Proceedings of SALT 32: 831–851, 2022

Weak necessity modals as homogeneous pluralities of worlds*

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Abstract We provide the first analysis of the apparent homogeneity effects displayed by weak necessity modals. Previous work on weak necessity modals like English *should* has focused on their other properties, namely their relatively "weak" truth conditions, and their morphological composition across languages. We add homogeneity effects to the list of desiderata, and present an account in which weak necessity modals crucially involve plural reference to worlds, rather than universal quantification, which captures all the relevant data. We then extend this account to weak necessity modals derived from strong necessity modals, as observed morphologically in French and Javanese, for which we provide a compositional analysis with interesting implications for typology.

Keywords: weak necessity modals, homogeneity, plurals, QUD

1 Introduction

In this work, we present a novel analysis of the semantics of weak necessity modals like *should* as definite pluralities of worlds. We contrast weak necessity modals with strong necessity modals, which are taken to be quantificational, following standard Kratzer modal semantics. In short, we propose that weak necessity *should* is to strong necessity *must* what the definite determiner *the* is to the universal quantifier *all*. Our account can be captured by the following paraphrases.

(1) a. *must p* = all worlds in a given domain are p-worlds
b. *should p* = the worlds in a given domain are p-worlds

This proposal relies on empirical parallels observed between weak necessity modals and definite pluralities of individuals on a number of properties that have previously been shown to characterize plural definites. The principal property we discuss is homogeneity: just like plural definites, weak necessity modals across languages systematically take obligatory apparent wide scope with respect to negation. Unlike previous prominent analyses of weak necessity, ours has homogeneity as

* We thank Vera Hohaus, Jozina Vander Klok, Lisa Matthewson, Rob Pasternak, Philippe Schlenker, and the audience members at NYU Semantics Group.

an intrinsic feature inseparable from the core semantics, which is desirable given this homogeneous cross-linguistic picture. In support of our analysis, we also show that weak necessity modals share other properties with plural definites, including homogeneity removal by quantifiers, QUD-sensitive exception tolerance, and truth value gaps diagnosed by responses to borderline sentences.

The second contribution of this paper is to propose a compositional account of the weak necessity modals that are morphologically derived from strong necessity modals (as observed in French, Javanese, and other languages), which builds on our new view of weak necessity modals as plural definites.

In Section 2, we present some background on weak necessity modals. In Section 3, we show the core data that provides support for our plural referential analysis of *should* and other weak necessity modals. In Section 4, we give our semantics for *should*, and in Section 5, we propose an extension to French and Javanese, in which weak necessity modals are compositionally built from strong necessity modals. In Section 6, we discuss a few previous analyses and compare them to our own, and in Section 7 we conclude. In the Appendix, we present the formal details of our analysis of exception-tolerance.

2 Background on weak necessity modals

The characterization of weak necessity meaning Weak necessity modals are necessity modals, like English *should*, that have been described to be weaker, or have a more negotiable feel, than strong necessity modals, like *must* or *have to*. This difference is illustrated below: *should* indicates a necessity that allows for exceptions, while *have to* does not allow for exceptions.

(2) a. If you want to go to Colegio de México, you should take a taxi. ...but you could also take a bus if you have all morning.
b. If you want to go to Colegio de México, you have to take a taxi. ...??but you could also take a bus if you have all morning.

This feeling of weakness compared to strong necessity has been taken to be the characteristic property of weak necessity that every account of it has tried to capture; ours is no exception. While this type of data has been the main data to be accounted for by previous analyses, we take a step back from it, and have our proposal driven by data of weak necessity modals under negation, that reveals their nature as homogeneity-inducing. Their weakness relative to strong necessity modals then falls out from it.

The expression of weak necessity cross-linguistically Weak necessity semantics is found to be morphologically encoded in two different ways across languages. The first way is as a primitive lexical item (morphologically non-decomposable), such as English *should* and *ought*. In other cases, a weak necessity modal can be built from a strong necessity modal by adding additional morphology, whose nature varies across languages. In some languages, weak necessity modality can be expressed by combining a strong necessity modal with counterfactual morphology, as found for example in French, shown in (3) (see discussion of this strategy in von Fintel & Iatridou 2008).

(3) a. Strong necessity modal *devoir*

Tu **dois** partir.
you must go
You must go.

b. Weak necessity modal *devoir*+CF

Tu **devrais** partir.
you must.CF go
You should go.

Other languages express weak necessity by adding to a strong necessity modal a dedicated morpheme (i.e. not obviously related to any other semantic category in the language), as seen in Javanese, shown in (4), and related Malayo-Polynesian languages (see discussion of this strategy in Vander Klok & Hohaus 2020).

