_{ij}(\tilde{S}) + \overline{I}_{ij}(\tilde{S}) + \overline{F}_{ij}(\tilde{S}) \leq 3$$

where, $i = 1, 2, \ldots m$, $j = 1, 2, \ldots, n$ and

$$0 \leq \underline{\alpha}_{ij}, \underline{\beta}_{ij}, \underline{\gamma}_{ij} \leq 1, 0 \leq \underline{\alpha}^+_{ij} + \underline{\beta}^+_{ij} + \underline{\gamma}^+_{ij} \leq 3$$

where, $\underline{\alpha}_{ij} \in \underline{T}_{ij}(\tilde{S})$, $\underline{\alpha}_{ij} \in \underline{T}_{ij}(\tilde{S})$, $\underline{\alpha}_{ij} \in \underline{T}_{ij}(\tilde{S})$ and $\underline{\alpha}^+_{ij} = sup\ \underline{T}_{ij}(\tilde{S})$, $\underline{\beta}^+_{ij} = sup\ \underline{I}_{ij}(\tilde{S})$, $\underline{\gamma}^+_{ij} = sup\ \underline{F}_{ij}(\tilde{S})$.

$$0 \leq \overline{\alpha}_{ij}, \overline{\beta}_{ij}, \overline{\gamma}_{ij} \leq 1, 0 \leq \overline{\alpha}^+_{ij} + \overline{\beta}^+_{ij} + \overline{\gamma}^+_{ij} \leq 3$$

where, $\overline{\alpha}_{ij} \in \overline{T}_{ij}(\tilde{S})$, $\overline{\alpha}_{ij} \in \overline{T}_{ij}(\tilde{S})$, $\overline{\alpha}_{ij} \in \overline{T}_{ij}(\tilde{S})$ and $\overline{\alpha}^+_{ij} = sup\ \overline{T}_{ij}(\tilde{S})$, $\overline{\beta}^+_{ij} = sup\ \overline{I}_{ij}(\tilde{S})$, $\overline{\gamma}^+_{ij} = sup\ \overline{F}_{ij}(\tilde{S})$.

If $\underline{T}_{ij}(\tilde{S}), \underline{I}_{ij}(\tilde{S}), \underline{F}_{ij}(\tilde{S})$ and $\overline{T}_{ij}(\tilde{S}) + \overline{I}_{ij}(\tilde{S}) + \overline{F}_{ij}(\tilde{S})$ has only one values in each element of matrix, therefore $0 \leq \underline{\alpha}_{ij} + \underline{\beta}_{ij} + \underline{\gamma}_{ij} \leq 3$, $0 \leq \overline{\alpha}_{ij} + \overline{\beta}_{ij} + \overline{\gamma}_{ij} \leq 3$ then the multi-valued rough neutrosophic matrix was simplified to a single-valued rough neutrosophic matrix.

Example 4.1. *Let $D_{ij}(\tilde{S})$ be a 2×2 MVRNM*

$$D_{ij}(\tilde{S}) = \begin{pmatrix} [(\{0.1\}, \{0.5, 0.6\}, \{0.8\}), & [(\{0.3\}, \{0.2\}, \{0.6, 0.7\}), \\ (\{0.2, 0.3\}, \{0.4\}, \{0.7, 0.8\})] & (\{0.2\}, \{0.1, 0.3\}, \{0.9\})] \\ [(\{0.5, 0.6\}, \{0.8\}, \{0.3\}), & [(\{0.4\}, \{0.7, 0.8\}, \{0.9\}), \\ (\{0.7\}, \{0.6\}, \{0.1\})] & (\{0.1\}, \{0.4, 0.5\}, \{0.6, 0.7\})] \end{pmatrix}$$

If each element of $D(\tilde{S})$ has one value in each membership function, then it will be reduced to a single-valued rough neutrosophic matrix.

$$D_{ij}(S) = \begin{pmatrix} \left\langle (\{0.1\}, \{0.5\}, \{0.8\}), (\{0.2\}, \{0.4\}, \{0.7\}) \right\rangle & \left\langle (\{0.3\}, \{0.2\}, \{0.6\}), (\{0.2\}, \{0.1\}, \{0.9\}) \right\rangle \\ \left\langle (\{0.5\}, \{0.8\}, \{0.3\}), (\{0.7\}, \{0.6\}, \{0.1\}) \right\rangle & \left\langle (\{0.4\}, \{0.7\}, \{0.9\}), (\{0.1\}, \{0.4\}, \{0.6\}) \right\rangle \end{pmatrix}$$

If each element of $D(\tilde{S})$ has been transformed into interval values, then it will be called an interval-valued rough neutrosophic matrix.

$$D_{ij}([S^l, S^u]) = \begin{pmatrix} [(\{0.1, 0.2\}, \{0.5, 0.6\}, \{0.8, 0.9\}), & [(\{0.3, 0.4\}, \{0.2, 0.3\}, \{0.6, 0.7\}), \\ (\{0.2, 0.3\}, \{0.4, 0.5\}, \{0.7, 0.8\})] & (\{0.2, 0.3\}, \{0.1, 0.3\}, \{0.9, 1\})] \\ [(\{0.5, 0.6\}, \{0.8, 0.9\}, \{0.3, 0.4\}), & [(\{0.4, 0.5\}, \{0.7, 0.8\}, \{0.9, 1\}), \\ (\{0.7, 0.8\}, \{0.6, 0.7\}, \{0.1, 0.2\})] & (\{0.1, 0.2\}, \{0.4, 0.5\}, \{0.6, 0.7\})] \end{pmatrix}$$

In particular, multi-valued neutrosophic matrices are subsets of hesitantly valued matrices. For example, take the rough neutrosophic matrix with hesitant values.

$$H_{ij}(S) = \begin{pmatrix} [(\{0.1\},\{0.5,0.6,0.7\},\{0.8\}), & [(\{0.3\},\{0.2,0.3\},\{0.6,0.7,0.8\}), \\ (\{0.2,0.3\},\{0.4\},\{0.7,0.8\})] & (\{0.2\},\{0.1,0.3,0.4\},\{0.9\})] \\ [(\{0.5,0.6,0.7\},\{0.8\},\{0.3\}), & [(\{0.4,0.5\},\{0.7,0.8,0.9\},\{0.9\}), \\ (\{0.7\},\{0.6\},\{0.1,0.2,0.3,0.4\})] & (\{0.1,0.2\},\{0.4\},\{0.6,0.7\})] \end{pmatrix}$$

Here, the elements of the rough neutrosophic matrix have hesitant values. So it is called a hesitant neutrosophic matrix.

Therefore, we can say that a multi-valued neutrosophic rough matrix is also called a hesitant rough neutrosophic matrix. But the converse need not be true. Because the elements of a multi-valued rough neutrosophic matrix have single or interval values in each, So this matrix plays an important role in every other type of rough neutrosophic matrix.

