

# Scope, monotonicity and maximal informativity *cannot be underestimated!* A compositional analysis of an apparent linguistic illusion\*

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**Abstract** We propose a compositional analysis of an apparent linguistic illusion that productively arises with sentences like *The importance of this position cannot be underestimated*, featuring the prefix *under-* and a negated possibility modal. On what we term the ‘intended’ interpretation, the sentence means that the position’s importance is very high, while on what we call the ‘pedantic’ interpretation, the sentence means that the position’s importance is very low. Although the intended reading has been claimed to be illusory, we demonstrate that it arises from an interaction between the scope of the negated modal, the monotonicity of the underlying degree set, and maximal informativity (Beck 2013; Hohaus & Bade 2022).

**Keywords:** linguistic illusions, degree semantics, monotonicity, maximal informativity

## 1 Introduction: Another semantic illusion

Cases of semantic illusion, where the initially perceived interpretation of a sentence does not appear to be an interpretation licensed by the grammar, have been taken to offer valuable insights into how human language is organised and processed. Our focus in this paper is on a type of inversion illusion (after terminology in O’Connor 2015). Inversion illusions are characterised by a flip in the interpretation of the comparison involved; they are also surprisingly robust when compared to other types of grammatical illusion. The most famous example of such an illusion is the so-called depth-charge sentence of Wason & Reich (1979), in (1), which under its prominent, inverted interpretation has the interpretation of its *enough*-counterpart. We productively observe a related flip with verbs prefixed by *under-*, as in (2), where the salient interpretation for many speakers is that of the corresponding *over*-sentence. We refer to this here as the ‘intended’ interpretation.

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- (1) Depth-charge sentences  
(Wason & Reich 1979: p. 541)

No head injury is too insignificant to be ignored.

⇔ No head injury is insignificant enough to be ignored.

Perceived, “inverted” interpretation: ✓ Don’t ignore any head injuries!

Pedantic interpretation: %Ignore all head injuries!

- (2) Under-over illusion  
(inspired by Gleitman & Papafragou 2013: p. 504)<sup>1</sup>

The importance of this position cannot be underestimated.

⇔ The importance of this position cannot be overestimated.

Intended, “inverted” interpretation: ‘This position is very important!’

Pedantic interpretation: ‘This position is not important at all!’

Under the inverted interpretation, it is not possible for the position in question to be taken to be more important than it actually is, because it is already considered to be of great importance. Compare this to what we refer to here as the ‘pedantic’ reading of the sentence, on which it is not possible for the position to be estimated to be less important than it actually is, because it has little importance attached to it already.

While the under-over illusion has not received as much attention in the research literature in linguistics and psychology as the infamous depth-charge examples, it certainly also has not gone unnoticed, especially in the popular media.<sup>2</sup> Barbara Wallraff’s “Word Court” for the June 2004 edition of *The Atlantic* is a case in point, from which the first excerpt below is taken. The second excerpt is from Ben Zimmer’s “On Language” column for *The New York Times* on the 21st January 2011.

Kent Tankersley, of Helsinki, Finland, writes: “I work in a European office of a multinational company. After I recently wrote the somewhat clichéd phrase *We cannot understate the importance of...*, some of my colleagues, who are not native English-speakers, pointed out that this didn’t make sense. Although the phrase sounded perfectly okay to me and the only other native English-speaker in the office, I had to admit that, logically, the literal meaning was the opposite of what I intended. Obviously, it should be *We cannot overstate the importance of...*” *Cannot understate* and *cannot overstate* are like architectural elements in an M. C. Escher drawing: if you like, you can flip-flop them in your mind.”

<sup>1</sup> Mark Libermann, “Multiplex negatio ferblondiat”, *Language Log*, 14th July 2007 (URL: <http://itre.cis.upenn.edu/~myl/languagelog/archives/004716.html>, last accessed: 11th November 2024).

<sup>2</sup> But see also Mark Liberman and Geoffrey Pullum’s *Language Log* (URL: <https://languagelog.ldc.upenn.edu/nll/>), which has several of posts on the topic, including “Multiplex negatio ferblondiat” (14th July 2007), “Cannot underestimate = must not underestimate” (6th November 2008), and “Underestimate, overestimate, whatever” (23rd March 2011).

Allen Pinsky e-mails: “Gov. Andrew M. Cuomo used the phrase *cannot be underestimated* twice in his inaugural address. My interpretation of this is the reader/listener couldn’t possibly think lower of the situation or person. As he was referring to the degree of economic suffering, shouldn’t he have said either *cannot be overestimated* or *should not be underestimated* in this context? . . . A straight, literal reading of the governor’s speech would imply that he’s saying the opposite of what he really means.” . . . This perplexing turn of phrase is extremely common, even among careful writers and speakers.

We propose here that this “flip-flop” and “perplexing turn of phrase” is not an illusion but rather the result of a structural ambiguity; the different interpretations of under-over sentences arise compositionally. Under the analysis developed in what follows, the ambiguity of under-over sentences results from an interaction between scope, negation, and maximal informativity. More specifically, the inversion reading arises when the maximally informative degree for the comparee in the comparison equals the minimal degree, rather than the maximal degree. In this respect, the alleged under-over illusion originates from the same source that Hohaus & Bade (2022) identify for the inversion reading in depth-charge sentences, that is, it is an effect of maximal informativity.

