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Research Article

Establishing the intensity of interventions in young beech (*Fagus sylvatica* L.) stands based on a spline regression mathematical model

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Abstract

The purpose of this work is to establish the optimal intensity of interventions in young beech stands. The study was carried out in the north-western part of Romania, in Zalău Forest District, Răstolț Production Unit. Statistical-mathematical inventories were carried out on circular sample plots of 300 m^2, adopting a coverage probability of p = 90% and a tolerance of t = 10%. The number of sample plots was established according to the area of the stand S(ha), volume variation coefficient s_v(%), p(%) and t(%). The optimal intensity of interventions is determined by using the average diameter of crowns on each stand. The average diameter of crown was determined for the stands under study based on a spline regression mathematical model which represents the main objective of the current study. The stands, wherein no forest interventions have been applied, have a relatively high stocking and a relatively small spacing. Consequently, the intensity of selective combined thinning must be weak to moderate, and the periodicity shorter of 4-5 years. Another objective of the present paper is the determination of the allometric relationships between the diameter of the crowns and the heights of trees, in three young beech stands with or without any forest interventions, specific to the stage of development. From the analysis of experimental data, it turned out that the most suitable law of dependence is the linear regression, with a positive slope, representing the allometric relationship between the diameter of the crowns and the height of trees.

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Keywords: average crown diameter; cubic spline regression; *Fagus sylvatica* L.; forest management practices; sustainable forest management

Introduction

Sustainable forest management represents the ability to manage forests in a way that preserves the forest habitat, its resources, and biodiversity, as well as meeting the current economic, social, and commercial needs of the human population without rendering them useless or incapable to be used by the future population, flora, and fauna (Siry *et al*., 2005). The same concept implies adopting a proper management according to national (regional) strategies, depending on the needs of society (Duncker *et al*., 2012), environmental protection, and, finally, landscape conservation at a given moment. The promotion of adequate, sustainable management of forests also requires the alignment of wood consumption with the preservation of the forest ecosystem (Yoshilda *et al*., 2017).

As a result, the analysis, study, and simulation of the stand structure are essential elements in the planning process of forest management for the sustainable exploitation of the forestry potential of the forests (Bica *et al*., 2017). At present, it is also possible to design a sustainable forest management plan (Martinez-Falero *et al*., 2017) with optimal efficiency with the help of various specialized software related to the forestry sector.

The European beech (*Fagus sylvatica* L.) represents the dominant tree species in the natural forests in Central Europe and one of the most important timber species on the continent. It is known that this highly competitive species is sensitive to drought and, therefore, can be increasingly threatened by heat waves associated with climate change and drought in parts of its distribution range. As a result, to limit the impact of climate change on beech stands, it is necessary to identify populations that demonstrate drought tolerance as a basis for selecting more adapted and drought-resistant provenances (Leuschner, 2020). The Suharu Forest (Dorohoi Forest District) is one of the species last compact beech forests in the eastern range. To the south, the beech areal extends to the Pyrenees and the shores of the Mediterranean Sea, from France to Greece, while to the north, it reaches the southern part of Scotland and the southern Scandinavian Peninsula.

According to Figure 1, beech is a plain and hilly species in the North-Western Europe, while in the South and South-Eastern Europe, it has become an exclusively mountainous species (Stănescu *et al*., 1997).

In the Alps, it reaches altitudes of up to 1,650 m; in the Pyrenees, up to 2,000 m; and on the slopes of Mount Etna in Sicily, it reaches 2,160 m. Beech forms extensive pure or mixed stands, especially in Central Europe. Thus, in Europe, the most extensive areas are found in the former Yugoslavia, with approximately 4,000,000 hectares, followed by Romania with approximately 2,000,000 hectares, France with 1,700,000 hectares, Germany with 1,150,000 hectares, and Bulgaria with 620,000 hectares (Stănescu *et al*., 1997).

European beech primary forests are also considered relatively stable forest ecosystems over time in their natural range, being characterized by active ecosystem (bio ecological) level processes (Stillhard *et al*., 2022). Therefore, European beech has been successfully used to rehabilitate degraded spruce stands in Norway and parts of Germany. Consequently, the positive influence of beech biogroups on humus composition and implicitly on soil fertility was quantified (Axer *et al*., 2022). Currently, the promotion of beech stands, naturally regenerated, under shelter, on the sites favorable to this way of regeneration, represents the main objectives of forest management from Romania because the beech species occupies approximately 32% of the surface of the national forest area (Romsilva, 2023). The beech is also the dominant species in the Southern Carpathians and Banat Mountains, which also form the oldest non-uniform, stands in Romania (Crişan *et al*., 2024).

In Romania, beech stands regenerate naturally, from seed, under shelterby applying progressive and successive cutting treatments, depending on vegetation conditions (Allensworth *et al*., 2021; MMAP, 2022). Natural regeneration is obtained after abundant fruiting, which has a periodicity of 6-7 years (MMAP, 2022).

In some situations, artificial regeneration works will be carried out to complete the regeneration in the optimal time on the entire proposed surface. These can be achieved by direct sowing or by planting seedlings. Promoting even-aged young beech stands, regenerated naturally or mixed, with standard (regular) structure, represents one of the national forestry strategies. As a result, appropriate forest management practices can be systematically (Allensworth *et al*., 2021) and rigorously applied, based on established mathematical models, thus ensuring the successful promotion and implementation of sustainable forest management (Figure 1).

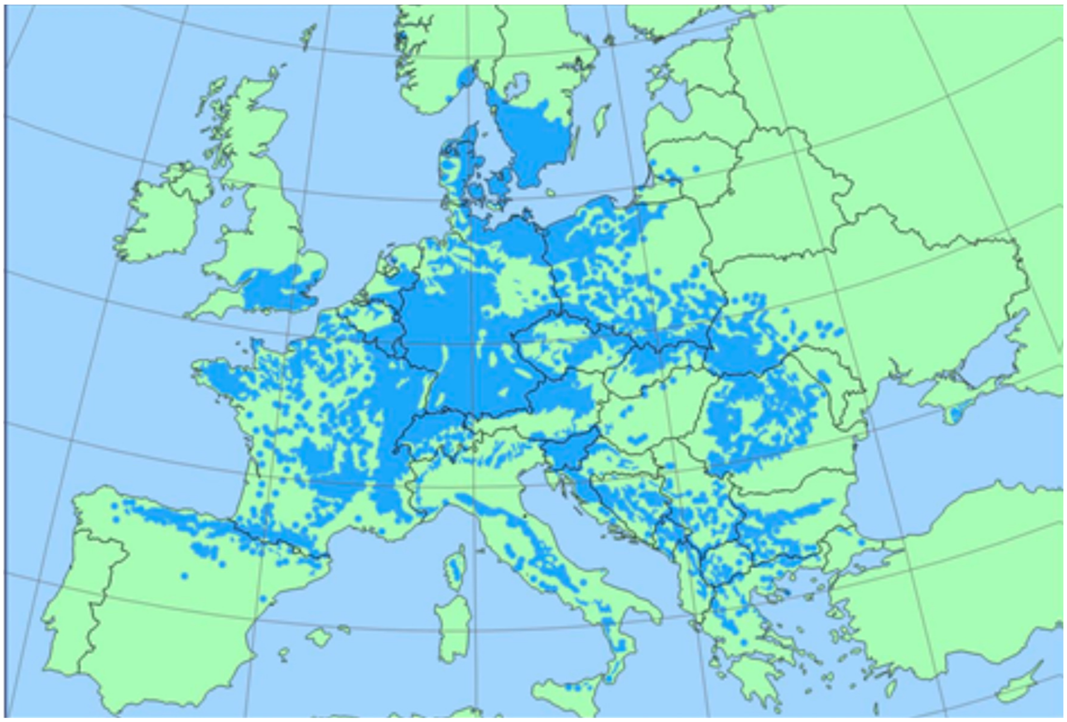


Figure 1. Natural distribution of the European beech *(Fagus sylvatica* L.) (EUROGEN, 2008)

In order to apply an appropriate stand management, it is necessary to carry out an appropriate forest management related analysis. Thus, a stand is characterized by its structural indices which are determined based on experimental data. This analysis must be carried out every time an assessment of the stand state is necessary, in order to establish the necessary management solutions. The results provided by the silvotechnical analysis are used to establish the specificity and intensity of forestry interventions, which represent the forestry diagnosis (Crainic and Tăut, 2008).

The objectives of the case study refer to the young beech stands, with or without any interventions, and are represented by: 1) determination of the average crown diameter by using a spline regression method and, as a consequence, establishing the optimal number of trees per surface unit which leads to the intensity of the intervention (the number of trees to be extracted); 2) establishing an allometric correlation between the average diameter of the tree crown and their average height for young beech stands; 3) determination of the stocking and spacing indices, and obtaining the profiles of the studied and analyzed stands (stand samples).

