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Prediction of Electricity Generation in Nigeria using Exponential Regression and Cobb-Douglas Models

Aneke Chikezie[1*], Harrison Osasogie Edokpolor[2], and Uwakwe Chikwado[2]

[1]Department of Electrical/Electronic and Computer Engineering, Faculty of Engineering, University of Uyo, Uyo, Nigeria
[2]Department of Electrical/Electronic Engineering Imo State Polytechnic, Umuagwo, Owerri, Nigeria
*Corresponding Author: AnekeChikezie_UNIUYO@yahoo.com

Abstract

This study presents prediction of electricity generation in Nigeria using two different statistical models, namely; exponential regression and Cobb-Douglas models. Rainfall and temperature were used as the explanatory variables. Data on electricity generation in Nigeria between 2002 and 2014 were obtained from the Central Bank of Nigeria Statistical Bulletin while Data on rainfall and temperature between 2002 and 2014 were extracted from the National Bureau of Statistics (NBS) abstract. Test of model fitness and forecasting accuracy were done using generic statistical approach which include coefficient of determination and root mean square error. The prediction accuracy of the two statistical models was compared and the best model was selected. The best model was then used to forecast electric power generation in Nigeria for the next fifteen years (2015-2029). Among the two statistical models, Cobb-Douglas model was selected as the best model as it gave the highest value of coefficient of determination (r^2=99.85%) and the least Root Mean Square Error (48.57%). Furthermore, the Cobb-Douglas model was used to forecast the electric power generation from 2015 to 2029. The forecasted data shows that power generation in Nigeria in 2029 will stand at 3446.85MWh as against the value of 3249MWh in 2014.

Keywords: Electricity; Electricity Generation; Cobb-Douglas Model; Exponential Regression Model; Electricity Forecasting

1. Introduction

Electricity generation is the process of producing electrical energy or the amount of electrical energy produced by transforming other forms of energy into electrical energy; commonly expressed in kilowatt-hours (kWh) or megawatt-hours (MWh). Electricity is generated from fossil fuels, nuclear power plants, hydro power plants, geothermal systems, solar panels, biofuels, wind and so forth. Presently, Nigeria mostly employs gas-fired and hydroelectric turbines for bulk generation, oil being too expensive and coal-fired stations having gone moribund [1-3].

Electricity is one of the most important commodity for national development. It plays a very important role in the socio-economic and technological development of every nation. The global economy is heavily dependent on energy [4-7]. Electricity is used for a number of purposes that include industrial, commercial and household purposes. The electricity demand in Nigeria far outstrips the supply and the supply is epileptic in nature [8-12]. It is widely accepted that there is a strong correlation between socio-economic development and the availability of electricity. Ayodele [13] asserted that in recognition of the consolidating linkage between the energy sector and the other sectors of the economy, electricity generation and utilization therefore, have pervasive impacts on a range of socio-economic activities and consequently, the living standard of citizens in the country.

Adequate power generation and regular supply is the prime mover of technological and social development. There is hardly any enterprise or indeed any aspect of human development that does not require energy in one form or the other- electric power, fuels [14].

Nigeria is richly endowed with various energy sources, crude oil, natural gas, coal, hydropower, solar energy, fissionable materials for nuclear energy, yet Nigeria consistently suffers from energy shortage, a major impediment to industrial and technological growth. The total generating capacity is about 3000MW, approximately one third of the current level of national demand.

The Nigerian power sector has persistently been erratic and running with a shortfall. The country is faced with acute electricity problems, which is hindering its development notwithstanding the availability of vast natural resources in the country. The electricity demand in Nigeria far outstrips the power generated. It is grossly inadequate to meet the demand of an ever increasing population. This is largely due to inadequate or lack of effective planning. The whole scenario lies on the absence of more informative and detailed data on electricity generation using statistical models. There is a need to understudy electric power generation in Nigeria using some of the factors that could be responsible for the variation in power generation and to forecast into the future for effective planning. Hence, this paper seeks to provide appropriate statistical model for electricity generation in Nigeria based on the available data for the year 2002 to 2014. The model will use rainfall and temperature as the explanatory variables.

