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Ordering combination for modal comparison*

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Abstract The well-known ordering semantics for modality has recently been challenged by a number of puzzles which may cast doubt on the viability of this approach. We investigate the properties of the ordering relations used in ordering semantics with a focus on how to derive appropriate orderings based on intuitively correct premises. We use these tools to propose solutions to two of the puzzles and discuss how they relate to other puzzles that have been raised in the literature, and to modal semantics more generally.

Keywords: Modality, modal comparatives, gradable modals, ordering

1 Introduction

Modal predicates are used to make claims not only about what is absolutely necessary, 100% possible, totally required, or definitely permitted, but also about what is more or less necessary, more or less possible, more or less required, or more or less permitted. This kind of modal comparison is familiar in the epistemic domain, as illustrated in (1a), but is not limited to it. As illustrated in (1b), deontic modals admit of comparison as well.[1]

(1) a. The breakdown of the European Union is now **more likely** than the collapse of the single currency that was supposed to bind it together.
b. In accentual syllabics it is **more permissible** to vary the syllable count than the number of accents, or stresses.

Modal expressions that admit of comparative uses and of degree modification—what we will call gradable modal expressions (GMEs)—span a broad range, both

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1 Examples from http://www.washingtonpost.com/gIQAuY6wZN_story.html and www.barefootmuse.com/archives/issue2/evans2.htm.

syntactically and semantically. Syntactically, there are modal adjectives (*likely, permissible*) and nouns (*chance, likelihood*), attitude verbs (e.g. *want, require*), and modal auxiliaries (particularly the weak-necessity auxiliaries *should* and *ought*):

(2) a. It is very likely that ....
b. There is a good chance that ....
c. We very much want to ....

(3) a. It is more likely that ... than that ....
b. There is a better chance that ... than that ....
c. We ought to ... more than ....

Semantically these expressions can express different modality types (such as epistemic, circumstantial, bouletic, and deontic, as seen in (2)-(3)). They also show properties of various classes of gradable predicates: *certain* but not *probable*, for example, can be modified by *completely*, suggesting that the former is a "maximum standard" predicate in Kennedy & McNally's (2005) terms. In order to properly understand GMEs, we must integrate the semantics of modal expressions with a theory of gradability (Yalcin 2006, 2010; Portner 2009; Lassiter 2010, 2011; Klecha to appear). In this paper, we take some initial steps towards this goal. While we will not propose a full account of modal comparison, we will discuss features of the ordering relations used by modal operators and the origins of these orderings, with the understanding that doing so is necessary as the foundation for developing a degree-based semantics for GMEs in the future.

Our focus in this paper will be on a number of problems that have been raised for the ordering semantics account of modality, specifically Kratzer's well-known theory (Kratzer 1981, 1991, 2012). As with other modal predicates, according to this theory comparative modals are interpreted with respect to two distinguished contextual factors: the *modal base*, which determines the possibilities that are relevant in the current context, and the *ordering source*, which provides a set of propositions encoding the priorities among these possibilities. The ordering source induces a "goodness" ordering on worlds, and this ordering serves as the basis for interpreting comparative modal sentences.

Recently, the viability of this ordering-based approach has been called into question (Portner 2009; Yalcin 2010; Lassiter 2011), partially on the basis of a number of puzzles which concern GMEs (Goble 1996; Jackson & Pargetter 1986; Levinson 2003; Kolodny & MacFarlane 2010). These puzzles have often been articulated in the context of discussions of practical reasoning—discussions of what ought to be done or what the best course of action is—and have the character that the optimal outcome does not necessarily determine what the most advisable course of action is. Rather, in these cases it is crucial to take into account the likelihood of different outcomes in addition to how good they are.

A good example of this kind is the "Medicine Problem", originally due to Goble (1996) and presented in the following simplified form by Lassiter (2011). Consider a doctor who needs to choose which of two medicines to give to a critically ill patient. One has a small chance of leading to a total cure, while the other is certain to save the patient but leave him or her slightly debilitated. In this scenario, the sentence *The doctor ought to administer the second medicine* can be judged true. This is a puzzle because the worlds which best represent the doctor's goals are worlds in which the patient is completely recovered, and thus has taken the first medicine. But in considering what action is advisable it isn't just the best outcome that should be considered. We should also consider the likelihood of the outcomes.

In this paper, we aim to better understand this and other challenges for the ordering approach to modality. Our main claim is that these problems can be solved by explicating the relationship between contextually-provided priorities and the world orderings that underly the interpretation of comparative modal sentences. In particular, we propose that this relationship is often mediated by mechanisms of ordering-source construction and combination. We introduce two such mechanisms, which model (i) how expectations or requirements "add up", and (ii) how ranked sets of expectations or priorities are combined. We see (i) and (ii) as helping to spell out the fine structure of modal interpretation, and playing an important role in the analysis of graded modality.

2 Formal Background

In the semantics of modality developed by Kratzer (1981, 1991, 2012), an ordering source induces a pre-order on worlds in which a set of modal base propositions hold. In (4), the ordering source is indicated as the function g (from worlds to sets of propositions), and the modal source as the function f.