Section 2 will discuss the role of maximal informativity in the interpretation of comparison constructions further. Section 3 spells out the analysis of the core example in (2), which we extend to a wider range of *under*-examples in Section 4. In Section 5, we briefly consider but ultimately discard an alternative analysis of the illusion, which locates its source in the modal base. Section 6 concludes and identifies some directions for further research.

## 2 Background: Maximal and maximally informative degrees

Under a textbook view, the interpretation of comparison constructions in English not only involves the notion of degrees, abstract elements of scales, but also that of maximality (see, for instance, the overview in Beck 2011).<sup>3</sup> A comparative like (3a), for instance, under this view, is true if and only if the maximal degree to which Arnim is old exceeds the maximal degree to which Sigrid is old. A degree question like (3b) enquires after the maximal age degree that Cornelius has.

- (3) a. Arnim is older than Sigrid.

$$\text{MAX}(\lambda d. \text{AGE}(A) \geq d) > \text{MAX}(\lambda d'. \text{AGE}(S) \geq d')$$

Informally: ‘The maximal degree  $d$  such that Arnim is  $d$ -old exceeds the maximal degree  $d'$  such that Sigrid is  $d'$ -old.’

<sup>3</sup> Degree-based analyses go back to at least Russell (1905) and Cresswell (1976). For maximality in comparison, see, in particular, Russell 1905, von Stechow 1984, and Rullmann 1995.

b. How old is Cornelius?

$$\{p : \exists d [p = \lambda w. \text{MAX}(\lambda d'. \text{AGE}(C) \geq d') = d]\}$$

Informally: ‘What is the maximal degree  $d$  such that Cornelius is  $d$ -old?’

Maximality here is required as a result of the monotonicity properties of the gradable adjective, in (4). We assume that *old* is downward scalar (see, for instance, von Stechow 1984): If Cornelius is nine months old, he is also eight months old, and so on.

$$(4) \quad \llbracket \text{old} \rrbracket = \lambda d. \lambda x. \text{AGE}(x) \geq d$$

Degree constructions however notably have been observed to allow for interpretations that appear to involve a minimal, rather than a maximal degree (Seuren 1979; Rullmann 1995; Beck & Rullmann 1999; Beck 2011, 2013; Heim 2007; Beck 2023).<sup>4</sup> The comparative in (5) is a comparison with the minimal amount of flour required for the cake recipe, and the degree question in (6) enquires after the minimal amount of arsenic that is lethal in humans. Notably, the maximum amount might not even be defined in both cases.

(5) [One can bake this cake if one has at least 500 g of flour.]

I have more flour than is sufficient to bake this cake.

(Beck 2013: p. 5, no. 9)

Informally: ‘I have more flour than the minimal amount that suffices to bake this cake.’

(6) How much arsenic is lethal in humans?

(after Beck & Rullmann 1999: p. 256, no. 16-c)

Informally: ‘What is the minimal degree  $d$  such that  $d$ -much arsenic can kill somebody?’

While the relevant degree thus is not the maximal one, it is the maximally informative one, given the entailment patterns involved: If 500 grams of flour will be sufficient to bake the cake, 550 grams will be as well. If 140 grams of arsenic are a lethal dose for humans, 141 grams will be as well. The underlying degree predicates are upward scalar. Rather than making recourse to minimality, a unified semantics for the relevant constructions can be maintained if we assume that comparison constructions are sensitive to maximal informativity, at the very least in degree questions and in the interpretation the standard-phrase (Beck & Rullmann 1999; Fox & Hackl 2006; Beck 2010, 2013; Krifka 2010; Lahiri 1991; Heim 1994; Rett 2015, 2020). We adopt here the definition in (7). Whether the maximal or the minimal degree is

<sup>4</sup> They may in fact even be ambiguous between a maximal and a minimal interpretation, a case which we set aside here, but see Rullmann (1995) and Beck (2013).

the most informative degree will be determined by the monotonicity of the degree predicate: For positive degrees, the maximally informative degree returned for a downward scalar predicate is the maximal degree. For upward scalar predicates, it is the minimal degree.

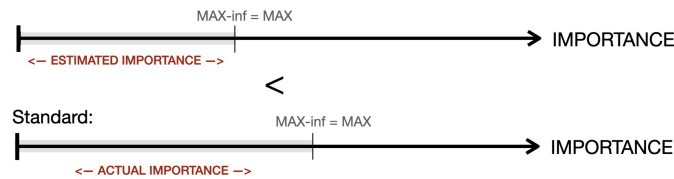
- (7) For any  $p \in D_{\langle d, \langle s, t \rangle \rangle}$  and world  $w$ , the maximally informative degree the- $\text{MAX}_{\text{inf}}(p)(w) = \iota d [p(d)(w) = 1 \ \& \ \neg \exists d' [d \neq d' \ \& \ p(d')(w) = 1 \ \& \ [\text{if } p(d')(w) = 1, \text{ then } p(d)(w) = 1]]]$   
(based on Beck 2013: p. 13, no. 36)

Looking ahead, our core observation will be that the two interpretations of under-over sentences are a result of a scope ambiguity with negation that affects the monotonicity of the degree predicate underlying the comparison (and thus maximal informativity). Under the pedantic interpretation, one of the degree sets involved is downward scalar, while under the intended interpretation, it is upward scalar, as we will discuss in more detail the next section.

