

# The Semantics of Relative Position\*

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Locative prepositions are probably one of the best studied lexical classes in linguistic semantics. As a result an impressive body of knowledge about their meaning and use has been accumulated in the past quarter of a century.<sup>1</sup> However, in comparison with the semantic literature about quantifiers, articles, and pronouns (to name some other well-studied parts of speech), a general formal framework for the representation of this knowledge is missing.<sup>2</sup> Most of the theoretical results are either informal or formulated in terms of unanalyzed semantic primitives (like 'interior' for *in* and 'proximate' for *near*). The purpose of this paper is to provide a framework for the study of locative prepositions, based on the mathematical notion of a *vector*, i.e. a directed line-segment pointing from one point in space to another. Section 1 of this paper gives the basic outlines of such a vector-based semantics, section 2 provides the general mathematical background, and section 3 shows how semantic definitions for individual locative prepositions can be given in a straightforward way. The fruitfulness of the vector-based approach is demonstrated in section 4, where a number of general, algebraic properties of regions are formulated that explain constraints on modification and allow the postulation of universals of prepositions.

## 1 Vectors as relative positions

The locative prepositions that will be discussed in this paper are given in (1):

- |     |                     |                  |         |
|-----|---------------------|------------------|---------|
| (1) | in front of         | next to / beside | in      |
|     | in back of / behind | between          | on / at |
|     | above / over        | inside           | near    |
|     | below / under       | outside          |         |

For the purposes of this paper, locative prepositions are to be understood as those prepositions that are primarily used to denote locations, when used as the complement of the copula *be*. Prepositions like *across*, *around*, *beyond*, *down*, *past*, *through*, and *up* are not classified as locative prepositions, because their locative use (in cases like *The postoffice is across the road / around the corner*) is derived from a more basic directional meaning (as argued in Bennett (1975), Cresswell (1978), and Jackendoff (1983)). Many other locative prepositions (like *against*, *among*, *opposite*, *throughout*) are absent from the list in (1) for the simple reason that their analysis involves complications that go beyond the basic proposal of this paper.<sup>3</sup>

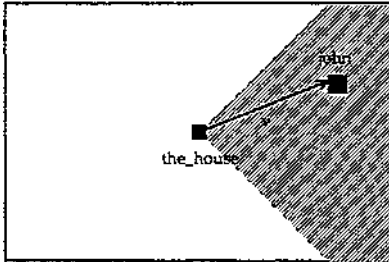
Traditionally, a locative preposition like *behind* in the sentence *John is behind the house* is analyzed as a relation between a theme (John) and a reference object (the house), as in (2):

- (2) behind ( john , the house )  
 (3) loc ( john , behind ( the house ) )

In much work this relation is broken down into two parts, as in (3): the preposition maps the reference object to a *region* or *place*, and the theme is located in this region by a general location relation.<sup>4</sup> The nature of the spatial region in (3) is usually left unspecified or assumed to be some set of points. This is sufficient as long as we are interested only in the global semantic structure of prepositions in general, but it falls short if we want to capture how individual prepositions define their region.

The proposal of this paper is to analyze a region as a set of *vectors*. The behind-the-house-region will then be the set of vectors emanating from the house that point backwards and the theme is located at the end point of one of these vectors, yielding the kind of existentially quantified conjunction in (4) familiar from Neo-Davidsonian verb semantics.<sup>5</sup> This situation can be graphically represented as in (5), where the shaded area represents the region behind the house and *v* is one of the vectors from this region.

- (4)  $\exists v [ v \in \text{behind}(\text{the house}) \wedge \text{loc}(\text{john}, v) ]$   
 (5) John is behind the house



Other prepositions can be analyzed vector-theoretically along the same lines:

- (6) a. *near the house*: the set of vectors with their origin at the house with a length smaller than a pragmatically determined number *r*  
 b. *above the house*: the set of vectors that point from the house in an upward direction  
 c. *between the house and the barn*: the vectors that point from the house towards the barn and vice versa

The general idea should be clear at an intuitive level. What the vector does is formalize the notion of a *relative position*, i.e. a position specified in relation to a reference object that functions as a spatial origin. Moreover, the vector concept provides us by its very nature with the parameters of *distance* and *orientation* that prepositions use to specify relative positions

An important argument for using vectors instead of points or primitive portions of space comes from the modification of prepositions. A preposition like *behind* can be modified by dimensional adjectives (7a), measure phrases (7b), and adverbs (7c):

- (7) a. *far* behind the house  
 b. *two meters* behind the house  
 c. *right* behind the house

Suppose that we take the region denoted by the modified PP *behind the house* to be a set of points. Then the modifiers have to be interpreted as functions mapping these sets to subsets by picking out those points that are 'far', 'two meters', or 'right'. In order to do this, the modifiers have to refer to the *distance* between the points and the reference object, which requires a two-place 'distance' function *dist* mapping pairs of a point and a reference object to a distance:<sup>6</sup>