The importance and specificity of this study lie in the fact that the structural and synthesis indicators of the stands, determined from experimental data, accurately characterize their actual state, facilitating a relevant silvotechnical analysis and diagnosis. Consequently, it will be possible to establish the specifics of forest management practices that are imposed and their intensity very precisely, depending on the reality in the field, respectively, depending on the actual requirements of the stands, thus trying to implement sustainable forestry as close to nature as possible.

According to some older research, a series of regression equations were obtained to determine the crown width of trees, based on the experimental data, related to some forest species in south-western Oregon (Paine and Hann, 1982). The prediction models of tree crown width for 53 species in the western part of the United States were developed by the USDA Forest Service within the Forest Health Monitoring Program in 2004 (Bechtold, 2004). A series of research and studies were also carried out in Maine, in the North East of the USA, which aimed to determine the width of the crowns of isolated and under shelter trees, for the American beech, seven species of deciduous trees, and respectively seven resinous species. In this context, mathematical models based on non-linear regressions were used to process and model the experimental data (Russell and Weiskittel, 2011). Relatively recent research, which used high-performance mathematical models, led to obtaining the value of the tree crown diameter, based on regression equations, using experimental data for beech, and spruce (Sharma *et al*., 2016).

The performance of forestry works in young, mixed, and naturally regenerated stands is generally based on knowing the optimal and the actual number of trees per surface unit (ha) to control interspecific and intraspecific competition. This has direct implications for the growing space and implicitly for the internal environment of forest phytocenosis.

Forest management practices in young stands represented by selective combined thinnings, generally follow ecosystem (bioecological) level processes that take place naturally in the forest ecosystem. The impact of these practices on the stand can be quantified by analyzing and comparing its structural and synthesis indices (Karamzadeh *et al*., 2023) (stocking index I_N, density indices I_G, I_V, slenderness coefficient $Z_{\%}$ and stand spacing index $S_{\%}$ Hart-Beking).

The number of trees per surface unit (ha) can be determined expeditiously, directly in the field, through full or partial statistical-mathematical inventories based on the correlations between their main tree measurements (average height, crown diameter, etc.) (Florescu and Nicolescu, 1998), and indirectly, using general yield tables (Giurgiu *et al*., 1972). Also, the number of trees can be determined by expeditious methods by using the known correlations between the height of the trees and the diameter of their crowns (depending on the degree of development of the crowns or the size of the crowns). Finally, the tree's size, the spacing index of the stand $S_{\%}$ (Florescu and Nicolescu, 1998), and the crown competition factor (CCF) (Hall, 1994; Pretzsch, 2009; Fernandez-Moya and Urbán-Martínez, 2020) can be evaluated.

The data referring to the crown of the trees influence the optimization of tree growth patterns. As a result, the size of the crown is associated with the growing space. In this context, based on the experimental data related to the dimensions of the crown, models were developed that can predict the growth space, the stocking, the basal area, the spacing of the stands and the intensity of the interventions (Foli *et al*., 2003). Tree crown diameter is one of their important variables influencing forest growth, forest yield and management modeling (Fu *et al*., 2017).

A series of studies have analyzed and highlighted the allometric relationships that can be established between the height of the trees, the breast-high diameter d_{bh}, their volume, and current growth. Recent studies show that the pruned height (of the crown base) is directly correlated with the total height of the tree, thus highlighting a close, obvious allometric relationship between the two essential morphometric variables of trees and stands (Raptis *et al*., 2021a).

Interesting studies for green spaces, urban forests, and a series of stands have addressed the correlations between the characteristics of the tree crown and the surface occupied by them (Foli *et al*., 2003; Pretzsch *et al*., 2015). The dependence between the different shapes of tree crowns and their status in the forest phytocenosis was also studied and analyzed (Hâruța, 2011). The sizes and shapes of tree crowns can also be determined by digital analysis of the images, recorded on vertical terrestrial photographs inside the stands, using specially designed software (Hâruța and Fodor, 2010), provided that the stocking of the stands allows the recording of field data. All these research and studies highlight the interest in a series of allometric relations between the various measurements of trees and stands for the successful implementation and promotion of adaptive forest management measures, which ensure the sustainable management of forest resources (Allensworth *et al.*, 2021).

Estimates of crown size are of vital importance in forest management practice. Non-linear models were developed for the prediction of black pine (*Pinus nigra* Arn.) crown diameter in natural, pure and even-aged stands. As a result, a generalized model for determining crown diameter was developed, depending on: diameter at breast height, total height, pruned height, basal area and relative spacing index (Raptis *et al.*, 2021b).

For the first objective, we use a spline regression mathematical model in order to establish the average crown diamater of the trees corresponding to the studied stands. The corresponding cubic spline will have a cubic Hermite polynomial expression, as in formula (6), where the experimental data are interpolated, and the values of the local derivatives remain free.

There are many procedures in the literature for calculating these local derivatives. Thus, it is worth mentioning the classical methods of Akima (1970), Catmul and Rom (1974) and Kobza (2002), as well as other recent works (Bica, 2012; Han and Guo, 2018; Aràndiga and Yáñez, 2019; Han and Yang, 2020; Aràndiga *et al.*, 2022). Quartic spline regression could also be used, as in Li *et al.* (2022), but this approach is left for a future study. In this paper, the local derivatives are calculated using a regression model so that the residual function to be minimized. Furthermore, the obtained cubic spline will be used to calculate the average crown diameter of the involved tree. The value of the average diameter of the crowns will be used to determine some structural and synthesis characteristics of the stands for the optimization of theforest management practicesand implicitly for the implementation of an appropriate management for the sustainable management of forest resources.

A series of models for determining the crown diameter of European beech (*Fagus sylvatica* L.) used a large data set from permanent research plots (PRP). A series of elements were used for these research, such as: diameter at breast height (DBH), dominant height (HDOM), height-diameter ratio (tree slenderness coefficient), pruned height (to the crown base). Consequently, it was found that the diameter of the crown increases with the increase of the dominant height, but decreases with the increase of the height-diameter ratio, the height at the base of the crown and the competition among the trees (Sharma *et al.*, 2016).

The implementation of forest management, which is based on the analysis of the spatial structure of stands, represents a scientific and easy-to-use method for the sustainable management of forests, which is currently relatively frequently used (Ye *et al.*, 2018). As a result, a series of structural characteristics of the stands are analyzed synthetically for the efficient establishment of silvotechnical decisions, with direct implications on the functions of the forests and, respectively, the services they offer.

It is essential to present the profiles of the studied stands, based on some simulated models, because a series of tree measurements (total and pruned height - up to the first green branch from the base of the tree crown, diameter of the crown in two perpendicular directions) and spatial positioning of the trees have been inventoried, elements that characterize the tree's horizontal and vertical structure.

The present study is different from that of other researchers due to the mathematical model used, the type of stands, the inventory method, the studied structural characteristics and the results obtained.

Materials and Methods

Study area

The case study was carried out within the Sălaj Forest Department, Zalău Forest District, Răstolț Forest Management Unit (FMU) in the beech stands, in plots 112C, 114C and 115B (Figure 2). The most representative characteristics of the stands in the plots under study, depending on the data from the forest management plan (Romsilva Răstolț management) and the field, are presented synthetically in Table 1.

Table 1. Characteristics of the stands under study

Crt. no.	Plot		Yield class	Age (years)	Stoking index	Forest management practices	
	No.	S (ha)				Proposed	Accomplished
1	112C	4.40	2	20	1.0	Cleanings	-
2	114C	1.70	2	45	1.0	Thinnings	Thinnings
3	115B	3.30	2	25	1.0	Cleanings	-

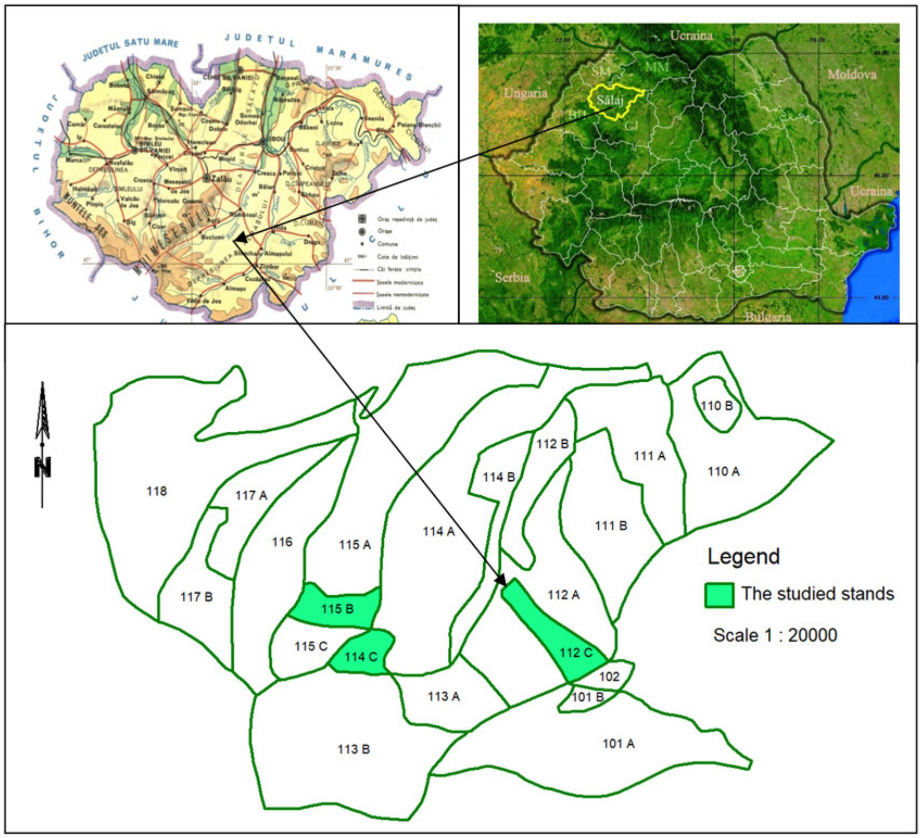


Figure 2. Location of research plots - Răstolț map, 2004 (Romsilva, 2023)

