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Measurements from *per* without complex dimensions*

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Abstract To what extent is the compositional structure of quantity terms in natural language aligned with the structure of the quantity calculus commonly used in scientific practice, a calculus that critically relies on mathematical operations like division and the computation of quotients? In pioneering work, Coppock (2021) addresses this general question through a case study on the English preposition *per*, as in *0.9 grams per milliliter*. Coppock proposes that *per* expresses the operation of quantity division, an operation that forms quantities like $0.9\frac{\text{g}}{\text{mL}}$ by using ratios of measurements from different dimensions. Here we show that this "division theory" of *per* makes the wrong prediction with respect to statements about measures of density and concentration. We argue that these types of expressions call for an "anaphoric theory" of *per*. On this analysis, anaphora allows for the composition to invoke multiple measurements in basic dimensions, creating the appearance of reference to complex quantities like $0.9\frac{\text{g}}{\text{mL}}$, even though no such quantities are actually composed nor denoted in the formal semantics.

Keywords: measurement, *per*, semantic composition, quantity calculus, quantity division

1 Introduction

Natural language offers a rich inventory of lexical items and constructions that can convey information about measurement (e.g., Lønning 1987; Krifka 1989; Schwarzschild 2002; Nakanishi 2007; Rothstein 2009, 2011; Partee & Borschev 2012). Similarly, the formal sciences have developed an intricate and precise calculus of quantities for the purposes of forming and evaluating scientific theories. In this paper, we ask to what extent lexical meanings and semantic composition in natural language are aligned with the conceptualization of measurement in science, the subject matter of measurement theory (e.g., Krantz, Luce, Suppes & Tversky 1971).

We will address a particular instance of this question emerging from Coppock (2021), regarding the meaning of the English preposition *per* and its relation to *quo-*

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tient quantities, a critical innovation within the quantity calculus (see the discussions in de Boer 1995, Raposo 2018, Roche 1998).

At their core, all measurements relate entities to quantities along a certain dimension. For example, one could measure a sample of oil in terms of its volume, yielding a specific quantity such as 10mL (ten millilitres). Alternatively, one could measure the same sample in terms of its weight/mass, yielding a quantity such as 9g (nine grams). In some ways, we can think of the dimensions of volume (VOL) and weight (WT) as *basic* in the sense that they are associated with simple (mono-morphemic) terms that represent quantities along these dimensions (e.g., *grams*, *pounds*, *litres*, *ounces*, etc.). However, the quantity calculus used in scientific practice not only permits such simplex measures, but also provides a compositional mechanism to build complex dimensions. For example, consider the sample of oil mentioned above, whose weight is 9g and whose volume is 10mL. Given these measurements along these basic dimensions, the quantity calculus permits a measure of density, namely $0.9\frac{\text{g}}{\text{mL}}$, a quantity within the dimension $\frac{\text{WT}}{\text{VOL}}$. To identify such complex measures, the quantity calculus uses the ratio between two separate measurements to establish an equivalence class. For example, $1\frac{\text{g}}{\text{mL}}$ represents an equivalence class that pairs weight quantities with volume quantities such that the number of grams constituting the weight quantity is in a one-to-one correspondence to the number of millilitres constituting the volume quantity. Similarly, $2\frac{\text{g}}{\text{mL}}$ represents an equivalence class of similar pairings, but where the number of grams is double the number of millilitres. $0.9\frac{\text{g}}{\text{mL}}$, which is equivalent to $\frac{9\text{g}}{10\text{mL}}$, represents an equivalence class where the ratio of grams to millilitres is nine to ten. The resulting algebra of equivalence-classes based on these ratios has many of the same characteristics as an algebra of rational numbers and division more generally (hence the use of the dividing line, see Raposo 2018 for a more thorough discussion). For this reason, we will call the operation that forms these complex measurements *quantity division*.

Although quantity division is a well-established operation within measurement theory, it is an open question whether similar compositional mechanisms are at play in natural language. In pioneering work, Coppock (2021) proposes that this is indeed the case. Specifically, Coppock argues that quantity division is encoded in the meaning of *per* in phrases such as *0.9 grams per millilitre*, the most straightforward English translation of the scientific quantity $0.9\frac{\text{g}}{\text{mL}}$.

In this paper, we will argue that *per* cannot be used to form quotient quantities as Coppock suggests and that the division theory more generally is on the wrong track. We will explore two relevant syntactic environments to make our case. One environment is illustrated by the sentence *The sample weighs 0.9 grams per milliliter*, where a measure phrase containing *per* is the argument of a so-called measurement verb (e.g., *weigh*). The other environment is illustrated by the sentence *The sample contains 0.1 grams of salt per milliliter*, where *per* is used as a modifier within a so-called pseudo-partitive (e.g., *grams of salt*) that, in turn, is the object of a verb of inclusion (e.g., *contain*). Although such sentences convey information about higher order dimensions such as density or concentration, we will argue that they do not do so through quantity division (e.g., there is no composition of *grams* and *milliliter* into the quotient $\frac{\text{g}}{\text{mL}}$). The observed truth conditions, we argue, instead arise by virtue of *per* being covertly anaphoric. We propose that this "anaphoric theory" of *per* enables a compositional derivation where the appearance of reference to a complex dimension is actually due to two separate measurements in basic dimensions.