- (8) a.  $\llbracket \text{far PP} \rrbracket = \{ p \in \llbracket \text{PP} \rrbracket \mid \text{dist}(p, \llbracket \text{NP} \rrbracket) > r \}$   
 b.  $\llbracket \text{two meters PP} \rrbracket = \{ p \in \llbracket \text{PP} \rrbracket \mid \text{dist}(p, \llbracket \text{NP} \rrbracket) = 2m \}$   
 c.  $\llbracket \text{right PP} \rrbracket = \{ p \in \llbracket \text{PP} \rrbracket \mid \text{dist}(p, \llbracket \text{NP} \rrbracket) \approx 0 \}$

These definitions face a serious problem: they are non-compositional. In order to get the relevant distances, the modifiers have to access the denotation of the object NP, which is not their sister in syntactic structure.<sup>7</sup>

The parallel definitions in a vector-based system on the other hand are both straightforward and compositional:

- (9) a.  $\llbracket \text{far PP} \rrbracket = \{ v \in \llbracket \text{PP} \rrbracket \mid |v| > r \}$   
 b.  $\llbracket \text{two meters PP} \rrbracket = \{ v \in \llbracket \text{PP} \rrbracket \mid |v| = 2m \}$   
 c.  $\llbracket \text{right PP} \rrbracket = \{ v \in \llbracket \text{PP} \rrbracket \mid |v| \approx 0 \}$

The PP on which the modifier operates denotes a set of vectors and the modifier selects from this set each vector *v* which has a particular *length*, represented by  $|v|$ . After this informal introduction to a vector-based semantics of prepositions and their modifiers, we will now turn to the formal foundation of this approach.

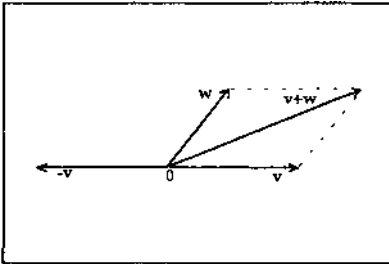
## 2 The algebra of vectors

The set of all vectors with the same origin corresponds to the algebraic notion of a *vector space*:

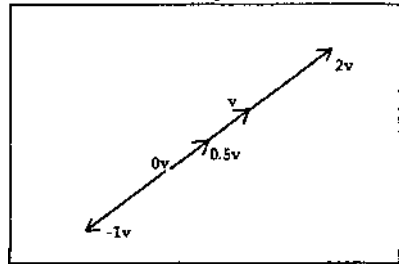
- (10) A vector space  $V$  over the set of real numbers  $R$  is a set that is closed under two operations:
- Vector addition* For every pair  $v, w \in V$  there is exactly one  $v + w \in V$ , the *vector sum* of  $v$  and  $w$
  - Scalar multiplication* For every  $v \in V$  and  $s \in R$  there is exactly one  $sv \in V$ , the *scalar product* of  $v$  by scalar  $s$

The operations of vector addition and scalar multiplication are graphically illustrated in (11) and (12), respectively:

(11) *Vector addition*



(12) *Scalar Multiplication*



A vector space has the following properties:

- (13)
- For all  $u$  and  $v \in V$ ,  $u + v = v + u$
  - For all  $u, v$ , and  $w \in V$ ,  $(u + v) + w = u + (v + w)$
  - There is an element  $0 \in V$ , the *zero vector*, such that  $v + 0 = 0 + v = v$  for all  $v \in V$
  - For every  $v \in V$  there is a  $-v \in V$ , the *inverse* of  $v$ , such that  $v + (-v) = 0$
  - For all  $u$  and  $v \in V$  and every  $c \in R$ ,  $c(u + v) = cu + cv$
  - For every  $v \in V$  and  $a$  and  $b \in R$ ,  $(a + b)v = av + bv$  and  $(ab)v = a(bv)$
  - For every  $v \in V$ ,  $1v = v$

In order to use vectors for model-theoretic interpretation of natural language expressions, we have to add vectors to the ordinary domain of objects  $E$ . One vector space  $V$  is not sufficient, however. The model will have to contain a much larger set

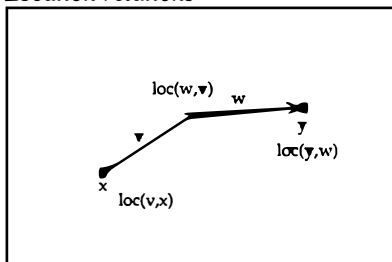
S of vectors, providing for each pair of points P and Q, a vector pointing from P to Q and a vector pointing from Q to P. S is then the union of an infinite set of vector spaces, one for each point in space and this S is added to the traditional domain of individual objects.<sup>8</sup>

In addition to E and S (with its algebraic structure), a general location relation loc is assumed that determines the spatial relationships between all these entities. Loc is a subset of the set of pairs  $(E \cup S) \times (E \cup S)$  that can be understood in the following way:

- (14) a.  $loc(x, y)$   $x = y$
- b.  $loc(v, x)$  the beginning point of vector v is located at object x
- c.  $loc(y, w)$  object y is located at the end point of vector w
- d.  $loc(w, v)$  the beginning point of vector w is located at the endpoint of vector v

The diagram gives a rough idea of the nature of this relation:<sup>9</sup>

(15) *Location relations*



The interpretation of a locative prepositional phrase has the general schematic form in (16). A locative PP denotes a set of vectors taken from a 'universe' of vectors that is determined by the reference object NP, as in (17). The reference object becomes the origin of this spatial universe by selecting only those vectors starting from it.<sup>10</sup> The preposition defines a subset of this space by imposing certain conditions on the length or orientation of vectors. When the PP is used as a predicate, it provides the range for an existential quantifier over vectors that picks out the vector at which the subject is located, as defined in (18).

