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THE EQUITABLE NON-SPLIT DOMINATION NUMBER OF GRAPHS

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Abstract. Finding a group of dominant servers is a necessary step towards optimising service provisioning to all clients in the complex world of server-client networks. This crucial decision rests on the non-split dominance number in the graph architecture. We present the idea of an equitable non-split domination number in graphs to address the possible problem of unequal task distribution among servers. When the vertices of a dominating set, D_{ens}, have the same colour in an equitable colouring scheme, the set is said to be equitable. The induced subgraph $\langle V - D\rangle$ also maintains its connectivity. In particular, the application of equitable non-split domination numbers to graphs like the Triangular Grid, Antiprism, Wheel, and Complete Graphs is explored in this study.

1. Introduction

In graph theory domination of graphs is the huge research area. The dominating set of the graph G is a subset D of a vertex set V, such that every vertex not in $V - D$ is adjacent to at least one vertex in the vertex subset D [2]. A dominating set D is a minimal dominating set if no proper subset of D is a dominating set. The number of elements in such a set is called the domination number of a graph [3]. In 1973, Meyer [18] introduced the concept of equitable vertex coloring. Though the idea of equitable coloring emerged a few decades back, it still remains an active area of research. Recently in 2019, Hanna and Obszarski [5] discussed the equitable chromatic number of hyper graphs. Vivik and Xavier [16] in 2021 applied equitable coloring to process the systematic way of assigning tasks to the servers. Equitable coloring of union of graphs were obtained by Loura and Michael [9] in 2022. The concept of non-split domination is introduced by Kulli [7] in the year 2000. There are kind of non-split were defined for various graphs and investigated by the researchers. [8] Global non-split domination concepts was introduced and results were discussed by Kulli and Janakiraman. In 2019 Sujatha and Manickam [15] analyzed non-split domination number of graphs and finalized with the application of non-split domination. Kiruba and Sunitha [6] in 2020 obtained the results on strong split domination polynomials for graphs and gave some properties on the coefficient of polynomial. The non-split domination number of fuzzy graphs were studied by Mohamed and Begum [12] in the year 2021.

The idea of equitable coloring, which has its roots in graph theory, has several real-world applications. Energy efficiency is a major challenge in the world of wireless sensor networks, where there is one noteworthy application. To ensure a balanced resource utilisation and encourage energy saving, sensors are assigned duties based on equitable colouring. Equitable colouring reduces energy imbalances by allocating colours in a way that minimises the differences in job assignments between neighbouring sensors, hence increasing the network's total lifespan. Furthermore, equitable colouring is important for vehicular network traffic control. In metropolitan settings, where traffic congestion is a common

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problem, equitable colouring helps to maximise flow of vehicles. Through equitable colouring schemes, routes are distributed among vehicles in an equitable manner, resulting in a fair and efficient distribution of transportation resources that reduces congestion and improves overall traffic efficiency. A graph theory notion known as non-split domination has numerous real-world applications, most notably in communication network design and optimisation. For effective routing and network management in the field of telecommunications, determining a minimal set of dominating vertices is essential. When it comes to arranging servers or routers strategically to provide complete coverage with the fewest possible components, non-split dominance is essential. This optimisation improves communication networks' scalability and reliability while also lowering infrastructure expenses. Modelling the transmission of disease and epidemiology are two more relevant applications. In order to restrict the transmission of infectious diseases, non-split domination is used to identify crucial nodes within a contact network. This process helps to build targeted intervention measures. Dominant nodes can be positioned in a way that best influences and controls the network's information or contagion flow. Distribution network optimisation can be achieved by using equitable colouring and non-split dominance to make sure that warehouses and distribution centres are positioned in the best possible way to allocate resources fairly and efficiently. More resilient, effective, and equitable systems in a variety of disciplines can be developed by comprehending and utilising equitable colouring and non-split dominance in these situations.