Our discussion begins with a review of Coppock's division theory.

2 The division theory

Coppock's (2021) division theory of *per* posits the lexical entry in (1), where *r* and *q* range over quantities. Effectively, then, this theory assumes that *per* denotes a function that successively inputs two quantities and creates a complex measurement through the operation of quantity division, essentially forming an equivalence class determined by the resulting ratio.

(1) $[\![\text{per}]\!] = \lambda \text{r}.\ \lambda \text{q}.\ \frac{\text{q}}{\text{r}}$

In the quantity calculus, units like g and mL represent particular quantities in their respective dimensions, namely unit quantities. Thus, g and mL are specific quantities of weight and volume, and 2g and 2mL are quantities that are twice as large. Coppock makes the natural assumption that such unit quantities serve as the denotations of the corresponding unit nouns. So *gram(s)* and *milliliter(s)* are assumed to denote g and mL, respectively.

Given (1) and this reasonable assumption regarding unit quantities, the prepositional phrase *per milliliter* would denote $\lambda \text{q}.\frac{\text{q}}{\text{mL}}$, a function that takes a basic quantity and maps it to a complex quantity. Furthermore, if the sequence *grams per milliliter* forms a constituent, as Coppock suggests, it would denote the quotient $\frac{\text{g}}{\text{mL}}$, a complex unit quantity in the dimension $\frac{\text{WT}}{\text{VOL}}$.

Regarding numerals like 0.9, Coppock assumes that they invoke a multiplication operation $*$, which maps a number and a quantity to another quantity. As shown in (2) (omitting $*$ for readability), the entire measure phrase *0.9 grams per milliliter* would then denote $0.9\frac{\text{g}}{\text{mL}}$, again a complex quantity in the dimension $\frac{\text{WT}}{\text{VOL}}$.

(2) $[\![\text{0.9 grams per milliliter}]\!] = 0.9\frac{\text{g}}{\text{mL}}$

Coppock demonstrates that this analysis of *per* supports the derivation of plausible truth conditions for a variety of different sentences. However, there are syntactic environments where the division theory makes the wrong prediction. To illustrate such environments, we begin with a discussion of measurement verbs.

3 Undergeneration challenge: measurement verbs

English features a class of transitive verbs (*measurement verbs*) whose complement position can be filled by a measure phrase (see, e.g., Kotek 2011). The verb *weigh* is a case in point, as demonstrated in (3).

(3) The sample weighs 0.9 grams.

Cases like (3) are amenable to a straightforward compositional analysis. Let μ_{D} be a measure function that maps entities to quantities in dimension D. For example μ_{WT} is a function that maps any input to its weight, a quantity in the dimension WT. With this notation in mind, plausible truth conditions for (3) are given in (4), where the sample's weight is equated with the quantity 0.9g. Assuming that the measure phrase *0.9 grams* denotes the quantity 0.9g, these truth conditions follow immediately from the lexical entry for *weigh* given in (5).[1]

(4) $\mu_{\text{WT}}(\text{the sample}) = 0.9\text{g}$

(5) $[\![\text{weigh}]\!] = \lambda \text{q}.\ \lambda \text{x}.\ \mu_{\text{WT}}(\text{x}) = \text{q}$

With this background in place, consider now the sentence in (6), which differs from (3) merely in the fact that the complement of *weigh* is the measure phrase *0.9 grams per milliliter*, rather than just *0.9 grams*.

(6) The sample weighs 0.9 grams per milliliter.

The division theory of *per*, committed to the equality in (2), assigns to the complement of *weigh* in (6) the denotation $0.9\frac{\text{g}}{\text{mL}}$. Assuming the entry for *weigh* in (5), the division theory accordingly delivers for (6) the truth conditions in (7).

(7) $\mu_{\text{WT}}(\text{the sample}) = 0.9\frac{\text{g}}{\text{mL}}$

But these truth conditions are obviously inadequate. After all, the function μ_{WT} outputs quantities in the dimension WT. Given that $0.9\frac{\text{g}}{\text{mL}}$ is a quantity in the dimension $\frac{\text{WT}}{\text{VOL}}$, the truth conditions in (7) are contradictory.

Thus as matters stand, the division theory undergenerates in that it fails to deliver adequate truth conditions for cases where a *per*-PP modifies a measurement verb's

1 It might be more adequate for '=' in (4) and (5) to be replaced with '≥', hence for the conventional meaning of *weigh* to only demand that the subject entity weigh *at least* the amount specified in the object. The intuited *exact* meaning would then arise from a strengthening external to the predication, e.g., from pragmatic reasoning. However, the issue of *exact* vs. *at least* meanings is immaterial for the present purposes, and we will maintain the lexical *exact* semantics in the exposition.

complement. To accommodate such cases, the division theory would need to be supplemented with the assumption that measurement verbs are polysemous. In addition to the meaning in (5), *weigh* would be required to permit the meaning in (8), where $\mu_{\frac{\text{WT}}{\text{VOL}}}$ has been substituted for μ_{WT}.

(8) $[\![\text{weigh}]\!] = \lambda q.\ \lambda x.\ \mu_{\frac{\text{WT}}{\text{VOL}}}(x) = q$

This additional meaning for *weigh* would allow for sentence (6) to be assigned the truth conditions in (9), a non-contradictory proposition that states that the complex quantity $0.9\frac{\text{g}}{\text{mL}}$ is equivalent to the measure of the sample's density. At first sight, at least, this captures what sentence (6) is intuited to convey.

