

OPTIMIZING MEDICAL DEVICE MANUFACTURING INVENTORY MODEL FOR DETERIORATING ITEMS IN TWO WAREHOUSES USING FIFO POLICY

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KEYWORDS

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ABSTRACT:

The Medical Device Manufacturing's two-warehouse inventory model for spoiled items in the event of shortages and inflation under the FIFO shipping policy addresses the challenges of inventory management in the Medical Device Manufacturing. This mathematical model combines theories of inventory management, spoilage, shortages and inflation to optimize inventory decisions. The model is based on the FIFO principle to ensure that the oldest inventory is shipped first, minimizing the risk of spoilage. Dairy expiration rates are considered to determine optimal restocking and shipping policies. The model also takes into account the possibility of bottlenecks and aims to minimize them through effective inventory management. In addition, inflation is taken into account to reflect changes in the economic environment and its impact on inventory costs. By applying mathematical techniques and optimization algorithms, the model helps to minimize total inventory costs and improve operational efficiency in the Medical Device Manufacturing. This summary provides an overview of the key elements and theoretical underpinnings of the Medical Device Manufacturing's two-store inventory model and highlights its importance in improving inventory management practices.

1. Introduction and related work

Effective inventory management is vital for the Medical Device Manufacturing to ensure product quality, minimize costs and meet customer demand. However, inventory management in the Medical Device Manufacturing poses unique challenges due to factors such as the expiration of perishables, the possibility of shortages, and the impact of inflation on costs. To address these challenges, the Medical Device Manufacturing's inventory model includes two warehouses for deteriorating items. congestion and inflation as part of the FIFO shipping policy, a systematic approach to optimizing inventory decisions. The model draws on the theories of inventory management, decomposition, shortages and inflation to provide a comprehensive framework for effective inventory management in the Medical Device Manufacturing. By integrating these theories, the model aims to find a balance between maintaining optimal inventory levels, minimizing the risk of product deterioration, reducing shortages and controlling costs in the face of inflation.

The FIFO shipping policy, which ensures that the oldest stock is shipped first, is introduced to prioritize the use of products with a shorter remaining shelf life. This policy reduces the risk of product spoilage and waste and improves overall product quality and customer satisfaction.

In addition, the model takes into account the spoilage of dairy products and their specific spoilage rates. By considering product shelf life and incorporating restocking and shipping policies accordingly, the model enables companies to optimize inventory turnover and minimize spoilage losses.

Bottlenecks in the Medical Device Manufacturing can lead to lost sales and dissatisfied customers. The model solves this problem by optimizing stock levels and reordering decisions to reduce

occurrences of stock-outs and improve customer service. By minimizing bottlenecks, companies can improve customer satisfaction and gain a competitive advantage in the marketplace.

In addition, the model takes into account the impact of inflation on inventory costs. By factoring in inflation, businesses can make informed decisions on pricing, order quantities, and inventory costs to adjust to changing economic conditions and maintain profitability.

The Medical Device Manufacturing Two-Warehouse Inventory Model for Spoiled Items with Shortages and Inflation under the FIFO Shipping Policy provides a theoretical foundation and practical guidance for dairy companies to optimize their inventory management practices. By using mathematical techniques and optimization algorithms, companies can increase operational efficiency, reduce costs, and improve overall business performance in the dynamic Medical Device Manufacturing.

However, there are a number of things whose significance does not remain the same over time. The deterioration of these substances plays an important role and cannot be stored for long {Yadav, et. al. (1 to 10)}. Deterioration of an object can be described as deterioration, evaporation, obsolescence and loss of use or limit of an object, resulting in lower stock consumption compared to natural conditions. When commodities are placed in stock as inventory to meet future needs, there may be deterioration of items in the system of arithmetic that may occur for one or more reasons, etc. Storage conditions, weather or humidity. {Yadav, et. al. (11 to 20)}. Inach it is generally claimed that management owns a warehouse to store purchased inventory. However, management can, for a variety of reasons, buy or give more than it can store in its warehouse and name it OW, with an additional number in a rented warehouse called RW located near OW or slightly away from it {Yadav, et. al. (21 to 53)}. Inventory costs (including holding costs and depreciation costs) in RW are usually higher than OW costs due to additional costs of handling, equipment maintenance, etc. To reduce the cost of inventory will economically use RW products as soon as possible. Actual customer service is provided only by OW, and in order to reduce costs, RW stocks are first cleaned. Such arithmetic examples are called two arithmetic examples in the warehouse {Yadav and swami. (54 to 61)}. Management of supply of electronic storage devices and integration of environment and nervous networks {Yadav and Kumar (62)}. Analysis of seven supply chain management measures in improving the inventory of electronic devices for storage by sending an economic load using GA and PSO and analysis of supply chain management in improving the inventory of storage and equipment using genetic calculation and model design and analysis of chain inventory from bi warehouse and economic difficulty of freight transport using genetic calculation {Yadav, AS (63, 64, 65)}. Inventory policies of inventory and inventory requirements and different storage costs under allowable payments and inventory delays An example of depreciation of goods and services of various types and costs of holding down a Business-Loan and an inventory model with sensitive needs of prices, holding costs in contrast to loans of business expenses under inflation {Swami, et. al. (66, 67, 68)}. The objectives of the Multiple Objective Genetic Algorithm and PSO, which include the improvement of supply and deficit inventory, inflation, and a calculation model based on a genetic calculation of scarcity and low inflation by PSO {Gupta, et. al. (69, 70,)}). An example with two warehouses depreciation of items and storage costs under particle upgrade and an example with two warehouses of material damage and storage costs in inflation and soft computer techniques {Singh, et. al. (71, 72)}. Delayed alcohol supply management and refinement of particles and green cement supply system and inflation using particle enhancement and electronic inventory calculation system and distribution center using genetic calculations {Kumar, et. al. (73, 74,75)}. Example of depreciation inventory with two warehouses and stock-based stocks using a genetic inventory and vehicle inventory system for demand and inflation of stocks with two distribution centers using genetic inventory {Chauhan and Yadav (76 , 77)}. Marble Analysis Improvement of industrial reserves