Definition 4.2. Determinant of MVRNM

The determinant of MVRNM $D_{ij}(\tilde{S})$ of order $n \times n$ is denoted by $det(D_{ij}(\tilde{S}))$ or $\left|D_{ij}(\tilde{S})\right|$ and it is defined as

$$\left|D_{ij}(\tilde{S})\right| = \sum_{\sigma = s_n} \left[\prod_{i=1}^{n} \left\langle \underline{d}(\tilde{s})^{\tilde{T}}_{i\sigma(i)}, \underline{d}(\tilde{s})^{\tilde{I}}_{i\sigma(i)}, \underline{d}(\tilde{s})^{\tilde{F}}_{i\sigma(i)} \right\rangle, \left\langle \overline{d}(\tilde{s})^{\tilde{T}}_{i\sigma(i)}, \overline{d}(\tilde{s})^{\tilde{I}}_{i\sigma(i)}, \overline{d}(\tilde{s})^{\tilde{F}}_{i\sigma(i)} \right\rangle\right]$$

where, $\underline{d}(\tilde{s})^{\tilde{T}}, \underline{d}(\tilde{s})^{\tilde{I}}, \underline{d}(\tilde{s})^{\tilde{F}}$ and $\overline{d}(\tilde{s})^{\tilde{T}}, \overline{d}(\tilde{s})^{\tilde{I}}, \overline{d}(\tilde{s})^{\tilde{F}}$ are multiple truth, indeterminacy and false values of lower $\underline{D}_{ij}(\tilde{S})$ and upper $\overline{D}_{ij}(\tilde{S})$ approximation matrices respectively and $i = 1, 2, \ldots, n$. s_n states the symmetric group of all permutations of $\{1, 2, \ldots, n\}$.

Example 4.2. *Take the matrix* $D_{ij}(\tilde{S})$

$$\begin{aligned} det(D_{ij}(\tilde{S})) = {} & [(\{0.1\},\{0.5,0.6\},\{0.8\}),(\{0.2,0.3\},\{0.4\},\{0.7,0.8\})] \cdot \\ & [(\{0.4\},\{0.7,0.8\},\{0.9\}),(\{0.1\},\{0.4,0.5\},\{0.6,0.7\})] + \\ & [(\{0.3\},\{0.2\},\{0.6,0.7\}),(\{0.2\},\{0.1,0.3\},\{0.9\})] \cdot \\ & [(\{0.5,0.6\},\{0.8\},\{0.3\}),(\{0.7\},\{0.6\},\{0.1\})] \end{aligned}$$

$$= \left[(\{.1\},\{.7,.8\},\{.9\}),(\{.1\},\{.4,.5\},\{.7,.8\})\right] + \left[(\{.3\},\{.8\},\{.6,.7\}),(\{.2\},\{.6\},\{.9\})\right]$$

$$det(D_{ij}(\tilde{S})) = \left[(\{0.3\},\{0.7,0.8\},\{0.6,0.7\}),(\{0.2\},\{0.4,0.5\},\{0.7,0.8\})\right]$$

Definition 4.3. Adjoint of MVRNM

The adjoint of a MVRNM $D_{ij}(\tilde{S})$ with order $n \times n$ is stated as $adj.D_{ij}(\tilde{S})$. It is also stated by $adj.D_{ij}(\tilde{S}) = \left|D_{ji}(\tilde{S})\right|$. where $D_{ji}(\tilde{S})$ is the transpose of MVRNM.

$$adj.D_{ij}(\tilde{S}) = \sum_{\sigma \in s_{n_i n_j}} \left[\prod_{t \in n_j} \left\langle \underline{d}(\tilde{s})^{\tilde{T}}_{t\sigma(t)}, \underline{d}(\tilde{s})^{\tilde{I}}_{t\sigma(t)}, \underline{d}(\tilde{s})^{\tilde{F}}_{t\sigma(t)} \right\rangle, \left\langle \overline{d}(\tilde{s})^{\tilde{T}}_{t\sigma(t)}, \overline{d}(\tilde{s})^{\tilde{I}}_{t\sigma(t)}, \overline{d}(\tilde{s})^{\tilde{F}}_{t\sigma(t)} \right\rangle\right]$$

where $s_{n_i n_j}$ is the set of all permutations of n_j over n_i and $n_j = 1, 2, \ldots, n$.

Operations on MVRNM

Let $D_{ij}(\tilde{S}) = \left\langle \left(\underline{d}_{ij}(\tilde{s})^{\tilde{T}}, \underline{d}_{ij}(\tilde{s})^{\tilde{I}}, \underline{d}_{ij}(\tilde{s})^{\tilde{F}}\right), \left(\overline{d}_{ij}(\tilde{s})^{\tilde{T}}, \overline{d}_{ij}(\tilde{s})^{\tilde{I}}, \overline{d}_{ij}(\tilde{s})^{\tilde{F}}\right) \right\rangle$ and

$C_{ij}(\tilde{S}) = \left\langle \left(\underline{c}_{ij}(\tilde{s})^{\tilde{T}}, \underline{c}_{ij}(\tilde{s})^{\tilde{I}}, \underline{c}_{ij}(\tilde{s})^{\tilde{F}}\right), \left(\overline{c}_{ij}(\tilde{s})^{\tilde{T}}, \overline{c}_{ij}(\tilde{s})^{\tilde{I}}, \overline{c}_{ij}(\tilde{s})^{\tilde{F}}\right) \right\rangle$ be two MVRNM with same order then,

(i) Transpose of $D_{ij}(\tilde{S})$

$$D_{ji}(\tilde{S}) = \left\langle \left(\underline{d}_{ji}(\tilde{s})^{\tilde{T}}, \underline{d}_{ji}(\tilde{s})^{\tilde{I}}, \underline{d}_{ji}(\tilde{s})^{\tilde{F}}\right), \left(\overline{d}_{ji}(\tilde{s})^{\tilde{T}}, \overline{d}_{ji}(\tilde{s})^{\tilde{I}}, \overline{d}_{ji}(\tilde{s})^{\tilde{F}}\right) \right\rangle$$

(ii) Trace of $D_{ij}(\tilde{S})$

$$Tr[D_{ij}(\tilde{S})] = \prod_{i=1}^{n} D_{ii}(\tilde{S})$$

(iii) Complement of $D_{ij}(\tilde{S})$

$$D_{ij}(\tilde{S})^{-1} = \left\langle \left(\underline{d}_{ij}(\tilde{s})^{\tilde{T}}, 1-\underline{d}_{ij}(\tilde{s})^{\tilde{I}}, \underline{d}_{ij}(\tilde{s})^{\tilde{F}}\right), \left(\overline{d}_{ij}(\tilde{s})^{\tilde{T}}, 1-\overline{d}_{ij}(\tilde{s})^{\tilde{I}}, \overline{d}_{ij}(\tilde{s})^{\tilde{F}}\right) \right\rangle$$