2. Motivation

In a power distribution network, substations are linked by power lines to create a network where each node represents a substation or distribution point, as shown in Figure.1 and Figure.2. Certain substations within this network function as non-split dominating sets, guaranteeing continuous operation even if some substations stop working. This resilience is essential for uninterrupted power supply, as it prevents network failures and ensures the distribution of electricity to connected areas. Essentially, non-split dominating sets act as pillars of stability, protecting against disturbances and upholding the network's operational integrity in the face of unforeseen events or failures.

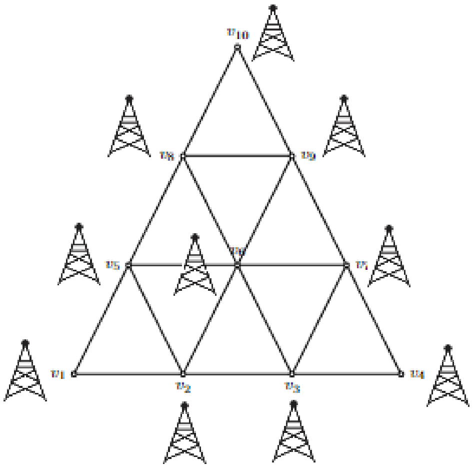


Figure 1.

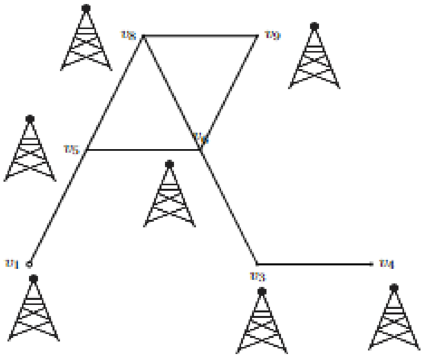


Figure 2.

3. Preliminaries

In this section, we present the key concepts and conclusions that form the foundation of our investigation. These proven ideas and outcomes provide the foundation for our work, which develops and grows upon it.

Definition 3.1. A **dominating set** [14] for graph $G = (V, E)$ is a subset D of V, such that every vertex not in D is adjacent to at least one number of vertex in D. The domination number $\gamma(G)$ is number of vertices in a smallest dominating set for G.

Definition 3.2. The dominating set D is said to be **non-split** [7] dominating set of graph $G = (V, E)$, if the induced subgraph $\langle V - D\rangle$ is connected. [7] **Non-split domination number** γ_{ns} of G is the minimum cardinality of the non-split dominating set.

Definition 3.3. A **vertex coloring** [19] of G is an assignment of colors to the vertices of G such that no adjacent vertices have the same color. Given a vertex coloring of G with m colors, let V_i denote the set of vertices colored i, for $i = 1, ..., m$.

Definition 3.4. We define a proper vertex coloring of a graph G as **equitable** [19], if the size of color classes differ by at most one. The equitable chromatic number of G, denoted by $\chi_{eq}(G)$, is the smallest integer m such that G is equitably m-colorable.

Definition 3.5. An **antiprism graph** [4] is a graph having $2n$ vertices on two cycles labelled with $\{v_i : i = 1, 2, \ldots, n\}$ in the inner cycle, $\{v_j : j = n + i\}$ in the outer cycle. Joining these two cycles with edges of the form $(v_iv_j : 1 \leq i \leq n, j = n + i)$, (v_1v_{2n}) and $(v_iv_{j-1} : 2 \leq i \leq n, j = n + i)$ such that v_i and $v_{i+1(\bmod n)}$ are adjacent. It is symbolized as Q_n.

Definition 3.6. For any integer $n \geq 4$, the **wheel graph** W_n [17] is the $n - 1$-vertex graph obtained by joining a vertex v_0 to each of the $n - 1$ vertices $\{v_1, v_2, .., v_n\}$ of the cycle graph.

Definition 3.7. A **triangular grid graph** [11] is the graph obtained by interpreting the order $(n+1)$ triangular grid as a graph, with the intersection of grid lines being the vertices and the line segments between vertices being the edges.