(9) $\mu_{\frac{\text{WT}}{\text{VOL}}}(\text{the sample}) = 0.9\frac{\text{g}}{\text{mL}}$

However, we will now argue that this is not the correct analysis of examples like (6). In certain cases, the entry in (8) predicts truth conditions that, while contingent and potentially usable, are nevertheless unattested. Consider the sentence in (10).

(10) This cube weighs what that cube weighs.

This sentence differs from those in (3) and (6) in that the complement of *weigh* in the matrix, *what that cube weighs*, is a free relative clause. This free relative hosts its own occurrence of *weigh* whose complement position that has been relativized, as sketched in (11), leaving a gap that is bound by the wh-phrase *what*.

(11) what_1 [that cubes weighs t_1]

Sentence (10) is naturally understood as stating that this cube has the same weight as that cube—i.e., the truth conditions stated in (12).

(12) $\mu_{\text{WT}}(\text{this cube}) = \mu_{\text{WT}}(\text{that cube})$

That this meaning can be intuited for (10) is in fact unsurprising. As discussed in Jacobson (1995) and Rullmann (1995), free relative clauses are often interpreted as a type of definite description, with the relativized predicates serving the same role as an NP restrictor. Given this, and assuming the basic meaning in (5) for the embedded occurrence of *weigh*, the free relative clause in (11) is expected to refer to that cube's weight, that is, the quantity $\mu_{\text{WT}}(\text{that cube})$. If the matrix occurrence of *weigh* is likewise interpreted as in (5), the truth conditions in (12) fall out immediately.

However, if *weigh* is taken to be polysemous, as required under the division theory, then (12) is not the only meaning predicted for (10). Given the lexical entry for *weigh* in (8), one could derive the alternative truth conditions for (10) in (13), which is almost identical to (12) except that $\mu_{\frac{\text{WT}}{\text{VOL}}}$ takes the place of μ_{WT}. In other words, sentence (10) is predicted to permit a meaning which states that this cube and that cube have the same measure of density.

(13) $\mu_{\frac{\text{WT}}{\text{VOL}}}(\text{this cube}) = \mu_{\frac{\text{WT}}{\text{VOL}}}(\text{that cube})$

It seems clear, however, that sentence (10) cannot actually be understood as conveying the meaning in (13). One way of confirming this is to imagine that "this cube" weighs 1kg while "that cube" weighs 2kg. Intuitively, this assumption is incompatible with (10) being true. Of course, this should not be so if (13) were a possible meaning of (10), since a 1kg cube may well have the same density as a 2kg cube. Intuitions about example (10), then, militate against the assumption that *weigh* can have the meaning in (8).

Cases with free relatives are just one type of example that illustrate the implausibility of the lexical interpretation in (8). The crucial feature of (10), setting it apart from (3) and (6), is that the complement of *weigh* does not constrain the dimension of measurement, and hence nothing should prevent *weigh* from being interpreted via (8) if indeed this were possible in principle. This feature of (10) is shared by the comparative sentence in (14a) and the wh-interrogative in (14b). In line with the intuitions regarding (10), the sentences in (14) can only be understood as comparing, or asking for, measures of absolute weight, not measures of density.

(14) a. This cube weighs more than that cube weighs.
b. What does this cube weigh?

We conclude that measurement verbs do not in general give rise to meanings that can be construed as reporting on measurements in complex dimensions like $\frac{\text{WT}}{\text{VOL}}$. Such meanings only arise in the presence of a suitable measure phrase, like *0.9 grams per milliliter*. We take this observation to teach us that measurement verbs do not have meanings like the one given for *weigh* in (8).

Hence the division theory faces a problem of undergeneration. With entries like (8) excluded, the theory does not provide a compositional interpretation for *any* measure phrases with *per* in the complement position of a measurement verb. How, then, are examples (6) to be analyzed? In the next section, we offer an answer to this question, one that critically depends on an anaphoric interpretation of *per*.

4 The anaphoric theory

In addition to permitting a division between quantities in different dimensions, the quantity calculus used in science also posits a notion of division that operates on two quantities within the *same* dimension. This operation doesn't output complex quantities but rather pure numbers. For example, the quantity calculus equates the quotient $\frac{9\text{mL}}{\text{mL}}$ with the number 9 and the quotient $\frac{9\text{g}}{10\text{g}}$ with the number 0.9. Within the quantity calculus, pure numbers can be thought of as a particular kind of quantity, viz. quantities in the *identity* dimension, ID.

Adopting this assumption, we propose that *per* derives measure functions that map input entities to quantities in the ID dimension, that is, to pure numbers. To illustrate, we propose that *per milliliter* denotes the function in (15). This function maps an input to the number that constitutes its volume in milliliters. A 9mL sample of oil, for example, would map to the number 9.

(15) $[\![\text{per milliliter}]\!] = \lambda\text{x}.\ \frac{\mu_{\text{vol}}(\text{x})}{\text{mL}}$

Extrapolating from (15), we propose the lexical entry for *per* in (16). Here dim(q) is the dimension of the quantity q. So according to (16), *per* denotes a function that maps an input quantity to a measure function, which for any input entity returns a number, viz. the number obtained by measuring the entity in the input quantity's dimension and dividing it by that same quantity.