Technical documentation and data collection

The growing space necessary for each tree for optimal growth and development is constituted by the overground volume, which is delimited by the average diameter of the crown (D_{crw}) and the average height of the tree (H), to which the volume of soil in which the trees have fixed their roots is added (Florescu and Nicolescu, 1998). Because the roots of the neighboring trees can use the same volume of soil through concrescence without being decisively influenced, it is conventionally considered that the growing space corresponding to each tree corresponds to the space in which the trunk and its crown develop and which can be calculated with the relation (Florescu and Nicolescu, 1998):

$$Sg(H) = \frac{\pi \cdot D_{crw}^2}{4} \cdot H \tag{1}$$

Thus, $Sg(H)$ represents the projection of the tree crown in the horizontal plane, which is similar to the potential area of the tree crown projection area (cpa) (Pretzsch, 2009). It represents the size of the land surface that must be provided to a tree to grow and develop normally in height and density and to achieve (develop) a well-proportioned crown with a length between 0.33 and 0.50 of the trunk height (Florescu and Nicolescu, 1998).

Since in the even-aged stands with a regular structure, it is considered that H is relatively the same (equal) for all the neighboring trees (from biogroups) from the upper ceiling (from the crown), having a relatively constant value, it will not influence the size of the growing space. Consequently, relation (1) becomes:

$$Sg = \frac{\pi \cdot D_{crw}^2}{4} \tag{2}$$

The potential area of the tree crown projection (cpa) can be calculated using the following formula (Pretzsch, 2009):

$$cpa = (a_0 + a_1 \cdot d_{bh})^2 \cdot \frac{\pi}{4} \tag{3}$$

where: cpa - potential area of the tree crown projection; a_0, a_1 - coefficients of the regression equation; d_{bh} - diameter at breast height (measured at 1.30 m from the ground).

The height of the trees represents an essential morphometric variable of the stand, which is found in all forest management inventories (Fernandez-Moya and Urbán-Martínez, 2020), because it is an indicator of the growth and development of the stand, of the production and productivity of the stand, of the synthetic structural indices, being used to evaluate the volume of the tree stand.

The literature in the field shows that in the case of deciduous species, the diameter of the crown d_c is considered to range between 0.16 and 0.20 of the average height. For oak species $D_{crw} = 0.20 \cdot H$, and $D_{crw} = 0.16 \cdot H$ for beech species. For coniferous species D_{crw} is between 0.25 - 0.33 of the average height (Florescu and Nicolescu, 1998).

As a result, the value of the growing space (potential crown projection area) S_g (cpa) can be determined according to the correlation between the average height of the trees and the diameter of the crowns and depending on the correlation between d_{bh} and the diameter of the tree crowns.

The notion of growing space (S_g) will be used when the correlation between the average height of the trees and the diameter of the crowns is analyzed.

To determine the number of trees per surface unit N_{crw} - respectively per hectare, depending on the diameter of the crown D_{crw}, the value of the growing space S_g will be used, according to the following relationship (Florescu and Nicolescu, 1998):

$$N_{crw} = \frac{10000}{S_g} \tag{4}$$

Because the crown development indices - respectively, the correlations between the average height of the trees H and the diameter of the related crowns D_{crw} vary depending on a series of factors (species, age, vegetation conditions, forest management practices) (Florescu and Nicolescu, 1998), an appropriate mathematical model is required to determine the number of trees N_{crw} with high precision, for the precise establishment of the correlation (allometric relations) between H and D_{crw}..

In the present study, the observations were made on the itinerary and, respectively, on the stationary. The observations on the itinerary were made while traveling to the studied and analyzed stands, recording a series of data related to their structural and qualitative particularities. Stationary observations were made within the experimental plots in the stand, where the data were recorded for the research, on the occasion of the statistical-mathematical inventories.

To carry out the case study, statistical-mathematical inventories were made on experimental areas following the algorithm presented by Giurgiu *et al.* (1972). Thus, experimental plots of circular shape, with an area of 300 square meters, were established and delimited in the field.

The number of experimental areas was established with the following relationship (Giurgiu *et al.*, 1972; Crainic and Tăut, 2008):

$$n = \frac{t^2 \cdot s^2{}_{v\%} \cdot F}{F \cdot \Delta_{\%}{}^2 + t^2 \cdot f \cdot s_{\%}} \qquad (5)$$

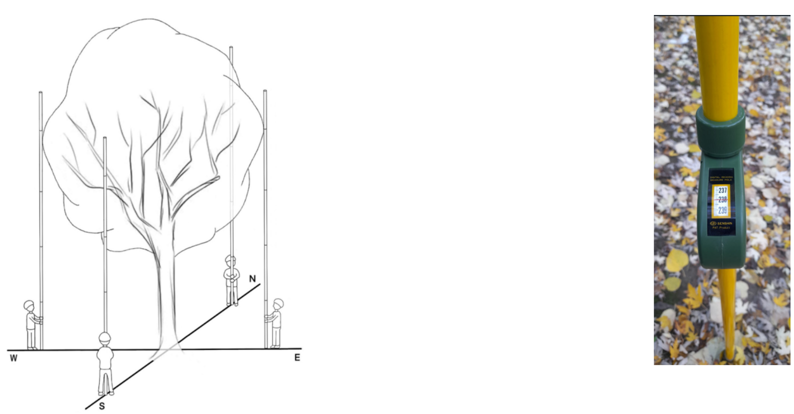
where: F - the surface of the stand; $\Delta_{\%}{}^2$ - admissible tolerance; t - tolerance coefficient (coefficient corresponding to the considered coverage probability); f - size of the experimental plot and $s_{v\%}$ - variation coefficient of the volume per stand.

The volume variation coefficient for the studied stands (determined according to the stand's homogeneity class) is $s_{v\%}$ = 25%, the tolerance used was t = 10%, and the coverage probability was p = 90% (Florescu and Nicolescu, 1998). Depending on the surface of each stand, the number of experimental plots was established - *n*, as well as the distance between their centers - *d*. These data can be found in Table 2.

Table 2. Elements of statistical inventories on the experimental plots in the stands under study

No.	Planimetric and altimetric coordinates of the plots' centers in the national reference system			*t* (%)	*p* (%)	*n* (pieces)	*d* (m)
	X(m)	*Y*(m)	*Z*(m)				
112C	612157,611	343555,470	602.500	10%	90%	16	50
114C	611913,330	342993,841	650.000	10%	90%	16	36
115B	612000,573	342844,437	590.000	10%	90%	16	43

Data from the field for each tree are represented by: the current number, species, crown diameter, total height, and pruning height. The average slope for each experimental plot was also determined. The total height of the trees in the experimental plots was measured with a Vertex 5, using a T3 model transponder. Due to the overstocking of the studied stands, the pruned height was measured (in cm) with a graded telescopic rod, which can be used for heights of up to 8.0 m.



(A) (B)

Figure 3. Measurement of the crown diameter for the inventoried trees: (A) The working principle; (B) Telescopic rod used

Although there are established methods (Preuhsler, 1979; Röhle, 1986; Pretzsch *et al*., 2015) for determining the radius of the tree crown and, implicitly, the crown diameter for young beech stands that are naturally regenerated and overstocked, these methods are relatively difficult to apply to the stands under study (see also Allensworth *et al*., 2021).

In order to make the process of collecting data from the field more efficient, the crown diameter of the inventoried trees was determined in two perpendicular directions. As a result, the transmission of the extreme points of the crown to the ground level was achieved in two perpendicular directions with the help of the graded rod, which was used in a vertical position (Figure 3B). Next, the inventoried trees' crown diameter was determined by a horizontally graded square ruler (in cm) in the two cardinal directions at ground level (Figure 3A).