### 3 The analysis: Scope, monotonicity and maximal informativity

We begin our analysis of under-over sentences by first considering the analysis of a simpler sentence, the one in (8), without the negated possibility modal. The sentence encodes a less-than comparison between the actual importance assigned to the position ( $P$ , for short) and its estimated importance. More specifically, it is true, we propose, if and only if the maximally informative degree  $d$  such that  $P$  is estimated to be  $d$ -important is less than the maximally informative degree  $d'$  such that  $P$  is  $d'$ -important in the actual world. We illustrate this interpretation in Figure 3.

- (8) The importance of this position is underestimated.

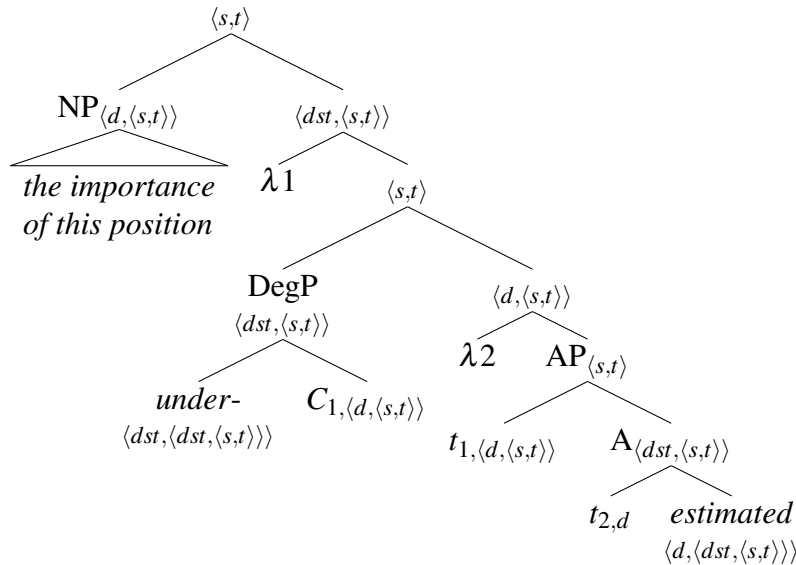


**Figure 1** Maximal informativity in comparison with *under-*

We derive this interpretation on the basis of the Logical Form in (9), where *underestimated* has been decomposed into a modal adjectival root *estimated* and a degree head *under-*, with a pronominal standard argument of type  $\langle d, \langle s, t \rangle \rangle$ ; more on which

below. This Logical Form is characterised by two movement operations: The Degree Phrase with *under-* undergoes Quantifier Raising due to a type mismatch, while the subject noun phrase (of type  $\langle d, \langle s, t \rangle \rangle$ ) undergoes Subject Raising and binds not only its trace but also the pronominal standard argument of *under-* (see Meier 2003 for a syntactically similar analysis of comparison with *too*). In line with the Lowest Possible Type Constraint (Beck 1996), Subject Raising here leaves behind a trace of type  $\langle d, \langle s, t \rangle \rangle$ .

(9) Logical Form for a simple *under-*sentence:



At the lexical level, under the decomposition assumed, *under-* encodes a less-than comparison between two intensional degree sets, as in (10). The comparison holds between the maximally informative degrees (Beck & Rullmann 1999; Beck 2013), as defined in (7) above. At the Logical Form, the first of these sets, the standard of the comparison, is syntactically represented by a covert proform of the relevant semantic type.

$$(10) \quad \llbracket \textit{under-} \rrbracket = \lambda D'_{\langle d, \langle s, t \rangle \rangle} \cdot \lambda D_{\langle d, \langle s, t \rangle \rangle} \cdot \lambda w. \\ \text{the-MAX}_{\text{inf}}(D)(w) < \text{the-MAX}_{\text{inf}}(D')(w)$$

Under this decomposition analysis, the root *estimated* serves two roles: It maps an intensional degree set, here of importance degrees for *P*, to its estimated counterpart in the world of evaluation and checks whether its degree argument is an element of said estimate, as spelled out in (11). While we will be using the simplified (11a) in what follows, we provide a more explicit semantics in (11b).<sup>5</sup> We propose that

<sup>5</sup> Note that we sometimes use a subscript notation for world arguments, to increase readability.

*estimated* existentially quantifies over those possible worlds compatible with the relevant estimate in the evaluation world.<sup>6</sup>

- (11) a.  $\llbracket \textit{estimated} \rrbracket =_{\text{simplified}} \lambda d. \lambda D_{\langle d, \langle s, t \rangle \rangle}. \lambda w. d \in \text{ESTIMATE}_w(D)$   
 b.  $\llbracket \textit{estimated} \rrbracket = \lambda d. \lambda D_{\langle d, \langle s, t \rangle \rangle}. \lambda w. \exists w' \in \text{ESTIMATE}(w) : D(d)(w') = 1$

Finally, we analyze the subject noun phrases *the importance of this position* as providing an intensional (and downward-entailing) degree set, where the degree is less than or equal to the importance of *P* in the respective world, as in (3). While not central to the analysis of under-over illusions, we thereby treat the noun as having a degree-based semantics (but see also, among others [Scontras 2014](#); [Hanink, Koontz-Garboden & Makasso 2019](#)), and the definite determiner as semantically vacuous. While we localize maximal informativity in *under-*, there certainly is an alternative view under which it is contributed by the definite determiner ([von Fintel & Iatridou 2014](#); [Beck 2023](#)).