- (16)  $[[PP P NP]]_M = \{ v \in space([[NP]]_M) \mid \dots v \dots \}$
- (17)  $space(x) = \{ v \in S \mid loc(v, x) \}$
- (18)  $[[S NP is PP]]_M = 1$  iff  $\exists v \in [[PP]]_M$  such that  $loc([[NP]]_M, v)$

### 3 Semantic definitions of prepositions

The preceding section has given the formal basis for precise definitions of locative prepositions. Following Landau and Jackendoff (1993), I will make a distinction between 'distance prepositions', that define a region on the basis of the length of its vectors, and 'direction prepositions', that select vectors that point in a particular direction.<sup>11</sup>

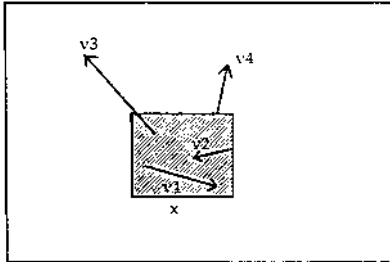
#### 3.1 'Distance prepositions'

The basic intuition on which the definitions in this section are based is that a distance preposition specifies the length of a vector pointing from the *surface* of the reference object, either *outward* (e.g. *outside* and *near*) or *inward* (e.g. *inside* and *in*). We want to be able to say that the length of outward pointing vectors is *positive* and the length of inward pointing vectors is *negative*. In other words: *outside* specifies a positive distance from its reference object and *inside* specifies a negative distance.<sup>12</sup> This notion of distance (i.e. vector length) requires some preliminary notions, defined in (19) and (20):

- (19) For any  $v \in \text{space}(x)$ ,
- a.  $v$  is *internal* to  $x$  iff  $\forall w [ \text{loc}(w,v) \rightarrow \text{loc}(w,x) ]$
  - b.  $v$  is *external* to  $x$  iff  $v$  is not internal to  $x$ .
- (20) For any  $v \in \text{space}(x)$ ,
- a.  $v$  is *extravert* w.r.t.  $x$  iff for every  $s$  ( $0 < s \leq 1$ ),  $sv$  is external to  $x$
  - b.  $v$  is *introvert* w.r.t.  $x$  iff  $v$  is internal to  $x$  and  $-v$  is extravert w.r.t.  $x$

A vector that has its beginning point in an object  $x$  can either be *internal* or *external* to  $x$ . (19) defines it as internal to  $x$  if in addition to its beginning point its end point is also in  $x$ , and external if this is not the case. The vectors  $v_1$  and  $v_2$  in (21) are internal vectors and  $v_3$  and  $v_4$  are external vectors. This distinction is used in (20) to define another distinction, that between *extravert* (outward pointing) and *introvert* (inward pointing) vectors. The definition implies that only vectors at the boundary of an object can be extravert or introvert, all other vectors are neither extravert nor introvert. In diagram (21),  $v_4$  is an extravert vector, but  $v_3$  is not, and  $v_2$  is an introvert vector,  $v_1$  is not.

(21) *Internal, external, introvert, and extravert*



I assume a function  $\| \cdot \|$  which assigns to each vector  $v$  in  $S$  a real number greater than or equal to zero, the length or *norm*  $\| v \|$  of  $v$ . The extended notion of length used here takes into account whether a vector is extravert or introvert by assigning a *polarity*: extravert vectors are positive and introvert vectors are negative.

(22) If  $v \in \text{space}(x)$ , then

- a.  $|v| = \| v \|$  if  $v$  is extravert w.r.t.  $x$
- b.  $|v| = - \| v \|$  if  $v$  is introvert w.r.t.  $x$
- c.  $|v| = 0$  if  $v = 0w$  for a vector  $w$  extravert w.r.t.  $x$

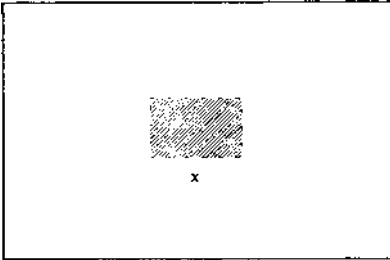
$\| \cdot \|$  is a total and absolute function, but  $| \cdot |$  is a function defined relative to a vector space and applying only to vectors located at the surface of an object. It assigns the value 0 to the zero vectors at the surface (i.e. those vectors that are the result of multiplying an extravert vector by 0).