(16) $[\![\text{per}]\!] = \lambda\text{q}.\ \lambda\text{x}.\ \frac{\mu_{\text{dim(q)}}(\text{x})}{\text{q}}$

Given this entry, semantic composition in a measure phrase like *0.9 grams per milliliter* requires the presence of an expression that denotes the second input to the function denoted by *per*, that is, the entity being measured. We therefore assume that, as sketched in (17), the *per*-PP hosts a silent pronoun *pro* in its specifier position.[2]

(17) $[_{\text{MP}}\ [_{\text{MP}}$ 0.9 grams] $[_{\text{PP}}$ pro [per milliliter]]]

As a result, the *per*-PP in (17) will then denote a number, viz. the volume of this entity denoted by *pro* in milliliters, $\frac{\mu_{\text{vol}}([\![\text{pro}]\!])}{\text{mL}}$.

The compositional analysis of the other elements in (17) only require one other assumption, namely that number terms combine with quantity denoting terms via the multiplication operation $*$. We share this assumption with Coppock's (2021) rendition of the division theory, reviewed in Section 2. In our analysis, however, multiplication applies twice. It applies in the composition of *0.9 grams*, yielding 0.9 $*$ g, which in a second step of multiplication further combines with the *pure* number

2 The parse in (17) manifests another departure from the specifics of the proposal in Coppock (2021). While Coppock assumed that in *0.9 grams per milliliter*, *per* merges with the two unit nouns to form the constituent *grams per milliliter*, we instead take the *per*-PP to modify an entire measure phrase, here *0.9 grams*, to form a larger measure phrase, here *0.9 grams per milliliters*. Such a parse is more consistent with syntactic evidence, including wh-movement data like *what the sample weighs per milliliter*, and so we opt for this parse in our exposition. However, the issue is orthogonal to present concerns, since due to associativity of $*$, (17) and the alternative structure $[_{MP}$ *0.9 [grams* $[_{PP}$ *pro [per milliliter]]]]* wind up being equivalent, both denoting $0.9 * \text{g} * \frac{\mu_{\text{vol}}([\![\text{pro}]\!])}{\text{ml}}$.

denoted by the *per*-PP. The resulting the denotation is stated in (18).

(18) $[\![[_{\text{MP}} [_{\text{MP}} \text{0.9 grams}] [_{\text{PP}} \text{pro [per milliliter]]]}]\!] = (0.9 * \text{g}) * \frac{\mu_{\text{vol}}([\![\text{pro}]\!])}{\text{ml}}$

Quantity calculus construes the product operation $*$ as commutative and associative. Under this unobjectionable assumption, and dropping one occurrence of $*$ for readability, (18) can be restated equivalently as in (19). So the measure phrase in (17) as a whole will denote the quantity α g, where α is 0.9 times the volume in milliliters of the denotation of *pro*.

(19) $[\![[_{\text{MP}} [_{\text{MP}} \text{0.9 grams}] [_{\text{PP}} \text{pro [per milliliter]]]}]\!] = (0.9 * \frac{\mu_{\text{vol}}([\![\text{pro}]\!])}{\text{ml}})\ \text{g}$

We are now ready to return to data that in Section 3 that we presented as a problem of undergeneration for the division theory. As we will see, the anaphoric theory offers a straightforward solution to this problem.

5 Challenge from measurement verbs met

Let's reconsider sentences with measurement verbs under the anaphoric theory, such as (6), repeated below. Such a theory would parse (6) as in (20), where critically there is a silent pronoun in the specifier of the *per*-PP.

(6) The sample weighs 0.9 grams per milliliter.

(20) [the sample] weighs $[_{\text{MP}}$ $[_{\text{MP}}$ 0.9 grams] $[_{\text{PP}}$ pro [per milliliter]]]

Now, given the equality in (19), and assuming the basic entry for *weigh* in (5), repeated below, semantic composition will deliver the meaning in (21).

(5) $[\![\text{weigh}]\!] = \lambda \text{q}. \lambda \text{x}. \mu_{\text{WT}}(\text{x}) = \text{q}$

(21) $\mu_{\text{WT}}(\text{the sample}) = (0.9 * \frac{\mu_{\text{vol}}([\![\text{pro}]\!])}{\text{ml}})\ \text{g}$

The truth conditions in (21) depend on the value assigned to *pro*. In (20), it is expected that *pro* can get its denotation from the subject *the sample*. The truth conditions derived for sentence (6) can then be restated as in (22). On this analysis, then, (6) conveys that the sample has a weight of α g, where α is 0.9 times the number of milliliters that represents the sample's volume.

(22) $\mu_{\text{WT}}(\text{the sample}) = (0.9 * \frac{\mu_{\text{vol}}(\text{the sample})}{\text{ml}})\ \text{g}$

The quantity calculus used in science treats (22) as equivalent to (9), repeated below, the truth conditions derived for (6) under the division theory of *per*. The anaphoric theory detailed here, then, offers an alternative compositional path to the very same meaning derived under the division theory—with some key differences. Under the division theory, *weigh* measures in the density dimension $\frac{\text{WT}}{\text{VOL}}$ and its complement denotes a quotient quantity. Crucially, neither is true under the anaphoric theory, where *weigh* measures in the weight dimension WT and its complement denotes a quantity of weight.