- (12)  $\llbracket \textit{the importance of this position} \rrbracket = \lambda d. \lambda w. \text{IMPORTANCE}_w(P) \geq d$

Putting these pieces together, we derive the truth conditions in (13), which require the maximally informative degree of importance the position has in the actual world to exceed the maximally informative degree of importance it is estimated to have. In this case, the maximally informative degrees are the maximal degrees in both sets.

- (13) Truth conditions for a simple under-sentence:

$$\begin{aligned} & \text{the-MAX}_{\text{inf}} \\ & (\lambda d. \lambda w. d \in \text{ESTIMATE}_w(\lambda d'. \lambda w'. \text{IMPORTANCE}_{w'}(P) \geq d'))(w_{@}) \\ & < \text{the-MAX}_{\text{inf}}(\lambda d''. \lambda w''. \text{IMPORTANCE}_{w''}(P) \geq d'')(w_{@}), \\ & \text{where } w_{@} \text{ is the actual world} \end{aligned}$$

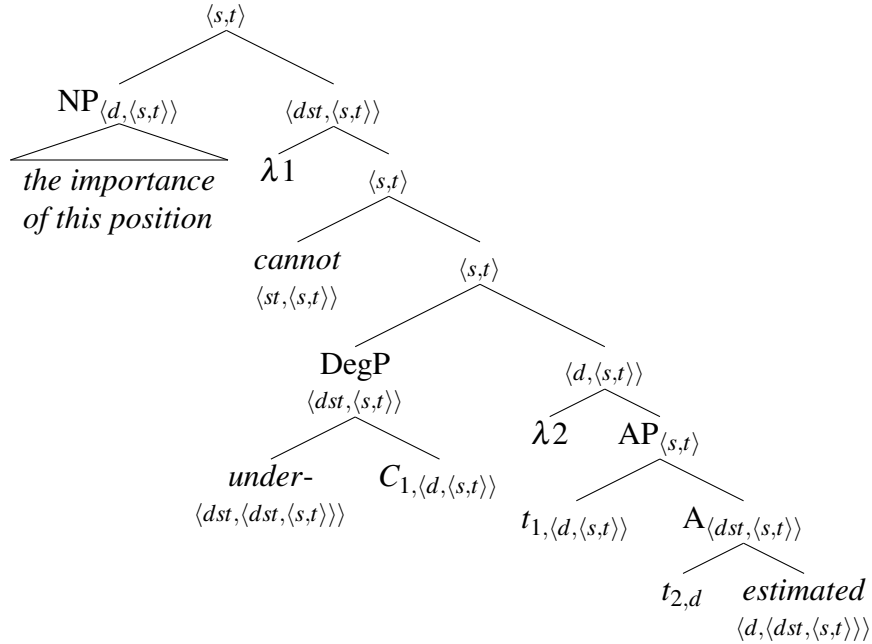
‘The maximally informative estimated degree of importance assigned to *P* is less than the maximally informative actual degree of importance *P* has.’

We are now in a position to spell out our analysis of the apparent illusion in under-over sentences: The two readings of our example from (2) rely on the different

<sup>6</sup> We make use of existential, rather than universal, quantification over worlds to ensure that the set of estimated degrees includes the maximum degree compatible with estimates across worlds. If we made use of universal quantification over worlds, the maximum degree in the set would correspond to the smallest degree in the set of estimates across worlds. This would result in incorrect truth conditions for the basic sentence in (16), where we want the maximum degree of the position’s actual importance to exceed the *largest* degree of importance across worlds compatible with the estimates. See [Schwarzschild & Wilkinson \(2002\)](#) and [Beck \(2010\)](#) for discussion of similar undesirable interactions between universal quantification and the maximality operator over degrees.

scope configurations of the negated possibility modal and the scopally mobile Degree Phrase on the one hand, and the distinct interaction of each configuration with maximal informativity on the other (modulated by monotonicity). If *underscopes* below *cannot*, as in the Logical Form in (14), we derive the pedantic reading, which is spelled out in (15) and visualised in Figure 3.

(14) Logical Form for the pedantic reading, with CANNOT » DegP:



(15) Truth conditions for the pedantic reading, with CANNOT » DegP:

$\neg \exists w^* [w^*$  is accessible from  $w@$  &  
the-MAX<sub>inf</sub>( $\lambda d. \lambda w. d \in \text{ESTIMATE}_w(\lambda d'. \lambda w'. \text{IMPORTANCE}_{w'}(P) \geq d')$ )( $w^*$ )  
 $<$  the-MAX<sub>inf</sub>( $\lambda d''. \lambda w''. \text{IMPORTANCE}_{w''}(P) \geq d''$ )( $w^*$ )]

‘There is no accessible world where the estimated importance of  $P$  is less than the importance  $P$  has in the actual world.’