Given this function we can define the 'distance-based' prepositions as follows:

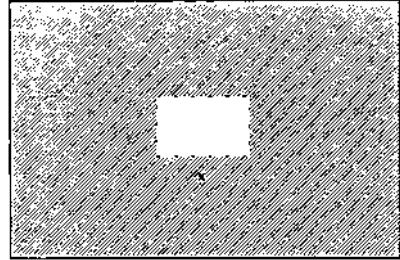
- (23) a.  $[[ \text{in NP} ]] = [[ \text{inside NP} ]] = \{ v \in \text{space}([ \text{NP} ]) \mid |v| < 0 \}$
- b.  $[[ \text{outside NP} ]] = \{ v \in \text{space}([ \text{NP} ]) \mid |v| > 0 \}$
- c.  $[[ \text{on NP} ]] = [[ \text{at NP} ]] = \{ v \in \text{space}([ \text{NP} ]) \mid |v| = 0 \}$
- d.  $[[ \text{near NP} ]] = \{ v \in \text{space}([ \text{NP} ]) \mid |v| < r \}$   
( $r > 0$ )

The diagrams (24) to (27) indicate what kind of regions are determined by these definitions. Notice however, that the diagram only gives a bounded two-dimensional cross-section of regions in unbounded three-dimensional space. The region is represented by the shaded area (unless the region is a point or line) and the dotted boundary of an area indicates that the area is topologically open.<sup>13</sup>

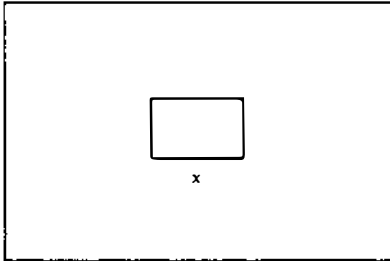
(24) in / inside x



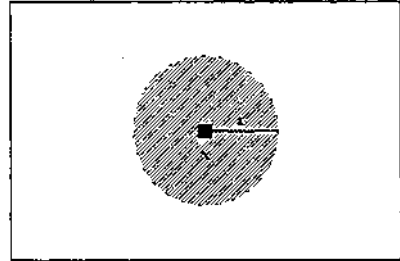
(25) outside x



(26) on/at x



(27) near x



Of course, this is only a very rough and idealized representation of the meaning of these items. It leaves much to be desired, but it captures the central intuitions about these prepositions expressed in most of the literature. An account of the meaning differences between *in* and *inside* and between *on* and *at* is left for further research. The reference object for *inside* might be the boundary of a bounded object instead of that object itself. *On* and *at* differ with respect to the dimension of their reference object, as observed by many authors (e.g. Leech 1969). The definition of *near* abstracts away from the fact that the 'radius'  $r$  may depend on factors like the size of the reference object, the structure of the environment, etc.

### 3.2 'Direction prepositions'

Prepositions like *behind* and *under* denote a region on a particular side of the reference object by selecting vectors that point in a certain direction. The literature about these prepositions often uses the notion of *axes* in this context, which are assigned to the reference object and the space around it on the basis of intrinsic, environmental, or deictic characteristics. There is a *frontal* axis pointing 'forward' from the face of an object, a *vertical* axis pointing 'upward' (in the opposite direction of gravity), and a *lateral* axis, perpendicular to both of these. Abstracting away from the complex factors that determine these axes, we assume that the model provides

three functions, each assigning a set of vectors to an object  $x$ .<sup>14</sup> The lateral axis is a one-dimensional vector space, extending infinitely in both directions (a line, geometrically), but the vectors of the frontal axis and vertical axis point in one direction only (these axes are half-lines, geometrically).

Two important notions for understanding the relations between direction prepositions are the *inverse* of an axis, which is simply the set of vectors pointing in the opposite direction and the so-called *orthogonal complement*  $\perp A$ , which is the set of vectors orthogonal to the vectors in  $A$ :

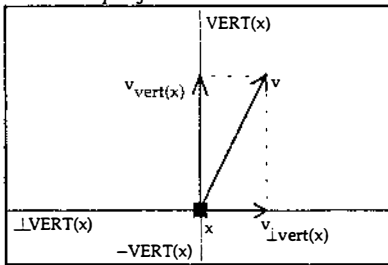
(28) the *inverse* of an axis  $A$  is  $-A = \{ v \in S \mid -v \in A \}$

(29) the *orthogonal complement* of an axis  $A$  is  $\perp A = \{ v \in S \mid \forall w \in A [ v \perp w ] \}$

The orthogonal complement of the vertical axis of  $x$  corresponds to the horizontal plane through  $x$ .

Every vector located at  $x$  can be decomposed into several *components* in the axes defined for  $x$ . This is illustrated in (30). The vector  $v$  is decomposed into a vertical component  $v_{\text{VERT}(x)}$  (its *projection* on the vertical axis) and a horizontal component  $v_{\perp \text{VERT}(x)}$  (its projection on the orthogonal complement of the vertical axis).