(4) **World ordering**:
$$\forall v,z \in \cap f(w) : v \leq_{g(w)} z \text{ iff } \{p \in g(w) : z \in p\} \subseteq \{p \in g(w) : v \in p\}$$

This definition can lead to incomparability among worlds, since worlds in which different sets of ordering-source propositions are true will not necessarily be ranked with respect to each other. The Hasse diagram in Figure 1 illustrates the world ordering for the simple case in which there are three ordering source propositions, $g(w) = \{p,q,r\}$. The nodes represent equivalence classes of worlds in which the indicated propositions hold, and the edges represent orderings among the worlds, with more highly ranked worlds placed above lower-ranked worlds. The upper rightmost node, for example, represents worlds in which q and r are true (and p is not). These are ranked above worlds in which only r is true (the lower rightmost node) and below worlds in which p, q and r are all true (the topmost node). The

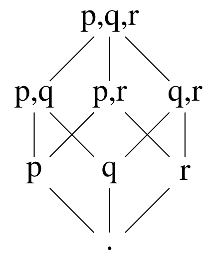

Figure 1 World ordering with $g(w) = \{p, q, r\}$.

q, r-worlds are not ranked with respect to the p-worlds.

Kratzer uses the ordering defined in (4) to provide truth conditions for basic modal predicates such as *must* and *can*, where (simplifying her analysis) *must* ϕ is true in a world w if all the most highly $\leq_{g(w)}$-ranked $f(w)$-worlds are ϕ-worlds, and *can* ϕ is true if some of the most highly $\leq_{g(w)}$-ranked $f(w)$-worlds are ϕ-worlds. Kratzer also proposes that $\leq_{g(w)}$ be used to interpret comparative modal expressions, and defines relations of comparative possibility for propositions, relations which are based on the more basic ordering of worlds. She stresses that many candidates for defining such relations should be considered, since "[n]otions of comparative possibility relating to probability are unlikely to be the same as notions of comparative possibility relating to desirability, for example" (Kratzer 2012: 40-41). In (5), we show the candidate discussed by Kratzer (2012: 41) in connection with comparative possibility relating to probability.[2]

(5) **Comparative possibility**:

a. A proposition p is **at least as good a possibility** as a proposition q in w with respect to f and g iff there is no accessible world in $q - p$ that is more ideal than every accessible world in $p - q$:
$\neg\exists v \in \bigcap f(w) \cap (q-p)[\forall z \in \bigcap f(w) \cap (p-q)[v \leq_{g(w)} z]]$

b. A proposition p is a **better possibility** than a proposition q in w with respect to f and g iff p is at least as good a possibility as q in w with respect to f and g, but not vice versa.

According to (5), determining whether a proposition is a **better possibility** with respect to another involves comparing the accessible (modal-base) worlds in which

2 This definition differs from the one in the original version of the paper (Kratzer 1981). It allows a connection to quantitative notions of epistemic probability and addresses certain difficulties raised by Yalcin (2010).

the first of the propositions holds but not the second to the accessible worlds in which the second proposition holds but not the first. For logically independent propositions, a proposition comes out as a **better possibility** than another if there is an accessible world in the first that is more highly ranked than all the accessible worlds in the second. For example, with $\leq_{g(w)}$ as in Figure 1, if a is a proposition that is true in some p,q,r-worlds, some p,q-worlds, and some p-worlds, and b is a proposition that is only true in some p,r-worlds and some r-worlds, then a is a better possibility than b. The notion of relative possibility in (5) has a long history, going back to Lewis 1973, and has been adopted in research related to probabilistic reasoning, e.g., Halpern 2003.

To give a concrete example, consider the case of (1b). Here the natural modal base is one that limits our discussion to poems of the familiar sort. The ordering source encodes priorities associated with the system of accentual syllabics (with propositions such as "the lines in the poem have the same number of syllables", "the lines in the poem have the same number of stressed syllables", and "the lines in the poem have the same pattern of stresses"). Ideal poems, then, are poems with perfectly regular length and stress patterns. This serves as the basis for the truth of the necessity statement in (6):

(6) In accentual verse, a poem must have a fixed number of stresses and syllables within a line.

But the ordering source also induces a **better-possibility** relation which is used to interpret the comparative statement in (7):

(7) In accentual verse, it is better for a poem to have lines made up of 5 iambic feet than to have lines made up of 5 stresses and a variable number of unstressed syllables.

Intuitively we might say that (7) is true because worlds in which "more" of the ordering-source propositions hold are better than those in which "fewer" hold (here, the nice regular-lined poems are those in which both the priority for having equal stresses and equal length hold, for example). This is not quite what Kratzer's **better-possibility** relation says, however. It isn't the number of ordering source propositions that matters, but rather the entailment relations among them. (7) is true, on this analysis, because having five iambic feet per line (asymmetrically) entails having five stresses per line.

On Kratzer's analysis, our intuitions about the truth conditions of comparative modality should follow from the entailment relations among the ordering-source propositions. There are, it turns out, cases in which our intuitions about the truth conditions of comparative modals appears to be related to the number of ordering-source propositions that are satisfied and not the entailment relations among them.

We discuss these in the next section. In the subsequent section, we consider another challenge: cases in which some ordering-source propositions are given priority over others. Once we have addressed these challenges, we will relate our analysis to other topical issues in ordering semantics and draw conclusions.