The spline regression mathematical model

Statistical-mathematical inventories were carried out in the field, with a series of data on the structural and qualitative indicators of the stands, of which the most relevant for the case study are crown diameter (D_{crw}) and height (H_{crw}) of the tree for the inventoried trees. The independent variable is the height denoted by x and the dependent variable is the crown diameter denoted by y. There were considered young beech stands in which some (in one stand) or no interventions (in two stands) were applied, as it can be seen in Table 1.

It is of interest to establish a functional dependence between the height of the tree and the diameter of its crown, leading to a correlation with the stocking and spacing index in the deciduous stand, using the experimental data. Such a correlation will suggest the necessary forest management practices and solutions to optimize the production process in the forestry sector. Moreover, on a specified stand, this functional dependence will be helpful for computing the crown diameter D_{crw} without measuring it, only by using the height H of the tree.

For obtaining the average crown diameter, a cubic spline regression function is designed which is defined on two intervals.

The intersection point of these intervals of the spline function is determined by the intersection of the linear regression line with the experimental data curve near the midpoint of the independent variable interval $[x_1, x_n]$.

Let v_2 be the abscissa of this intersection point. In this way, the Hermite cubic spline is defined on the intervals $[v_1, v_2]$ and $[v_2, v_3]$ such that $v_1 = x_1$ and $v_3 = x_n$. If there is k such that $x_k < v_2 < x_{k+1}$, then for the subinterval $[x_k, x_{k+1}]$ there is $q_2 = 0.5\ (y_k + y_{k+1})$, and $q_1 = y_1, q_3 = y_n$, and, consequently, the expression of Hermite cubic polynomials (see Akima *et al*., 1970 and *Ahlberg* et al*., 1967*) for i = 2 and i = 3, will be:

$$S(x) = \frac{(v_i - x)^2\left[2(x - v_{i-1}) + h_i\right]}{h_i^3} \cdot q_{i-1} + \frac{(x - v_{i-1})^2\left[2(v_i - x) + h_i\right]}{h_i^3} \cdot q_i + \\ + \frac{(v_i - x)^2 (x - v_{i-1})}{h_i^2} m_{i=1} - \frac{(x - v_{i-1})^2 (v_i - x)}{h_i^2} \cdot m_i = \\ = A_i(x) \cdot q_{i-1} + B_i(x) \cdot q_i + C_i(x) \cdot m_{i-1} + D_i(x) \cdot m_i \tag{6}$$

Here $[y_1, y_n]$ is the dependent variable interval, $h_2 = v_2 - v_1$, $h_3 = v_3 - v_2$ and the local derivatives m_1, m_2 and m_3 remain free.

This cubic spline is helpful for obtaining the average crown diameter involved in the decision on appropriate forest management practices.

The purpose is to determine the local derivatives m_1, m_2 and m_3 and to obtain the cubic spline function, representing the solution of the spline regression problem. Different from (Akima, 1970; Kobza, 2002; Bica, 2012; Bica, 2014; Han and Guo, 2018; Aràndiga and Yáñez, 2019; Han and Yang, 2020; Han and Yang, 2021; Aràndiga *et al.*, 2022; Barrera *et al.*, 2022), here the local derivatives m_1, m_2 and m_3 are computed by using the least squares method in order to minimize the residual function.

$$R(m_1, m_2, m_3) = \sum_{j=1}^{n} \left(S(x_j) - y_j \right)^2 \tag{7}$$

For this purpose, it is noted that the points $x_1, \ldots; x_k$ belong to the interval $[v_1, v_2]$, while $x_{k+1}, \ldots; x_n$ range within the interval $[v_2, v_3]$. By using the least squares method it is obtained the following result:

The values (m_1, m_2, m_3) that minimize the residual function are uniquely determined, obtaining the unique cubic spline S as the solution of the regression spline problem of the data $(x_i, y_i),\ i = \overline{1,n}$.

Indeed, by applying the least squares method to the residual function R(m_1, m_2, m_3), the values (m_1, m_2, m_3) will be the solution of the system of normal equations:

$$\frac{\partial R}{\partial m_1} = 0,\ \frac{\partial R}{\partial m_2} = 0,\ \frac{\partial R}{\partial m_3} = 0 \tag{8}$$

Let $l = n - k$, and after elementary calculus, this system has the following three-diagonal form:

$$\begin{cases} f_0 \cdot m_1 + g_0 \cdot m_2 = r_0 \\ e_1 \cdot m_1 + f_1 \cdot m_2 + g_1 \cdot m_3 = r_1 \\ e_2 \cdot m_2 + f_2 \cdot m_3 = r_2 \end{cases} \tag{9}$$

Here it was denoted:

$$f_0 = \sum_{j=1}^{k} \left[C_1(x_j) \right]^2,\quad f_1 = \sum_{j=1}^{k} \left[D_1(x_j) \right]^2 + \sum_{j=1}^{l} \left[C_2(x_{k+j}) \right]^2,\quad f_2 = \sum_{j=1}^{l} \left[D_2(x_{k+j}) \right]^2; \tag{10}$$

$$g_0 = \sum_{j=1}^{k} C_1(x_j) \cdot D_1(x_j),\quad g_1 = \sum_{j=1}^{l} C_2(x_{k+j}) \cdot D_2(x_{k+j}); \tag{11}$$

$$e_1 = \sum_{j=1}^{k} C_1(x_j) \cdot D_1(x_j),\quad e_2 = \sum_{j=1}^{l} C_2(x_{k+j}) \cdot D_2(x_{k+j}); \tag{12}$$

$$r_0 = -q_1 \cdot \sum_{j=1}^{k} A_1(x_j) \cdot C_1(x_j) - q_2 \cdot \sum_{j=1}^{k} B_1(x_j) \cdot C_1(x_j) + \sum_{j=1}^{k} y_j \cdot C_1(x_j) \tag{13}$$

$$r_1 = -q_1 \cdot \sum_{j=1}^{k} A_1(x_j) \cdot D_1(x_j) - q_2 \cdot \sum_{j=1}^{k} B_1(x_j) \cdot D_1(x_j) - q_2 \cdot \sum_{j=1}^{l} A_2(x_{k+j}) \cdot \tag{14}$$

$$\cdot C_2(x_{k+j}) - q_3 \cdot \sum_{j=1}^{l} B_2(x_{k+j}) \cdot C_2(x_{k+j}) + \sum_{j=1}^{k} y_j \cdot D_1(x_j) + \sum_{j=1}^{l} y_{k+j} \cdot C_2(x_{k+j}),$$

(15)

$$r_2 = -q_2 \cdot \sum_{j=1}^{l} A_2(x_{k+j}) \cdot D_2(x_{k+j}) - q_3 \cdot \sum_{j=1}^{l} B_2(x_{k+j}) \cdot D_2(x_{k+j}) + \sum_{j=1}^{l} y_{k+j} \cdot D_2(x_{k+j})$$

By an elementary computation, the diagonal minors of the Hessian matrix of R will be positive and as a consequence, the unique solution of the linear system (9) is the unique point of minimum of R. Consequently, the above-determined point (m_1, m_2, m_3) minimizes the residual function R. Thus, the cubic spline S with these local derivatives m_1, m_2, m_3 is the unique solution of the cubic spline regression problem.

Now, the resulting cubic spline, with m_1, m_2 and m_3 previously calculated, is used to determine the average crown diameter by the following formula:

$$d = \frac{1}{v_3 - v_1} \int_{v_1}^{v3} S(x)dx \qquad (16)$$

To implement the algorithms described in this paper, the following software were used: Matlab (Etter, 1993; Ghinea and Fireţeanu, 2004) (for solving the linear system (9)), Scientific Workplace (Bagby, 2005) (for computing the average crown diameter according to formula (16)), Statistical Product and Service Solutions (SPSS) (Gunarto, 2019) for obtaining the allometric relationships.

The formulae for the elements e_1, e_2, f_0, f_1, f_2, g_0, g_1, r_0, r_1, r_2 corresponding to the linear system (9), as well as the solution of this linear system are computed by using the Matlab programming language.

Stand structural indices

Stocking index for the stands under study. Since the number of trees is determined by the three methods, it is possible to determine the value of the stocking index I_N for the stands under study, using for this purpose the optimal (theoretical) number of trees, recommended by the yield tables, drawn up for the stands of the national forest area. The formula used to determine the tree stocking index is as follows (Nicolescu, 1995; Crainic and Tăut, 2008):

$$I_N = \frac{N_{field}}{N_{table}} \qquad (17)$$

where: I_N - stocking index; N_{field} - number of trees in the field per surface area (ha) and N_{table} - number of trees in the yield tables, per surface unit (ha), for the same species, yield class and age.

The tree stand spacing index can be determined using the values of *the* $S_{\%}$ spacing index, proposed by Hart-Beking (Florescu and Nicolescu, 1998; Crainic and Tăut, 2008), using the established formula:

$$s_{\%} = \frac{a}{h_{dom}} \cdot 100 \qquad (18)$$

where: a - theoretical distance between the trees and h_{dom} - dominant height of the stand.

To determine the dominant height - h_{dom} in the studied stands, the following relationship is used:

$$h_{dom} = 1.15 \cdot \bar{h} \qquad (19)$$

where: $\bar{h}$ - average height of the stand.