(30) *Axes and projection*



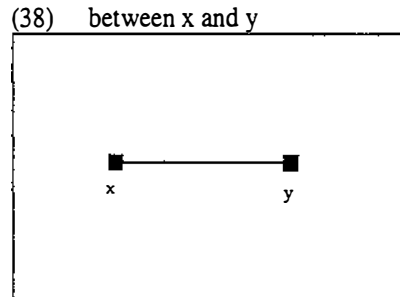
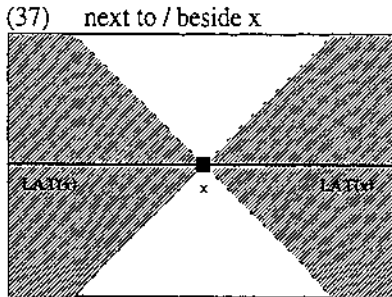
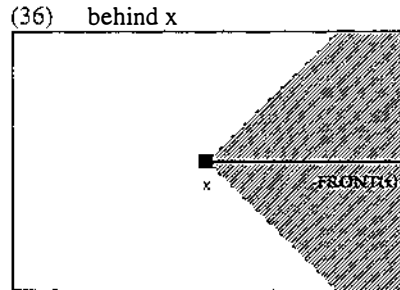
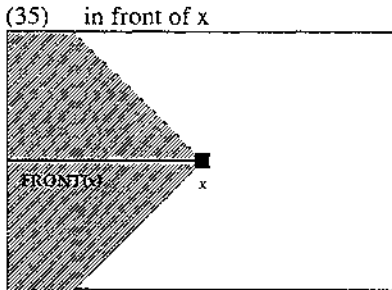
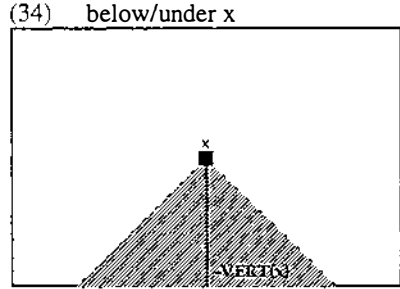
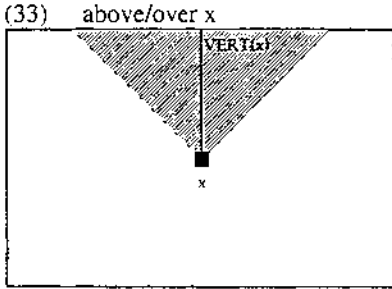
The notion of projection used here is defined in (31):

(31) The *projection*  $v_A$  of vector  $v$  on axis  $A$  is that vector  $u \in A$  for which there is a  $w, w \perp u$  and  $u + w = v$

This provides the necessary formal apparatus to define the most important direction-based prepositions. The central idea is that a vector is 'over/above'  $x$ , for example, if its projection on the vertical axis is has a greater length than the projection on the orthogonal horizontal plane. The vector  $v$  in (30) is therefore taken to be 'above/over'  $x$ . In all the definitions in (32), the length of the component in a relevant axis is compared with the component orthogonal to that axis:

- (32) a.  $[[ \text{above/over NP} ]] = \{ v \in \text{space}([ [ \text{NP} ] ]) \mid |v_{\text{VERT}}| > |v_{\perp \text{VERT}}| \}$
- b.  $[[ \text{below/under NP} ]] = \{ v \in \text{space}([ [ \text{NP} ] ]) \mid |v_{\text{-VERT}}| > |v_{\perp \text{-VERT}}| \}$
- c.  $[[ \text{in front of NP} ]] = \{ v \in \text{space}([ [ \text{NP} ] ]) \mid |v_{\text{FRONT}}| > |v_{\perp \text{FRONT}}| \}$
- d.  $[[ \text{behind NP} ]] = \{ v \in \text{space}([ [ \text{NP} ] ]) \mid |v_{\text{-FRONT}}| > |v_{\perp \text{-FRONT}}| \}$
- e.  $[[ \text{next to/beside NP} ]] = \{ v \in \text{space}([ [ \text{NP} ] ]) \mid |v_{\text{LAT}}| > |v_{\perp \text{LAT}}| \}$

These definitions yield the cone-shaped regions in (33)-(36) and the diabolo-shaped region in (37).<sup>15</sup> (Remember that the diagrams only give finite cross-sections of these regions.)



In order to keep things simple, the region for *between* is assumed to be the axis between the two reference objects, as shown in (38), but it is possible to define a 'diamond-shaped' region by taking all the vectors departing from  $x$  and  $y$  that have a particular projection on the axis. The simple definition is given in (39):

$$(39) \quad \llbracket \text{between NP}_1 \text{ and NP}_2 \rrbracket = \\ \{ \mathbf{v} \in \text{space}(\llbracket \text{NP}_1 \rrbracket) \mid \exists s [ s \geq 1 \wedge \text{loc}(\llbracket \text{NP}_1 \rrbracket, s\mathbf{v}) ] \} \cup \\ \{ \mathbf{v} \in \text{space}(\llbracket \text{NP}_2 \rrbracket) \mid \exists s [ s \geq 1 \wedge \text{loc}(\llbracket \text{NP}_2 \rrbracket, s\mathbf{v}) ] \}$$

Given these definitions of distance-based and direction-based prepositions, we can now turn to a range of properties of regions that we can define vector-theoretically.