Conjecture 3.1. [18] $\chi_{eq}(G) \geq \Delta(G)$ *for any connected graph* G *which is neither a complete graph nor an odd cycle*

Theorem 3.2. [10] *If* G *is a connected simple graph,* $G \neq K_n$ *and* $\Delta \geq 3$*, then* G *is* $\Delta(G)-$ *colorable in the classical sense.*

Definition 3.8. The dominating set D of a graph G is said to be **equitable non-spilt dominating set** D_{ens} of G , if the vertices $v_i \in D$ assign the same color on equitable coloring and the induced subgraph $\langle V - D\rangle$ is connected.

Theorem 3.3. *For any graph* $G = (V, E)$*, the bound for equitable non-split domination number is given by* $\gamma(G) \leq \gamma_{ens}(G) \leq \left\lceil \frac{|V|}{\chi_{eq}} \right\rceil$.

Proof. According to definition 3.3 of $\gamma_{ens}(G)$ the equitable non-split dominating set D_{ens} must be the dominating set. For any graph G, it is true that the domination number of G is less than or equal to its equitable non-split domination number, hence $\gamma(G) \leq \gamma_{ens}(G)$. Also the cardinality of equitable color class of any G is $\left\lceil \frac{|V|}{\chi_{eq}} \right\rceil$. Hence $\gamma(G) \leq \gamma_{ens}(G) \leq \left\lceil \frac{|V|}{\chi_{eq}} \right\rceil$. □

In the next section we use this inequality to find the equitable non-split domination number of antiprism, complete, wheel and triangular grid graphs.

4. COMPUTATION OF EQUITABLE NON-SPLIT DOMINATION NUMBER

In this section, we investigate and determine the fair non-split dominance numbers for the antiprism, complete, wheel, and triangular grid graphs in particular. We explore the complex topologies of these graphs and offer a thorough understanding of the fair distribution of dominating sets and their importance in network setup optimisation for different purposes.

Theorem 4.1. *The equitable non-split domination number of antiprism graph Q_n is given by*

(i) $\gamma_{ens}(Q_n) = \frac{2n}{3}$, *if* $n \equiv 0 \mod 3$;
(ii) $\gamma_{ens}(Q_n) = \frac{n}{2}$, *if* $n \equiv 0 \mod 2$ *and* $n \not\equiv 0 \mod 3$;
(iii) $\gamma_{ens}(Q_n) = \lfloor \frac{n}{2} \rfloor$, *if* $n \equiv 0 \mod 2$ *and* $n \not\equiv 0 \mod 3$.

Proof. Let Q_n be the antiprism graph with vertex set V and edge set E. The number of vertices in Q_n , $|V| = 2n$ given by $V = \{v_j\} \cup \{u_j\}$ where $j = 1, 2, \ldots, n$ and the number of edges $|E| = 4n$.

Case 1. When $n \equiv 0 \mod 3$, assign color 'a' to the vertices of the sets

$$\{v_i : i = 1, 4, \ldots, n-2\} \cup \{u_j : j = 2, 5, \ldots, n-1\},$$

color 'b' to the vertex set

$$\{v_k : k = 2, 5, \ldots, n-1\} \cup \{u_l : l = 3, 6, \ldots, n\}$$

and finally color 'c' to the vertices

$$\{v_p : p = 3, 6, \ldots, n\} \cup \{u_q : q = 1, 4, \ldots, n-2\}.$$

From this process, it is true that the nodes are assigned colors in an equitable manner. The cardinal number of each color class is given by $\left\lceil \frac{2n}{3} \right\rceil$. Each color class is a dominating set and the induced subgraph obtained by deleting one of the color classes is the connected graph. See Figure 3. Hence there holds

$$\gamma_{ens}(Q_n) = \frac{2n}{3}.$$

By theorem 3.3,

$$\gamma(Q_n) \leq \gamma_{ens}(Q_n) = \frac{2n}{3} \leq \left\lceil \frac{2n}{3} \right\rceil.$$