(4) a. Strong epistemic necessity *mesthi*

Yu Dur **mesthi** nek omah.
sister Dur EPIS.NEC at house
'Dur must be at home.'

b. Weak epistemic necessity *mesthi*+NE

Yu Dur **mesthi-ne** nek omah.
sister Dur EPIS.NEC-NE at house
'Dur should be at home.'

3 Weak necessity modals pattern like plural definites

3.1 Homogeneity: a hallmark property of plural predication

A distinctive property of plural definites is their interaction with negation, in which they display *homogeneity effects*, meaning that they take obligatory apparent wide scope with respect to negation, even if they are syntactically lower (Fodor 1970;

Löbner 1985).

We first show that the basic meaning of a plural definite is comparable to that of a universal quantifier, in that if a predicate is true of a definite plurality, it is generally infelicitous to state there there exists individuals among that plurality that make the predicate false.

(5) a. The guests are here, # but some of them aren't.
b. Every guest is here, # but some of them aren't.

In contrast with universal quantifiers, plural definites take obligatory apparent wide scope with respect to negation.

(6) a. The guests are not here, # but some of them are.
b. Every guest is not here, ✓ but some of them are.

Again, this pattern persists when the negation is located in a higher clause.

(7) a. I don't think the guests are here, # but some of them are.
b. I don't think every guest is here, ✓ but some of them are.

We diagnose the wide scope universal meaning by testing the felicity of an existential quantifier over the same domain: while a narrow scope reading with respect to negation would be compatible with an existential claim, a wide scope one is not.

Since this wide scope persists when negation is located in a higher clause, we analyze it as only apparent (following most authors), where plural definites do not actually take scope, but are interpreted directly as the arguments of predicates. For this reason, we refer to the apparent "wide scope" interpretation as *scopeless*.

We now turn to weak necessity modals, which display similar homogeneity effects as plural definites. First, just like plural definites, they appear to have universal force, as revealed by their incompatibility with existential quantifiers over the same domain. We use the adverbial *according to the rules* to fix the ordering source so that both *should* and *allowed* are evaluated on the same domain of worlds.

(8) a. According to the rules, you should go,
but you are allowed not to go.
b. According to the rules, you have to go,
but you are allowed not to go.

Under negation, weak necessity modals pattern like plural definites, where they must be interpreted as taking apparent wide scope, in contrast with strong necessity modals like *have to*, which can take scope below negation.

(9) a. According to the rules, you shouldn't go,

but you are allowed to go.

b. According to the rules, you don't have to go,
✓ but you are allowed to go.

Again, we diagnose the apparent wide scope behavior by testing the felicity of the existential modal quantifier *allowed* evaluated with respect to the same ordering source of *should*, where a narrow scope with respect to negation would be compatible with it, but not a wide scope.

Like with plural definites, the apparent wide scope persists when negation is extra-clausal. *Should* must have a wide scope interpretation, in contrast with *have to*, which takes scope below.

(10) a. According to the rules, I don't think you should go,
but you are allowed to go.

b. According to the rules, I don't think you have to go,
✓ but you are allowed to go.

(11) a. According to the rules, it's not the case that you should go,
but you are allowed to go.

b. According to the rules, it's not the case that you have to go,
✓ but you are allowed to go.

Note that some strong necessity modals like English *must* are also known to have wide scope readings with respect to negation, as in (12a) (Jeretič 2021, Homer 2015, Iatridou & Zeijlstra 2013). However, they differ from weak necessity modals in that they can be interpreted with low scope under extra-clausal negation, as shown in (12b).

(12) a. According to the rules, you mustn't go,
but you are allowed to go.

b. According to the rules, I don't think you must go,
✓ but you are allowed to go.

We take these examples as evidence that *must*'s wide scope in matrix sentences should not receive the same analysis as the wide scope of weak necessity modals (see Jeretič 2021 for an analysis of *must* that captures the difference between clausemate and non-clausemate negation).

As far as we know, all weak necessity modals have this apparent wide scope with respect to negation. We have checked this for: English *should*, French *falloir* and *devoir* with counterfactual (CF) marking, Javanese modals with NE marking, Russian *sledovat'/stoit'*, Swedish *bör*, Spanish *deber/tener que/hay que*+CF, Hungarian *kell*+CF, Portuguese *dever/haver de*+CF, Italian *occorrerre/bisognare/dovere*+CF,

Dutch *moeten*+CF, Greek *prepi*+CF.

Since homogeneity effects are systematically found for weak necessity modals across languages, a suitable analysis of the meaning of weak necessity should be able to automatically capture these effects. However, as we will show in Section 6, this is not the case for the most prominent previous analyses of weak necessity, where one has to account for homogeneity in an independent way, e.g. through positive polarity, as analyzed by Iatridou & Zeijlstra 2013 and Homer 2015. In contrast, the analysis proposed in this paper of weak necessity modals as plural definites has homogeneity as an intrinsic part of the meaning of a plurality, thus making the right prediction for the cross-linguistic picture.