#### 4 Properties of regions

In this section we will study closure properties of regions under certain operations on their vectors (multiplication by a positive or negative scalar and rotation) and continuity properties of regions. Some of these properties seem to be universals of prepositions, others single out classes of prepositions with particular modification possibilities.<sup>16</sup>

The first property that the region denoted by a locative PP does or not have is closure under lengthening:<sup>17</sup>

- (40) *Closure under lengthening*  
A region  $R$  is closed under lengthening iff  
for every non-zero  $\mathbf{v} \in R$ ,  $s\mathbf{v} \in R$  for every  $s > 1$ .

Given an arbitrary vector  $\mathbf{v}$  in a region that is closed under lengthening, one can stretch this vector and the result will still be in the region. Intuitively, a region that is closed under lengthening is unbounded in the direction in which the vectors point. A region that is not closed under lengthening is bounded by the boundaries of the reference object or by modifiers like *close* and *at most two meters*:

- (41) a. *closed*: in front of, behind, above, under, beside, outside, far behind, at least two meters behind  
b. *not closed*: near, in, at, on, inside, between, close behind, at most two meters behind

The prepositions that are closed under lengthening are the ones that can be modified by measure phrases like *two meters*. This is illustrated by the following examples from Dutch:<sup>18</sup>

- (42) a. twee meter voor / achter / boven / onder / naast / buiten NP  
two meters in front of / behind / above / under / beside / outside NP
- b. \* twee meter tussen / bij / in / op / binnen NP  
two meters between / near / in / on / inside NP

Measure phrases modifying PPs can now be treated as functions on sets of vectors that are closed under lengthening. The vector-based approach to regions allows us to make this generalization, although we do not know yet why measure phrases have this selectional restriction.

The counterpart of closure under lengthening is closure under shortening:

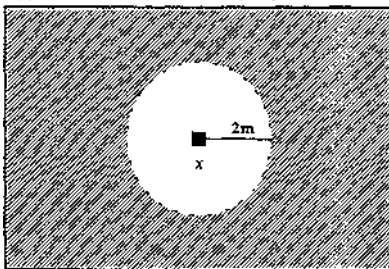
- (43) *Closure under shortening*  
A region R is closed under shortening iff  
for every  $v \in R$ ,  $sv \in R$  for every  $0 < s < 1$ .

This property says that one can take an arbitrary vector from the region, make it shorter, and the result will again be in the region. Intuitively, the regions that are closed under shortening make contact with the reference object. When we consider the set of simple and modified prepositions, we get the following result:

- (44) a. *closed*: all simple prepositions, prepositions modified with e.g. *close*,  
*at least two meters*
- b. *not closed*: prepositions modified with e.g. *high*, *far*, *more than two meters*

The diagram in (45) shows the region of *more than two meters outside x*. The region does not make contact with the reference object, because there is a two meters wide gap created by the modifier *more than two meters*.

- (45) more than two meters outside x



Notice that in (44) all simple prepositions (i.e. prepositions not modified or conjoined) are closed under shortening. This might be taken as a coincidence, but it

would be far more interesting to interpret it as a real universal about prepositional semantics:

- (46) *Universal 1* All *simple* prepositions are closed under shortening.

Intuitively, this universal implies that simple prepositions denote regions that are spatially connected to the reference object. One clear and easily falsifiable prediction drawn from this universal is that there are no *distal* prepositions. This seems to be true for English (and other languages I know of): the proximate preposition *near* does not have a distal counterpart *far* (*\*far the house*). Of course, this gap is filled by the expression *far from*, but this is not a simple locative preposition, but a combination of the *directional* preposition *from* with the distal adjective *far*.<sup>19</sup> The universal would only be falsified by a distal expression which would be a genuine simple preposition.

Closure under lengthening and shortening are defined with respect to the operation of scalar multiplication that changes the *length* of a vector while keeping its orientation fixed. *Rotation* is an operation that keeps the length of a vector fixed while changing its orientation. A rotation  $r$  in a vector space  $V$  is a function from  $V$  to  $V$ , rotating each vector in a particular plane over a particular angle.<sup>20</sup> When we restrict our attention to point-sized reference objects, it makes sense to investigate the closure under rotation property of regions:<sup>21</sup>

- (47) *Closure under rotation*  
A region  $R$  is closed under rotation iff  
for every  $v \in R$ ,  $rv \in R$ , for every rotation  $r$ .