based on genetic engineering and multi-particle improvement {Pandey, et. al. (78)}. White wine industry in supply chain management using nervous networks {Ahlawat, et. al. (79)}. Best policy for importing damaged items immediately and payment of conditional delays under the supervision of two warehouses {Singh, et. al. (80)}. The research by {Yadav et al. (81)} centers on improving inventory management for perishable commodities through the lens of green technology investments, considering factors such as selling price, carbon emissions, and time-sensitive demand. In another analysis, {Yadav, Yadav, and Bansal (82)} utilize an interval number technique to explore a two-warehouse inventory management model for perishable goods, addressing demand and cost uncertainties. Their optimization methods highlight how investing in preservation technology can reduce waste and enhance inventory efficiency. Focusing on a two-warehouse approach to optimize inventory levels, {Yadav, Yadav, and Bansal (83)} present a model that addresses the deterioration of goods during storage, emphasizing the importance of managing degradation costs to improve overall inventory performance. In order to maximize inventory levels while lowering expenses related to degradation and backlog, {Negi and Singh (86)} investigate an inventory system that controls uncertainty, takes into account time-varying demand patterns, and assesses inventory control tactics in the event of stockouts.

2. Assumptions and Notations:

In developing the mathematical model of the Medical Device Manufacturing inventory system the following assumptions are being made:

1. A single item is considered over a prescribed period T units of time.
2. The Medical Device Manufacturing demand rate $D(t)$ at time t is deterministic and taken as a ramp

type function of time i.e. $D(t) = (\beta_0 + \beta) e^{-\beta_2 \{t - (t_\beta + t_F)\} H[t - (t_\beta + t_F)]}$, $\beta > 0$, $\beta_0 > 0$, $\beta_2 > 0$ where

$$H[t - (t_\beta + t_F)] \text{ is the Heaviside's function defined as } H[t - (t_\beta + t_F)] = \begin{cases} 0, & t < (t_\beta + t_F) \\ 1, & t \geq (t_\beta + t_F) \end{cases}$$

3. The replenishment rate is infinite and lead-time is zero.
4. When the demand for goods is more than the supply. Shortages will occur. Customers encountering shortages will either wait for the vender to reorder (backlogging cost involved) or go to other vendors (lost sales cost involved). In this model shortages are allowed and the backlogging rate is $\exp(-\beta_3 t)$, when Medical Device Manufacturing inventory is in shortage. The backlogging parameter β_3 is a positive constant.
5. The variable rate of deterioration in both Medical Device Manufacturing warehouse is taken as $\beta_1(t) = \beta_1 t$. Where $0 < \beta_1 \ll 1$ and only applied to on hand Medical Device Manufacturing inventory.
6. No replacement or repair of deteriorated items is made during a given cycle.
7. The Medical Device Manufacturing owned warehouse (OW) has a fixed capacity of W units; the Medical Device Manufacturing rented warehouse (RW) has unlimited capacity.
8. The goods of Medical Device Manufacturing OW are consumed only after consuming the goods kept in Medical Device Manufacturing RW.

In addition, the following notations are used throughout this paper:

$\Pi_{ow}^{fifo}(t)$ The Medical Device Manufacturing inventory level in OW at any time t .

$\Pi_{rw}^{fifo}(t)$ The Medical Device Manufacturing inventory level in RW at any time t .

β_w The capacity of the Medical Device Manufacturing own warehouse.

Q The Medical Device Manufacturing ordering quantity per cycle.

- T Planning horizon.
- β_4 Inflation rate.
- β_{hcow} The holding cost per unit per unit time in Medical Device Manufacturing OW.
- β_{hcrw} The holding cost per unit per unit time in Medical Device Manufacturing RW. where $C_1 < C_2$
- β_{dc} The deterioration cost per unit.
- β_{sc} The shortage cost per unit per unit time.
- β_{opc} The opportunity cost due to lost sales.
- β_{OC} The replenishment cost per order.