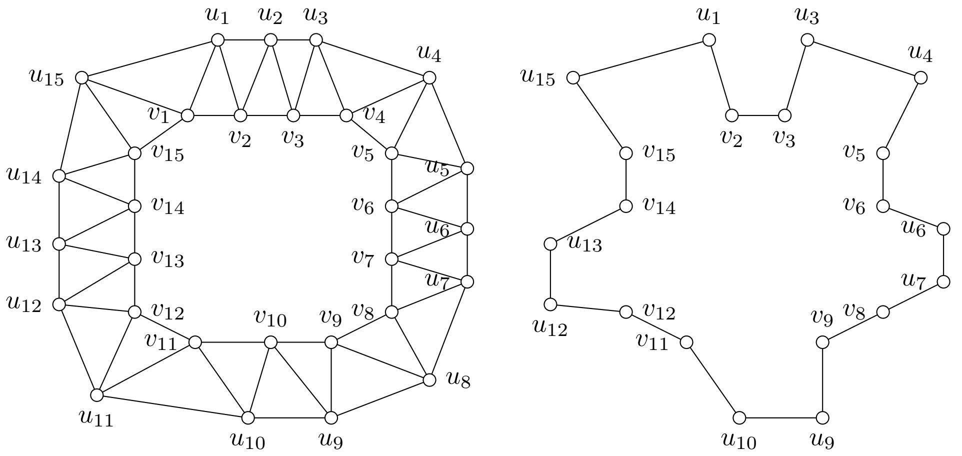


FIGURE 3. Antiprism graph Q_{15} and its connected subgraph.

Case 2. If $n \not\equiv 0 \mod 3$, the equitable chromatic number is $\chi_{eq}(Q_n) = 4$. This case further breaks down into two sub-cases as below.

Subcase 2.1. $n \equiv 0 \mod 2$.

Subcase 2.1.1. When $n \equiv 0 \mod 4$, let 'a' be the color assigned to the vertex set

$$\{v_i : i = 1, 5, \ldots, n-4\} \cup \{u_j : j = 3, 7, \ldots, n-1\},$$

color 'b' assigned to the vertices

$$\{v_k : k = 2, 6, \ldots, n-2\} \cup \{u_l : l = 4, 8, \ldots, n\},$$

'c' be the color assigned to the vertex set

$$\{v_p : p = 3, 7, \ldots, n-1\} \cup \{u_q : q = 1, 5, \ldots, n-4\}$$

and color 'd' to the vertex set

$$\{v_r : r = 4, 8, \ldots, n\} \cup \{u_s : s = 2, 6, \ldots, n-2\}$$

such that the graph is equitably 4-colorable. The cardinal number of each color class is given by $\frac{n}{2}$. Each equitable color class is the dominating set of graph Q_n and the induced subgraph obtained by deleting one of the color classes is connected. Hence, there holds

$$\gamma_{ens}(Q_n) = \frac{n}{2}.$$

Subcase 2.1.2 When $n \equiv 2 \mod 4$, let 'a', 'b', 'c' and 'd' are the colors assigned to the vertex sets

$$\{v_i : i = 1, 5, \ldots, n-1\} \cup \{u_j : j = 3, 7, \ldots, n-3\},$$
$$\{v_k : k = 2, 6, \ldots, n-4, n\} \cup \{u_l : l = 4, 8, \ldots, n-2\},$$
$$\{v_p : p = 3, 7, \ldots, n-3\} \cup \{u_q : q = 1, 5, \ldots, n-1\} \text{ and}$$
$$\{v_r : r = 4, 8, \ldots, n-2\} \cup \{u_s : s = 2, 6, \ldots, n-4, n\} \text{ respectively.}$$

It is fact that the graph Q_n is equitably 4-colorable. Each equitable color class contains $\frac{n}{2}$ number of vertices and also each color class is the dominating set. The induced subgraph obtained by deleting one of such color class is the connected graph. Therefore,

$$\gamma_{ens}(Q_n) = \frac{n}{2}.$$

From the inequality in theorem 3.3, it implies

$$\gamma(Q_n) \le \gamma_{ens}(Q_n) = \frac{n}{2} \le \left\lceil \frac{2n}{4} \right\rceil.$$

Subcase 2.2. $n \equiv 1 \mod 2$.