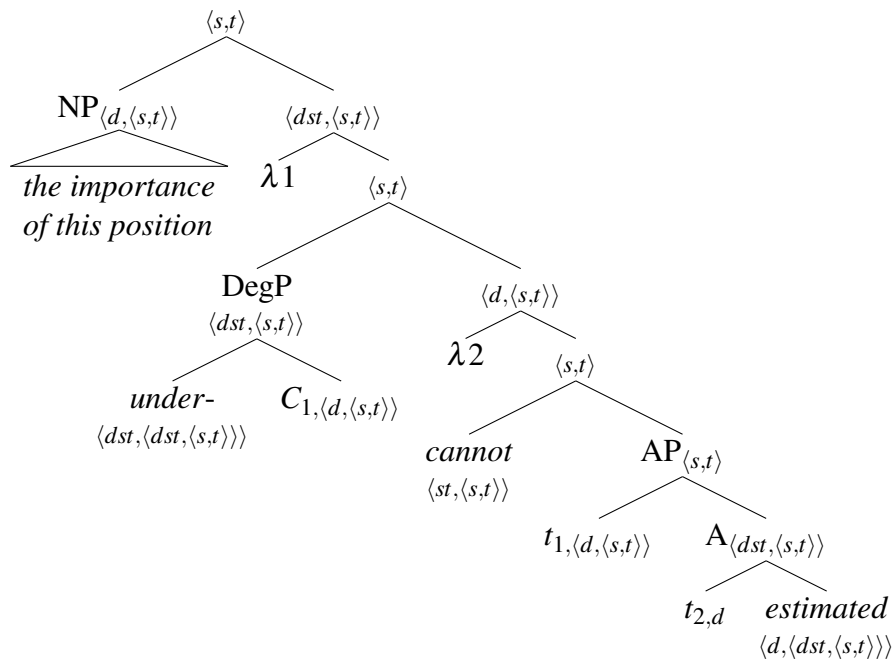


**Figure 2** Maximal Informativity and the pedantic interpretation

The resulting degree sets involved in the comparison are both downward entailing, and the-MAX<sub>inf</sub> returns the maximal degree in each set. Under this scopal configuration, the sentence is thus true if and only if there is no accessible world where the estimated importance of the position is less than its importance in that world. Hence, the position is always overestimated. This is pedantic reading, which is often claimed to be the true literal interpretation of the sentence.

However, our analysis also admits for the Degree Phrase to scope above the modal. This possibility is captured in the Logical Form provided in (16), with the resulting interpretation in (17).

(16) Logical Form for the intended reading, with DegP » CANNOT:



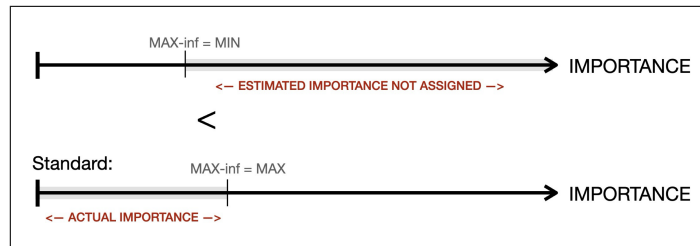
(17) Truth conditions for the intended reading, with DegP » CANNOT:

the-MAX<sub>inf</sub>( $\lambda d. \lambda w. \neg \exists w^* [w^*$  is accessible from  $w$  &  
 $d \in \text{ESTIMATE}_{w^*}(\lambda d'. \lambda w'. \text{IMPORTANCE}_{w'}(P) \geq d')](w@)$   
 $< \text{the-MAX}_{\text{inf}}(\lambda d''. \lambda w''. \text{IMPORTANCE}_{w''}(P) \geq d'')(w@)$ )

‘The maximally informative degree of estimated importance that  $P$  lacks in any world is less than the importance  $P$  has in the actual world.’

Put differently, the sentence is then true if and only if the actual importance of  $P$  exceeds the highest importance degree that  $P$  has been assigned across worlds (and  $P$  hence is very important). Crucially, as shown in Figure 3, the intensional degree set in the first part of the comparison here is upward entailing: If the position lacks a degree of importance  $d$ , then for any  $d' > d$ , the position also lacks  $d'$ . In this

case, the maximally informative degree is the minimal degree in the set, that is, the smallest degree that the position is not estimated to have across possible worlds.



**Figure 3** Maximal Informativity and the intended interpretation

A consequence of these truth conditions is that the position’s actual importance must be very high to exceed this degree. This, we submit, is the source of the inference that the position’s importance is very high on the intended reading. Why is this reading then often characterised as an illusion? We posit with [Hohaus & Bade 2022](#) that minimal degrees appear to be dispreferred in comparisons, especially when the other degree involved is a maximal one. Note also that under the intended reading, the scope of *under-* with respect to the negated modal does not correspond to the surface scope, which may be another reason why it is perceived as an illusion, assuming a preference for interpretations that reflect the linear word order.

To conclude this section, let us briefly summarize the key points of the analysis: Under the view presented here, *under-over* sentences are ambiguous, rather than receiving an interpretation that is not licensed by the grammar. The alleged semantic illusion arises from the interaction of scope, monotonicity, and informativity. The inverted reading is based on an inverse scope configuration, one of the degree sets involved in the comparison is upward monotonic, and the minimal degree is maximally informative. We explore how widespread inverted readings are with *under-*prefixation next.

#### 4 Additional predictions: Inverted readings with *under-*prefixation

With *under-*prefixation, we find that the inverted reading appears with other modal auxiliaries, but, unsurprisingly, not without the presence of negation. This section discusses three such cases and shows how they are covered by the analysis proposed above. First, consider the variant in (18), with a necessity rather than a possibility modal auxiliary. There are two possible interpretations, one on which the importance of the relevant position is very high (our intended reading), and one in which it is not permissible to estimate the position’s importance to be less than its actual importance.

- (18) The importance of this position must not be underestimated.  
 Intended, inverted interpretation: ‘This position is very important!’  
 Additional interpretation: ‘It is not permitted to give an estimate of the position’s importance that is less than its actual importance.’