Distance prepositions have this property, direction prepositions lack it:

- (48) a. *closed*: in, inside, at, on, near, outside  
b. *not closed*: in front of, behind, above, under, beside, between

In this way, the intuitive distinction that we made earlier and that was also reflected in the form of the definitions, has now received a clear algebraic formalization. The use of directional modifiers like *recht* (straight) and *schuin* (diagonally) in Dutch seems to be sensitive to this algebraic property when it modifies a PP:<sup>22</sup>

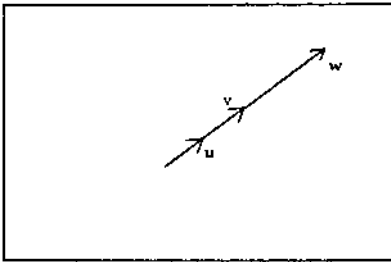
- (49) a. \* *recht/schuin in / binnen / op / bij / buiten NP*  
straight/diagonally in / inside / on / near / outside NP  
b. *recht/schuin voor / achter / boven / onder / naast / tussen NP*  
straight/diagonally in front of / behind / above / under / beside / between NP

The reason that closure under rotation is relevant for directional modification is the following. The regions that are closed under rotation are not oriented in a particular

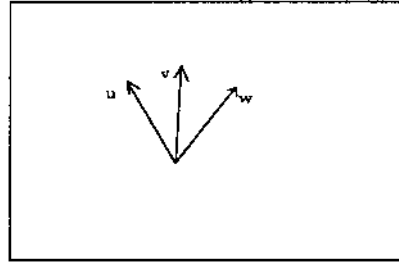
direction because they are not defined on the basis of an axis. But a vector in a region can only be said to be 'straight' or 'diagonal' with respect to a reference axis or reference plane. Therefore, it is impossible to define vectors as 'straight' or 'diagonal' in a region that is closed under rotation.

The final property or cluster of properties to be discussed in this paper concerns the *continuity* of regions denoted by locative PPs. The two relevant notions of continuity are based on the two ways in which a vector  $v$  can be 'between' two other vectors  $u$  and  $w$ :

(50)  $v$  linearly between  $u$  and  $w$



(51)  $v$  radially between  $u$  and  $w$

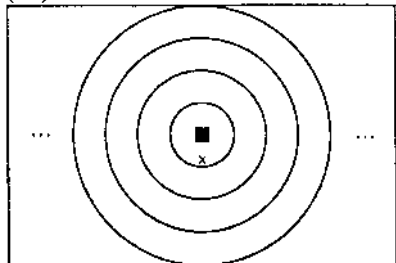


A vector  $v$  is *linearly between*  $u$  and  $w$  if  $v$  is a lengthening of  $u$  and  $w$  is a lengthening of  $v$ . A vector  $v$  is *radially between* two vectors  $u$  and  $w$  that form an acute angle if the shortest rotation of  $u$  into  $w$  passes over  $v$ . Both of these notions of 'betweenness' correspond to a form of continuity:

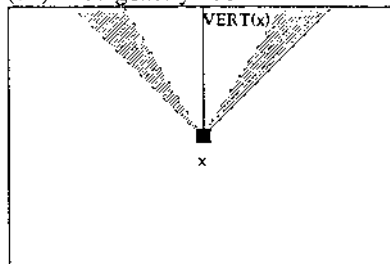
- (52) a. *Linear continuity*  
A region  $R$  is linearly continuous iff  
for all  $u, w \in R$ , if  $v$  is linearly between  $u$  and  $w$ , then  $v \in R$
- b. *Radial continuity*  
A region  $R$  is radially continuous iff  
for all  $u, w \in R$ , if  $v$  is radially between  $u$  and  $w$ , then  $v \in R$

The best way to grasp these continuities is by looking at two cases that each lack one of these properties. The region in (53) denoted by *an even number of meters outside x* consists of an infinite set of concentric shells around the reference object  $x$ . This region is *radially* continuous but *linearly discontinuous*. The PP *diagonally above x* in (54) denotes a kind of cup (in three-dimensional space at least), that one can get by scooping out the cone-shaped region of *above x*. This region is *linearly* continuous, but *radially discontinuous*.

(53) an even nr of meters outside x



(54) diagonally above x



Two continuity universals can now be formulated, a stronger one for *simple* prepositions and a weaker one for all prepositions, whether they are modified or not:

- (55) *Universal 2* All simple prepositions are linearly *and* radially continuous.  
 (56) *Universal 3* All prepositions are linearly *or* radially continuous.<sup>23</sup>

## Conclusion

This paper has laid out the first principles of a semantics of locative prepositions based on vectors. It integrates the insights from the literature about prepositions in a general, formal framework, but it adds an important algebraic dimension to the study of spatial meaning. This makes it possible to study the meanings of prepositions, both simple and modified, in a way that is reminiscent of the Generalized Quantifier Theory of determiners: in addition to precise definitions of individual prepositions, algebraic properties can be formulated that either single out empirically relevant subclasses of prepositions or that provide universals of prepositions. It cannot be stressed enough, of course, that such a semantic theory of locative prepositions captures only a part of the bewildering richness of prepositional meanings and uses explored in the literature, but it is undoubtedly a central part.

## Endnotes

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<sup>1</sup> A bibliography of relevant literature would equal the size of this paper.