(9) $\mu_{\frac{\text{WT}}{\text{VOL}}}(\text{the sample}) = 0.9\frac{\text{g}}{\text{ml}}$

This is why the anaphoric theory immediately solves the undergeneration challenge for the division theory identified in Section 3, the challenge from data with free relatives and other cases without measure phrases. We saw that those data are in conflict with the assumption that *weigh* can measure in the dimension $\frac{\text{WT}}{\text{VOL}}$. On the anaphoric theory, such data are unproblematic precisely because the theory is compatible with *weigh* measuring in the basic WT dimension only.

In sum, we conclude that the anaphoric theory of *per* offers a theoretically parsimonious account of intuitions about data where measure phrases with *per* serve as the complement of a measurement verb like *weigh*.

6 Overgeneration challenge: pseudo-partitives

In Section 3, we demonstrated that the division theory failed to deliver adequate truth conditions for cases with measurement verbs. However, this finding does not rule out the possibility that the division theory might yield the best analysis for other instances of *per*. One might propose that *per* is ambiguous, having one meaning that computes a quotient quantity via quantity division and another that yields a basic measurement via anaphora and pure-number multiplication. In this section we present an argument that *per* never denotes quantity division. We show that such an interpretation overgenerates, deriving meanings that are not actually attested.

Our argument critically relies on data where measure phrases appear within pseudo-partitive constructions that serve as an object of a verb of inclusion. To begin, consider the sentences in (23), where the complements of *contain* are the pseudo-partitives *0.9 grams of salt* and *0.8 milliliters of salt*.

(23) a. The mixture contains 0.9 grams of salt.
b. The mixture contains 0.8 milliliters of salt.

As discussed by Schwarzschild (2006), the measure terms in such constructions restrict the denotation of the NP, in this case *salt*. Thus, *0.9 grams of salt* denotes the set of quantities of salt that measure 0.9g and similarly *0.8 milliliters of salt* denotes the set of quantities of salt that measure 0.8mL. To implement such a restriction, one needs to employ a measure function, however as demonstrated by the examples in (23), this measure function is underspecified—(23a) requires a measurement of weight whereas (23b) requires one of volume.

For the purposes of our argument, the details of how to semantically encode this type of restriction and underspecification are not critical. However, for the sake of concreteness, we will adopt the analysis proposed by Nakanishi (2007) where a covert element MUCH mediates the relationship between the measure term and the noun phrase. Under such an analysis, the sentences in (23) would have the syntactic structures in (24), with the interpretation of MUCH given in (25).

(24) a. The mixture contains [$_{DP}$ ∃ [$_{AP}$ [$_{MP}$ 0.9 grams] MUCH] of salt]
b. The mixture contains [$_{DP}$ ∃ [$_{AP}$ [$_{MP}$ 0.8 milliliters] MUCH] of salt]

(25) $[\![\text{MUCH}]\!] = \lambda q.\ \lambda x.\ \mu(x) = q$

Critically, μ in (25) represents an underspecified measure function. Given these structures and interpretation of MUCH, the AP in (24a) would denote $\lambda x.\ \mu(x) = 0.9g$ whereas the one in (24b) would denote $\lambda x.\ \mu(x) = 0.8mL$. In the end, assuming an existential interpretation of the pseudo-partitive DP (which the structures in (24) for concreteness attribute to a silent determiner ∃), one would derive the plausible truth conditions for the sentences in (23) that are given in (26).

(26) a. $\exists x[\text{salt}(x) \wedge \mu(x) = 0.9g \wedge \text{contain(the mixture, x)}]$
b. $\exists x[\text{salt}(x) \wedge \mu(x) = 0.8mL \wedge \text{contain(the mixture, x)}]$

Such truth conditions capture speaker intuitions as long as μ is identified with μ_{WT} in (26a) and with μ_{VOL} in (26b).

With this background about pseudo-partitives in place, consider the sentence in (27), which is equivalent to the sentences in (23), but where the complex measure phrase *0.1 grams per milliliter* has replaced the more basic measure terms.

(27) The mixture contains 0.1 grams per milliliter of salt.

In a more colloquial rendition, the *per*-PP may instead appear at the sentence's right edge, as in *The mixture contains 0.1 grams of salt per milliliter*. Such a variation is not surprising given that PPs are known to routinely undergo extraposition while being interpreted as though they had not (e.g., Büring & Hartmann 1995). Critically, semantic composition under either the division theory or the anaphoric theory requires that, regardless of its surface position, the *per*-PP at logical form merge with an expression that denotes a quantity, as in the syntactic structure in (28).

(28) [the mixt.] contains [$_{DP}$ ∃ [$_{AP}$ [$_{MP}$ 0.1 grams [per millil.]] MUCH] of salt]

Recall now that under the division theory, the measure phrase argument of MUCH in (28) denotes the quotient quantity $0.1\frac{\text{g}}{\text{mL}}$. Therefore, in parallel to the analysis of the baseline in (23), the division theory assigns to (28) the truth conditions in (29).