To summarize, weak necessity modals can be compared to plural definites in their scopeless behavior with respect to negation, as shown in (11a) for *should* and (7a) for a plural definite. This scopeless behavior is to be contrasted with strong necessity modals and universal nominal quantifiers, which do not have such scopeless behavior, as shown in (11b) and (12b) for *have to* and *must*, and in (7b) for *every*.

These parallels suggest a unified analysis for weak necessity modals and plural definites, where the difference between 'should' and 'must' is the same as the one between 'the' and 'all'. On our analysis, weak necessity modals are referring expressions that denote pluralities of worlds, just like plural definite nominals denote pluralities of individuals. In the next sections, we provide additional support for this parallel by showing that weak necessity modals pass additional diagnostics for plural predication.

3.2 Homogeneity removal

We now look at a phenomenon called *homogeneity removal*, where a universal quantifier over the domain of individuals denoted by a plural definite "removes" its truth value gap. This is the case in (13b), where 'all' quantifies over the set of relevant guests (here 'all' is floating, but the effect is the same if it is not).

(13) a. The guests are not here, # but some of them are.
b. The guests are not **all** here, ✓ but some of them are.

Notice that, strictly speaking, nothing is being "removed" in (13). Quantifiers just interact with negation as expected, while plural definites are referring expressions, and therefore do not take scope with respect to negation.

There is a comparable phenomenon occurring with *should*, where introducing the modal quantifier adverb 'necessarily' removes homogeneity.

(14) a. The advice was that you shouldn't go, # but that you can go.

b. The advice was that you shouldn't **necessarily** go,
✓ but that you can go.

(15) a. According to the rules, you should not work from home...
but it is allowed.
b. According to the rules, you should not **necessarily** work from home...
✓ but it is allowed.

In both (14) and (15), we get a strong scopal contrast between the sentence with the quantifier 'necessarily' and the one without.

3.3 Exception tolerance

Plural predication tolerates exceptions in the right discourse contexts, in contrast with universal quantifiers. This kind of exception tolerance is also known as *non-maximality* (Dowty 1987; Schwarzschild 1996; Brisson 2003; Križ 2016; Malamud 2012). Plural predication structures are accepted as true when the exceptions are not relevant to the QUD, or in other words, when an existential and a universal claim fall in the same cell in the partition made by the QUD.

In (16), we give an example of a context in which universal quantification is false, regardless of the QUD. In contrast, plural predication becomes true or false depending on whether the QUD asks about the truth of an existential claim (thus making irrelevant whether existential or universal quantification is true), like QUD1, or a universal claim, like QUD2.

(16) *C: 4 out of 6 students asked questions.*
QUD1: Was the class lively?/Did any students ask questions?
QUD2: Who all asked questions?/How many students asked questions?
a. The students asked questions. (QUD1: ✓; QUD2: #)
b. All the students asked questions. (QUD1: #; QUD2: #)

Weak necessity modals display the same pattern. While for plural definites, the "exceptions" are irrelevant individuals, for weak necessity modals the "exceptions" are irrelevant possibilities—worlds whose inclusion or exclusion in the plurality does not matter for the QUD.

(17) *C: One can get a perfect grade by doing most exercises correctly; doing all gives extra credit.*
QUD1: What is a way to get a perfect grade?
QUD2: What are the minimal requirements to get a perfect grade?
a. To get a perfect grade, you should do every exercise.
(QUD1: ✓; QUD2: #)

b. To get a perfect grade, you have to do every exercise.
(QUD1: #; QUD2: #)

In this context, a necessary condition to get a perfect grade is to do most exercises. Doing *all* of the exercises is not necessary. We observe that the strong necessity claim (17b) is infelicitous under any QUD. On the other hand, (17a) can be rescued by by a QUD like QUD1 above, which does not distinguish between cases in which the addressee does every exercise, and cases in which the addressee does most exercises, as long as they get a perfect grade.

3.4 Responses to indeterminate sentences

Another characteristic property of homogeneity is that outright denials of indeterminate sentences are infelicitous. Križ observes that in borderline cases (i.e. cases in which existential but not universal quantification is true), it is preferable to respond with *well*, rather than denial.

(18) *C: Mary talked to only some of the girls.*

a. A: #Mary talked to the girls.
B: #{No, That's not true}, only to some.
B: Well, only to some.

b. A: #Mary talked to all of the girls.
B: {No, That's not true}, only to some.
B: #Well, only to some.

The same pattern applies to borderline cases with *should.*

(19) *C: Two doors lead to the living room; both are equally good options.*

a. A: #You should take the right door to go to the living room.
B: #{No, That's not true}, you don't have to, but you can.
B: Well, you don't have to, but you can.

b. A: #You must take the right door to go to the living room.
B: {No, That's not true}, you don't have to, but you can.
B: #Well, you don't have to, but you can.