Subcase2.2.1. When $n \equiv 1 \mod 4$, assign color 'a' to the vertex set

$$\{v_i : i = 1, 5, \ldots, n-4\} \cup \{u_j : j = 3, 7, \ldots, n-2\},$$

color 'b' to the vertex set

$$\{v_k : k = 2, 6, \ldots, n-3, n\} \cup \{u_l : l = 4, 8, \ldots, n-5\},$$

color 'c' to the vertices

$$\{v_p : p = 3, 7, \ldots, n-2\} \cup \{u_q : q = 1, 5, \ldots, n-4, n-1\}$$

and color 'd' to the vertex set

$$\{v_r : r = 4, 8, \ldots, n-5, n-1\} \cup \{u_s : s = 2, 6, \ldots, n-3, n\}$$

such that the graph is assigned with colors in an equitable manner. The number of elements in the equitable color classes of 'a' and 'b' is equals to $\left\lfloor \frac{n}{2} \right\rfloor$, and the cardinal number of equitable color classes of 'c' and 'd' is given by $\left\lceil \frac{n}{2} \right\rceil$. All the four equitable color classes are the dominating sets and removing one of such color class results in the desired connected induced subgraph. Here the equitable color classes of 'c' and 'd' have the minimum cardinality. Thus,

$$\gamma_{ens}(Q_n) = \left\lfloor \frac{n}{2} \right\rfloor.$$

Subcase 2.2.2. When $n \equiv 3 \mod 4$, assign color 'a' to the vertex set

$$\{v_i : i = 1, 5, \ldots, n-2\} \cup \{u_j : j = 3, 7, \ldots, n-4\},$$

color 'b' to the vertex set

$$\{v_k : k = 2, 6, \ldots, n-1\} \cup \{u_l : l = 4, 8, \ldots, n-3, n\},$$

color 'c' to the vertices

$$\{v_p : p = 3, 7, \ldots, n-4, n\} \cup \{u_q : q = 1, 5, \ldots, n-2\}$$

and color 'd' to the vertices

$$\{v_r : r = 4, 8, \ldots, n-3\} \cup \{u_s : s = 2, 6, \ldots, n-1\}.$$

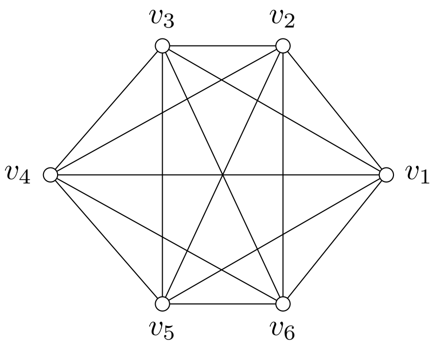
Henceforth the graph is assigned with colors in an equitable way. The number of elements in color classes 'a' and 'd' is given by $\left\lfloor \frac{n}{2} \right\rfloor$, and the cardinal number of color classes 'b' and 'c' is $\left\lceil \frac{n}{2} \right\rceil$. Each equitable color class is the dominating set and the induced subgraph obtained by deleting one of the equitable color classes will be a connected graph. The minimum dominating set is the equitable color classes of 'a' and 'd'. Therefore,

$$\gamma_{ens}(Q_n) = \left\lfloor \frac{n}{2} \right\rfloor.$$

By theorem 3.3, we have $\gamma(Q_n) \leq \gamma_{ens}(Q_n) = \left\lfloor \frac{n}{2} \right\rfloor \leq \left\lceil \frac{2n}{4} \right\rceil$. ☐

Theorem 4.2. *For complete graph K_n where $n \geq 1$, the equitable non-split domination number is given by $\gamma_{ens}(K_n) = 1$.*