(29) $\exists \text{x}[\text{salt(x)} \wedge \mu(\text{x}) = 0.1\frac{\text{g}}{\text{mL}} \wedge \text{contain(the mixture, x)}]$

Does (29) represent the actually perceived meaning of sentence (27)? The answer depends on the setting of μ. If μ were set to the measure function in (30), then (29) would express the proposition in (31), which under the quantity calculus is equivalent to (32)

(30) $\mu := \lambda \text{x}.\ \frac{\mu_{\text{WT}}(\text{x})}{\mu_{\text{VOL}}(\text{the mixture})}$

(31) $\exists \text{x}[\text{salt(x)} \wedge \frac{\mu_{\text{WT}}(\text{x})}{\mu_{\text{VOL}}(\text{the mixture})} = 0.1\frac{\text{g}}{\text{mL}} \wedge \text{contain(the mixture, x)}]$

(32) $\exists \text{x}[\text{salt(x)} \wedge \mu_{\text{WT}}(\text{x}) = (0.1 * \frac{\mu_{\text{vol}}(\text{the mixture})}{\text{ml}})\ \text{g} \wedge \text{contain(the mixture, x)}]$

The resulting truth conditions specify that the sentence is true if and only if the mixture contains an aggregate of salt whose weight in grams is 0.1 times the mixture's volume in milliliters. On the surface, such truth conditions seem to capture what the sentence in (27) conveys.

Note that the proposed setting in (30) implies that MUCH can be anaphoric, referencing in its denominator a measure of the referent of the subject DP *the mixture*. While such an anaphoric setting of an underspecified measure function may seem unexpected, there is independent evidence for its availability. Bale & Schwarz (2019) posit such settings for quantity constructions featuring overt *much*, such as the question in (33) (see also Bale, Schwarz & Shanks 2021).

(33) How much salt does the mixture contain?

Although this question can be read as asking for the salt's absolute weight or volume, it can also be read as asking for the degree concentration of salt in the mixture—a reading that can be brought to the fore by appending "proportionally speaking" at the beginning or end of the question. Correspondingly, although (34) can be taken to convey that Amy knows the salt's absolute weight or volume, it can also be understood to assert that Amy knows the degree concentration of salt in the mixture. Once again, such a reading becomes more salient when "proportionlly speaking" is added to the beginning or end of the statement.

(34) Amy knows how much salt the mixture contains.

Building on Cresswell (1976) and Bale & Barner (2009), Bale & Schwarz conclude from such data that *much*, like its covert counterpart, introduces an underspecified measure function. Accordingly, the lexical entry for covert MUCH in (25) can be applied to overt *much* as well. The question in (33) then has the interpretation rendered informally in (35), where the measure function μ is again underspecified. The range of readings that (33) permits then indicates that μ can be valued not only by basic measure functions like μ_{WT} and μ_{VOL}, but also by anaphorically determined measure functions that output quotients, like the one in (30).

(35) for which q: $\exists$x[salt(x) $\wedge$ μ(x) = q $\wedge$ contain(the mixture, x)]

We take this independent evidence for the viability of (30) to confirm that the division theory can assign to sentence (27) the truth conditions in (31). This establishes that the division theory has the benefit of capturing the intuition that sentences like (27) report on the concentration of one substance in another.

However, although the division theory seems to get the correct truth conditions for pseudo partitives such as (27) at first blush, closer examination of such constructions reveals some problems. To begin, note that in the quantity calculus, quantity division can map different pairs of quantities to the same output. For example, given that kg = 1000g and L = 1000mL, the quantity calculus necessarily equates the quotient quantities $\frac{\text{g}}{\text{mL}}$ and $\frac{\text{kg}}{\text{L}}$. Since under the division theory, *0.1 kilograms per liter* can denote $0.1\frac{\text{kg}}{\text{L}}$, the division theory therefore predicts that (27) and (36a), with its predicted truth conditions in (36b), can be read as equivalent.

(36) a. The mixture contains 0.1 kilograms per liter of salt.
b. $\exists$x[salt(x) $\wedge$ $\frac{\mu_{\text{WT}}(\text{x})}{\mu_{\text{VOL}}(\text{the mixture})} = 0.1\frac{\text{kg}}{\text{L}}$ $\wedge$ contain(the mixture, x)]

But this prediction is not borne out. Intuitions seem clear that (27) and (36a) carry different implications about the mixture's volume. Sentence (36a) implies that the mixture's volume is at least 1L, whereas (33) merely implies that it is at least 1mL. Apparently, then, the unit noun in the *per*-PP sets a lower bound on the measurement of an entity referenced in the sentence, here the entity denoted by the subject.

We will refer to this effect as *unit sensitivity*. To confirm the effect's existence, we can offer data where it gives rise to contrasts in acceptability, due to the subject denotation's measure being fixed. The examples in (37) and (38) are cases in point. In these cases, implications about the samples' volumes are introduced by attribute measure phrases within the subject phrases. In (37), the expected unit sensitivity implication, 5mL $\geq$ mL, is true. In contrast, the corresponding implications in (38), 5mL $\geq$ L and 0.1mL $\geq$ mL, are contradictory. It is those contradictions that are intuited to yield infelicity in these cases.

(37) The 5 milliliter sample of mixture contained 0.1 grams of salt per milliliter.

(38) a. #The 5 milliliter sample of mixture contained 0.1 grams of salt per liter.
b. #The 0.1 milliliter sample of mixture contained 0.1 grams of salt per

milliliter.