<sup>2</sup> But see Bierwisch (1988) and Landau and Jackendoff (1993), for two interesting studies from a model-theoretic semantics and a cognitive psychology point of view, respectively.

<sup>3</sup> The definitions of this paper capture idealized geometric uses of prepositions. Many other perceptual, pragmatic, and functional aspects will have to be taken into account for a more realistic account of the meaning and actual use of prepositions. See also Herskovits (1986).

<sup>4</sup> Jackendoff (1983), Creary et al. (1989), Wunderlich (1991).

<sup>5</sup> Dowty (1989) and Parsons (1990), among others. Zwarts (1992) suggests an extension of the Neo-Davidsonian approach to all lexical categories, including adjectives and prepositions.

<sup>6</sup> In these definitions  $r$  is a pragmatically provided number and  $\approx$  is an informal representation of 'almost ='.

<sup>7</sup> I am making the (standard) assumption that PP modifiers combine with a P + NP constituent, i.e. that the structure is [ Mod [ P NP ] ]. A compositional semantics of modification might be possible in the alternative structures [[Mod P ] NP ] and [ Mod P NP ], but as far as I know, no syntactician has ever seriously defended these structures.

<sup>8</sup> In this paper vectors are taken as primitive spatial entities. Whether they are really basic or defined as pairs of points or tuples of real numbers would have to be made clear in a more extensive exposition of the vector-based semantics of space. Note moreover that the model does not contain spatial points in the ordinary sense, because they do not play an independent role. However, we can *define* points as zero vectors or vector spaces, if the concept of a point should turn out to be useful.

<sup>9</sup> Note that the objects  $x$  and  $y$  are usually not points, but objects with a spatial extension. In that case,  $\text{loc}(\mathbf{v}, x)$  and  $\text{loc}(y, \mathbf{v})$  denote situations in which the beginning point of  $\mathbf{v}$  is located *in* object  $x$  and the end point of  $\mathbf{v}$  is located *in*  $y$ , respectively.

<sup>10</sup> I am restricting myself here to *singular* reference objects. An interesting question is how regions are defined when the preposition has a *plural* reference object, like in *inside the boxes* or *behind the trees*. Some prepositions, like *inside* have to distribute over their plural reference object, defining a region for each reference object, other prepositions, like *behind* seem to be able to treat the plural reference object as a unit, defining one region for it.

<sup>11</sup> There is a partial correspondence with the distinction between *topological* and *projective* prepositions made in Herskovits (1986).

<sup>12</sup> The length of an inward pointing vector may be modified: e.g. *deep inside the forest* and *two miles inside the forest*. These examples clearly show that the modified distance is measured from the edge of the forest.

<sup>13</sup> From the topological point of view the interior and exterior regions of an object are open sets, the surface region is a closed set. Two objects touch if their surface regions overlap. See also Hayes (1985).

<sup>14</sup> In order not to make things too complicated, I will assume that the reference object of these prepositions is idealized as a point in space, because its internal structure is irrelevant.

15 Several people have pointed out to me that the cone-shaped regions might be right for *over* and *under*, but that *above* and *below* simply compare the altitude levels of two objects. The conditions will then be  $|v_{\text{VERT}(x)}| > 0$  for *above* and  $|v_{-\text{VERT}(x)}| > 0$  for *below* and the resulting regions will be the entire areas above/below the horizontal plane through the reference object.

16 Zwarts (1994) shows the relevance of these properties for the characterization of valid and invalid inferences with prepositions.

17 A PP can be said to be closed under lengthening when the region it denotes is closed under lengthening in every model. Similarly for the other properties. We will often talk about PPs or prepositions having a particular property when it is strictly speaking a property of the corresponding region.

18 The pattern in Dutch is clearer than in English, where *beside* and *next to* behave as being *not* closed under lengthening, which might be due to a meaning component of contact or proximity, which is absent in Dutch. *Binnen* (inside) and *in* may become closed under lengthening when the reference object itself is open-ended (e.g. *two meters inside the area*).

19 That *far* is really an adjective in this construction and not a preposition is shown by its modification possibilities: *very far from the house*, *too far from the house*, but *\*right far from the house*, *\*two miles far from the house*.

20 The set of all possible rotations in  $\mathbf{V}$  together with function composition constitutes a *group* with the zero rotation as an identity element and each rotation having an inverse rotation bringing a vector back into its original position.

21 The region of *in* and *inside* are empty in this case (points do not have an interior), *at* and *on* correspond to singleton regions with the zero vector and all regions are subsets of one single vector space.

22 In English only some of the prepositions that are not closed under rotation can be modified in this way, e.g. *straight behind* and *diagonally above*. I have no idea why these modifiers have a more restricted use in English.

23 Yoad Winter pointed out to me that this universal does not seem to be valid for those cases where a preposition is modified by a conjunction of two modifiers: e.g. *diagonally and an even number of meters above the door*. This PP seems to denote a region which is neither linearly nor radially continuous. However, it is not clear to me yet whether conjunction of *and* *inside* PPs really creates coherent regions, or whether the conjunction applies at another level.

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