(iv) Addition of $D_{ij}(\tilde{S}) + C_{ij}(\tilde{S}) =$

$$\left(\bigcup_{\underline{d}^{\tilde{T}},\underline{c}^{\tilde{T}}} max\{\underline{d}_{ij}(\tilde{s})^{\tilde{T}}, \underline{c}_{ij}(\tilde{s})^{\tilde{T}}\}, \bigcup_{\underline{d}^{\tilde{I}},\underline{c}^{\tilde{I}}} min\{\underline{d}_{ij}(\tilde{s})^{\tilde{I}}, \underline{c}_{ij}(\tilde{s})^{\tilde{I}}\}, \bigcup_{\underline{d}^{\tilde{F}},\underline{c}^{\tilde{F}}} min\{\underline{d}_{ij}(\tilde{s})^{\tilde{F}}, \underline{c}_{ij}(\tilde{s})^{\tilde{F}}\} \right),$$
$$\left(\bigcup_{\overline{d}^{\tilde{T}},\overline{c}^{\tilde{T}}} max\{\overline{d}_{ij}(\tilde{s})^{\tilde{T}}, \overline{c}_{ij}(\tilde{s})^{\tilde{T}}\}, \bigcup_{\overline{d}^{\tilde{I}},\overline{c}^{\tilde{I}}} min\{\overline{d}_{ij}(\tilde{s})^{\tilde{I}}, \overline{c}_{ij}(\tilde{s})^{\tilde{I}}\}, \bigcup_{\overline{d}^{\tilde{F}},\overline{c}^{\tilde{F}}} min\{\overline{d}_{ij}(\tilde{s})^{\tilde{F}}, \overline{c}_{ij}(\tilde{s})^{\tilde{F}}\} \right)$$

(v) Multiplication of $D_{ij}(\tilde{S}).C_{ij}(\tilde{S}) =$

$$\left(\bigcup_{\underline{d}^{\tilde{T}},\underline{c}^{\tilde{T}}} min\{\underline{d}_{ij}(\tilde{s})^{\tilde{T}}, \underline{c}_{ij}(\tilde{s})^{\tilde{T}}\}, \bigcup_{\underline{d}^{\tilde{I}},\underline{c}^{\tilde{I}}} max\{\underline{d}_{ij}(\tilde{s})^{\tilde{I}}, \underline{c}_{ij}(\tilde{s})^{\tilde{I}}\}, \bigcup_{\underline{d}^{\tilde{F}},\underline{c}^{\tilde{F}}} max\{\underline{d}_{ij}(\tilde{s})^{\tilde{F}}, \underline{c}_{ij}(\tilde{s})^{\tilde{F}}\} \right),$$
$$\left(\bigcup_{\overline{d}^{\tilde{T}},\overline{c}^{\tilde{T}}} min\{\overline{d}_{ij}(\tilde{s})^{\tilde{T}}, \overline{c}_{ij}(\tilde{s})^{\tilde{T}}\}, \bigcup_{\overline{d}^{\tilde{I}},\overline{c}^{\tilde{I}}} max\{\overline{d}_{ij}(\tilde{s})^{\tilde{I}}, \overline{c}_{ij}(\tilde{s})^{\tilde{I}}\}, \bigcup_{\overline{d}^{\tilde{F}},\overline{c}^{\tilde{F}}} max\{\overline{d}_{ij}(\tilde{s})^{\tilde{F}}, \overline{c}_{ij}(\tilde{s})^{\tilde{F}}\} \right)$$

Definition 4.4. The energy of multi-valued rough neutrosophic matrix

Let $D(\tilde{S})$ be a multi-valued rough neutrosophic matrix with $n \times n$ order. The element of the matrix is uneven, so take maximum value of truth membership function and take the minimum value of indeterminacy and false membership function. Now the matrix is transformed into a single-valued rough neutrosophic matrix.

$$D(\tilde{S}) = \left\langle \left(\bigcup_{max} \underline{d}_{ij}(\tilde{s})^{\tilde{T}}, \bigcup_{min} \underline{d}_{ij}(\tilde{s})^{\tilde{I}}, \bigcup_{min} \underline{d}_{ij}(\tilde{s})^{\tilde{F}} \right), \left(\bigcup_{max} \overline{d}_{ij}(\tilde{s})^{\tilde{T}}, \bigcup_{min} \overline{d}_{ij}(\tilde{s})^{\tilde{I}}, \bigcup_{min} \overline{d}_{ij}(\tilde{s})^{\tilde{F}} \right) \right\rangle$$
$$D(S) = \left\langle \left(\underline{d}_{ij}(s)^{T}, \underline{d}_{ij}(s)^{I}, \underline{d}_{ij}(s)^{F}\right), \left(\overline{d}_{ij}(s)^{T}, \overline{d}_{ij}(s)^{I}, \overline{d}_{ij}(s)^{F}\right) \right\rangle_{n\times n}$$

The matrix $D(S)$ is separated into 6 different matrices: the first 3 are truth, indeterminacy, and false lower approximation matrices; the next 3 are truth, indeterminacy, and false upper approximation matrices. where, the elements of 6 matrices are $\underline{x}_{ij} \in \underline{d}_{ij}(s)^{T}$, $\underline{y}_{ij} \in \underline{d}_{ij}(s)^{I}$, $\underline{z}_{ij} \in \underline{d}_{ij}(s)^{F}$ and $\overline{x}_{ij} \in$

$\overline{d}_{ij}(s)^T$, $\overline{y}_{ij} \in \overline{d}_{ij}(s)^I$, $\overline{z}_{ij} \in \overline{d}_{ij}(s)^F$. Then the multi-valued rough neutrosophic matrix's energy is described as

$$E[D_{ij}(S)] = \left(E[\underline{d}_{ij}(s)^T], E[\underline{d}_{ij}(s)^I], E[\underline{d}_{ij}(s)^F]\right), \left(E[\overline{d}_{ij}(s)^T], E[\overline{d}_{ij}(s)^I], E[\overline{d}_{ij}(s)^F]\right)$$

$$E[D_{ij}(S)] = \left(\sum_{i=1}^{n}\left|\underline{\delta}_i - \mu_{\underline{\delta}}\right|, \sum_{i=1}^{n}\left|\underline{\xi}_i - \mu_{\underline{\xi}}\right|, \sum_{i=1}^{n}\left|\underline{\psi}_i - \mu_{\underline{\psi}}\right|\right), \left(\sum_{i=1}^{n}\left|\overline{\delta}_i - \mu_{\overline{\delta}}\right|, \sum_{i=1}^{n}\left|\overline{\xi}_i - \mu_{\overline{\xi}}\right|, \sum_{i=1}^{n}\left|\overline{\psi}_i - \mu_{\overline{\psi}}\right|\right)$$

where, $\underline{\delta}_i$, $\overline{\delta}_i$ are the eigenvalues of truth membership lower and upper approximation matrices, $\underline{\xi}_i$, $\overline{\xi}_i$ are the eigenvalues of indeterminacy membership lower and upper approximation matrices, and $\underline{\psi}_i$, $\overline{\psi}_i$ are the eigenvalues of false membership lower and upper approximation matrices. $\mu_{\underline{\delta}}$, $\mu_{\overline{\delta}}$, $\mu_{\underline{\xi}}$, $\mu_{\overline{\xi}}$, $\mu_{\underline{\psi}}$, and $\mu_{\overline{\psi}}$ are the mean values of respective eigenvalues.