Similarly, unit sensitivity is also detectable in the contrasts between examples in (39) and the one in (40), examples that reference measures of concentration in a complex dimension built from the dimensions of pure numbers and text length.

(39) a. The monograph by Dole contained more than 3 typos per page.
b. The paragraph by Dole contained more than 3 typos per line.

(40) #The paragraph by Dole contained more than 3 typos per page.

Again illustrating unit sensitivity, the felicity contrast between the examples in (39) and the one in (40) is intuitively rooted in the expectation that, while a monograph has at least one page and a paragraph has at least one line, a paragraph comprises less than a whole page.

Under the division theory, unit sensitivity comes as a surprise. We saw that the effect discriminates between examples that the division theory predicts to be semantically equivalent, such as (27) and (36a). More generally, for the cases we have seen, the theory is ill-equipped to capture unit sensitivity semantically. The division theory parses *per* as structurally distant to one of the two expressions whose denotations unit sensitivity relates. In structures like (28), *per* and its unit noun complement are deeply embedded within the subject's scope. A compositional derivation of unit sensitivity from this sort of structure is therefore not viable. A semantic account of unit sensitivity, and hence a semantic account of contrasts like those in (37) to (40), are accordingly not within the division theory's reach.

Let us take stock. Given the underspecification of the predicate MUCH, pseudo-partitives would be expected to be hospitable to measure phrases denoting quotient quantities. Under the division theory, therefore, measure phrases with *per* in pseudo-partitives are predicted to yield interpretable structures. This is problematic because, failing to capture unit sensitivity, the meanings so derived are not fully adequate. In one manifestation of the problem, the division theory fails to capture infelicity due to contradictory unit sensitivity implications. What this means is that apart from the problem of undergeneration seen with measurement verb data, the division theory also faces a problem of overgeneration. Crucially, this problem extends to the hypothesis, entertained in the beginning of this section, that quantity division is merely one of two possible lexical meanings of *per*.

What are possible solutions to this problem? One might hope to capture unit sensitivity in terms of pragmatic inference, thereby relieving semantics from the burden of deriving the effect. If successful, this would restore the division theory's compatibility with pseudo-partitive data. However, while such an approach is conceivable, we are not aware of an independently motivated path of pragmatic reasoning that would have the intended effect. As matters stand, therefore, the pseudo-partitive data furnish an argument that *per* never denotes quantity division.

7 Unit sensitivity under the anaphoric theory

How can pseudo-partitive data be analyzed under the anaphoric theory introduced in Section 4? Consider again (27), repeated below. With regard to syntax, the anaphoric theory posits much the same logical form as the division theory, with the crucial difference that the *per*-PP hosts the covert pronoun *pro* in its specifier, as in (41).[3]

(27) The mixture contains 0.1 grams per milliliter of salt.

(41) [the mixture] contains $[_{DP}\ \exists\ [_{AP}\ [_{MP}\ [_{MP}$ 0.1 grams] $[_{PP}$ pro [per milliliter]]] MUCH] of salt]

Given the entry for *per* in (16), repeated below from Section 4, the anaphoric theory assigns to the measure phrase argument of MUCH in (41) the denotation in (42). Suppose now that, just as in the measurement verb data, *pro* is anaphoric to the matrix subject and hence refers to the mixture. Then the entry for MUCH in (25), also repeated below, delivers for (41) the truth conditions in (32), repeated from the last section.

(16) $[\![\text{per}]\!] = \lambda q.\ \lambda x.\ \frac{\mu_{\text{dim(q)}}(x)}{q}$

(42) $[\![\ [_{MP}\ [_{MP}\ 0.1\ \text{grams}]\ [_{PP}\ \text{pro}\ [\text{per milliliter}]\ ]\ ]\ ]\!] = (0.1 * \frac{\mu_{\text{vol}}([\![\text{pro}]\!])}{\text{ml}})\ \text{g}$

(25) $[\![\text{MUCH}]\!] = \lambda q.\ \lambda x.\ \mu(x) = q$

(32) $\exists x[\text{salt}(x) \wedge \mu_{\text{WT}}(x) = (0.1 * \frac{\mu_{\text{vol}}(\text{the mixture})}{\text{ml}})\ \text{g} \wedge \text{contain}(\text{the mixture}, x)]$

That is, the anaphoric theory as stated in Section 4 derives for (27) the very same truth conditions derived under the division theory. This means that the anaphoric theory correctly captures the information that (27) conveys about the degree of concentration of salt in the mixture. It also means, of course, that the anaphoric theory as stated in Section 4 does not capture unit sensitivity any more than the division theory does.

This does not mean, however, that the two theories are on a par with regard to unit sensitivity. Unlike the division theory, the anaphoric theory lends itself to a natural amendment under which unit sensitivity is captured. Crucially, under the anaphoric theory, the two denotations referenced in unit sensitivity are arguments of *per*. For the denotation of the measured entity, here the mixture, this is made possible by the covert pronoun in the *per*-PP's specifier. Given its anaphoric link to the matrix

3 For reasons given in footnote 2, we keep assuming here that the *per*-PP modify not just the adjacent unit noun (as suggested in Coppock 2021), but an entire measure phrase, here *0.1 grams*.

subject *the mixture*, this pronoun ensures that *per* has access to both denotations that unit sensitivity relates. This makes it possible to build unit sensitivity into the denotation of *per*. Updating (16), we are led to the revised lexical entry in (43).