3 Expectations adding up

In order to motivate a probability-based approach to modality, Lassiter (2011) argues that Kratzer's approach yields unintuitive results in cases in which the number of expectations satisfied seems central to determining the ordering associated with a comparative modal judgment. He presents the case in (8) as one exemplifying this.

(8) [Context: Bill is extremely predictable. He almost always drives to and from work, arrives home by 6 p.m., and has macaroni for dinner.]
It is more likely that Bill will have something other than macaroni for dinner than it is that he will both fail to be home by 6 p.m. *and* fail to drive his car.

The **better-possibility** relation in (5) fails to predict the truth of (8) with an ordering $\leq_{g(w)}$ based on $g(w)$ = {**Bill drives, he is home by 6, he has macaroni**}. Recall that a world is only ordered with respect to another when the set of ordering-source propositions that are satisfied in one is a subset of the set of ordering-source propositions satisfied in the other. Thus $g(w)$ does not rank worlds in which Bill has macaroni but doesn't drive or arrive home by 6 p.m. with respect to worlds in which he is home by 6 p.m. and drives his car but fails to have macaroni. Because these worlds are not ranked with respect to one another, neither of the propositions compared in (8) is better than the other, and the sentence is predicted to be false.

This prediction appears to be problematical. Many speakers have the intuition that (8) is true in the context described. We believe that this intuition is based on a derived ordering that encodes the notion that, other things being equal, the more expectations that are satisfied, the better. In other words, we sometimes rank worlds by the number of ordering-source propositions that are true in them. We propose that this effect is achieved by computing the truth conditions of (8) with respect to an ordering source derived from the set of expectations mentioned above, as in (9).

(9) **Premises adding up**: For any set of propositions A,
$Premise_{add\text{-}up}(A) =_{def.} \bigcup_{i=0}^{|A|} \{p_i\}$, where p_i ={w: at least i propositions in A are true in w}.

(10) **Ordering source add-up**: For any ordering source g, $OS_{add\text{-}up}(g) = g'$, where for any world w, $g'(w) = Premise_{add\text{-}up}(g(w))$.

The **better-possibility** relation induced by the ordering source $OS_{add\text{-}up}(g)$ correctly models truth judgments about (8), since worlds in which only two expectations

$$\begin{array}{c} \text{p,q,r} \\ | \\ \text{(p,q)} \vee \text{(q,r)} \vee \text{(p,r)} \\ | \\ \text{p} \vee \text{q} \vee \text{r} \\ | \\ \cdot \end{array}$$

Figure 2 World ordering of $OS_{add\text{-}up}(g)(w)$, where $g(w) = \{p, q, r\}$.

are satisfied are more highly ranked according to $\leq_{OS_{add\text{-}up}(g)(w)}$ than those in which only one is. As we see in Figure 2, worlds that were previously incomparable are made comparable. This is because the ordering-source propositions—those that actually induce the world ordering via (4)—are not simply the propositions that we originally thought of when we articulated the expectation at work in the context.

What we take this example to show, then, is that there is not always a clear, direct relationship between the propositions that we are using to order worlds and those that we refer to when we describe the context for a modal utterance, with phrases like *my expectations are that* .... In many cases the ordering-source propositions (those that fundamentally determine our intuitions about the truth conditions of comparative modality), are derived (in more or less straightforward ways) from those we refer to in describing the context. Sometimes, for example, the ordering-source propositions encode additional "higher order" features of the context which are implicit, such as the notion that meeting more of the expectations or priorities leads to a higher ranking. In the next section we will see that (9) is only one way in which the priorities we talk about are related to ordering sources.

4 Multiple orderings

The truth of sentences expressing comparative modality often appears sensitive not just to a set of priorities, but to ranked sets of priorities (Kratzer 1981; von Fintel & Iatridou 2008). Consider a simple deontic case: it is prohibited to murder, to jaywalk, and to trespass. If we were to treat each of these priorities on par we would generate an ordering on worlds in which (without add-up), worlds in which there is murder but no jaywalking and trespassing are incomparable with worlds in which there is no murder, but there is trespassing and jaywalking. With add-up things are even worse, however. We would generate an ordering on worlds in which the worlds in which there is no trespassing or jaywalking, but there is murder, are deontically better than the worlds in which there is trespassing and jaywalking, but no murder. But of course murder is far worse than jaywalking and trespassing—worlds without murder should to be ranked higher than worlds with jaywalking and trespassing, no matter what. This phenomenon is pervasive, and it is one of the sources of the criticisms that have been leveled at the premise-theoretic approach to modality.[3] Consider, for example (11), the Medicine Problem discussed earlier (Goble 1996; Lassiter 2011).

(11) [Context: A doctor has the choice of two medicines—A or B—to administer to a critically ill patient. A has a small chance of producing a total cure and a large chance of killing the patient. B is sure to save the patient's life, but will leave him slightly debilitated. Doing nothing will certainly lead to death.]
It is better to administer medicine B than to administer medicine A.

The puzzle is that although the most desirable worlds are those in which A is administered and the patient is lucky, survives, and is totally cured, we do not judge that it is better to administer A than it is to administer B. In other words, we take (11) to be true in the context described. This is, of course, because the truth conditions are sensitive both to the desirability ordering and to the likelihood ordering. In this case, (11) is true because the most desirable of the most likely worlds are worlds in which medicine B and not medicine A is administered.