To determine the theoretical distance between the trees, two variants of the arrangement of the trees in the stand are considered respectively, the trees arranged in the corners of a square of a_4 side and the trees arranged in the center and corners of a regular hexagon (quincunx) of side a_6. The formulae for determining the value of the theoretical distance between the trees in the two variants are presented below.

$$a_{4}=\sqrt{\frac{10000}{N}} \tag{20}$$

where: N - number of trees per hectare.

$$a_{6}=\sqrt{\frac{10000}{N\cdot\left(\frac{\sqrt{3}}{2}\right)}} \tag{21}$$

As a result, the relations for determining the value of the spacing index in the two variants become:

$$s_{4(\%)}=\frac{a_4}{h_{dom}}\cdot 100=\left(\frac{\sqrt{\frac{10000}{N}}}{h_{dom}}\right)\cdot 100 \tag{22}$$

$$s_{6(\%)}=\frac{a_6}{h_{dom}}\cdot 100=\left(\frac{\sqrt{\frac{10000}{N\cdot\left(\frac{\sqrt{3}}{2}\right)}}}{h_{dom}}\right)\cdot 100 \tag{23}$$

Establishing the number of trees to extract and the intensity of the intervention

In order to optimize the growth and development of the naturally-regenerated cultivated stands, within which maintenance cuts are systematically applied, in the context of the application of sustainable management of the forest resources, it is necessary to analyze some characteristics (structural) related to some stands in virgin or quasi-virgin forests (with regular and irregular structure), to be used as a reference model (Govedar *et al.*, 2018). Although the young specimens of beech tolerate shading well, in the case of young beech stands, regenerated naturally under shelter, through repeated regeneration cuts, with an average regeneration period, it is necessary to apply specific maintenance cuts at the right time, for achieving balanced tree measurements, obtaining optimal growth, and achieving a stable dynamic balance (Pavlin *et al.*, 2023).

Due to the age and tree measurements of the three stands under study, it is necessary to apply maintenance cuts related to young stands, respectively, combined selective thinning. The intensity of forest management practices in the stands under study is established by considering the specifics of the interventions, age, actual and optimal number of trees per surface unit, consistency, the stoking index, and the spacing of the stands.

To determine the amount of intervention intensity according to the number of trees, it is necessary to know the number of trees in the field and, respectively, the number of trees in the yield tables (Giurgiu *et al.*,

1972) for the same species, yield class, and age. The calculation algorithm for the number of extracted trees and the intensity of the intervention is presented in the following relations.

$$N_{extracted} = N_{field} - N_{table} \tag{24}$$

where: $N_{extracted}$ - number of extracted trees (piece/ha); N_{field} - real number of trees in the field, after intervention (piece/ha) and N_{table} - number of trees in the yield tables (piece/ha).

$$I_n = \frac{N_{extracted}}{N_{field}} \cdot 100 \tag{25}$$

where: I_n - intensity of the intervention depending on the number of extracted trees (%).

Since the number of trees in the field was determined in the three variants presented in Table 4, and for the amount of intervention intensity (depending on the number of trees), three corresponding variants will be determined. For the stands under study, the intensity of the interventions will be analyzed for each stand, depending on the presented elements.

Simulation of stand profiles

Along with the regional ecological evaluations, it is necessary to inform on the presentation of some related models of the stand profiles under study as well as on the forest ecosystem management activities, which are based on the evaluation of the functional relationships between ecosystem-level processes and forest ecosystem services (Pomara and Lee, 2021). In this context, the PROARB 2.0 application (Popa, 1999) was used for the plane representation and perspective of some representative samples from the studied and analyzed stands (on rectangular experimental plots) and implicitly for the realization of some models of the related profiles, the data from the field being recorded in standardized field sheets. This free software is specific to the forestry sector and is relatively easy to be used. In the same time, the generated profile models are representative for the stands under study.

In order to create the model of the stand's spatial structure, it is usually necessary to measure the total height of the trees and their pruned height, elements that are frequently used as input data for their growth and development models (Allensworth *et al.*, 2021).

The experimental plots used to create the profiles are rectangular, and the necessary data, collected (inventoried) from the field, are represented by: Crt. no - current number; L_{ep} - the length of the experimental plot (m); l_{ep} - the width of the experimental plot (m); X and Y - coordinates in the horizontal plane in a local rectangular system of reference (m); H - the total height of the tree (m); H_p - the pruned height of the tree, respectively the height from which the crown begins (m); D_{crw1} - diameter of the crown on the X axis (South-North direction) (m); D_{crw2} - diameter of the crown on the Y axis (West-East direction) (m); total number of trees inventoried in the experimental plot; species and average slope of the land (grades).

The right angles related to the rectangular experimental surfaces were traced in the field with the arpentor's square, or as the case may be, depending on the land, using the properties of Pythagorean numbers.

The horizontal, vertical, and three-dimensional profile model of the stand can currently be simulated with various specialized programs, offering a representative (suggestive) image of the various structural characteristics, which constitute edifying arguments for the establishment and effective application of forest management practices, in various stands. The simulation of the profile models for the stands under study in plots 112C, 114C and 115B was performed with PROARB 2.0 (Popa, 1999) software due to its specificity.

Results

The data recorded through statistical-mathematical inventories in the experimental areas located in the three analysed stands were processed separately for each stand. The data on the trees inventoried in the experimental plots from the studied stands were processed with the EXCEL program.

In the stand from plot 112C, following statistical-mathematical inventories in the 16 experimental plots (see Table 2), with a total (cumulated) area of 4,800 m^2, a number of 2,448 beech trees were inventoried. In the stand from plot 114C, a number of 608 beech trees were also inventoried. In the same manner, a number of 2,080 trees of the beech species were inventoried in the stand from plot 115B. For each stand under study, the trees were divided into whole value height classes, with the size of the interval between the classes of one meter, for which the corresponding average values of the crown diameter were optimally determined by using the above presented mathematical model.

Next, the records of the relative frequencies related to the height classes and their graphic representation for each studied stand are presented.

Results for the stand in plot 112C

Determination of tree crown diameter

The arithmetic mean of the crown diameters is calculated, D_{amcrw} = 1.409 m, and the average crown diameter D_{scrw} is obtained by applying formula (16). Here, the experimental data set is divided into two intervals for tree height (with $l = 3$, $k = 4$) having $v_1 = 4$, $v_2 = 7$, $v_3 = 10$, with corresponding diameter values $q_1 = 1$, $q_2 =$ 1.24, $q_3 =$1.83, and the local derivatives $m_1 = 0.4558$, $m_2 = 0.6194$ and $m_3 = -0.2049$ are obtained. The average crown diameter D_{scrw} = 1.410 m is obtained (calculated with spline model) using formula (16).

Furthermore, it is noticed that the values of D_{scrw} = 1.410 m and D_{amcrw} = 1.409 m are very close.

Establishing the allometric relationship

Figure 4A shows the distribution of the relative frequencies related to the tree heights for the stand in plot 112C. Figure 4B shows the correlation between the diameter of the crowns and the height of the trees, with the related regression equation and the coefficient R^2, for the stand in plot 112C. It is noticed that the linear regression equation is the most appropriate dependence relation of the crown diameter according to the height of the trees.

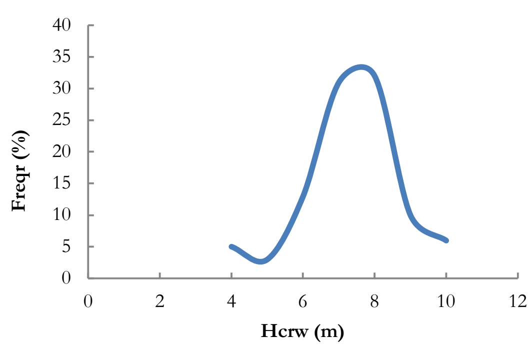


Figure 4A. Distribution of relative frequencies (Freqr) by height classes for the stand in plot 112C

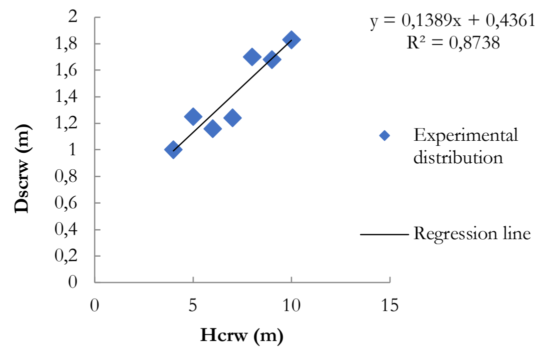


Figure 4B. Correlation between crown diameter (D_{scrw}) and height of the trees (H_{crw}), for the stand in plot 112C

Distribution of relative frequencies (Freqr) by height classes for the stand in plot 112C (Figure 4A), is closely akin to the Gauss type normal distribution (Leahu, 2001; Dorog, 2007; Dorog, 2008; Duduman and Drăgoi, 2019). The linear regression equation and R^2 value in Figures 4B, 5B and 6B were obtained with SPSS software, which is used for statistical analysis and data interpretation. In the Regression dialog window, the Linear option has been selected, as well as the dependent and independent variables used by the regression.