Recall from the previous section that indeterminate sentences are felicitous in contexts where their exceptions are irrelevant to the QUD. Since this is equally true of sentences with negation, responses with *no* can be made felicitous by a friendly QUD. In those cases, the *well*-responses are also fine. So the cases that are crucial for distinguishing the present theory from bivalent theories are all cases with a strict QUD (i.e. where an existential and universal claim correspond to different answers).

4 Analysis

We have shown that the difference between weak and strong necessity modals appears to be empirically parallel to the difference between universal quantifier expressions and definite descriptions in the nominal domain. We base our analysis on this parallel, where weak necessity modal statements can be exactly paraphrased by plural nominals, with the same truth and falsity conditions, as shown in (20).

(20) a. *You should go.* $\simeq$ *You go in the best worlds.*
b. *You shouldn't go.* $\simeq$ *You don't go in the best worlds.*

(21) a. *You have to go.* $\simeq$ *You go in all best worlds.*
b. *You don't have to go.* $\simeq$ *You don't go in all best worlds.*

Homogeneity removal by quantifiers is also parallel:

(22) *You shouldn't necessarily go.* $\simeq$ *You don't go in all the best worlds.*[1]

We implement this analysis by directly translating the proposal by Križ (2016) of plural definite nominals as referential pluralities of individuals into the equivalent for worlds as paraphrased above.

4.1 Homogeneity for plural nominals

Križ (2016) follows standard accounts of plural reference where pluralities are mereological sums of individuals, and predicates have sums in their extension, as shown in (23).

(23) a. The windows are open. $\text{open}(\bigoplus \text{window})$
b. The windows are not open. $\neg\text{open}(\bigoplus \text{window})$

Plural predication is then assumed to have a homogeneity property, which amounts to requiring non-overlap between the positive and negative extensions of the predicate, as defined in (24).

(24) **Homogeneity:** A plurality in the extension of a predicate $\lambda x.P(x)$ must not overlap with any plurality in $\lambda x.\neg P(x)$.

This means that in (23a), $\ulcorner\bigoplus \text{window}\urcorner$ (the sum of all windows in the domain) must not overlap with anything that is not-open, which entails that *every* window is open. For (23b), $\ulcorner\bigoplus \text{window}\urcorner$ must not overlap with anything open, which entails

1 With the caveat that *all* can quantifier raise, in contrast with *necessarily*, and *have to* from the previous example. Here the target meaning is of course the narrow scope of the quantifier, revealing its lack of homogeneity.

that *no* window is open. When *some but not all* of the windows are open in w, the plural predication structure is indeterminate in w—neither true nor false. We write $[\![\text{open}(\bigoplus \text{window})]\!](w) = \star$.

The semantics is trivalent. The indeterminate truth value is taken to be the source of *well*-responses, in cases where *yes* and *no* would be infelicitous. In discourse, this third truth value can be resolved to *true enough* in certain contexts, which is the source of the exception tolerance mentioned in Section 3.3 (see Appendix for formal details). Finally, homogeneity removal occurs when a quantifier is introduced and operates on the set of individuals given by the plurality. This is no longer a case of plural predication but of universal quantification, which explains why we no longer observe homogeneity properties and other properties of pluralities.

4.2 Weak necessity modals denote definite pluralities of worlds

Following standard Kratzer modal semantics (Kratzer 1981), we assume a contextually supplied modal base and ordering source, which provides the modal domain D. D is the set of "best" worlds among the modal base, according to the ordering source. To reduce notational clutter, we do not explicitly separate the modal base and ordering source here, and just write D. Following Kratzer, a strong necessity modal is a universal quantifier: it asserts that the prejacent p (type st) is true in all the worlds in that domain.

(25) $\text{must}_D := \lambda p_{\mathtt{st}}.\forall w.D(w) \rightarrow p(w)$

(26) $\ulcorner \text{you must}_D \text{ go} \urcorner = \text{must}_D(\lambda w'.\text{go}(\text{you})(w)) = \forall w.D(w) \rightarrow \text{go}(\text{you})(w)$

We take a weak necessity modal to be non-quantificational, instead it is the sum of all worlds in the modal domain D. We then assume that the prejacent proposition has sums of worlds in its extension, which allows D to satiate the proposition's world argument, as shown in (28).