2. Methodology

In this study, exponential regression model and Cobb- Douglas model are developed for predicting electricity generation in Nigeria. Their prediction performance are compared. The source of data collection for this study is secondary. Data on electricity generation in Nigeria between 2002 and 2014 are obtained from the Central Bank of Nigeria statistical bulletin [15] while data on rainfall and temperature were extracted from the National Bureau of Statistics (NBS) abstract [16].

2.1. Exponential Regression Model

The exponential regression model 17,18] can be expressed mathematically as:

$$\ln G_i = \ln \beta_0 + \beta_1 R + \beta_2 T + e_i \qquad (1)$$

Let g = $\ln G_i$ and $\beta_0 = \ln \beta_0$. Where G = electricity generation (MWh), R = Amount of rainfall (mm), T = Temperature (^{0}C). e_i is the residual term. The residual term is assumed to be normally distributed with mean 0 and variance σ^2. To estimate the parameters $\beta_0, \beta_1, \beta_2$, an expression for each of the parameter is derived by the least square method of estimation. Then, equation (1) becomes;

$$g_i = \beta_0 + \beta_1 R + \beta_2 T + e_i \qquad (2)$$

$$e_i = (g_i - \beta_0 - \beta_1 R - \beta_2 T) \qquad (3)$$

The sum of square of the residual is given as;

$$S_1 = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (g_i - \beta_0 - \beta_1 R_i - \beta_2 T_i)^2 \qquad (4)$$

To estimate the parameters of the equation (4), partial differentiation is done with respect to each of the parameter. Hence,

$$\frac{\partial S_1}{\partial \beta_0} = 2(-1)\sum_{i=1}^{n} (g_i - \beta_0 - \beta_1 R_i - \beta_2 T_i) \qquad (5)$$

$$\frac{\partial S_1}{\partial \beta_1} = 2(-1)\sum_{i=1}^{n} [R_i (g_i - \beta_0 - \beta_1 R_i - \beta_2 T_i)]$$

$$= 2(-1)\sum_{i=1}^{n} (R_i g_i - \beta_0 R_i - \beta_1 R_i^2 - \beta_2 R_i T_i) \qquad (6)$$

$$\frac{\partial S_1}{\partial \beta_2} = -2\sum_{i=1}^{n} T_i (g_i - \beta_0 - \beta_1 R_i - \beta_2 T_i)$$

$$= -2\sum_{i=1}^{n} (T_i g_i - \beta_0 T_i - \beta_1 T_i R_i - \beta_2 T_i^2) \qquad (7)$$

To find estimate of the parameters $(\beta_0, \beta_1, \beta_2)$, Equations (5), (6) and (7) are set to zero. Then

$$\sum_{i=1}^{n} g_i - n\beta_0 - \beta_1 \sum_{i=1}^{n} R_i - \beta_2 \sum_{i=1}^{n} T_i = 0 \qquad (8)$$

$$\sum_{i=1}^{n} R_i g_i - \beta_0 \sum_{i=1}^{n} R_i - \beta_1 \sum_{i=1}^{n} R_i^2 - \beta_2 \sum_{i=1}^{n} R_i T_i = 0 \qquad (9)$$

$$\sum_{i=1}^{n} T_i g_i - \beta_0 \sum_{i=1}^{n} T_i - \beta_1 \sum_{i=1}^{n} T_i R_i - \beta_2 \sum_{i=1}^{n} T_i^2 = 0 \qquad (10)$$

Rearranging Equations (8), (9) and (10)

$$n\beta_0 + \beta_1 \sum_{i=1}^{n} R_i + \beta_2 \sum_{i=1}^{n} T_i = \sum_{i=1}^{n} g_i \qquad (11)$$

$$\beta_0 \sum_{i=1}^{n} R_i + \beta_1 \sum_{i=1}^{n} R_i^2 + \beta_2 \sum_{i=1}^{n} R_i T_i = \sum_{i=1}^{n} R_i g_i \qquad (12)$$

$$\beta_0 \sum_{i=1}^{n} T_i + \beta_1 \sum_{i=1}^{n} T_i R_i + \beta_2 \sum_{i=1}^{n} T_i^2 = \sum_{i=1}^{n} T_i g_i \qquad (13)$$

Solving the systems of Equations (11 - 13) with three unknowns $(\beta_0, \beta_1, \beta_2)$ give the parameters of the models.