The question is how to integrate this into the ordering semantics. Our proposal is that this is another case of a derived ordering source, where here the ordering source relevant to interpretation is one that is derived from a prioritized sequence of sets of priorities. In the next section we describe one way in which this derived ordering source can be constructed.

4.1 Ordered merging

In the framework we are considering, a set of propositions induces an ordering on worlds, as in (4). The world ordering that reflects the intuitions that we have about prioritized sets of priorities is one in which, intuitively, the primary priorities are taken into consideration first, and then the secondary priorities. If "no murder" is the primary priority and "no trespassing" is the secondary priority, then worlds in which there is no murder and no trespassing should be the most highly ranked, with worlds in which there is no murder and some trespassing next, followed by worlds in which there is no trespassing but some murder, and finally the worst worlds of all, those with some murder and some trespassing. This is illustrated in Figure 3 on the left.

3 See von Fintel & Iatridou 2008, Rubinstein 2012, and Silk this volume for discussion in the context of the semantics of weak necessity modals.

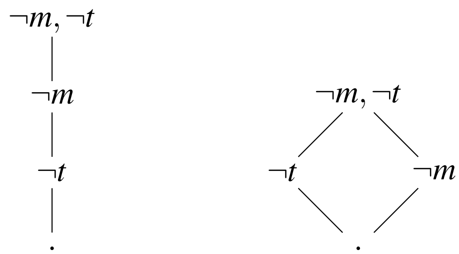

Figure 3 World ordering for ranked ordering sources (left), compared to unranked union (right).

A set of propositions that will induce this ordering on worlds is the following, where $\neg m$ is no murder and $\neg t$ is no trespassing: $\{\neg m, \neg m \wedge \neg t, \neg m \vee \neg t\}$. (It is easy to see that all these propositions are true in the best worlds, none of them in the worst worlds, at least two of them in the no murder worlds, and no more then one in the murder worlds.)[4] However, the question arises of why the ordering source chosen should be one which yields precisely this set of premises. As von Fintel & Iatridou (2008) have noted, simply taking the union of ordering sources representing the more basic priorities (in this case, $\{\neg m\}$ and $\{\neg t\}$) yields incorrect results. Keeping the rest of the mechanism constant, the ordering of worlds that results from taking $\{\neg m, \neg t\}$ as the combined ordering source is illustrated in Figure 3 on the right. Here it is clear that the notion that one ordering source takes precedence over another is lost, as the worlds in which there is murder but no trespassing are not ranked with respect to the worlds in which there is trespassing but no murder.

What we would like, then, is a general way of combining ordered sequences of ordering sources which yields the intuitively correct world orderings. We define a general binary merging operation $*$ that does just that; it operates on two sets of premises, giving priority to the considerations encoded in the first:

(12) **Ordered merging of premise sets**: For non-empty sets of propositions A_1 and A_2: $A_1 * A_2 =_{def.}$

$$A_1 \cup \bigcup_{y \in A_2} \left\{ y \wedge \bigwedge_{z \in A_1} z \right\} \cup \bigcup_{y \in A_2} \left\{ y \vee \bigvee_{z \in A_1} z \right\} \cup \bigcup_{\substack{0<n<|A_1| \\ y \in A_2}} \left\{ (y \wedge \bigvee_{\substack{x \subseteq A_1 \\ |x|=n}} \bigwedge_{z \in x} z) \vee \bigvee_{\substack{x \subseteq A_1 \\ |x|=n+1}} \bigwedge_{z \in x} z \right\}$$

4 This context also illustrates why, when comparing two propositions, we need to ignore worlds in which both hold. Consider (i), which we take to be true in a context with the priorities specified above.

(i) It is better not to murder than not to trespass.

The $\neg m$-worlds are ranked higher than the $\neg t$-worlds, if we ignore the $\neg m, \neg t$-worlds.

$$
\begin{aligned}
A_1 &= \{p, q\} \\
A_2 &= \{r, s\} \\
\\
A_1 * A_2 &= \{p, q, \\
&\quad r \wedge p \wedge q, \\
&\quad s \wedge p \wedge q, \\
&\quad (r \wedge (p \vee q)) \vee (p \wedge q), \\
&\quad (s \wedge (p \vee q)) \vee (p \wedge q), \\
&\quad r \vee p \vee q, \\
&\quad s \vee p \vee q\}
\end{aligned}
$$

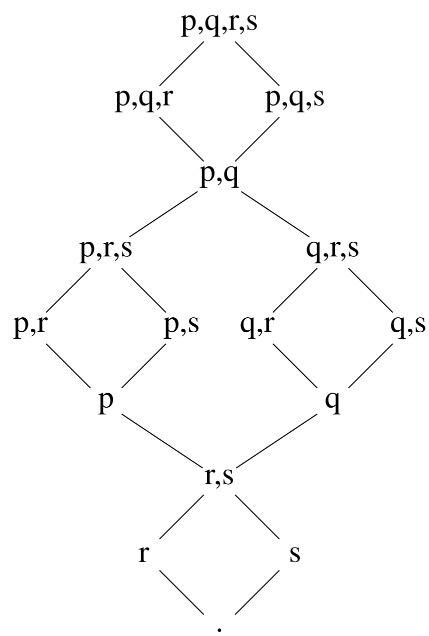

Figure 4 Ordered merging.