(27) $\text{should}_D := \bigoplus D$

(28) $[\![\text{you should}_D \text{ go}]\!]^g = [\![\text{go}(\text{you})(\text{should}_D)]\!]^g = [\![\text{go}(\text{you})(\bigoplus D)]\!]^g$

$$= \begin{cases} 1 & [\![\text{go}(\text{you})(w)]\!]^g = 1 \text{ for all } w \in g(D) \\ 0 & [\![\text{go}(\text{you})(w)]\!]^g = 0 \text{ for all } w \in g(D) \\ \star & \text{otherwise} \end{cases}$$

This analysis is based on trivalent semantics for plural predicates and is completely parallel to the case of plural predication of individuals. It therefore immediately gives us the properties discussed in Section 3 common to individual and world pluralities: homogeneity and homogeneity removal, preference for *well*-responses to

indeterminate weak necessity modals, as in (19), and exception tolerance of weak necessity modals (see Appendix for formal details).

5 Deriving weak necessity from strong necessity

As shown in Section 2, in many languages, weak necessity is derived from a strong necessity modal and an additional morpheme, with some differences in the meaning of this morpheme across languages. We propose a compositional analysis that derives weak necessity from strong necessity, and is equipped to capture the differences between languages.

5.1 Picking out a witness set of a quantifier

With an analysis of weak necessity as a plurality of worlds, we can derive it from combining a strong necessity modal with an additional operator X, defined in (29). We appeal to the notion of 'minimal witness set' (inspired by, but not identical to, the analysis of weak necessity in Vander Klok & Hohaus 2020), defined as a minimal set that makes a quantifier true, in (29b).

(29) $X := \lambda M_{<st,st>}.\bigoplus \iota W[W \in WIT(M)]$
where:
a. For a set of sets $\mathscr{S}$, $\iota S[S \in \mathscr{S}]$ picks out the unique set in $\mathscr{S}$ if $\mathscr{S}$ is a singleton, and is undefined otherwise.
b. W is a *minimal witness set* of M iff $W \in M$ and $\neg\exists W' \subset W : W' \in M$.
c. $WIT(M)$ is defined as the set of minimal witnesses for the modal M.

In words, X is an operator that picks out the unique smallest set that makes a quantifier true and takes the mereological sum of its elements. Applied to a universal quantifier, it simply picks out the domain of that quantifier, which is exactly what we need to derive a weak necessity modal, which is a domain of worlds, from a strong necessity modal, which is a universal quantifier over that domain.

Therefore, we apply X to a strong necessity modal, defined in (30a), to derive a plurality of worlds as shown in (30b).

(30) a. $\text{must}_D := \lambda p.\forall w \in D.p(w)$
b. $X(\text{must}_D) = \bigoplus \iota W[W \in WIT(must)] = \bigoplus D$

Thus, X is an operator that allows us to derive a sum of worlds from a universal quantifier. We now show how this operator, or versions of it, predicts the distribution of the weak necessity-forming morpheme in specific languages.

5.2 Cross-linguistic variation in the morpheme deriving weak necessity

Vander Klok & Hohaus 2020 show that the Javanese morpheme NE, which forms weak necessity modals, is only found on necessity modals, but not possibility. This restriction can be seen in the following example (31).

(31) Vander Klok & Hohaus 2020: pg. 2: (3)

a. Aku iso ngelangi.
1SG CIRC.POS AV.swim
'I can swim.'

b. *Aku iso-ne ngelangi.
1SG CIRC.POS-NE AV.swim

Our proposal captures this restriction: X picks out the unique witness set of a quantifier, and returns the plurality associated with that set. Indeed, an existential quantifier has many minimal witness sets, as many as there are elements in its domain. However X is only defined if there is one minimal witness set. Therefore X can apply to universal quantifiers. This captures the observed distribution for Javanese NE.

Additional support for NE in Javanese picking out a unique minimal witness set is that the morpheme NE is also used as a definiteness marker for nominals. This syncretism is found in many related Malayo-Polynesian languages, including Madurese and Indonesian (see Vander Klok & Hohaus (2020) and references therein). We might therefore expect that NE expresses definiteness applying to an individual or sum of individuals, and that the minimal witness set operator is a null morpheme that arises in the case of modals in order to avoid a type clash. We leave investigation of this idea to future work.

In other languages (e.g. French, and many others, see von Fintel & Iatridou (2008)), the morpheme deriving weak necessity is the morpheme used for counterfactual statements, and can also apply to possibility modals, as in the following French examples.

(32) a. Tu devrais partir.
you must.CF leave
You should leave.

b. Tu pourrais partir.
you can.CF leave
You could leave.

The current definition of X does not capture the felicity of counterfactually-marked possibility modals. However, while we defined X to be parallel to definite plural nominals, i.e. defined for a *unique* minimal witness set, definiteness is in fact not a

necessary component of homogeneity in Križ's analysis, and all that is in fact needed for homogeneity is plural reference.

Therefore, in order to capture the typology, we propose that this counterfactual marking in French and analogous languages picks out a witness set, but without the uniqueness requirement (and arguably without the minimality requirement; we leave further refinements to future work).