Results for the stand in plot 114C

Determination of tree crown diameter

The arithmetic mean of the crown diameters was obtained as in the case of the stand in plot 112C and has the value, D_{amcrw} = 3.420 m. Here, the experimental data set is divided into two intervals for tree height by taking l = 3, k = 4, v_1 = 10, v_2 = 13, v_3 = 16, with the corresponding diameter values q_1 = 1.75, q_2 = 3.20, q_3 = 5.25, and m_1 = -0.3110, m_2 = 0.2963, m_3 = 0.5203 are obtained, the average crown diameter D_{scrw} = 3.246 m being determined with formula (16).

Establishing the allometric relationship

Figure 5A presents the distribution of the relative frequencies related to tree heights for the stand in plot 114C. Figure 5B presents the correlation between the crowns' diameter and the trees' height, with the related elements, for the stand in plot 114C.

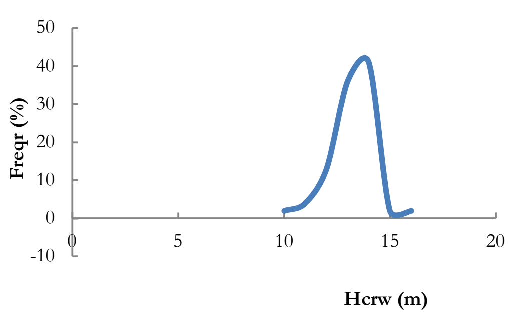


Figure 5A. Distribution of relative frequencies, depending on the height classes for the stand in plot 114 C

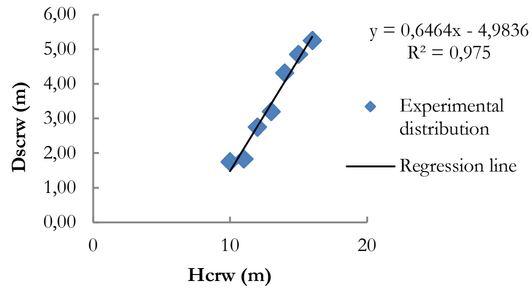


Figure 5B. Correlation between crown diameter (D_{scrw}) and height of the trees (H_{crw}), for the stand in plot 114C

Distribution of relative frequencies, depending on the height classes for the stand in plot 114C (Figura 5A), is closely akin to the Gauss type normal distribution, with right skewness (Leahu, 2001; Dorog, 2007; Dorog, 2008; Duduman and Drăgoi, 2019). The law of linear regression dependence between the crown diameter and the tree height can be observed in Figure 5B.

Results for the stand in plot 115B

Determination of tree crown diameter

For the stand in plot 115B, the arithmetic mean of the crown diameter is D_{amcrw} = 1.755 m. The experimental data set is divided into two intervals for tree height, taking l = 3, k = 3, v_1 = 7, v_2= 9.5, v_3 = 12, with the corresponding diameter values q_1 = 1.05, q_2 = 1.52, q_3 = 2.73, and m_1 = 0.2536, m_2 = -0.1787, m_3 = 0.8094 are obtained, while the average crown diameter is D_{scrw} = 1.635 m according to formula (16). We see again closed values for D_{amcrw} = 1.755 m and D_{scrw} = 1.635 m.

Establishing the allometric relationship

Figure 6A shows the distribution of the relative frequencies related to tree heights for the stand in plot 115B. Figure 6B shows the correlation between the diameter of the crowns and the height of the trees, with the related linear regression equation and the coefficient R^2, for the stand in plot 115B.

The distribution of the relative frequencies, depending on the height classes for the stand in plot 115B, presented in Figure 6A, looks like a bimodal distribution. The law of linear regression dependence between crown diameter and tree height can be seen in Figure 6B.

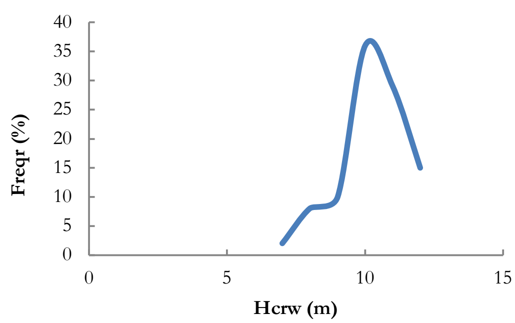


Figure 6A. Distribution of relative frequencies, depending on the height classes for the stand in plot 115B

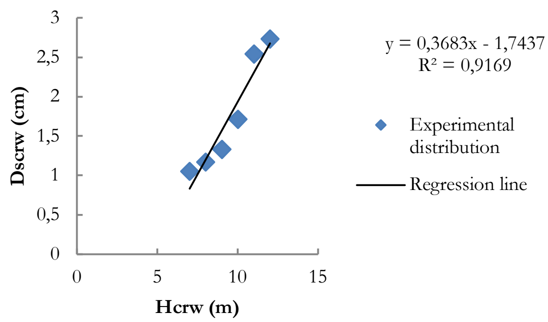


Figure 6B. Correlation between crown diameter (Dscrw) and height of the trees (Hcrw), for the stand in plot 115B

The determination coefficients R^2 have the following values: $R^2 = 0.8738$ in the stand in plot 112C, $R^2 = 0.975$ in the stand from plot 114C, and $R^2 = 0.9169$ in the stand from plot 115B. Consequently, the corresponding correlation coefficients R are presented as follows: R = 0.9348 in the stand from plot 112C, R = 0.9874 in the stand from plot 114C, and R = 0.9575 in the stand from plot 115B, revealing a good appropriateness of the linear regression law for these stands. Therefore, in the case of young beech stands, it is inferred that the most suitable allometric relation between the tree height and the crown diameter is the linear regression with positive slope.

Determination of the actual number of trees per surface unit (ha)

After determining the value of the average crown diameter D_{scrw} with the above presented mathematical model, it will be possible to determine the actual number of trees per surface unit (N_{scrw}) using the formulae (2) and (4). Another possibility of determining the number of trees per surface unit is represented by the use of the arithmetic mean diameter of the crowns D_{amcrw}, related to the trees distributed by height classes.

As standard reference in the present study, the actual number of trees can be determined by extrapolating the number of inventoried trees (N_{extrp}), cumulated from all the experimental surfaces, for each tree to the surface unit (ha).

The values related to the number of trees, determined by the three methods (N_{scrw}, N_{amcrw}, N_{extrp}), are presented in Table 3.

Table 3. The number of trees per hectare, determined in the three variants

Crt. no.	D_{crw} (m)	S_n (m²)	N_{real} (pc/ha)
	Trees in the plot 112C		
1	$D_{scrw} = 1.410$	$S = 1.561$	$N_{scrw} = 6404$
2	$D_{amcrw} = 1.409$	$S = 1.559$	$N_{amcrw} = 6413$
3	Trees inventoried on experimental areas and extrapolated/hectare		$N_{extrp} = 5092$
	Trees in the plot 114C		
4	$D_{scrw} = 3.246$	S = 8.275	$N_{scrw} = 1208$
5	$D_{amcrw} = 3.420$	$S = 9.186$	$N_{amcrw} = 1089$
6	Trees inventoried on experimental areas and extrapolated/hectare		$N_{extrp} = 1265$
	Trees in the plot 115B		
7	$D_{scrw} = 1.635$	$S = 2.100$	$N_{scrw} = 4763$
8	$D_{amcrw} = 1.755$	$S = 2.419$	$N_{amcrw} = 4134$
9	Trees inventoried on experimental areas and extrapolated/hectare		$N_{extrp} = 4326$

By analyzing the results presented in Table 3, it can be noticed that the number of trees $N_{scrw} = 1208$ determined by using the average crown diameter D_{scrw} for the plot 114C (where thinning were made) is much closer to the value N_{extrp}

= 1265 than N_{amcrw} = 1089, which proves the superiority of the proposed spline regression method in the computation of the average crown diameter for young beech stands.

Determination of stocking index for the stands under study

As a result, the values of the stocking indices for the studied stands will be presented in three variants (Table 4), using the actual number of trees in the field, which was determined with the presented algorithms.

From the analysis of the data in Table 4, it is found that the stocking index in the stand, where there were no interventions, is more than one, and in the stands where there were interventions tends to one. Again, comparing the values of I_N for the stand 114C, it is noticed a closed value to one for I_N = 0.95 determined with N_{scrw} =1208, comparing to I_N = 0.85 (obtained with N_{amcrw} = 1089).