It is straightforward to see why the intended reading is available on our approach: Since *must not* in English always exhibits wide scope of the necessity modal with respect to negation, that is, MUST » NOT (see, for instance, Iatridou & Zeijlstra 2013), the truth conditions of this reading, given below, are equivalent to the truth conditions in (17), due to the interdefinability of necessity and possibility modality. In (19), the comparison is between the lowest degree of importance that *P* lacks across worlds, which is the highest degree of importance it has been assigned in a world. This degree will also be the maximally informative degree, given that if the position lacks said degree, it will also lack any higher degree. The standard of the comparison is the maximally informative (= maximal) degree of importance the position has in the actual world. The sentence is thus true if and only if the actual importance of the position is higher than any estimate.

- (19) Truth conditions for the inverted reading, with DegP » MUST NOT:

$$\text{the-MAX}_{\text{inf}}(\lambda d. \lambda w. \forall w^* [w^* \text{ is accessible from } w \rightarrow d \notin \text{ESTIMATE}_{w^*}(\lambda d'. \lambda w'. \text{IMPORTANCE}_{w'}(P) \geq d')])(w_{@})$$

$$< \text{the-MAX}_{\text{inf}}(\lambda d''. \lambda w''. \text{IMPORTANCE}_{w''}(P) \geq d'')(w_{@})$$

‘The maximally informative degree of estimated importance that *P* lacks across worlds is less than the actual importance of *P*.’

The truth conditions for the other interpretation of the sentence, with *must not* scoping over DegP are given below. Here, every accessible world is one in which the maximally informative estimated importance assigned to the position is at least as high as the importance it actually has. In other words, there is no world where we give too low of an estimate.<sup>7</sup>

- (20) Truth conditions for the non-inverted reading, with MUST NOT » DegP:

$$\forall w^* [w^* \text{ is accessible from } w_{@} \& \text{the-MAX}_{\text{inf}}$$

<sup>7</sup> As we discuss in Section 5, this is an interpretation available to *the importance of this position cannot be underestimated* as well. To the extent that the sentence with *cannot* differs from its counterpart in (18), it appears to us that (18) lacks our pedantic reading, where the position’s importance is too low. This, on our view, stems from an independent difference between *cannot* and *must not* with respect to the kinds of modal flavors with which they are compatible: While *cannot* may express an epistemic, deontic, teleological, and circumstantial impossibility, *must not* is most naturally interpreted as encoding deontic impossibility (Iatridou & Zeijlstra 2013). The pedantic reading, available with *cannot*, requires circumstantial impossibility, an option unavailable with *must not*.

Scope, monotonicity and maximal informativity

$$(\lambda d. \lambda w. d \in \text{ESTIMATE}_w(\lambda d'. \lambda w'. \text{IMPORTANCE}_{w'}(P) \geq d'))(w^*) \\ \geq \text{the-MAX}_{\text{inf}}(\lambda d''. \lambda w''. \text{IMPORTANCE}_{w''}(P) \geq d'')(w^*)]$$

‘In every world accessible from  $w^*$   
the maximally informative estimated degree of importance for  $P$   
is at least as high as the actual importance of  $P$ .’

We may now consider cases where either a modal or negation is not present. Notice that the examples in (21) are both unambiguous, too: The sentence in (21a) can only mean that the position’s actual importance is not greater than the relevant estimate, and (21b) can only be taken to mean that it is necessary that the position’s importance is greater than the estimated importance.

- (21) a. The importance of this position is not underestimated.  
b. The importance of this position must be underestimated.

These two examples have in common that they can only be interpreted with surface scope. We propose that the other scope configuration is blocked here as it is undefined. To see this, consider the truth conditions for the putative inverted interpretation of (21a) below.

- (22) Truth conditions for  $\#(\text{DegP} \gg \text{NOT})$ :  
the-MAX<sub>inf</sub>( $\lambda d. \lambda w. d \notin \text{ESTIMATE}_w(\lambda d'. \lambda w'. \text{IMPORTANCE}_{w'}(P) \geq d'))(w_{@})$   
< the-MAX<sub>inf</sub>( $\lambda d''. \lambda w''. \text{IMPORTANCE}_{w''}(P) \geq d'')(w_{@})$   
‘The maximally informative degree that is not included in the estimates  
of the importance of  $P$  is less than the actual importance of  $P$ .’

Here, we need to compare the position’s actual importance with the maximally informative degree  $d$  that is not an element of the set of estimates of the position’s importance. Due to the negation, this is an upward-entailing set of degrees, and thus the maximally informative degree, if it exists, would be the minimum. However, we are dealing with the set of degrees that are not in the set of estimates of the position’s importance, where the latter set has a defined maximum value. There is thus no minimum degree, and thus no maximally informative degree. The narrow scope of negation with respect to *under-* thus gives rise to undefinedness, explaining its absence.

A similar account can be applied to the inverse scope reading of (21b). Consider the truth conditions corresponding to this reading in (23).

- (23) Truth conditions for  $\#(\text{DegP} \gg \text{MUST})$ :  
the-MAX<sub>inf</sub>( $\lambda d. \lambda w. \forall w^* [w^* \text{ is accessible from } w \rightarrow$   
 $d \in \text{ESTIMATE}_{w^*}(\lambda d'. \lambda w'. \text{IMPORTANCE}_{w'}(P) \geq d')]) (w_{@})$

$$< \text{the-MAX}_{\text{inf}}(\lambda d'' . \lambda w'' . \text{IMPORTANCE}_{w''}(P) \geq d'')(w_{@})$$

‘The maximally informative degree of importance that  $P$  is estimated to have across worlds is less than the actual importance of  $P$ .’