Example 4.3. *Let the multi-valued rough neutrosophic matrix $D_{ij}(\tilde{S})$ be converted to a single-valued rough neutrosophic matrix $D_{ij}(S)$ by the max-min procedure. Here we convert the $D_{ij}(\tilde{S})$ matrix into $D_{ij}(S)$ by taking the max value of truth and the min value of indeterminacy and false.*

$$D_{ij}(S) = \begin{pmatrix} [(\{0.1\},\{0.5\},\{0.8\}), & [(\{0.3\},\{0.2\},\{0.6\}), \\ (\{0.3\},\{0.4\},\{0.7\})] & (\{0.2\},\{0.1\},\{0.9\})] \\ [(\{0.6\},\{0.8\},\{0.3\}), & [(\{0.4\},\{0.7\},\{0.9\}), \\ (\{0.7\},\{0.6\},\{0.1\})] & (\{0.1\},\{0.4\},\{0.6\})] \end{pmatrix}$$

The matrix can be separated into six matrices, that include truth, indeterminacy, and false membership lower and upper matrices:

$$\underline{d}_{ij}(s)^T = \begin{pmatrix} 0.1 & 0.3 \\ 0.6 & 0.4 \end{pmatrix}, \quad \underline{d}_{ij}(s)^I = \begin{pmatrix} 0.5 & 0.2 \\ 0.8 & 0.7 \end{pmatrix}, \quad \underline{d}_{ij}(s)^F = \begin{pmatrix} 0.8 & 0.6 \\ 0.3 & 0.9 \end{pmatrix}$$

$$\overline{d}_{ij}(s)^T = \begin{pmatrix} 0.3 & 0.2 \\ 0.7 & 0.1 \end{pmatrix}, \quad \overline{d}_{ij}(s)^I = \begin{pmatrix} 0.4 & 0.1 \\ 0.6 & 0.4 \end{pmatrix}, \quad \overline{d}_{ij}(s)^F = \begin{pmatrix} 0.7 & 0.9 \\ 0.1 & 0.6 \end{pmatrix}.$$

The Eigenvalues of $\underline{d}_{ij}(s)^T$ are $\underline{\delta}_i = -0.2, 0.7$ and the mean of its eigenvalues is $\mu_{\underline{\delta}} = 0.25$. The energy of $\underline{d}_{ij}(s)^T$ is $E[\underline{d}_{ij}(s)^T] = |-0.2 - 0.25| + |0.7 - 0.25| = 0.9$ In the same way, the energy can be calculated for each matrix, and leading to the energy of $D_{ij}(S)$:

$$E[D_{ij}(S)] = \left[(0.9, 0.8246, 0.8544), (0.7746, 0.4899, 0.6083)\right].$$

5. Multi-criteria decision making approach using separation measure for multi-valued rough neutrosophic set

In this section, we present the new multi-criteria decision-making method for solving the problem in a decision-making environment. The proposed method includes the separation measure and its ranking coefficient formula for a multi-valued rough neutrosophic set. The method shows the importance of the multi-valued rough neutrosophic set and matrix in solving MCDM problems. The procedure for the proposed method is given below.

We take i alternatives $(P_1, P_2, \ldots, P_i)$ and j criteria $(C_1, C_2, \ldots, C_j)$ for solving MCDM problem. The ratings are taken by the performance of the every alternative over each criterion. The weights of each criteria are asigned by an expert $(c_1, c_2, \ldots, c_j)$. In this process a single person evaluated the alternatives. The ratings are given by the linguistic variable then it transformed into multi-valued neutrosophic numbers. The decision matrix is given below,

$$\begin{array}{c} \\ P_1 \\ P_2 \\ \vdots \\ P_i \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_j \end{array} \\ \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1j} \\ a_{21} & a_{22} & \dots & a_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & \dots & a_{ij} \end{pmatrix} \end{array}$$

Step 1: Build the multi-valued rough neutrosophic decision matrix.

From the above step, we form a multi-valued rough neutrosophic set for each alternative over each criterion. Here, $U = \{a_{11}, a_{12}, \dots, a_{1j}, \dots, a_{2j}, \dots, a_{ij}, c_1, c_2, \dots, c_j\}$ and
$U/R = \{\{a_{11}, c_1\}, \{a_{12}, c_2\}, \dots, \{a_{1j}, c_j\}, \dots, \{a_{2j}, c_j\}, \dots, \{a_{i1}, c_1\}, \dots, \{a_{ij}, c_j\}\}$
we take a multi-valued neutrosophic set $N(\tilde{S})$ = the values of every element in U.
By this we built the following matrix,

$$d_{\tilde{N}} = \left\langle \underline{N}_{ij}(\tilde{S}), \overline{N}_{ij}(\tilde{S}) \right\rangle = \begin{array}{c} \\ P_1 \\ P_2 \\ \vdots \\ P_i \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_j \end{array} \\ \begin{pmatrix} \left\langle \underline{\tilde{N}}_{11}, \overline{\tilde{N}}_{11} \right\rangle & \left\langle \underline{\tilde{N}}_{12}, \overline{\tilde{N}}_{12} \right\rangle & \dots & \left\langle \underline{\tilde{N}}_{1j}, \overline{\tilde{N}}_{1j} \right\rangle \\ \left\langle \underline{\tilde{N}}_{21}, \overline{\tilde{N}}_{21} \right\rangle & \left\langle \underline{\tilde{N}}_{22}, \overline{\tilde{N}}_{22} \right\rangle & \dots & \left\langle \underline{\tilde{N}}_{2j}, \overline{\tilde{N}}_{2j} \right\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \left\langle \underline{\tilde{N}}_{i1}, \overline{\tilde{N}}_{i1} \right\rangle & \left\langle \underline{\tilde{N}}_{i2}, \overline{\tilde{N}}_{i2} \right\rangle & \dots & \left\langle \underline{\tilde{N}}_{ij}, \overline{\tilde{N}}_{ij} \right\rangle \end{pmatrix} \end{array}$$

where, $\underline{N}_{ij}$ is lower approximation values and $\overline{N}_{ij}$ upper approximation values of the multi-valued neutrosophic set $\tilde{S}$.
Step 2: Determining the weights of criteria (see [5]).
We use a variety of methods to determine the weights of the criterion. Let $c_i = \left\langle \tilde{t}_{i\alpha}, \tilde{i}_{i\beta}, \tilde{f}_{i\gamma} \right\rangle$ taken as a multi-valued neutrosophic number. The evaluated values are given by the expert for the criteria. Then the n^{th} criterion's weight can be defined as