2.2. *Cobb - Douglas Model*

Cobb - Douglas Model [19-21]can be represented as:

$$G_i = aR_i^b T_i^c + e_i \tag{14}$$

R = yearly mean rainfall and T = yearly mean temperature. Linearize the equation (14) by taking the logarithm of both sides of the equation.

$$\log(G_i) = \log(a) + b\log(R_i) + c\log(T_i) + e_i \tag{15}$$

Let $G_i^1 = \log(G_i), R_i^1 = \log(R_i)$, $T_i^1 = \log((T_i)$. Then, the Cobb - Douglas model in equation (15) becomes ;

$$G_i^1 = a + bR_i^1 + cT_i^1 + e_i \tag{16}$$

The residual of the equation (16) is;

$$e_i = G_i^1 - (a + bR_i^1 + cT_i^1) \tag{17}$$

$$e_i = G_i^1 - a - bR_i^1 - cT_i^1 \tag{18}$$

The sum of square of the residual (S_3)

$$S_3 = \sum_{i=1}^{n} e_i^2 = (G_i^1 - a - bR_i^1 - cT_i^1)^2 \tag{19}$$

$$S_3 = \sum_{i=1}^{n} (G_i^1 - a - bR_i^1 - cT_i^1)^2 \tag{20}$$

$$\frac{\partial S_3}{\partial a} = 2(-1)\sum (G_i^1 - a - bR_i^1 - cT_i^1) \tag{21}$$

$$\frac{\partial S_3}{\partial b} = 2(-1)\sum_{i=1}^{n} [R_i^1 (G_i - a - bR_i^1 - cT_i^1)]$$

$$= -2\sum_{i=1}^{n} (R_i^1 G_i^1 - a R_i^1 - bR_i^{1^2} - cR_i^1 T_i^1) \tag{22}$$

$$\frac{\partial S_3}{\partial c} = -2\sum_{i=1}^{n} [T_i^1 (G_i^1 - a - bR_i^1 - cT_i^1)]$$

$$\frac{\partial S_3}{\partial c} = -2\sum_{i=1}^{n} (T_i^1 G_i^1 - aT_i^1 - bR_i^1 T_i^1 - cT_i^{1^2}) \tag{23}$$

To find estimate of the parameters of the model in equation (15) that are (a, b, c), equations (21), (22) and (23) are set to zero. Then,

$$\sum (G_i^1 - a - bR_i - cT_i^1) = 0 \tag{24}$$

$$\sum_{i=1}^{n} (R_i G_i^1 - aR_i^1 - bR_i^{1^2} - cR_i^1 T_i^1) = 0 \tag{25}$$

$$\sum_{i=1}^{n} (T_i^1 G_i^1 - aT_i^1 - bR_i^1 T_i^1 - cT_i^{1^2}) = 0 \tag{26}$$

Expanding the bracket in the equations (24-26), it gives:

$$na + b\sum_{i=1}^{n} R_i^1 + c\sum_{i=1}^{n} T_i^1 = \sum_{i=1}^{n} G_i^1 \quad (27)$$

$$a\sum_{i=1}^{n} R_i^1 + b\sum_{i=1}^{n} R_i^{1^2} + c\sum_{i=1}^{n} R_i^1 T_i^1 = \sum_{i=1}^{n} R_i^1 G_i^1 \quad (28)$$

$$a\sum_{i=1}^{n} T_i^1 + b\sum_{i=1}^{n} T_i^1 R_i^1 + c\sum_{i=1}^{n} T_i^{1^2} = \sum_{i=1}^{n} T_i^1 G_i^1 \quad (29)$$

Solving the systems of the equations, 27-29, give the estimate of the parameters of the model.