(43) $[\![\text{per}]\!] = \lambda q.\ \lambda x{:}\ \mu_{\text{dim(q)}}(x) \geq q.\ \frac{\mu_{\text{dim(q)}}(x)}{q}$

According to (43), *per* triggers the presupposition that its quantity argument sets a lower bound on the individual argument's measure in the relevant dimension. With respect to (27), for example, this amounts to the intended implication, μ_{VOL}(the mixture)≥1mL, that the mixture's volume is at least 1mL. Assuming this presupposition projects globally, the perceived unit sensitivity in this case is captured. Likewise, assuming global projection, the intended unit sensitivity implications in the other pseudo-partitive examples above, including those rendered infelicitous by the implication's contradictoriness, are derived as presuppositions triggered by *per*.

We note that with the revised entry for *per* in (43), the anaphoric theory predicts unit sensitivity to arise in *all* sentences with *per*, including the measure verb data in Section 3. To test this prediction, let us briefly return to sentence (6) and its structure under the anaphoric theory in (20), both repeated below. Given that we took *pro* in (20) to refer to the sample, (43) now leads to the prediction that (6) implies μ_{VOL}(the sample)≥1mL, that the sample's volume is at least 1mL.

(6) The sample weighs 0.9 grams per milliliter.

(20) [the sample] weighs [$_{\text{MP}}$ [$_{\text{MP}}$ 0.9 grams] [$_{\text{PP}}$ pro [per milliliter]]]

This prediction is consistent with intuitions. To confirm the presence of unit sensitivity in measurement verb data, we can moreover consider acceptability contrasts in cases where the matrix subject's measure in the relevant dimension is fixed, say by an attributive measure phrase. Paralleling the pseudo-partitives data in (37) and (38) above, the contrast between (44) and the examples in (45) is a case in point.

(44) The 5 milliliter portion of solution weighed 0.9 grams per milliliter.

(45) a. #The 5 milliliter portion of solution weighed 0.9 kilograms per liter.
b. #The 0.1 milliliter portion of solution weighed 0.9 grams per milliliter.

The sentence in (44) sounds completely natural while those in (45) are extremely odd. The lexical entry in (43) can explain this contrast. While the unit sensitivity presupposition in (44), triggered by (43), is true (e.g., 5mL ≥ mL), the corresponding presuppositions in (45) are contradictory (e.g., 5mL ≥ L and 0.1mL ≥ mL).

In sum, under the anaphoric theory, the unit sensitivity effect in *per* data can be understood as a presupposition triggered by *per*, as in (43). We suggest, moreover, that this presupposition is conceptually natural. It encodes the intuition that the quantity in the numerator must be divided into parts by the quantity specified in the denominator. For example, a quantity of 5mL can be divided into parts consisting of sub-quantities of 1mL, however it cannot be divided into parts consisting of sub-quantities of 1L. If this is on the right track, then the anaphoric theory is not merely compatible with unit sensitivity, but the theory in fact derives this effect for free in the semantic composition.

8 Conclusion

We have argued that the division theory of *per* proposed in Coppock (2021) both undergenerates and overgenerates, and that the anaphoric theory offers a theoretically parsimonious solution to both of these problems. Based on the data examined, the natural conclusion is that *per* is always anaphoric and never expresses quantity division. However, before concluding this paper, we would like to draw attention to data that appear to be in conflict with this idea. It seems obvious enough that the copular sentence in (46) reports on the sample's density.

(46) The sample's density is 0.9 grams per milliliter.

Under the anaphoric theory, this meaning comes as a surprise. To see why, recall that the anaphoric theory assigns to the post-copular measure phrase in (46) the structure shown in (17), repeated below.

(17) $[_{MP}$ $[_{MP}$ 0.9 grams] $[_{PP}$ pro [per milliliter]]]

Recall also that the *per*-PP is taken to denote a pure number, which composes with the denotation of the lower measure phrase, *0.9 grams*, via product formation. As a consequence, the quantity denoted by (17) as a whole will be in the same dimension as the quantity denoted by the lower measure phrase, viz. the weight dimension WT. According to the anaphoric theory, then, sentence (46) should be understood as equating the sample's density with a quantity in the weight dimension WT, rather than $\frac{\text{WT}}{\text{VOL}}$, the dimension of density. Hence (46) should not be any more felicitous than (47), whose oddness is transparently due to mismatching dimensions in the portrayal of a weight quantity as the sample's measure of density.

(47) #The sample's density is 0.9 grams.

Under the division theory, in contrast, the semantic composition for the sentence in (46) looks straightforward, given that the post-copular measure phrase is assigned the denotation $0.9\frac{\text{g}}{\text{ml}}$. That said, we would like to point out that the data is not as clear cut as it would first appear. Consider the variant of (46) in (48), or the naturally occurring example in (49).

(48) The sample's density is 0.9 grams for every cubic centimeter.

(49) London's population density is 968 people for every km^2.
(https://blackburnnews.com)

These sentences also express notions of density. Yet they do not feature vocabulary that has an obvious association with quantity division. This leads one to wonder whether cases like (46) are correctly analyzed in terms of quantity division. We hope to further explore such data and their theoretical implications in future work.

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