(13) **Ordered merging of ordering sources**: For any ordering sources g_1 and g_2, $(g_1 * g_2) = g'$, where, for any world w, $g'(w) = g_1(w) * g_2(w)$.

The basic idea of definition (12) is that subsets of propositions in the primary premise set specify “layers” at which the secondary premise set acts. For example, a primary premise set with three propositions specifies four ranked layers—the worlds at which all three propositions hold, the worlds at which two propositions hold, the worlds at which only one proposition holds, and the worlds at which none hold. Within these four layers, the secondary premise set further ranks the worlds.

As an illustration, in Figure 4 we show the result of this operation applied to the two premise sets A_1 and A_2 given (think of these as sets of propositions determined by two ordering sources g_1 and g_2 at an evaluation world), each of them consisting of two propositions. The derived premise set $A_1 * A_2$ induces the world ordering indicated in the Hasse diagram on the right via the definition in (4). Here we see the p and q layer, the p or q layer, and the neither p nor q layer; within each layer A_2 (i.e., $g_2(w)$) orders the worlds, with $r \wedge s$-worlds above both r-worlds and s-worlds, and the latter above the neither r nor s-worlds. We note that this combination operation is analogous to the lexicographical product for posets (Neggers & Kim 1999): a secondary ordering source g_2 only plays a role in the ordering of worlds when the primary g_1 doesn’t determine a linear ordering.

4.2 Combination and Goble's puzzle

Let us now turn to showing how ordered merging can be used to predict the truth of the sentence *It is better to administer medicine B than to administer medicine A* in the context of Goble's (1996) medicine scenario (11).

We propose that the truth judgment arises because the context makes salient two relevant ordering sources which are combined: a stereotypical ordering source that models the likelihood of outcomes (which we will call $\mathbf{OS}_1$), and an ordering source encoding desirability of outcomes (which we will call $\mathbf{OS}_2$), with the former taking priority over the latter.[5] In other words, we interpret (11) using the standard mechanism of interpretation proposed by Kratzer (1981, 1991, 2012), but with respect to an ordering which is derived by way of the $*$-combination defined in the previous section.

To be more concrete, let us consider one plausible context for (11), that given in (14). Here we make explicit the propositions that induce the orderings and the propositions that are contributed by the modal base. Recall that the modal base tells us about the relevant facts that we take to hold in all the worlds in which we are interpreting the modal sentence. In order to be entirely explicit, we need to talk about the propositions characterizing certain biological processes, e.g., "the patient's endocrine system produces the normal variant of gutsophine" (L1 below) or "the patient's immune system reacts to medicine A" (L2).

(14) **Modal base (MB)** :
Taking B leaves the patient alive but debilitated,
Taking A when L2 occurs leads to death,
Taking A when ¬L2 and ¬L1 occur leads to death,
Taking A when ¬L2 and L1 occur leads to complete healthy recovery,
Taking neither A nor B leads to death, . . .
$\mathbf{OS}_1$: {**L1**, **L2**}
(The most likely worlds are those in which both L1 and L2 happen; the least likely are those in which neither happen.)
$\mathbf{OS}_2$**:** {**The patient lives**, **The patient is perfectly healthy**}
(The most desirable worlds are those in which the patient lives a perfectly healthy life; the least desirable are those in which the patient dies.)

The context tells us about the facts in the relevant worlds and about the two orderings—what is most likely and what is most desirable. The most likely worlds are those in which both L1 and L2 hold (these are worlds in which taking medicine A, for example, leads to death). The least likely worlds are those in which neither

5 For simplicity, we occasionally abstract away from the world argument of an ordering source, identifying it with the premise set it gives at the world of evaluation.

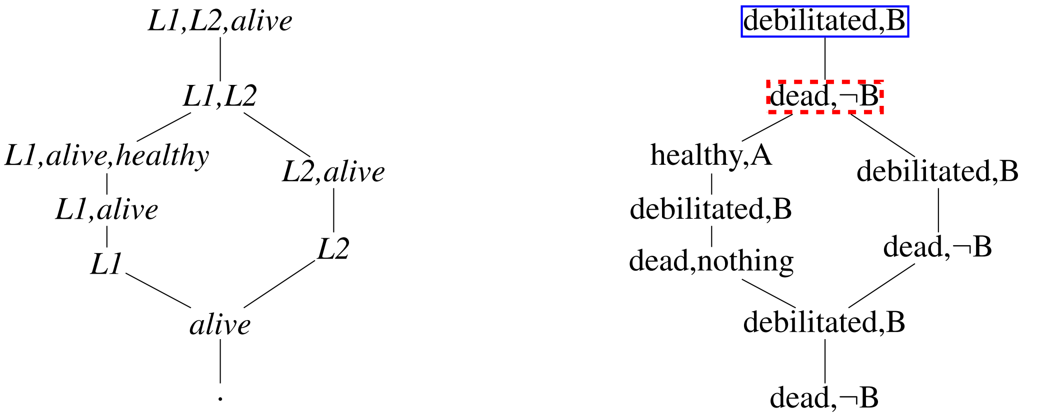

Figure 5 Derived ordering for example (11).

holds (these are also worlds in which taking A leads to death). The most desirable worlds, of course, are those in which the patient survives and is perfectly healthy. None of those are worlds in which the patient is administered medicine B, and none of them are among the most likely worlds.