2.3. Test of Model Fitness and Forecasting Accuracy.

2.3.1. Coefficient of Determination.

SST is the sum of square total which is given as

$$\text{SST}= \sum_{i=1}^{n} (G_i - \overline{G})^2 \quad (30)$$

G_i is the electricity generated during the year

$$\text{SS (Error)} = \sum_{i=1}^{n} (G_i - \hat{G})^2 \quad (31)$$

$$\text{SS (Regression)} = \sum_{i=1}^{n} \left(\hat{G}_i - \overline{G}\right)^2 \quad (32)$$

Coefficient of Determination (r^2)

$$r^2 = \frac{SSR}{SST} \quad (33)$$

Where $\overline{G}$ and $\hat{G}$ are the mean and estimated electricity generation.

2.3.2. Root Mean Square Error (RMSE)

The root mean square error was used to assess the forecasting accuracy of the four models. The RMSE is defined as:

$$RMSE = \sqrt{MSE} \quad (34)$$

$$MSE = \frac{1}{n}\sum_{i=1}^{n} (G_i - \hat{G}_i)^2 \quad (35)$$

Where G_i and $\hat{G}$ are the electricity generated during the year and estimated electricity generation and n is the number of observation.

3. Results and Discussion

The data on electricity generation obtained from central bank of Nigeria statistical bulletin and the two climatic variables (rainfall and temperature) extracted from National Bureau of statistics abstract between 2002 and 2014 were analysed based on the two different statistical models. Graphpad prism 5.0 Econometric View (E-View) software was used to plot the graph of actual and predicted power generation in Nigeria between 2002 and 2014 for the two statistical models. Generic statistical approach was used to assess the goodness of fit and forecasting accuracy of the four models. The best model was used to predict electricity generation in Nigeria for the next fifteen years (2015-2029).

3.1. Exponential Regression

From Table 1 the parameters of the exponential model are , β_1 = 0.000158, β_2 = 0.049154, $\beta_0 = e^{5.9952} = 401.4901$. Then

$$G = 401.4901 \exp(0.000158(R) + 0.049154(T)) \tag{36}$$

Table 1 presents the power generation model result using exponential regression model. Based on the result in Table 1, rainfall (β_1=0.00015) contribute positively to electricity generation. This means that as the amount of rainfall increases, power generation also increases. The r- square of 0.9975 was obtained which means that the rainfall and temperature explained 99.75% of the variation in power generation.

Table 1: Summary result of the estimates of the parameters of the exponential regression model

Model Parameters	β	SSR	SST	r- square	RMSE (%)
Constant	5.9952	20782094	20833613	0.9973	62.9525
Rainfall	0.000158				
Temperature	0.049154				

β = coefficients, SSR = regression sum of square, SST = Total Sum of Square, RMSE = Root Mean Square Error.

Table 2: Actual and predicted power generation in Nigeria between 2002 and 2014 using exponential regression model

Year	Actual generation (MWh)	Predicted generation (MWh)
2002	2564.3	2584.30
2003	2660.0	2694.05
2004	2663.6	2627.77
2005	2569.3	2611.58
2006	2871.3	2860.00
2007	2706.3	2727.30
2008	2698.1	2725.45
2009	2701.4	2720.33
2010	2703.5	2714.87
2011	2650.0	2592.98
2012	2719.9	2729.29
2013	3225.0	3335.34
2014	3249.0	3077.04