(33) $\text{CF} := \lambda M_{<st,st>}.\bigoplus W$ for some $W \in WIT(M)$

This allows for the morpheme to apply to both possibility and necessity modals, and still derive homogeneity effects with weak necessity modals. This proposal is in line with Schlenker (2004), who takes counterfactual marking to be a distal demonstrative pointing to a set of worlds.

6 Previous analyses don't capture homogeneity effects

We now go over prominent previous analyses of weak necessity modals and show that none are able to account for the basic homogeneity diagnostic sentence, repeated below in (34), where a negated weak necessity modal is incompatible with an existential claim.

(34) According to the rules, you shouldn't go, # but you can.

We also discuss other properties of weak necessity and how they are better captured with the pluralities of worlds approach rather than alternative analyses.

6.1 Domain restriction

A first prominent analysis of weak necessity is the domain restriction approach, most notably represented by von Fintel & Iatridou 2008. This type of analysis is based in a standard Kratzerian modal framework, where strong necessity modals like *must* are universal quantifiers over the set of best worlds according to an ordering source. Weak necessity modals are also universal quantifiers, but quantify over a subset of what the domain of strong necessity modal would be, by picking out the best worlds according to a second ordering source. In particular, if *must* quantifies over the best accessible worlds according to ordering source g_1, *should* quantifies over the best according to g_2 of the best according to g_1. Without any additional assumptions, this analysis does not capture the basic homogeneity effects: *should* is a universal quantifier, so its negation is wrongly predicted to be compatible with an existential quantifier.

Stipulating homogeneity is not enough. One could argue that the homogeneity of *should* is not a direct result of the presence of a secondary ordering source theory, and that the explanation of *should*'s particular interaction with negation is due to some other property (e.g. a lexical encoding of an excluded middle requirement as other neg-raising predicates, or a requirement to take syntactic wide scope). However, we argue that even with that additional assumption, the particular sentence in (34) poses a problem for the secondary ordering source theory. The adverbial "according to the rules" is taken to supply an ordering source and fix it for both modals present in the sentence. If *should* is taken to be restricted by two ordering sources, does the adverbial supply just one of them, or both of them at once? The latter option seems to be highly problematic for the two ordering source theory of weak necessity, as it would essentially erase the difference between strong and weak necessity.[2] Therefore, the adverbial would only refer to one ordering source, presumably the primary one. Further, it supplies the sole ordering source of the existential modal *allowed*. This entails that *should* quantifies over a strictly smaller set than *allowed*. So, even if *should not* is stipulated to have a strong wide scope reading, it will not be strong enough to contradict *allowed*, because *allowed* can be verified by worlds outside of the modal domain of *should*. In sum, even if the secondary ordering source theory is enriched with an independent mechanism accounting for homogeneity, (34) is not predicted to be infelicitous.

If homogeneity is not stipulated, homogeneity removal data is problematic. We might imagine that there is a way of deriving homogeneity from the addition of a secondary ordering source; this would address the desideratum for a theory of weak necessity to explain why weak necessity modals are always homogeneous. We show however that if that is the case, the domain restriction approach to weak necessity modals would have a hard time accounting for the homogeneity removal data, repeated in (35).

(35) According to the rules, you should not **necessarily** work from home...
✓ but it is allowed.

If the secondary ordering source is what triggers homogeneity, one could stipulate that *necessarily* functions as a modifier that removes the secondary ordering source. But *necessarily* has its weak necessity independent life as a modal quantifier, and

2 This point is related to a more general problem for this theory, since two ordering sources can always be combined into one (Katz, Portner & Rubinstein 2012). The difference between strong and weak necessity would therefore only lie in the number of separate contextually relevant parameters. A simple deontic ordering source like a set of rules introduced by an adverbial is not obviously decomposable, and therefore should not be able to refer to the combination of two ordering sources, at least not without additional contextual support.

it functions as a homogeneity remover in cases having nothing to do with weak necessity modals, as we will show in section 6.4. It is unclear why *necessarily* should function both as a secondary ordering source remover and a homogeneity remover in cases that are far from being analyzed as having something comparable to a secondary ordering source (e.g. conditionals, habituals).

6.2 Proportional quantifier approaches

A second approach is to treat *should* as analogous to *most* (Horn 1989). Under this approach, *should* is to *must* what *most* is to *all*. In other worlds, *should*(p) says that most of the best accessible worlds are p-worlds.

(36) a. $[\![$You must go$]\!] =$ *In all of the best accessible worlds, you go.*
b. $[\![$You should go$]\!] =$ *In most of the best accessible worlds, you go.*

In this analysis, the negation of *should* should again be compatible with an existential quantifier (as shown by a felicitous continuation of the paraphrase in (36b) *but in some, you don't*). This analysis therefore does not capture *should*'s homogeneity.