Given the background characterized in (14), the merged ordering source $\mathbf{OS}_1 * \mathbf{OS}_2$ looks like Figure 4, except that some of the nodes in the figure are empty due to logical relations among the ordering source propositions (there are no worlds where the patient is healthy but not alive) or incompatibility with the modal base (for example, it is impossible for all of the propositions in both $\mathbf{OS}_1$ and $\mathbf{OS}_2$ to be true in the same world). This ordering is illustrated in Figure 5. On the left, we see how the ordering source propositions order the worlds, and on the right we show what the doctor does and the outcome for the patient in each group of worlds. (¬B marks worlds in which the doctor either administers A or administers no medicine at all.)

The derived ordering in Figure 5 is used to interpret (11). As illustrated in Figure 5, the most highly ranked "administer B"-worlds (in the cell marked in solid blue) are better than the most highly ranked "administer A"-worlds (included in the cell marked in dashed red). Since the merged ordering source ranks worlds primarily on the basis of likelihood and only secondarily on the basis of desirability, we can say that (11) is true essentially because the most desirable of the most likely worlds are worlds in which the doctor administers B and not A. More formally, the set of worlds in which the doctor administers B is a better possibility than the set of worlds in which she administers A, according to (5).

4.3 The complication of ties

While combining likelihood and desirability into a single ordering yields a straightforward analysis of (11), more needs to be said in case the compared propositions are not distinguished at the top of the ordering, only lower down. For example, (15) also appears to be true in the medicine scenario explicated in (14).

(15) It's better to administer medicine A than it is to do nothing.

The problem is that (15) is not predicted to be true given what we have said so far, as is apparent in the graphic representation in Figure 5: the highest ranked "do nothing"-worlds are equally ranked with respect to the highest ranked "administer A"-worlds under the $\leq_{\mathrm{OS}_1 * \mathrm{OS}_2}$ ordering (they are in the dashed red box). This would suggest that it is equally good to do nothing and to administer medicine A. But it seems to us that (15) is true. Although most likely doing nothing and administering A will both result in the patient's death, if it happens that L1 holds but not L2, the two options come apart, and administering A will result in the patient's healthy recovery. However, under no circumstance relevant in the conversation will doing nothing lead to a happy outcome. So it is better to administer A than to do nothing.

To account for this judgment about (15), we consider a generalization of the **comparative-possibility** relation proposed by Kratzer (2012) and given in (5) above. Definition (5) was designed to correct for the problem of ties by removing from consideration worlds in which the propositions compared were both true. We suggest a modification which goes further and also removes from comparison worlds that are not distinguished by $\leq_{g(w)}$, even though they may be found outside the intersection of the propositions compared. Specifically, we propose that to compare propositions p and q, we can consider only those worlds in p which are not ranked equally with a q-world, and only those worlds in q which are not ranked equally with a p-world. If we write $p\backslash_o q$ to refer to the p-worlds which are not equivalent to any world in q under the ordering o (i.e., $\{w_1 \in p : \neg\exists w_2 \in q : w_1 \leq_o w_2$ and $w_2 \leq_o w_1\}$), we are comparing $p\backslash_{g(w)}q$-worlds to $q\backslash_{g(w)}p$-worlds.[6]

(16) A proposition p is **at least as good a possibility** as a proposition q in w with respect to f and g iff there is no accessible world in q which is both (i) not equivalent to any world in p and (ii) more ideal than every accessible world in p which is not equivalent to any world in q:
$(\neg\exists v \in \bigcap f(w) \cap (q\backslash_{g(w)}p) : [\forall z \in \bigcap f(w) \cap (p\backslash_{g(w)}q)[v \leq_{g(w)} z]])$

6 Note that this definition excludes the intersection of p and q, as on definition (5), because a world in this intersection is clearly equivalent to a world in q.

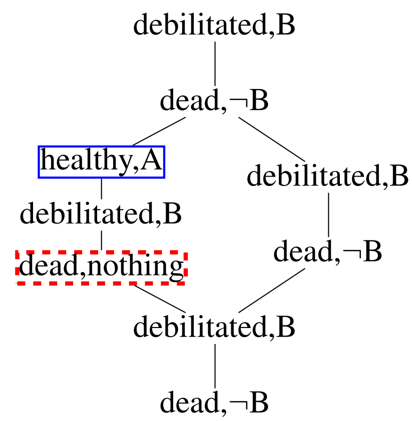

Figure 6 Comparing propositions with the modified comparative possibility.

In order to compare "administer medicine A" with "do nothing" with respect to an ordering source g we consider "administer medicine A"-worlds and "do nothing"-worlds that are not equivalently ranked by $\leq_{g(w)}$. This is illustrated for $\mathbf{OS}_1 * \mathbf{OS}_2$ in Figure 6. The node marked in solid blue represents the only set of accessible worlds in which medicine A is administered which can be considered for comparison (because the worlds in this node are not equally ranked with respect to any "do nothing"-worlds); the node marked in dashed red represents the only set of accessible worlds in which the doctor does nothing which can be considered, because they are not equally ranked with respect to any "administer A"-worlds. Since the former worlds are ranked higher than the latter, it is better to administer A than do nothing. The diagram represents the intuition that (15) is true because under no circumstances is doing nothing expected to turn out better than administering A, and there is a circumstance (when L1 holds but not L2, i.e., the three nodes on the left) where administering A is the better choice.