In order to evaluate (23), we need to find the maximally informative degree  $d$  such that for all accessible worlds,  $d$  is in the set of estimates at that world. Intuitively, this will end up being the smallest degree that is in the set of estimates across worlds.<sup>8</sup> On the reasonable assumption that the set of degrees at issue is very fine-grained, corresponding to the full range of estimates one could give for the position's importance across possible worlds, for any degree  $d$  in the set, there will always be a smaller degree  $d'$  that is held across a larger number of worlds. As a consequence, there is no maximally informative degree in the set of estimates of the position's importance in all possible worlds. This leads us to the same state of affairs that we observed with negation: interpreting *must* in the scope of *under-* leads to undefinedness, and this scope configuration is ruled out, as desired.

We close our discussion of the predictions of our analysis with an examination of the interpretation of non-negated possibility modals with *under-*prefixed verbs, as in (24). Note that this sentence is also unambiguous: It only has a reading corresponding to wide scope of the modal with respect to *under-*. Again, the unavailability of a second reading is plausibly due to one of the maximally informative degrees involved in the comparison being undefined. Let us consider what truth conditions this reading would be associated with, given below in (25).

(24) The importance of this position can be underestimated.

(25) Truth conditions for  $\#(\text{DegP} \gg \text{CAN})$ :

$$\begin{aligned} & \text{the-MAX}_{\text{inf}}(\lambda d . \lambda w . \exists w^* [w^* \text{ is accessible from } w \ \& \\ & d \in \text{ESTIMATE}_{w^*}(\lambda d' . \lambda w' . \text{IMPORTANCE}_{w'}(P) \geq d')])(w_{@}) \\ & < \text{the-MAX}_{\text{inf}}(\lambda d'' . \lambda w'' . \text{IMPORTANCE}_{w''}(P) \geq d'')(w_{@}) \end{aligned}$$

‘The maximally informative degree  $P$  is estimated to have in some world is less than the actual importance of  $P$ .’

Here, the position's maximally informative importance in the actual world is compared to the maximally informative degree  $d$  such that for some accessible world,  $d$  is an estimate of the position's importance in that world. Notice that the set of

<sup>8</sup> As discussed in footnote 6, this is reminiscent of a problem in the literature on the interaction of maximality and universal quantification in *than*-clauses, where, without resorting to a wide-scope analysis of the quantifier or an interval-based semantics of the *than*-clause, sentences like *John is taller than every girl* are predicted to be true if John is taller than the shortest girl, since the shortest girl's height corresponds to the largest degree of height that every girl has (Schwarzschild & Wilkinson 2002; Beck 2010). While this is a problem for the analysis of comparatives, we can capitalize on this effect to deliver a correct prediction for our analysis, as we discuss in the main text.

degrees we are considering for our estimate in some possible world is very similar to the one considered in our analysis of in (17), differing only in the absence of negation in front of the existential quantifier over worlds. Recall that we assumed for (17) that there is some minimal degree that exceeds all estimates of the position's importance across all worlds. If there is some particular minimal degree that the position lacks across worlds, then there can be no maximal degree that it possesses across worlds, because of the density of the set of degrees (Fox & Hackl 2006). This means that, in the absence of negation in the set considered in (25), no maximal informative degree exists, thus ruling out this scope configuration.

This section explored the predictions of the scope-based analysis of inversion illusions for the availability of the inverted reading with *under-* in different configurations. Not only does the analysis proposed in Section 3 derive the two interpretations of the original under-over example, but it also opens up a pathway to a unified explanation for the absence of the inverse scope reading with bare possibility and necessity modals with *underestimated*, as well as the absence of such a reading with negation in the absence of a modal: Such a scopal configuration results in one of the maximally informative degrees being undefined. If this approach is reasonable, then we have a full account of why both negation and a modal are necessary for the inverted reading to arise. Before we conclude with some further directions for research on under-over sentences in Section 6, we briefly review an alternative approach to the alleged illusion.

## 5 A alternative analysis? Locating the ambiguity in the modal base

In this section, we turn to an assessment of an alternative analysis to our own. On this analysis, the intuitive reading would still be derived compositionally, but not via a scope ambiguity, as we suggested in Section 3. Instead, the alternative would locate the ambiguity in the modal base of the negated possibility modal.<sup>9</sup> Under such an approach, the pedantic reading corresponds to an epistemic modal base, on which, given our knowledge, it is not possible for the position's actual importance to be less than our estimates. The intended reading, on the other hand, corresponds to a deontic modal base, on which it is not permissible for estimates to be lower than the position's actual value. This perspective initially seems plausible, at least on intuitive grounds, as the intended reading seems to be paraphrasable as in 26.

- (26) The importance of this position cannot be underestimated.  
'The importance of this position should not be underestimated.'

<sup>9</sup> This very perspective is represented in Mark Liberman, "Cannot underestimate = must not underestimate", *Language Log*, 6th November 2008 (URL: <https://languagelog.ldc.upenn.edu/nll/?p=813>, last accessed 14th November 2024). We would also like to thank Jessica Rett for discussion.