$$w_n = \bigcup_{\tilde{t}_{n\alpha}, \tilde{i}_{n\beta}, \tilde{f}_{n\gamma}} \frac{1 - \sqrt{\{(1 - \tilde{t}_{n\alpha}(x))^2 + (\tilde{i}_{n\beta}(x))^2 + (\tilde{f}_{n\gamma}(x))^2\}/3}}{\sum_{n=1}^{j}(1 - \sqrt{\{(1 - \tilde{t}_{n\alpha}(x))^2 + (\tilde{i}_{n\beta}(x))^2 + (\tilde{f}_{n\gamma}(x))^2\}/3})} \tag{5.1}$$

and $\sum_{n=1}^{j} w_n = 1$

Step 3: Transforming multi-valued rough neutrosophic decision matrix into MVNN.
Let $\left\langle \underline{N}_{ij}(\underline{\tilde{t}}_{ij}, \underline{\tilde{i}}_{ij}, \underline{\tilde{f}}_{ij}), \overline{N}_{ij}(\overline{\tilde{t}}_{ij}, \overline{\tilde{i}}_{ij}, \overline{\tilde{f}}_{ij}) \right\rangle$ be a multi-valued neutrosophic rough set. We use the accumulated geometric operator to transform the multi-valued rough neutrosophic number to multi-valued neutrosophic number: $N_{ij} \left\langle \tilde{t}_{ij}, \tilde{i}_{ij}, \tilde{f}_{ij} \right\rangle = N_{ij} \left\langle (\underline{\tilde{t}}_{ij}, \overline{\tilde{t}}_{ij})^{0.5}, (\underline{\tilde{i}}_{ij}, \overline{\tilde{i}}_{ij})^{0.5}, (\underline{\tilde{f}}_{ij}, \overline{\tilde{f}}_{ij})^{0.5} \right\rangle$.

Step 4: Determine the larger and smaller ideal solution for multi-valued rough neutrosophic number.

$$d_j^+ = (\tilde{t}_{j\alpha}^+, \tilde{i}_{j\beta}^+, \tilde{f}_{j\gamma}^+) = \bigcup_{\tilde{t}_\alpha, \tilde{i}_\beta, \tilde{f}_\gamma} (max_i(\tilde{t}_{ij\alpha}), min_i(\tilde{i}_{ij\beta}), min_i(\tilde{f}_{ij\gamma}))$$

$$d_j^- = (\tilde{t}_{j\alpha}^-, \tilde{i}_{j\beta}^-, \tilde{f}_{j\gamma}^-) = \bigcup_{\tilde{t}_\alpha, \tilde{i}_\beta, \tilde{f}_\gamma} (max_i(\tilde{t}_{ij\alpha}), min_i(\tilde{i}_{ij\beta}), min_i(\tilde{f}_{ij\gamma}))$$

	C_1	C_2	$\dots$	C_j
P_1	$\left\langle \tilde{t}_{11}, \tilde{i}_{11}, \tilde{f}_{11} \right\rangle$	$\left\langle \tilde{t}_{12}, \tilde{i}_{12}, \tilde{f}_{12} \right\rangle$	$\dots$	$\left\langle \tilde{t}_{1j}, \tilde{i}_{1j}, \tilde{f}_{1j} \right\rangle$
P_2	$\left\langle \tilde{t}_{21}, \tilde{i}_{21}, \tilde{f}_{21} \right\rangle$	$\left\langle \tilde{t}_{22}, \tilde{i}_{22}, \tilde{f}_{22} \right\rangle$	$\dots$	$\left\langle \tilde{t}_{ij}, \tilde{i}_{ij}, \tilde{f}_{ij} \right\rangle$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
P_i	$\left\langle \tilde{t}_{i1}, \tilde{i}_{i1}, \tilde{f}_{i1} \right\rangle$	$\left\langle \tilde{t}_{i2}, \tilde{i}_{i2}, \tilde{f}_{i2} \right\rangle$	$\dots$	$\left\langle \tilde{t}_{ij}, \tilde{i}_{ij}, \tilde{f}_{ij} \right\rangle$

Step 5: Calculate the separation measures between the multi-valued neutrosophic numbers of every alternative from larger ideal solution and smaller ideal solution.

Definition 5.1. Separation measure between two multi-valued neutrosophic numbers
Let $S_1 = (\tilde{s}_{\alpha 1}, \tilde{s}_{\beta 1}, \tilde{s}_{\gamma 1})$ and $S_2 = (\tilde{s}_{\alpha 2}, \tilde{s}_{\beta 2}, \tilde{s}_{\gamma 2})$ be 2 multi-valued neutrosophic numbers. Therefore, the following definition applies to the separation measure between S_1 and S_2.

$$D(S_1, S_2) = \bigcup_{\alpha,\beta,\gamma} |s_{\alpha 1} - s_{\alpha 2}| + |s_{\beta 1} - s_{\beta 2}| + |s_{\gamma 1} - s_{\gamma 2}|$$

Separation measure within the larger solution D^+ is

$$D^+(d_j, d_j^+) = \bigcup_{\alpha,\beta,\gamma} |\tilde{t}_\alpha - \tilde{t}_\alpha^+| + |\tilde{i}_\beta - \tilde{i}_\beta^+| + |\tilde{f}_\gamma - \tilde{f}_\gamma^+| \tag{5.2}$$

Separation measure within the smaller solution D^- is

$$D^-(d_j, d_j^-) = \bigcup_{\alpha,\beta,\gamma} |\tilde{t}_\alpha - \tilde{t}_\alpha^-| + |\tilde{i}_\beta - \tilde{i}_\beta^-| + |\tilde{f}_\gamma - \tilde{f}_\gamma^-| \tag{5.3}$$

Step 6: Determine the multi-valued rough neutrosophic ranking coefficient by the following formula

$$R_i^+ = \sum_{n=1}^{j} w_n D^+ \qquad\qquad R_i^- = \sum_{n=1}^{j} w_n D^- \tag{5.4}$$

Step 7: The final ranking is calculated using the formula below.