Proof. Let $K_n = (V, E)$ be the complete graph with vertex set $V = \{v_1, v_2, \ldots, v_n\}$ such that $|V| = n$ and $|E| = n(\frac{n-1}{2})$. For any complete graph $\chi_{eq}(K_n) = 1$ and also the domination number $\gamma(K_n) = 1$. Each color class have exactly one vertex of v_i and such vertex is the subset of V, which is adjacent to all other vertices of the graph. See Figure.4 The removal of vertex v_i results in the desired connected induced subgraph. Hence $\gamma_{ens}(K_n) = 1$. By theorem 3.3, $\gamma(K_n) \leq \gamma_{ens}(K_n) = 1 \leq n$. ☐



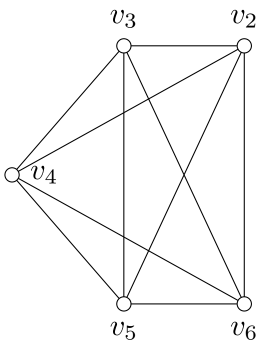


FIGURE 4. Complete graph K_6 and its connected subgraph.

Theorem 4.3. *The equitable non-split domination number of wheel graph* W_n, $n \geq 3$ *is given by* $\gamma_{ens}(W_n) = 1$.

Proof. Let $W_n = (V, E)$ be the wheel graph with vertex set $\{v_0, v_1, v_2, \ldots, v_n\}$ having $|V| = n + 1$ and $|E| = 2n$. Assign color 'a' to the vertex v_0. The colors for the remaining n vertices require $\frac{n+2}{2}$ colors if $n \equiv 0 \mod 2$ and $\frac{n+3}{2}$ colors if $n \equiv 1 \mod 2$, so that the graph is colored in an equitable way. Hence the equitable chromatic number of W_n is given by

$$\chi_{eq}(W_n) = \begin{cases} \frac{n+2}{2}, & \text{if } n \text{ is even} \\ \frac{n+3}{2}, & \text{if } n \text{ is odd} \end{cases}$$

The vertex v_0 is adjacent with all the other vertices henceforth $\gamma(W_n) = 1$ and removing the vertex v_0 gives the connected induced subgraph. Therefore,

$$\gamma_{ens}(W_n) = 1.$$

By theorem 3.3,

$$\gamma(W_n) \leq \gamma_{ens}(W_n) = 1 \leq \left\lceil \frac{(n+1)(n+3)}{2} \right\rceil.$$

□

In the context of cloud computing, which is defined by a triangular grid architecture, the use of equitable non-split domination provides a smart way to balance server load. Through equitable assignment of processing duties among servers, this methodology improves system performance and efficient use of resources. By reducing the possibility of server overload or underutilization, equitable non-split dominance promotes a fair distribution of workloads. This approach contributes to a more durable and resilient cloud computing infrastructure by improving scalability, responsiveness, and resource efficiency in triangular grid-based cloud settings.

Theorem 4.4. *For any triangular grid graph* T_n *the equitable non-split domination number is*

$$\gamma_{ens}(T_n) = \left\lfloor \frac{(n+1)(n+2)}{6} \right\rfloor.$$

Proof. Let $T_n = (V, E)$ be the triangular grid graph with $|V| = \frac{(n+1)(n+2)}{2}$. The vertices are represented as follows

$$\{v_{11}, v_{12}, \ldots, v_{1(n+1)}\} \cup \{v_{21}, v_{22}, \ldots, v_{2n}\} \cup \ldots \cup \{v_{n1}\}.$$