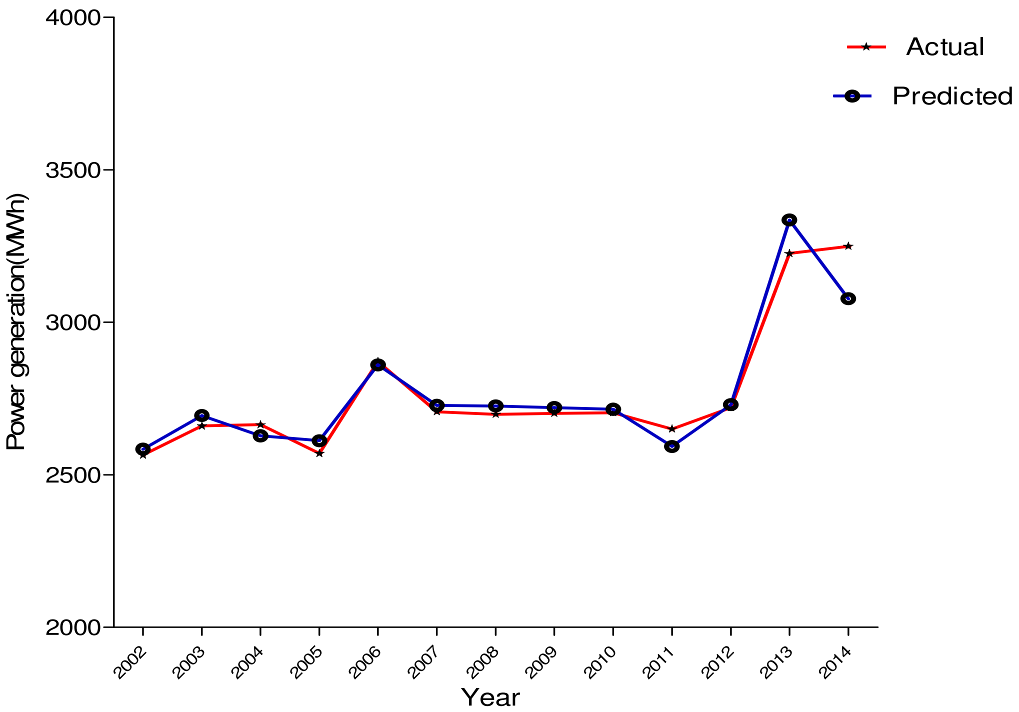

Figure 1: Graph of actual and predicted power generation in Nigeria between 2002 and 2014 using exponential regression model.

3.2. Cobb - Douglas Model

From the results in Table 3, the model specification based on Cobb - Douglas model is:

$$\text{Log } G = \log(0.198025) + 0.302795\log(R) + 1.486336\log(T) \tag{37}$$

In Table 4 , the Cobb-Douglas model was fitted to power generation data between 2002 - 2014 using rainfall and temperature as the independent variables. The result in Table 3 shows that the two independent variables explained for 99.85 percent of the variation in power generation within the period under study. The coefficient of rainfall (0.302795) was positive meaning that based on the Cobb – Douglas model, power generation tends to increase as rainfall increases.

Table 3: Summary result of the estimates of the Cobb - Douglas model

Model Parameters	β	SSR	SST	r- square	RMSE (%)
Constant	0.198025	20802945	20833613	0.9985	48.5704
Rainfall	0.302795				
Temperature	1.486336				

β = coefficients, SSR = regression sum of square, SST = Total Sum of Square, RMSE = Root Mean Square Error.

Table 4: Actual and predicted power generation in Nigeria between 2002 and 2014 using Cobb- Douglas model

Year	Actual generation (MWh)	Predicted generation (MWh)
2002	2564.3	2575.24
2003	2660.0	2671.44
2004	2663.6	2613.76
2005	2569.3	2575.37
2006	2871.3	2887.97
2007	2706.3	2723.69
2008	2698.1	2710.94
2009	2701.4	2736.78
2010	2703.5	2717.96
2011	2650.0	2615.47
2012	2719.9	2734.37
2013	3225.0	3300.90
2014	3249.0	3112.88