4.4 Other puzzles

We have spent some time discussing a variant of Goble's Medicine Problem, yet even assuming that what we have said about it is satisfactory, the question naturally arises of whether we have just developed an ad hoc way of dealing with a few puzzles. To the contrary, we believe that the ideas presented will be helpful in solving other problems in modal semantics. For example, although considerations of space prevent us from showing this in detail, we suspect that ordered merging is an important piece of the solution to the Miners' Paradox (Kolodny & MacFarlane 2010). In the context of (17), informants usually judge (18a) and (18b) true:

(17) Ten miners are trapped in a shaft—A or B—we do not know which. Flood waters threaten to flood the shafts. We can block one shaft or neither, but not both. If we block the wrong shaft, all ten will die. If we do nothing, both shafts will fill halfway and only one miner will die.

(18) a. We should/ought to block neither shaft.

b. If the miners are in shaft A, we should/ought to block shaft A.

A number of authors have suggested that these intuitions can be explained in terms of what outcomes are feasible given our knowledge; see Kolodny & MacFarlane 2010 ("chooseable"), Charlow to appear ("actionable"), and Cariani, Kaufmann & Kaufmann 2011. Given what we know, we can take an action that guarantees saving nine miners (doing nothing), but there is no action which guarantees saving ten. If we knew that the miners were in shaft A, we could guarantee saving ten (by blocking that shaft). The problem requires us to combine two kinds of factors: the preference to save as many miners as possible, and the likelihood that our actions will lead to the outcome we choose, given the information assumed in the conversation. We suggest that an ordered merging of two ordering sources representing these two factors gives the correct results. Here's a sketch. We have $\mathbf{OS}_1(w)$, which takes a set of relevant outcomes and a set of relevant actions and groups the outcomes into subsets based on how feasible they are.

(19) $\mathbf{OS}_1(w) = \{\bigcup\{p \in \{\text{Save 10 miners, Save 9 miners, Save 0 miners}\}$: there is an action $a \in \{\text{Block A, Block B, Block neither}\}$ such that, if a is performed, p has probability at least $i\} : 0 < i \leq 1\}$

In the basic scenario, $\mathbf{OS}_1(w) = \{\text{Save 9, Save 10 or 9 or 0}\}$, assuming that blocking either shaft is just as likely to kill ten as it is to save ten. We also have the preference to save as many miners as possible:

(20) $\mathbf{OS}_2(w) = \mathit{Premise}_{add\text{-}up}(\{\text{Save miner 1, Save miner 2, \ldots}\})$

The combined ordering source $\mathbf{OS}_1 {*} \mathbf{OS}_2$ groups outcomes by how "actionable" they are, and differentiates within equally actionable outcomes according to how many lives are saved. This makes (18a) true, because the only outcome that can be guaranteed is to save 9. For (18b), assuming that the antecedent of the conditional ultimately affects the probability distribution in (19) so that the miners are in shaft A with probability 1, the outcomes that can be guaranteed are to save zero, 9, or 10 miners, and the merged ordering prefers the last. It is interesting to note that $\mathbf{OS}_1$ incorporates probability into the analysis, as suggested by Lassiter (2011), but at the level of the construction of the ordering source, rather than in the core semantics of the modal. The ordering source $\mathbf{OS}_1 {*} \mathbf{OS}_2$ is also similar to the premise set discussed by von Fintel (2012, suggested by Kratzer in unpublished notes), but we construct it in an explicit way from the true underlying preference to save miners' lives.

5 Premise sets and ordered merging: comments on conditional priorities

Our approach to ordered merging can be described as an attempt to explain how complex priorities are constructed in contexts where there is an assumption that certain "simple" priorities take precedence over others. A question that arises is whether ordered merging is the only mechanism for combining priorities, or whether there are other general recipes that are used for this purpose, perhaps when precedence is not assumed. We suspect that the latter may be the case. In other words, there are likely to be various operators which derive conversational backgrounds from other conversational backgrounds, each reflecting an intuition about how the premises in the input backgrounds work together. However, careful investigation is needed in order to determine the range of cases for which such operators would be helpful. It might well be that certain patterns of modal reasoning can only be captured within ordering semantics by the ad hoc selection of appropriate sets of premises. In this section, we discuss cases involving priorities conditional on other priorities, where it is not clear whether the mechanism of ordered merging leads to a better analysis.

Suppose we believe strongly in non-corrupt government, and we also prefer to help developing countries when we can, which we can do by giving aid if the country's government is not corrupt. The following set of propositions produces an ordering which represents these desires.

(21) $\{q, q \wedge a\}$, where q is the proposition that government is not corrupt, and a is the proposition that aid is given to developing countries.