Case 1. When $n \equiv 0 \mod 3$ assign color 'a' to the vertex set

$$\{v_{11}, v_{14}, \ldots, v_{1(n+1)}\} \cup \{v_{22}, v_{25}, \ldots, v_{2(n-1)}\}$$
$$\cup \{v_{33}, v_{36}\}, \ldots, v_{3(n-3)}\} \cup \ldots \cup \{v_{(n-1)2}\} \cup \{v_{(n+1)1}\}$$

assign color 'b' to the set

$$\{v_{12}, v_{15}, \ldots, v_{1(n-1)}\} \cup \{v_{23}, v_{26}, \ldots, v_{2n}\}$$
$$\cup \{v_{31}, v_{34}, \ldots, v_{3(n-2)}\} \cup \ldots \cup \{v_{(n-1)3}\} \cup \{v_{(n+1)1}\}$$

and color 'c' to the vertex set

$$\{v_{13}, v_{16}, \ldots, v_{1n}\} \cup \{v_{21}, v_{24}, \ldots, v_{2(n-2)}\}$$
$$\cup \{v_{32}, v_{35}, \ldots, v_{3(n-1)}\} \cup \ldots \cup \{v_{(n-1)1}\} \cup \{v_{n2}\}$$

such that the graph T_n is equitably 3-colorable. The cardinality of color class of 'a' is given by $\left\lceil \frac{(n+1)(n+2)}{6} \right\rceil$ and the cardinality of color classes of 'b' and 'c' is given by $\left\lfloor \frac{(n+1)(n+2)}{6} \right\rfloor$. Each equitable color class is the dominating set and deleting one of the color class will give the connected induced subgraph. The minimum cardinality is the number of elements in the color classes 'b' and 'c' Therefore,

$$\gamma_{ens}(T_n) = \left\lfloor \frac{(n+1)(n+2)}{6} \right\rfloor.$$

Case 2. When $n \equiv 1 \bmod 3$ assign color 'a' to the vertex set

$$\begin{aligned}&\{v_{11}, v_{14}, \ldots, v_{1n}\} \cup \{v_{22}, v_{25}, \ldots, v_{2(n-2)}\}\\ &\cup \{v_{33}, v_{36}, \ldots, v_{3(n-1)}\} \cup \ldots \cup \{v_{(n-1)3}\} \cup \{v_{n1}\}\end{aligned}$$

assign color 'b' to the set

$$\begin{aligned}&\{v_{12}, v_{15}, \ldots, v_{1(n+1)}\} \cup \{v_{23}, v_{26}, \ldots, v_{2(n-1)}\}\\ &\cup \{v_{31}, v_{34}, \ldots, v_{3(n-3)}\} \cup \ldots \cup \{v_{(n-1)1}\} \cup \{v_{n2}\}\end{aligned}$$

and color 'c' to the vertex set

$$\begin{aligned}&\{v_{13}, v_{16}, \ldots, v_{1(n-1)} \ \cup \{v_{21}, v_{24}, \ldots, v_{2n}\}\cup\\ &\cup \{v_{32}, v_{35}, \ldots, v_{3(n-2)} \cup \ldots \cup \{v_{(n-1)2}\} \cup \{v_{(n+1)1}\}\end{aligned}$$

such that the graph T_n is equitably 3-colorable. The cardinality of each equitable color class is given by $\left\lfloor \frac{(n+1)(n+2)}{6} \right\rfloor$. Each equitable color class is the dominating set and deleting one of the color class will give the connected induced subgraph. Hence,

$$\gamma_{ens}(T_n) = \left\lfloor \frac{(n+1)(n+2)}{6} \right\rfloor.$$

Case 3. When $n \equiv 2 \bmod 3$ assign color 'a' to the vertex set

$$\begin{aligned}&\{v_{11}, v_{14}, \ldots, v_{1(n-1)}\} \cup \{v_{22}, v_{25}, \ldots, v_{2n}\}\\ &\cup \{v_{33}, v_{36}\}, \ldots, v_{3(n-2)}\} \cup \ldots \cup \{v_{(n-1)1}\} \cup \{v_{n2}\}\end{aligned}$$