Table 4. Values of the stocking index, in the three variants, for each stand

Plot	N_{real} (pc/ha)	N_{table} (pc/ha)	I_N	Observation
112C	N_{scrw} = 6404	5003	1.3	overstocked stand
	N_{amcrw} = 6413	5003	1.3	overstocked stand
	N_{extrp} = 5092	5003	1.2	overstocked stand
114C	N_{scrw} = 1208	1268	0.95	fully-stocked stand
	N_{amcrw} = 1089	1268	0.85	relatively fully stoked stand
	N_{extrp} = 1265	1268	0.99	fully-stoked stand
115B	N_{scrw} = 4763	3560	1.33	overstocked stand
	N_{amcrw} = 4134	3560	1.16	overstocked stand
	N_{extrp} = 4326	3560	1.21	overstocked stand

Determination of the spacing index $S_{\%}$

The tree spacing can be determined using the values of the $S_{\%}$ spacing index, proposed by Hart-Beking (Florescu and Nicolescu, 1998; Crainic and Tăut, 2008), using the established formula (18) - (23). The values of the spacing indices of the studied stands are presented in Table 5, for each stand.

Table 5. Values of the spacing index, in the three variants, for the studied stands

Crt. no.	Plot	N (pc/ha)	a_4 (m)	a_6 (m)	h (m)	h_{dom} (m)	S_4 (%)	S_6 (%)
1	112C	N_{scrw} = 6404	1.25	1.34	7.37	8.48	14.7	15.8
2		N_{acrw} = 6413	1.25	1.34	7.37	8.48	14.7	15.8
3		N_{extrp} = 5092	1.40	1.51	7.37	8.48	16.5	17.8
4	114C	N_{scrw} = 1208	2.88	3.09	13.24	15.23	18.9	20.3
5		N_{acrw} = 1089	3.03	3.26	13.24	15.23	19.9	21.4
6		N_{extrp} = 1265	2.81	3.02	13.24	15.23	18.5	19.8
7	115B	N_{scrw} = 4763	1.45	1.56	10.21	11.74	12.3	13.3
8		N_{acrw} = 4134	1.56	1.67	10.21	11.74	13.2	14.2
9		N_{extrp} = 4326	1.52	1.63	10.21	11.74	12.9	13.9

Establishing the specificity and intensity of the interventions in the studied stands

The values of the intensity of the intervention per number of trees in the three variants for the stand in plot 112C and plot 115B are presented in Table 6.

On site, the intensity of intervention will be guided by the value of N_{scrw} in each stand, obtained by using the values of D_{scrw}, being based on the cubic spline regression model. From the data analysis in the Table 6, it can be seen that the number of trees proposed to be extracted is $N_{extracted}$ = 1401 and the intensity of the intervention per number of trees, in the stand in plot 112C, in the three variants, ranges between 2 - 22%, being considered a strong intensity.

A selective thinning at the age of 45 was performed in the stand in plot 114C, the stocking index, and spacing index confirming this aspect, and as a result, at the time of this study, no intervention is necessary - respectively, a selective thinning.

From the analysis of the results presented in Table 6, it can be seen that the proposed number of trees to be extracted is $N_{extracted}$ = 1203 and the value of the intensity of the intervention, determined per number of trees, for the stand in plot 115B, in the three variants ranges between 18% and 25%, which can be assimilated with strong intensity.

Table 6. Values of intervention intensity per number of trees, in the three variants, for the two stands studied

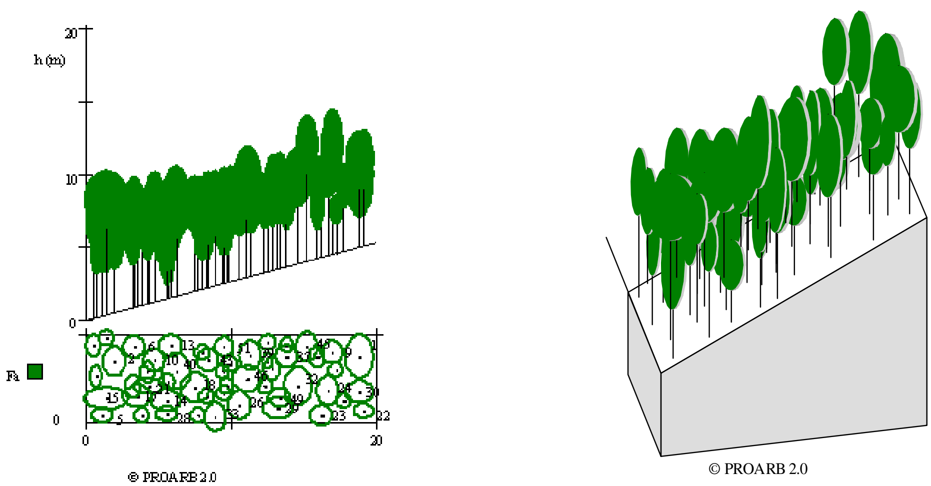
Plot	N_{real} (pc/ha)	N_{table} (pc/ha)	$N_{extracted}$ (pc/ha)	I_n (%)
112C	N_{scrw} = 6404	5003	1401	22
	N_{acrw} = 6413	5003	1410	22
	N_{extrp} = 5092	5003	89	2
115B	N_{scrw} = 4763	3560	1203	25
	N_{acrw} = 4134	3560	583	14
	N_{extrp} = 4326	3560	766	18

Profiles of the stands under study

Figures 7A,B present the profiles of a representative sample for the stand in plot 112C.

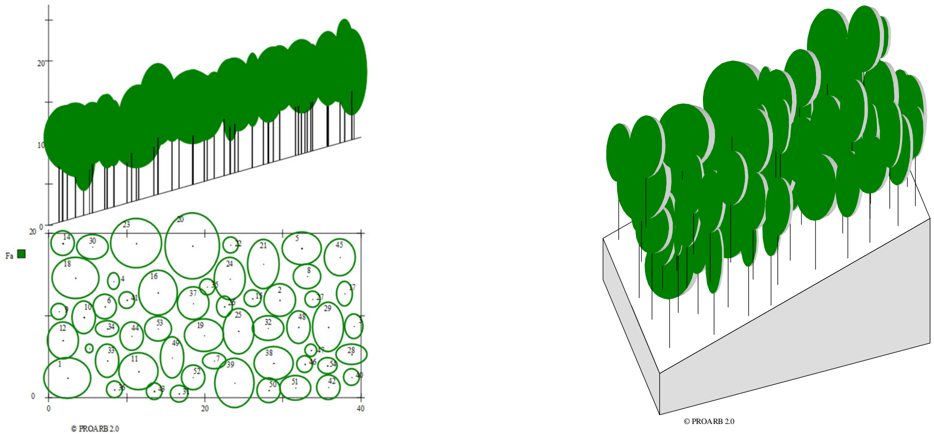
From the analysis of the profiles in Figures 7A and 7B, it can be seen that the stand in plot 112C is fully stocked, characterized by a stocking index k ≥ 1. This aspect is due to the fact that the forest management practices corresponding to the stage of development, respectively the combined selective thinning, still need to be carried out in this young beech stand.

Figures 8A, B present the profiles of a representative sample for the stand in plot 114C.



(A) (B)

Figure 7. The spatial profiles of the trees in the plot 112C, made with the PROARB 2.0 program: (A) Horizontal and vertical profile; (B) Tridimensional profile

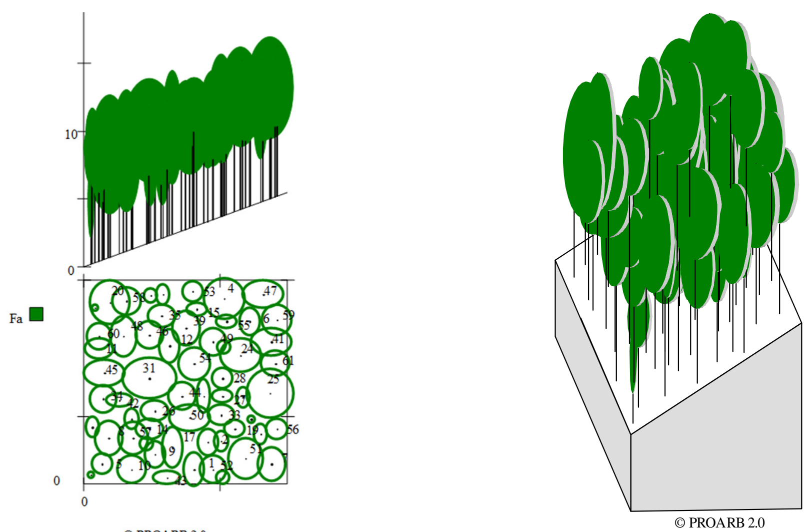


(A) (B)

Figure 8. The spatial profiles of the trees in the plot 114C, made with the PROARB 2.0 program: (A) Horizontal and vertical profile; (B) Tridimensional profile

Analyzing the profiles in Figures 8A and 8B, it can be noticed that the stand in plot 114C is understocked, the stocking index being $k < 1$. Consequently, it can be seen that a combined selective thinning was carried out in this stand a short time ago, and the stocking has not yet reached an optimal state, i.e.; the stocking index should have the value $k = 1$.