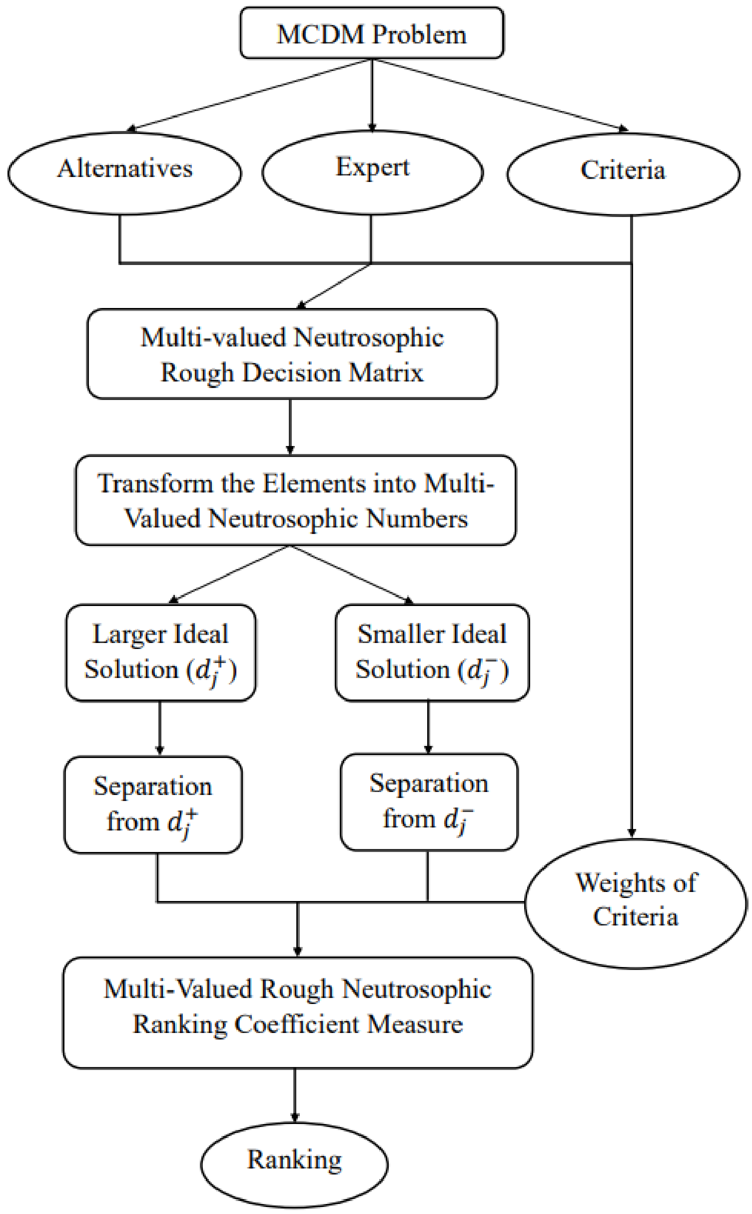
$$R_i = \frac{R_i^-}{R_i^+ + R_i^-} \tag{5.5}$$

The choices are listed from lowest to highest based on this value.

The method is applicable in all application areas. When the multi-valued neutrosophic set is included in the rough matrix, it becomes more efficient to solve the MCDM Problems.

Figure 1 shows the flowchart of the given method. This flowchart makes it simple for understanding the procedure. It describes the MCDM problem with selective alternatives among certain criteria that are assessed by an expert. The expert assigns weights to each criterion. Each alternative's rating values for each criterion are first constructed as a matrix, such as linguistic expressions, and they will then be converted into rough neutrosophic multi-values. Choose the larger ideal and smaller ideal solutions, then calculate the separation values from these ideal solutions. Finally, sorting the alternatives will be helped by the formula for ranking coefficient measures for multi-valued neutrosophic sets.

FIGURE 1. Flowchart of the given approach



6. Numerical Illustration

In a particular hospital, some patients come with the symptoms of dengue. Each one has a different condition. Doctors treat their condition along with their symptoms. The problem is to find out the condition of the patients and which ones have serious dengue stages. By applying the proposed method, we will solve the problem and rank the patients according to their conditions. There are four patients (P_1, P_2, P_3, P_4) to take as alternatives. We take 3 symptoms to identify the condition of dengue. Here symptoms are taken as criteria

C_1 - Fever,
C_2 - Nausea vomiting
C_3 - Aches and pains.

The doctor gives his report by linguistic variables, then we change it into multi-valued neutrosophic numbers. Table 2 presents the values of linguistic variables [16].

TABLE 2. Linguistic terms for MVNNs

Symptoms	**MVNNs**
Very Low - VL	$\langle\{0,.1\},\{.9,1\},\{1\}\rangle$
Low - L	$\langle\{.2\},\{.8\},\{.9,1\}\rangle$
Medium Low - ML	$\langle\{.3,.4\},\{.6,.7\},\{.8\}\rangle$
Fair - F	$\langle\{.5\},\{.5\},\{.5\}\rangle$
Medium High - MH	$\langle\{.6,.7\},\{.3,.4\},\{.2\}\rangle$
High - H	$\langle\{.8\},\{.2\},\{0,.1\}\rangle$
Very High - VH	$\langle\{.9,1\},\{0,.1\},\{0\}\rangle$

Symptoms reported by a doctor: Fever (c_1) - Very High, Nausea vomiting (c_2) - High, and Aches and pains (c_3) - Medium High. When it matches a patient's symptoms, we can identify the patient's condition. Table 3 shows the identified report of each patient given by doctor.

TABLE 3. Report of doctor for each patients

	C_1	C_2	C_3
P_1	H	F	MH
P_2	F	VH	L
P_3	VH	MH	F
P_4	MH	L	VL

Here, $U = \{a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}, a_{41}, a_{42}, a_{43}, c_1, c_2, c_3\}$

$$U/R = \begin{Bmatrix} \{a_{11}, c_1\}, & \{a_{12}, c_2\}, & \{a_{13}, c_3\} \\ \{a_{21}, c_1\}, & \{a_{22}, c_2\}, & \{a_{23}, c_3\} \\ \{a_{31}, c_1\}, & \{a_{32}, c_2\}, & \{a_{33}, c_3\} \\ \{a_{41}, c_1\}, & \{a_{42}, c_2\}, & \{a_{43}, c_3\} \end{Bmatrix}$$

$$N(\tilde{S}) = \begin{Bmatrix} (a_{11}, H), & (a_{12}, F), & (a_{13}, MH) \\ (a_{21}, F), & (a_{22}, VH), & (a_{23}, L) \\ (a_{31}, VH), & (a_{23}, MH), & (a_{33}, F) \\ (a_{41}, MH), & (a_{42}, L), & (a_{43}, VL) \\ (c_1, VH), & (c_2, H), & (c_3, MH) \end{Bmatrix}$$

Step 1: From the above, we defined the upper approximation and lower approximation of the multi-valued neutrosophic set for each alternative over criteria. The multi-valued rough neutrosophic decision matrix is built in this process. It is shown in Table 4.