assign color 'b' to the set

$$\begin{aligned}&\{v_{12}, v_{15}, \ldots, v_{1n}\} \cup \{v_{23}, v_{26}, \ldots, v_{2(n-2)}\}\\ &\cup \{v_{31}, v_{34}, \ldots, v_{3(n-1)}\} \cup \ldots \cup \{v_{(n-1)2}\} \cup \{v_{(n+1)1}\}\end{aligned}$$

and color 'c' to the vertex set

$$\begin{aligned}&\{v_{13}, v_{16}, \ldots, v_{1(n+1)}\} \cup \{v_{21}, v_{24}, \ldots, v_{2(n-1)}\}\\ &\cup \{v_{32}, v_{35}, \ldots, v_{3(n-3)}\} \cup \ldots \cup \{v_{(n-1)3}\} \cup \{v_{n1}\}\end{aligned}$$

such that the graph T_n is equitably 3-colorable. See Figure.5. The cardinality of each equitable color classes is given by $\left\lfloor \frac{(n+1)(n+2)}{6} \right\rfloor$. Each equitable color class is the dominating set and deleting one of the color class will give the connected induced subgraph. Hence,

$$\gamma_{ens}(T_n) = \left\lfloor \frac{(n+1)(n+2)}{6} \right\rfloor.$$

By theorem 3.3,

$$\gamma(T_n) \leq \gamma_{ens}(T_n) = \left\lfloor \frac{(n+1)(n+2)}{6} \right\rfloor \leq \left\lceil \frac{(n+1)(n+2)}{6} \right\rceil.$$

□

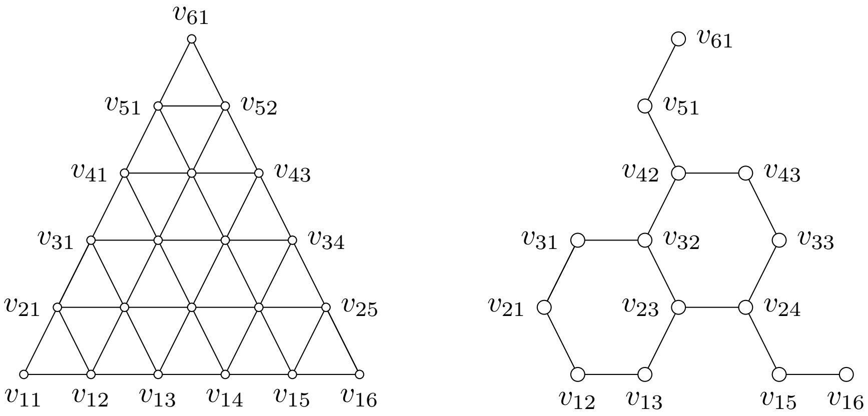


FIGURE 5. Triangular grid graph T_5 and its connected subgraph.

5. CONCLUSION

Distribution centres and warehouses can be positioned strategically to maximise coverage and minimise the number of facilities required by using equitable non-split dominance. This optimisation improves the effectiveness of supply chain networks while also lowering operating expenses. The boundaries of the domination number in graph theory have been the subject of extensive study over the course of several decades. This ongoing question has prompted studies aimed at comprehending the dominance numbers in various graph configurations. In this work, we add to this continuing discussion by presenting the novel idea of a non-split domination number that is equitable. We also study the fair distribution of dominating sets in graphs, going beyond traditional domination numbers. The results of our careful analysis and presentation of the boundaries of equitable non-split domination numbers for various graph types—such as antiprism, complete, wheel, and triangular grid graphs—are presented in this study. As a unique metric, the suggested equitable non-split domination number clarifies the equitable assignment of dominating sets in different graph topologies. Bounds on equal non-split dominance numbers are found, which provides important information about the intrinsic properties of these graphs and their possible uses in network optimisation. In the future, bounds on equitable non-split domination numbers can be investigated over a wider range of graph types. Subsequent investigations could reveal novel trends, connections, and uses, enhancing our comprehension of fair non-split dominance in many graph-theoretical scenarios. This work represents a major advancement in the theoretical underpinnings of domination theory and its applicability in a variety of fields.

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