Figures 9A,B present the profiles of a representative sample for the stand in plot 115B.



(A) (B)

Figure 9. The spatial profiles of the trees in the plot 115B, made with the PROARB 2.0 program: (A) Horizontal and vertical profile; (B) Tridimensional profile

From the analysis of the profiles in Figures 9A and 9B, it can be seen that the stand in plot 115B is overstocked, characterized by a stocking index $k \geq 1$. As a result, it is found that no forest management practices corresponding to the stage of development, respectively, with the combined selective thinning were applied in this young stand.

Discussion

The main result of this study is the introduction of a new method to determine the average crown diameter D_{scrw} in young beech stands, a method that is not only efficient and accurate but also practical. This method, based on the spline regression technique, was tested on three young beech stands, and the obtained numerical results confirmed its effectiveness. As a consequence, the optimal number of trees per surface unit N_{scrw} can be obtained for each stand, indicating the appropriate number of trees to be extracted $N_{extracted}$. This practical method establishes the intensity of optimal forest management interventions for each stand, ensuring the sustainable management of forest resources. As it can be observed in Table 3, for the stand 114C where suitable thinnings were applied followed by natural regeneration, the values of N_{scrw} obtained by using the average crown diameter D_{scrw} are closer to the number of trees after this natural regeneration N_{extrp} than N_{amcrw}, which was obtained by using the average crown diameter D_{amcrw} provided by the arithmetic mean. This argues the appropriateness and effectiveness of the spline regression method for computing the average crown diameter.

In the third variant, the number of trees per surface unit was also calculated based on the number of trees inventoried on the experimental plots, which was extrapolated from 4,800 m^2 to 10,000 m^2 (Table 5). This working method involves a large workload and requires a team of at least three people. However, the determination of the number of trees using the average value of the diameter of the tree crowns using the cubic spline regression is not only expeditious but also characterized by high efficiency, making it a practical and applicable method in forest management.

Considering the D_{scrw} values, determined as a function of H_{crw}, based on the experimental data, it is found that the hypothesis of the research is verified: $D_{scrw} \cong 0.20 \cdot H_{crw}$ for the stand in plot 112C and $D_{scrw} \cong 0.17 \cdot H_{crw}$ for the stand in plot 115B. In the case of the stand in plot 114C, $D_{scrw} \cong 0.25 \cdot H_{crw}$, because combined selective thinnings were carried out in it.

Determination of the arithmetic average diameter of the crowns D_{acrw} requires the accomplishment of some statistical-mathematical inventories on the experimental areas, which in young and dense stands is relatively difficult due to both the specifics of the data that must be recorded and the work process. The analysis of the two possible options for determining the average diameter of the crowns highlights the advantage of using the cubic spline regression, compared to the determination of the arithmetic average diameter of the crowns, taking into account the previously mentioned aspects, but also the insignificant differences between the values of the calculated average diameters.

Another task of this work was to establish the allometric correlations between the average diameters of the crowns and the heights of the related trees in a prescribed young beech stand. It was found that the most suitable dependence relation is the linear regression with positive slope, obtaining for studied stands the following equations as in Figures 4B, 5B, and 6B:

- $y = 0.1389 \cdot x + 0.4361$ ($D_{scrw} = 0.1389 \cdot H_{crw} + 0.4361$) in the stand in plot 112C;
- $y = 0.6464 \cdot x - 4.9836$ ($D_{scrw} = 0.6464 \cdot H_{crw} - 4.9836$) in the stand in plot 114C;
- $y = 0.3683 \cdot x - 1.7437$ ($D_{scrw} = 0.3683 \cdot H_{crw} - 1.7437$) in the stand in plot 115B.

From the analysis of the stocking index values, calculated in the three variants, for each stand under study, it is found that regardless the method of determining the number of trees, the index values are close or even identical. Consequently, the three variants of determining the number of trees by experimental methods ensure accurate results, although they are relatively different in terms of working algorithm.

As a result, two variants of three values of the $S_{\%}$ spacing index were determined for each stand (Table 5). From the analysis of the results presented in Table 5, it is found that in the stands where no combined selective thinning was applied, the spacing indices have values in the range 12.3-16.5%, respectively 13.3-17.8%. In the stands where thinning was performed, the spacing indices have values in the range 18.5-19.9%, respectively 19.8-21.4%.

The intensity of the intervention in the analyzed stands can be established according to the value of the spacing factor $S_{\%}$ (Table 5), or it can be calculated according to the number of trees to be extracted, formula (24) and formula (25) (Table 6). The values of the spacing indices can suggest the state of the stand's stocking and the limits of the amount of the intervention intensity per number of trees. In this context: if $S_{\%} > 20\%$ and the stand is spaced, with a normal stocking, a high intensity thinning can be applied; if $S_{\%}$ = 15.1-20% and the stand is relatively spaced, with a relatively normal stocking, a thinning with moderate intensity can be applied; if $S_{\%}$ = 10.1-15%, the stand has a reduced spacing and it is relatively dense and unstable, a thinning with reduced intensity can be applied; if $S_{\%}$ < 10%, the stand is excessively dense and very unstable, then the intensity of interventions ≤ 5% (Florescu and Nicolescu, 1998; Crainic and Tăut, 2008).

From the analysis of the data in Tables 4, 5 and 6, and considering the correlations between the spacing factor of the stands and the intensity of selective thinning depending on the number of trees (Florescu and Nicolescu, 1998), which are proposed in the stands under study, intervention intensity is presented in a differentiated manner. Therefore, in the stand from plot 112C, the intervention intensity can be moderate up to 15%, and in the stand from plot 115B, the intervention intensity can be low by up to 10% (MMAP, 2022). Considering these aspects regarding the recommended intensity of combined selective thinning in the studied stands, a consolidated ecosystemic stability can be ensured, with direct implications on the continuity of the forest in space and time and, respectively, on the sustainable and optimal valorization of products and services provided by the forest.

The promotion of some forest management practices with low to moderate intensities in young naturally-regenerated beech stands will also positively influence carbon sequestration, an important aspect in the context of current climate change mitigation strategies (Paluš *et al.*, 2020). As a consequence, the valorization of the wood resulting from the application of forest management practices in young beech stands will allow the optimization of the heating costs for various users in the analyzed adjacent area, aspects that have also been reported on the occasion of relatively recent studies (Hayashi *et al.*, 2017).

In the future, there is the intention to extend the research to various beech stands, naturally-regenerated, with even-aged structure, from all yield classes, in the stage of youth and maturity, within which combined selective thinning have been or will be performed. The research area will be also extended to young even-aged oak and resinous stands, regenerated naturally and mixed, because the area occupied by them represents a considerable share of the area of the national forest area. Consequently, for the studied stands, allometric correlations between D_{scrw} and H_{crw} will be determined. Moreover, spline regression techniques will be used in order to be able to establish in the end, expeditiously and with high precision, the intensity per number of trees of the necessary forest management practices, based on some relevant experimental data.

Conclusions

The study's importance lies in the fact that allometric correlations between crown diameter and tree height can be deduced relatively simply and accurately in young, naturally regenerated beech stands. The obtained allometric correlation is a linear regression with positive slope. Moreover, according to the spline regression model, the number of trees to be extracted can be determined expeditiously, with relatively high precision, based on experimental data.

The accuracy of the spline regression model is tested by the fact that the number of trees per hectare obtained with this model is closer to the number of trees obtained by statistical-mathematical inventory (partial, on the sample plots) than to the number suggested by the use of the arithmetic mean. This aspect is better highlighted in plot 114C, where combined selective thinnings were carried out. More precisely, the number N_{scrw} = 1,208 is closer to N_{extrp} = 1,265 than the value N_{acrw} = 1,089.

Consequently, it is possible to correctly evaluate in the field, the number of trees that must be extracted when combined selective thinning is applied in young beech stands.

In conclusion, the implementation of the working algorithm presented in this paper (spline regression) allows the adoption and implementation of solutions based on experimental data in the chosen area instead of presumptive solutions with a theoretical aspect suggested by the technical norms. Consequently, promoting tree-centred forestry will be possible, with a direct and positive implication for the sustainable management of forest resources.

Authors' Contributions

Conceptualization: G.C.C. and A.M.B.; Methodology: G.C.C. and A.M.B.; Software: M.C. and S.C.; Validation: G.C.C., S.C., A.S., A.M.B. and M.C.; Formal analysis: A.M.B. and M.C.; Investigation: G.C.C., F.M.C., F.I., C.O., and E.Ș.; Writing - original draft: G.C.C.; A.M.B.; M.C., A.S., C.O., E.Ș., F.M.C., and F.I.; Writing - review and editing: G.C.C.; A.M.B.; M.C., S.C., A.S., E.Ș., F.M.C. and F.I. .

All authors read and approved the final manuscript.

Ethical approval (for researches involving animals or humans)

Not applicable.

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Conflict of Interests

The authors declare that there are no conflicts of interest related to this article.

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