$a_{11} = H = \langle\{0.8\},\{0.2\},\{0,0.1\}\rangle$ and $c_1 = VH = \langle\{0.9,1\},\{0,0.1\},\{0\}\rangle$

$$\underline{N}(P_1C_1) = \left\langle min\ (\tilde{T}), max\ (\tilde{I}), max\ (\tilde{F})\right\rangle = \langle\{0.8\},\{0.2\},\{0,0.1\}\rangle$$

$$\overline{N}(P_1C_1) = \left\langle max\ (\tilde{T}), min\ (\tilde{I}), min\ (\tilde{F})\right\rangle = \langle\{0.9,1\},\{0,0.1\},\{0\}\rangle$$

Table 4. Multi-valued Rough Neutrosophic Decision Matrix

	C_1	C_2	C_3
P_1	$(\underline{N}\langle\{.8\},\{.2\},\{0,.1\}\rangle$ $\overline{N}\langle\{.9,1\},\{0,.1\},\{0\}\rangle)$	$(\underline{N}\langle\{.5\},\{.5\},\{.5\}\rangle$ $\overline{N}\langle\{.8\},\{.2\},\{0,.1\}\rangle)$	$(\underline{N}\langle\{.6,.7\},\{.3,.4\},\{.2\}\rangle$ $\overline{N}\langle\{.6,.7\},\{.3,.4\},\{.2\}\rangle)$
P_2	$(\underline{N}\langle\{.5\},\{.5\},\{.5\}\rangle$ $\overline{N}\langle\{.9,1\},\{0,.1\},\{0\}\rangle)$	$(\underline{N}\langle\{.8\},\{.2\},\{0,.1\}\rangle$ $\overline{N}\langle\{.9,1\},\{0,.1\},\{0\}\rangle)$	$(\underline{N}\langle\{.2\},\{.8\},\{.9,1\}\rangle$ $\overline{N}\langle\{.6,.7\},\{.3,.4\},\{.2\}\rangle)$
P_3	$(\underline{N}\langle\{.9,1\},\{0,.1\},\{0\}\rangle$ $\overline{N}\langle\{.9,1\},\{0,.1\},\{0\}\rangle)$	$(\underline{N}\langle\{.6,.7\},\{.3,.4\},\{.2\}\rangle$ $\overline{N}\langle\{.8\},\{.2\},\{0,.1\}\rangle)$	$(\underline{N}\langle\{.5\},\{.5\},\{.5\}\rangle$ $\overline{N}\langle\{.6,.7\},\{.3,.4\},\{.2\}\rangle)$
P_4	$(\underline{N}\langle\{.6,.7\},\{.3,.4\},\{.2\}\rangle$ $\overline{N}\langle\{.9,1\},\{0,.1\},\{0\}\rangle)$	$(\underline{N}\langle\{.2\},\{.8\},\{.9,1\}\rangle$ $\overline{N}\langle\{.8\},\{.2\},\{0,.1\}\rangle)$	$(\underline{N}\langle\{0,.1\},\{.9,.1\},\{1\}\rangle$ $\overline{N}\langle\{.6,.7\},\{.3,.4\},\{.2\}\rangle)$

Similarly, we can find the MVRNS for each pair in U/R. (each patient on each symptom)

Step 2: Transforming multi-valued rough neutrosophic decision matrix into MVNN.

$$P_1C_1 = \left\langle \{(0.8\times 0.9)^{0.5}, (0.8\times 1)^{0.5}\}, \{(0.2\times 0)^{0.5}, (0.2\times 0.1)^{0.5}\}, \{(0\times 0)^{0.5}, (0\times 0.1)^{0.5}\} \right\rangle$$
$$P_1C_1 = \langle\{.8485, .8944\}, \{0, .1414\}, \{0\}\rangle$$

Remaining values are calculated in the same way. The values are shown in Table 5.

Table 5. Transformation of MVRNS into MVNNs

	C_1	C_2
P_1	$\langle\{0.8485, 0.8944\}, \{0, 0.1414\}, \{0\}\rangle$	$\langle\{0.6324\}, \{0.3162\}, \{0, 0.2236\}\rangle$
P_2	$\langle\{0.6708, 0.7171\}, \{0, 0.2236\}, \{0\}\rangle$	$\langle\{0.8485, 0.8944\}, \{0, 0.1414\}, \{0\}\rangle$
P_3	$\langle\{0.9, 1\}, \{0, 0.1\}, \{0\}\rangle$	$\langle\{0.6928, 0.7483\}, \{0.2449, 0.2828\}, \{0, 0.1414\}\rangle$
P_4	$\langle\{0.7348, 0.8366\}, \{0, 0.2\}, \{0\}\rangle$	$\langle\{0.4\}, \{0.4\}, \{0, 0.3162\}\rangle$

	C_3
P_1	$\langle\{0.6, 0.7\}, \{0.3, 0.4\}, \{0.2\}\rangle$
P_2	$\langle\{0.3464, 0.3742\}, \{0.4899, 0.5657\}, \{0.4243, 0.4472\}\rangle$
P_3	$\langle\{0.5477, 0.5916\}, \{0.3873, 0.4473\}, \{0.3102\}\rangle$
P_4	$\langle\{0, 0.2646\}, \{0.5196, 0.6324\}, \{0.4472\}\rangle$

Step 3: Weights of symptoms C_1 - VH, C_2 - H and C_3 - MH. Calculate the values for each symptom.

$$w_1 = \frac{1 - \left(\sqrt{(1-0.9)^2 + (0) + (0)/3} + \sqrt{(1-1)^2 + (0.1)^2 + (0)/3}\right)}{3 - (0.1149 + 0.3365 + 0.6218)} = 0.459$$

Similarly, we get the weight values $w_2 = 0.345$ and $w_3 = 0.196$

Step 4: Larger (d^+) ideal solution and smaller (d^-) ideal solution. For each symptom, we find the maximum ideal value and minimum ideal value. These values are shown in Table 6.

Step 5: Using the equations (2) and (3) calculate the values of D^+ and D^-.

$$\begin{aligned} D^+(P_1C_1, d_j^+) &= |0.9-0.8485| + |1-0.8944| + |0-0| + |0.1-0.1414| + |0-0| \\ &= 0.1985 \end{aligned}$$

$$\begin{aligned} D^-(P_1C_1, d_j^-) &= |0.6708-0.8485| + |0.7171-0.8944| + |0.2236-0| + |0.2236-0.1414| + |0-0| \\ &= 0.6708 \end{aligned}$$

TABLE 6. Larger and smaller ideal solutions

	C_1	C_2
d^+	$\langle\{.9,1\},\{0,.1\},\{0\}\rangle$	$\langle\{.8485,.8944\},\{0,.1414\},\{0\}\rangle$
d^-	$\langle\{.6708,.7071\},\{.2236\},\{0\}\rangle$	$\langle\{.4\},\{.4\},\{0,.3162\}\rangle$
	C_3	
d^+	$\langle\{0.3464,0.3742\},\{0.4899,0.5657\},\{0.4243,0.4472\}\rangle$	
d^-	$\langle\{0,.2646\},\{.5196,.6324\},\{.4472\}\rangle$	

Similarly, calculated the separation measure value for each patient on each symptom. The values of D^+ and D^- shown in Table 7 and 8.