Furthermore, one advantage of this approach, at least at first glance, is that it appears to be able to capture the exception tolerance of *should*. However, it does not allow the degree of exception-tolerance to depend on the QUD. Instead, it depends on the exact proportion of p-worlds to $\neg p$-worlds in an abstract modal space, and it is unclear how a proportional quantifier could change its meaning relative to the QUD.

6.3 Degree-based approaches

Finally, we find a class of analyses that can be qualified as degree-based, found in Lassiter 2011, 2017; Portner & Rubinstein 2016. Under these approaches, weak necessity modals introduce a probability (for epistemics) or utility (for priority modals) function μ over propositions, whose result is then compared to some contextually-supplied standard d.

(37) $[\![$should p$]\!]$ iff $\mu(p) > d$

Again, such an analysis does not capture homogeneity effects: the negation of *should* as defined in (37) is compatible with an existential quantifier, as long as $d \neq 0$. The defender of the degree-based approach could push back on this. For example, the value of the contextual standard could shift under negation. We do not find this response entirely implausible. However we leave its development to future work on

this topic.

6.4 Weak necessity and its friends

The analyses above make weak necessity modals seem special or unique. However, we would like to suggest the homogeneity pattern observed with weak necessity modals is part of a very general phenomenon in natural language beyond modal verbs and plural definite DPs, and is shared with bare conditionals, generics, habituals, and frequency adverbs. Schlenker 2004 studies exception-tolerance in *if*-clauses and temporal *when*-clauses, analyzing both as plural definites. For further developments of Schlenker's hypothesis, see Križ 2015: Ch. 7. Agha 2021 argues that habituals also exhibit homogeneity effects characteristic of pluralities.

(38) Future conditionals
   a. It's not true that I'll be happy if I get a sodastream... # but I might be.
   b. It's not **necessarily** true that I'll be happy if I get a sodastream... ✓ but I might be.

(39) Bare past conditionals
   a. The ice cream was not spared if the fridge was left open... # but it might be have been.
   b. The ice cream was not **necessarily** spared if the fridge was left open... ✓ but it might have been.

(40) Generics
   a. Ticks don't carry lyme disease... # but some of them do.
   b. Ticks don't **necessarily/all/always** carry lyme disease... ✓ but some of them do.

(41) Habituals (bare)
   a. I don't smoke... # but I do sometimes.
   b. I don't **necessarily/always** smoke... ✓ but I do sometimes.

(42) Habituals (with frequency adverbs lacking quantifiers)
   a. The postal workers don't come on Sunday... # but they do sometimes.
   b. The postal workers don't **necessarily/always** come on Sunday... ✓ but they do sometimes.
   c. The postal workers don't come on **every** Sunday... ✓ but they do sometimes.

All of these constructions show signs of homogeneity. By hypothesis, all of them involve plural predication, not universal quantification. In all these cases,

explicit quantifiers "remove" homogeneity, allowing non-contradictory followups. And all of these constructions tolerate exceptions depending on what is relevant in the discourse context. The apparent generality of this pattern provides support for a non-stipulative unified analysis of all these different phenomena: one based on homogeneous pluralities, straightforwardly applicable to different semantic types, is such an analysis. In contrast, it is unclear whether the analyses cited above for weak necessity would be appropriate to the other phenomena under consideration.

7 Conclusion

Weak necessity modals show behavior parallel to plural nominals, and should receive a parallel analysis. On the basis of homogeneity diagnostics shown above, we conclude that weak necessity modals denote pluralities of worlds, in contrast to strong necessity modals, which are genuine quantifiers. Our analysis uniquely captures homogeneity and homogeneity removal. It also provides an account of exception tolerance, which can be favorably compared to previous theories that address exceptions. Finally, we provide the most explicit compositional account of the derivation of weak necessity from strong necessity, which can capture cross-linguistic systematicities, as well as variations.

Appendix 1: The analysis of exception tolerance for plurals

Weak necessity modals express modal generalizations that are more flexible than those of strong necessity modals. We analyze the exception tolerance of weak necessity modals as a special case of exception tolerance in plural predication structures, as in (43).

(43) *Context: The house has five windows. All are closed except for one on the second floor.*
The alarm system will go off if any windows are left open.
QUD1: *Will we get cold?*
QUD2: *Will the alarm go off?*
The windows are open.

In this context, the plural predication structure $[\![\text{open}(\bigoplus \text{window})]\!](w) = \star$, i.e. it is indeterminate. Indeterminate sentences can be repaired to *true enough* if the exceptions are irrelevant to the QUD. Formally, "the exceptions are irrelevant" means that every world w where $[\![\text{open}(\bigoplus \text{window})]\!](w)$ equals $\star$ is **contextually equivalent** to a world u where $[\![\text{open}(\bigoplus \text{window})]\!](u) = 1$. Equivalence is determined by Issues, which are partitions over the set of possible worlds.