We argue, however, that a compositional analysis making use of this approach, combined with our proposal for the meaning of *underestimate*, does not deliver the interpretation that we are after. Recall that the intended interpretation of an under-over sentence is one in which the degree of a scalar property held by an individual (in our running example, the importance of the position) is very high. On an analysis that casts the intended reading as arising from the choice of a deontic modal base, we would however arrive at the truth conditions in (27). Note that these are identical to the truth conditions in (15) for the pedantic reading, except for the accessibility relation appealed to.

(27) Alternative truth conditions for the inverted reading:

$$\neg \exists w^* [w^* \text{ compatible with what is permissible in } w_{@} \& \text{ the-MAX}_{\text{inf}}(\lambda d. \lambda w. d \in \text{ESTIMATE}_w(\lambda d'. \lambda w'. \text{IMPORTANCE}_{w'}(P) \geq d'))(w^*) < \text{the-MAX}_{\text{inf}}(\lambda d''. \lambda w''. \text{IMPORTANCE}_{w''}(P) \geq d'')(w^*)]$$

‘There is no world compatible with what is permitted where the maximally informative estimated degree of importance for  $P$  is less than the actual importance of  $P$ .’

These truth conditions require that no world compatible with our permissions be one in which the position’s importance is less than the estimates. However, this doesn’t correspond to the intended reading of our running example. All (27) says is that it is not permissible to give an estimate of the position’s importance that is less than its true importance in the world in question. This is compatible with a situation in which the position’s importance is actually quite low; we just aren’t allowed to provide an even lower estimate! While this is a possible interpretation of our sentence, an interpretation it shares with one interpretation of (18), derived in (20), it is not the same interpretation as the intended reading. The intended reading expresses a high degree of importance, and would be false in the aforementioned scenario.

Our approach, on the other hand, derives the high degree inference associated with the intended reading straightforwardly. What’s more, our approach also explains the intuitive equivalence in (26): As discussed in the previous section, English *cannot* only permits wide scope of negation with respect to the possibility modal, thus rendering it equivalent to *must not*, which is only compatible with narrow scope of negation with respect to the universal modal. *Should not* exhibits the same scopal properties as *must not*, and we therefore expect to be able to get the intended reading using *should not* instead of *cannot*. In sum, the intended reading falls out from our approach with modals encoding NOT » CAN and SHOULD » NOT precisely because they are equivalent, and exhibit the same interaction with the scope of abstraction over degrees and the maximal informativity operator.

## 6 Concluding remarks

In this paper, we have shown that the intended interpretation of under-over sentences is not a case of semantic illusion. Rather, the interpretation can be derived in a principled, compositional fashion from the interaction of independently motivated phenomena, namely the scope of a degree quantifier (*under-*) with respect to a negated modal on the one hand, and maximal informativity operating on the resulting upward-entailing degree set on the other. The intended reading is, contra the perception in the popular press not “perplexing” but the result of scope ambiguity. In this way, the interpretations of under-over sentences are very similar to that of depth-charge sentences, and have essentially the same underlying explanation: The different available interpretations arise from the the varying monotonicity of the set of degrees under consideration (Hohaus & Bade 2022). Taken together, our work, along with Hohaus & Bade 2022, demonstrates the usefulness of rigorous formal analysis in providing systematic grammar-based explanations for phenomena that appear at first blush to be illusory and processing-related.

There are many avenues for future research to consider. For one, the prefixes *under-* and *over-* have received very little semantic attention themselves, despite appearing productively with a great many verbal and adjectival roots. Future work may focus on the sorts of expressions that may appear with these prefixes, and what properties such expressions have in common.

Another area for future research concerns *over-*prefixation. Note that the sentence in (28) does not give rise to the alleged under-over illusions.

(28) The importance of this position cannot be overestimated.

A plausible analysis of the meaning of *overestimated* would be to adopt the same decomposition we had for *underestimated*, but reverse the direction of comparison, as in the analysis of *over-* below. The inverse scope reading, corresponding to the unavailable interpretation, would correspond to the expression in (30).

(29)  $\llbracket \textit{over-} \rrbracket = \lambda D'_{\langle d, \langle s, t \rangle \rangle} . \lambda D_{\langle d, \langle s, t \rangle \rangle} . \lambda w . \text{the-MAX}_{\text{inf}}(D)(w) > \text{the-MAX}_{\text{inf}}(D')(w)$

(30) Truth conditions for  $\#(\textit{over-DegP} \gg \text{CANNOT})$ :

$\text{the-MAX}_{\text{inf}}(\lambda d . \lambda w . \neg \exists w^* [w^* \text{ is accessible from } w \ \& \ d \in \text{ESTIMATE}_{w^*}(\lambda d' . \lambda w' . \text{IMPORTANCE}_{w'}(P) \geq d')])(w_{@})$   
 $> \text{the-MAX}_{\text{inf}}(\lambda d'' . \lambda w'' . \text{IMPORTANCE}_{w''}(P) \geq d'')(w_{@})$

‘The maximally informative degree that *P* lacks in importance across worlds exceeds the actual importance of *P*.’

In words, this says that the position’s actual importance is less than the estimated importance that it lacks in all possible worlds. While in the case of *underestimated*

this places the position's importance very high in the actual world, it is not clear what we can conclude about the position's importance from the truth conditions in (30). One possibility, then, is that the inverse scope reading of the modal with respect to *over-* is unavailable because it is uninformative. While this may be a promising intuition to follow, we would need to be more precise about the way in which (30) is uninformative, and whether this reading can nevertheless be detected in certain controlled circumstances. We leave a thorough investigation of this and other possibilities to future research.

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