The relevant premise set is $\{q, q \wedge a\}$, not $\{q, a\}$, since giving aid is only a priority with respect to countries with non-corrupt government.[7] As an ordering source, (21) ranks $q \wedge a$-worlds as more ideal than $q \wedge \neg a$-worlds, which in turn are more ideal than worlds in which either $\neg q \wedge a$ or $\neg q \wedge \neg a$ are true.

Although (21) gives the correct ordering in the scenario, it is not entirely clear whether the second proposition $q \wedge a$ really reflects a premise. Isn't it more natural to say "We want to give aid to a developing country, if it is not corrupt" than "We want to give aid to non-corrupt developing countries"? Thus, the question arises of whether we could derive an appropriate ordering source from a more natural premise, namely $q \rightarrow a$. Although simply adding this preference to $\{q\}$ (yielding the set $\{q, q \rightarrow a\}$) does not yield the right result,[8] it turns out that an ordered merging of $\{q\}$ with $\{q \rightarrow a\}$ does. Following (12), we have (22):

7 See Kratzer's (1977: §2.3) striding and flying example for discussion closely related to this point.

8 The premise set $\{q, q \rightarrow a\}$ ranks $q \wedge a$-worlds above both $q \wedge \neg a$-worlds and $\neg q$-worlds, with the latter two unranked with respect to each other. This is not correct in the assumed scenario, where we always prefer lack of corruption to corruption.

(22) $\{q\} * \{q \rightarrow a\} = \{q, q \wedge (q \rightarrow a), q \vee (q \rightarrow a)\} = \{q, q \wedge a, \top\}$

The tautology $\top$ has no effect on the ordering, so this is equivalent to (21).

While this relation between (21) and (22) is interesting, it is not clear how much progress we have made. On the one hand, we wonder if the truly intuitive second premise would be $q \leftrightarrow a$, rather than $q \rightarrow a$ (we don't want to give aid to corrupt countries). Crucially, $\{q\} * \{q \leftrightarrow a\}$ does not give the correct ordering of worlds. And on the other hand, perhaps the whole discussion is misguided in that we do not deeply prefer giving aid at all, but rather the prospect of a better life for the citizens of the country. If this is the case, we have a much less difficult scenario: a high-ranked preference for the citizens of the poor country to have better lives, a lower-ranked preference not to give money, and assumptions in the modal base about relations between government and economy.

A slightly different example might better exemplify a truly conditional preference. Suppose our religion requires us to go on a pilgrimage if we have the money. We also prefer simply to have money, but it's much worse to violate the religious requirement by not going on the pilgrimage if we have the money to do so, than it is to be poor. Neither the ordering source $\{p, \$\}$ nor the ordering source $\{\$ \rightarrow p, \$\}$ yields the correct ordering (where p represents going on a pilgrimage and \$ represents having money). What we want is an ordering where $\$ \wedge p$-worlds are highest ranked, $\$ \wedge \neg p$-worlds are lowest ranked, and $\neg\$$-worlds are in between.

Obviously one can construct a set of premises which will yield the desired ordering. For example, $\{\$ \wedge p, (\$ \wedge p) \vee \neg\$\}$ will do the trick, but these premises are not how we naturally describe the situation. Note, though, that the ordered merging of $\{\$ \rightarrow p\} * \{\$\}$ gives the correct ordering, and is based both on natural premises and on their intended relative importance: $\{\$ \rightarrow p\} * \{\$\} = \{\$ \rightarrow p, (\$ \rightarrow p) \wedge \$, (\$ \rightarrow p) \vee \$\} = \{\$ \rightarrow p, \$ \wedge p, \top\}$, Setting aside $\top$, both propositions in the merged ordering source are true in the best ($\$ \wedge p$) worlds, only one is true in the intermediate ($\neg\$$) worlds, and neither is true in the lowest-ranked ($\$ \wedge \neg p$) worlds. So, in this case ordered merging functions as a recipe for deriving the correct ordering source from simpler, natural premises.

6 Conclusion

We have shown how two problems for ordering semantics can be solved through the use of ordering sources derived by adding up and merging more basic ordering sources. We have also suggested that the same operations may have utility for solving some other problems, in particular the Miners problem and certain cases involving conditional priorities, though it is not clear that the technique yields clear gains in all cases.

We have speculated that there may be other operations for combining and manipulating conversational backgrounds which would help make it clear how the set of premises used for modal reasoning is arrived at in particular cases. From our perspective, it would be nice if all examples involving complex ordering sources could be derived from natural premises through general operations, but we do not wish to claim yet that this is how language always works.

Finally, we have shown that the complex ordering sources which are derived by the operations (especially ordered merging) are useful in explaining difficult examples involving comparative modality. We hope that these results will provide a basis for addressing the foundations of the semantics of GMEs within a degree-based scale semantics. Figure 6 gives an intuition for why we might say the degree of "goodness" of giving medicine A is higher than the degree of "goodness" of doing nothing. The ingredients which would go into constructing modal degrees would be conversational backgrounds, operations like ordered merging, and the definition of comparative possibility. Given that conversational backgrounds are context dependent, and that the operations and notion of comparative possibility might be defined in multiple ways, we would expect for modal degrees to be somewhat harder to pin down than more concrete degrees like heights and weights. And this seems to be the case. The question is whether they can be pinned down to a sufficient extent such that the overall approach explains the consistent semantic properties of GMEs.

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