(44) **Sufficient Truth (ST)** (based on Križ 2016)
We write $u \simeq_I v$ iff u and v are in the same cell of an issue I. A sentence S is **true enough** in world w with respect to I iff
(i) S is not false in w, and ($[\![S]\!](w) \neq 0$)
(ii) there is some I-equivalent world u where S is true at u. ($w \simeq_I u$ and $[\![S]\!](u) = 1$)

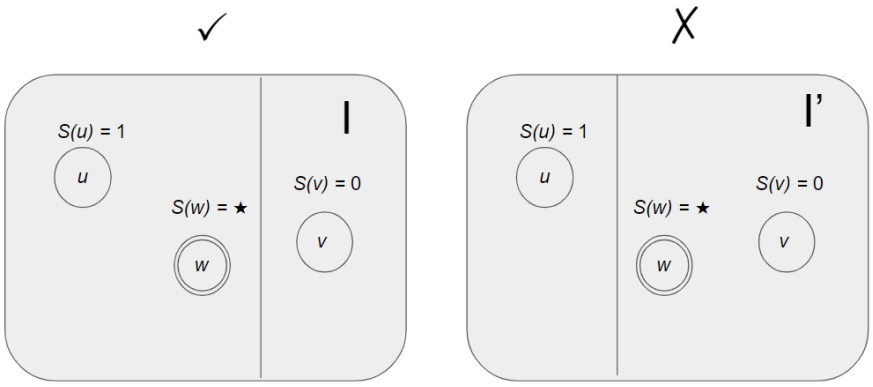

Figure 1 S is true enough at w relative to issue I, but not issue I'.

Križ (2016) weakens the Maxim of Quality to require Sufficient Truth, rather than strict truth.

(45) **Weak Maxim of Quality** (Križ 2016)
Say only sentences which you believe to be true enough.

Moreover, assertions must be both true enough and relevant, in the sense that they Address the Current Issue. (This can be seen as a consequence of Grice's Maxim of Relation.)

(46) **Addressing an Issue** (rephrased from Križ 2016)
A sentence S cannot be used to address an issue I if any cell $i \in I$ overlaps with both the positive and the negative extension of S. In other words, no cell $i \in I$ is such that S is true in some worlds in i and false in others.

Appendix 2: Exception-tolerant modals

It is straightforward to extend this notion to worlds. We make the following assumptions:

i. Every modal comes with a variable that points to its modal domain (type `st`). The modal denotes the sum of its set.

ii. All verbs take world arguments, which can be saturated by weak necessity modals, *if*-clauses, or variables.

iii. A verb can be homogeneous in its world argument, similar to how the verb *open* was homogeneous in its individual argument.

Contrasts between weak and strong necessity modals, like the one in (47), have been discussed before in the literature (e.g. von Fintel & Iatridou 2008: 127).

(47) *Context: To get to 10 Washington Place from Astoria, you can take the N or the R, but the N is faster.*
QUD: Which train is best?
a. To get to 10WP from Astoria, you should take the N.
b. #To get to 10WP from Astoria, you have to take the N.

von Fintel & Iatridou (2008) argues that (47) should be explained by adding a secondary ordering source to *should*, so that its domain of quantification is smaller than that of strong necessity modals. On our account, (47a) is acceptable, though not literally true. The plural-denoting expression *should* picks out the same set of worlds as *has to*, but *should*, being a plural definite, is exception-tolerant.

In the evaluation world w, the N is faster than the R, but you can take either. Consider another world u, where only the N train works. In this world, the N is still better than the R, but the R is not an option. If the QUD is *Which train is best?* then the worlds w and u are equivalent, since the N is best in both worlds.

Strong necessity modals are not QUD sensitive in this way, since they are universal quantifiers. Notice that in (47), taking the N and taking the R are assumed to be mutually exclusive. This is not an essential feature of modal exception-tolerance. Example (48) has a different structure.

(48) *C: One can get a perfect grade by doing most exercises; doing all increases the chances.*
QUD1: How do I increase my chances to get a perfect grade?
QUD2: What are the minimal requirements to get a perfect grade?
a. To get a perfect grade, you should do every exercise. (QUD1: ✓; QUD2: #)
b. To get a perfect grade, you have to do every exercise. (QUD1: #; QUD2: #)

In (48), the "exceptions" are the worlds where you do most (but not all) of the

exercises, and still get a perfect grade. Under QUD1, the exception-worlds are equivalent to the worlds where all exercises are completed, but this equivalence does not hold under QUD2.

Notice that in example (48), there is a required action (doing most exercises) and a preferred action (doing all), and the preferred action entails performing the required one—they are not mutually exclusive. The weak necessity modal can be felicitously used to highlight the preferred action, but the SN modal cannot.

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