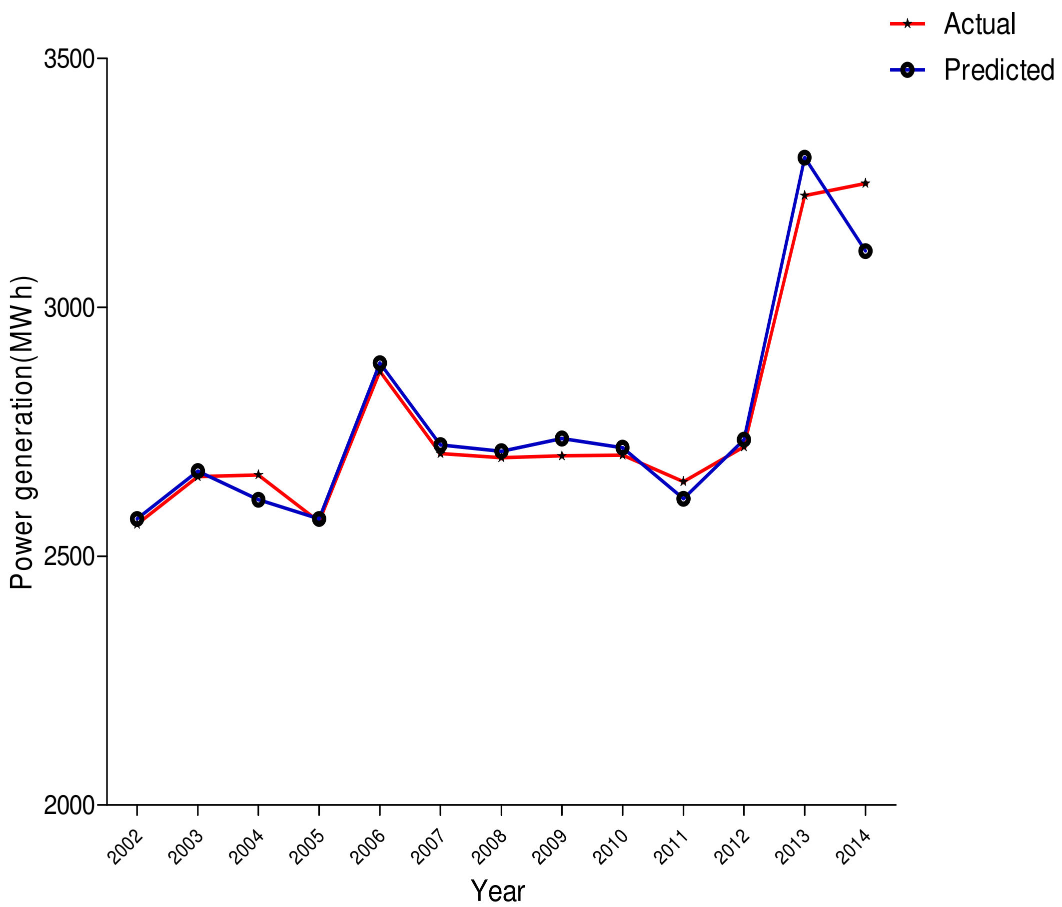

Figure 2: Graph of actual and predicted power generation in Nigeria between 2002 and 2014 using Cobb Douglas model.

3.3. Comparison of the Forecasting Accuracy of the Two Competing Models

Based on the result in Table 5, the Cobb- Douglas model has the highest coefficient of determination (r- square = 99.85%) and least root mean square error (RMSE = 48.57%). Hence, the Cobb- Douglas model is recommended as the best of the two competing models. Thus, the Cobb- Douglas model was used to forecast power generation for the next 15 years (2015-2029).

Table 5: Comparison of the forecasting accuracy of the two competing models.

S/N	Models	r- square (%)	RMSE (%)	Rank
1	Exponential regression	99.75	62.95	2
2	Cobb- Douglas Model	99.85	48.57	1

The model is

$$\text{Log } G = 0.198025 + 0.302795\log(R) + 1.486336\log(T).$$

To obtain the forecast value of rainfall and temperature, the method of trend estimation using time as the only independent variable was carried.

Results obtained are summarized in the Table 6.

Table 6: Forecast values of power generation in Nigeria

S/N	Year	Electricity generation(MWh)
1	2015	3047.38
2	2016	3080.29
3	2017	3112.37
4	2018	3143.68
5	2019	3174.25
6	2020	3204.12
7	2021	3233.34
8	2022	3261.93
9	2023	3289.93
10	2024	3317.36
11	2025	3344.25
12	2026	3370.62
13	2027	3396.49
14	2028	3421.90
15	2029	3446.85

4. Conclusion

This study presented the prediction of electricity generation in Nigeria based on the available data for the year 2002 to 2014. Exponential regression and Cobb-Douglas models were applied to model electricity generation in Nigeria and to forecast into the future for effective planning using rainfall and temperature as the explanatory variables. From the results, the Cobb-Douglas was selected as the best model as it gave the highest value of coefficient of determination and the least root mean square error as compared to the other model considered. The forecasted data shows that power generation in Nigeria in 2029 will stand at 3446.85MWh as against the value of 3249MWh in 2014.

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