TABLE 7. Separation from d^+

D^+	C_1	C_2	C_3
P_1	0.1985	1.1927	0
P_2	0.6457	0	1.4065
P_3	0	0.8295	0.4114
P_4	0.4286	1.9177	1.7346

TABLE 8. Separation from d^-

D^-	C_1	C_2	C_3
P_1	0.6708	0.4088	1.7346
P_2	0	1.9177	0.5753
P_3	0.8693	1.0882	1.3232
P_4	0.4407	0	0

Step 6: Calculate the multi-valued rough neutrosophic ranking coefficient by equation (4)

$$R_1^+ = 0.1985 \times 0.459 + 1.4115 \times 0.345 + 0 \times 0.196 = 0.5026$$

$$R_1^- = 0.6708 \times 0.459 + 0.4088 \times 0.345 + 1.7346 \times 0.196 = 0.7888$$

Similarly, the remaining values are calculated.

Step 7: Calculate the final ranking by the equation (5)

$$R_1 = \frac{0.7888}{0.5026 + 0.7888} = 0.6108$$

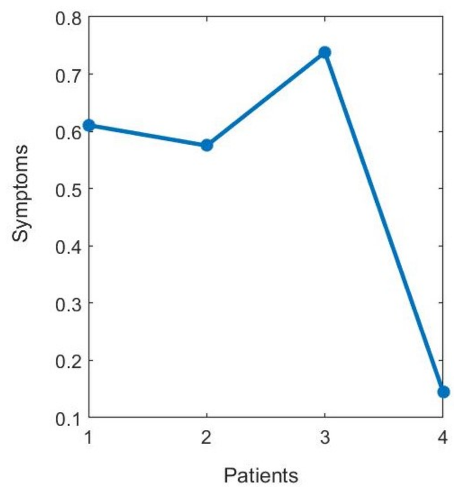
The remaining values are caculated by same way.

TABLE 9. Order of patient

	Ranking coefficient	Order
P_1	0.6108	II
P_2	0.5751	III
P_3	0.7381	I
P_4	0.1444	IV

Table 9 shows each patient's ranking values and their order. The order of the patient's condition is $P_3 > P_1 > P_2 > P_4$. Patient 3 suffers from severe dengue symptoms. Patient 4 was affected less than others. The outcome of the problem is displayed in Figure 2. It shows the plotting of alternatives, which was done by MATLAB. Each plot represents a patient. Patient 3 got a high position.

Figure 2. Plotting by Matlab



7. Comparison Analysis

The TOPSIS method was one of the best methods for solving MCDM situations. So we solve the problem with multi-valued neutrosophic numbers by the neutrosophic simplified TOPSIS method [16]. Then we compare our proposed method result to the existing TOPSIS method result to show our proposed method is valid when applied to decision-making problems. The ranking order is the same for these two methods. It is shown in Table 10. This comparison of the results demonstrates the accuracy of the suggested procedure and the stability of the ranking. Both findings are ranked in the following order: $P_3 > P_1 > P_2 > P_4$. The TOPSIS method's final ranking formula (R) is provided below, and the results are shown in Table.

$$R = \frac{F^-}{F^+ + F^-}$$

where, F^+ is maximum ideal solution and F^- is minmum ideal solution.

Table 10. Comparative Results

	TOPSIS Result				Proposed Method Result	
Patients	F^+	F^-	R	Order	Ranking coefficient	Order
P_1	3.07	5.57	0.6447	II	0.6108	II
P_2	4.6	4.66	0.5032	III	0.5751	III
P_3	1.98	6.64	0.7703	I	0.7381	I
P_4	7.74	0.88	0.1020	IV	0.1444	IV

8. Results and Discussion

The previous sections provided a simple numerical illustration of our suggested approach and its comparison results with the TOPSIS method. The chosen numerical example contains three criteria and four alternatives, but it can be solved using the suggested method with many more criteria and more alternatives. The multi-valued rough neutrosophic set thus contributes significantly to decision-making. The advantages and disadvantages of the multi-valued neutrosophic rough set are displayed in Table 11. The future studies will benefit from the presented set and matrix.

Table 11. Advantages and Disadvantages of the rough neutrosophic set in a multi-valued structure

Adavantages
1) The multi-valued rough neutrosophic set can easily be transformed into single-valued or interval-valued rough neutrosophic sets.
2) The steps of the given method are easy to applicable for this structure.
3) Useful for MCDM issues in daily life and used to rank more possibilities.
4) When applied to MCDM issues, this structure is a specific case compared to other structures.
5) Its utilization in the medical field is more valuable.
Disadvantages
1) This structure makes it difficult to directly access the alternatives. With the help of linguistic terms, evaluating will be more feasible.
2) Manually calculating the values for the steps is challenging.
3) Compared to other structures, it has fewer effective environments.

The multi-valued rough neutrosophic set contains the membership of multiple numbers. Every membership function of the set contains the values of single-valued or interval-valued numbers, as shown in Example 3.2 This form of a set is known as a multi-valued rough neutrosophic set. Further the idea will be expanded into a rough neutrosophic hesitant set. It contains the membership function of hesitant neutrosophic numbers. For example, take the set

$$\tilde{H} = \Big\langle \big(\{0.21, 0.42\}, \{0.1, 0.3, 0.4, 0.6\}, \{0.4, 0.8, 1\}\big), \big(\{0.9\}, \{0.45, 0.65\}, \{0.77, 0.35, 0.8\}\rangle\big)$$

In this set, each membership function contains one or more parameters. This form of a set $\tilde{H}$ is called a rough neutrosophic hesitant set. From this, we can say the multi-valued rough neutrosophic set is the subset of the rough neutrosophic hesitant set.

$$MVRNS \subset RNHS$$

Similarly, the multi-valued neutrosophic matrix in rough structure is the subset of the rough neutrosophic hesitant matrix.

9. Conclusion

A rough neutrosophic set aims to improve quality in various applications, and a rough neutrosophic matrix is used in decision-making problems. In this paper, we showed that a multi-valued neutrosophic rough set and matrix were used in MCDM situations. A multi-valued neutrosophic set has membership values including both single and interval values, so the multi-valued set is more effective at solving uncertainty problems than a neutrosophic set. Our proposed method for the multi-valued neutrosophic rough set successfully solves the task of identifying the condition of dengue patients. As a result, patient

3 has serious symptoms, and MVNRS demonstrates that it is more useful for handling indeterminacy situations. The suggested method's outcomes and comparison analyses are discussed in detail. Rough set theory is achieved in many fields like pattern recognition, machine learning, image processing, expert systems, knowledge discovery in databases, and so on. Our proposed method helps to solve MCDM issues in many applications, not only in the medical field. Furthermore, we will apply this approach using a multi-valued neutrosophic rough matrix and separation measure for multi-valued neutrosophic rough sets in various MCDM circumstances and extend the idea of a rough matrix to other types of neutrosophic environments, including Bipolar, Complex, and Pythagorean neutrosophic sets.

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