3. Formulation and Solution of the Model:

The Medical Device Manufacturing inventory levels at OW are governed by the following differential equations:

$$\frac{d\Pi_{ow}^{fifo}(t)}{dt} = \left[-\beta_1(t) I^{fifo}(t) \right] \quad 0 \leq t < (t_\beta + t_F) \quad (1)$$

$$\frac{d\Pi_{ow}^{fifo}(t)}{dt} + \beta_1(t) I^{fifo}(t) = -(\beta_0 + \beta) e^{-\beta_2(t_\beta + t_F)}, \quad (t_\beta + t_F) \leq t \leq (t_1 + t_F) \quad (2)$$

And

$$\left[\frac{d\Pi_{ow}^{fifo}(t)}{dt} \right] = \left[-(\beta_0 + \beta) e^{-\beta_2(t_\beta + t_F)} e^{-\beta_3 t} \right] \quad (t_1 + t_F) \leq t \leq (T + t_F) \quad (3)$$

with the boundary conditions,

$$\Pi_{ow}^{fifo}(0) = \beta_w \text{ and } \Pi_{ow}^{fifo}(t_1 + t_F) = 0 \quad (4)$$

The solutions of equations (1), (2) and (3) are given by

$$\Pi_{ow}^{fifo}(t) = \beta_w e^{-\beta_1 t^2 / 2}, \quad 0 \leq t < (t_\beta + t_F) \quad (5)$$

$$\Pi_{ow}^{lifo}(t) = \left[(\beta_0 + \beta) e^{-\beta_2(t_\beta + t_F)} \left\{ \frac{\{(t_1 + t_F) - t\} + \frac{\beta_1(t_1 + t_F)^3 - t^3}{6}}{\beta_1(t_1 + t_F)^3 - t^3} \right\} e^{-\beta_1 t^2 / 2} \right], \quad (t_\beta + t_F) \leq t \leq (t_1 + t_F) \quad (6)$$

$$\text{And } \Pi_{ow}^{fifo}(t) = \left[\frac{(\beta_0 + \beta)}{\beta_3} e^{-\beta_2(t_\beta + t_F)} \left\{ e^{-\beta_3 t} - e^{-\beta_3(t_1 + t_F)} \right\} \right] \quad (t_1 + t_F) \leq t \leq (T + t_F) \quad (7)$$

respectively.

The Medical Device Manufacturing inventory level at RW is governed by the following differential equations:

$$\frac{d\Pi_{rw}^{fifo}(t)}{dt} + \beta_1(t) \Pi_{ow}^{fifo}(t) = -(\beta_0 + \beta) e^{-\beta_2 t}, \quad 0 \leq t < (t_\beta + t_F) \quad (8)$$

With the boundary condition $\Pi_{rw}^{fifo}(0) = 0$ the solution of the equation (8) is

$$\Pi_{rw}^{fifo}(t) = \left[(\beta_0 + \beta) \left\{ \frac{\beta_2}{2} \left((t_\beta + t_F)^2 - t^2 \right) + \frac{\beta_1}{6} \left((t_1 + t_F)^3 - t^3 \right) \right\} e^{-\beta_1 t^2 / 2} \right] \quad (t_\beta + t_F) \leq t \leq (t_1 + t_F) \quad (9)$$

Due to continuity of $\Pi_{ow}^{Fifo}(t)$ at point $t = (t_\beta + t_F)$ it follows from equations (5) and (6), one has

$$\beta_w e^{-\beta_1 (t_\beta + t_F)^2 / 2} = \left[(\beta_0 + \beta) e^{-\beta_2 (t_\beta + t_F)} \left\{ \frac{\left\{ (t_1 + t_F) - (t_\beta + t_F) \right\} + \beta_1 \left\{ (t_1 + t_F)^3 - (t_\beta + t_F)^3 \right\}}{6} \right\} e^{-\beta_1 (t_\beta + t_F)^2 / 2} \right] \quad (10)$$

The total average cost consists of following elements:

- (i) Ordering cost per cycle = β_{oc} (11)
- (ii) Holding cost per cycle in Medical Device Manufacturing OW

$$C_{HO} = \left[\beta_{hcow} \left\{ \int_0^{(t_\beta + t_F)} \Pi_{ow}^{fifo}(t) e^{-\beta_4 t} dt + \int_{(t_\beta + t_F)}^{(t_1 + t_F)} \Pi_{ow}^{fifo}(t) e^{-\beta_4 \{(t_\beta + t_F) + t\}} dt \right\} \right]$$

Appendix A (12)

- (iii) Holding cost per cycle (C_{HR}) in RW

$$C_{HR} = \left[\beta_{hcrw} \left\{ \int_0^{(t_\beta + t_F)} \Pi_{rw}^{fifo}(t) e^{-\beta_4 t} dt \right\} \right]$$

$$C_{HR} = \beta_{hcrw} (\beta_0 + \beta) \left\{ \begin{aligned} & \left[\frac{(t_\beta + t_F)^2}{2} - \frac{(3\beta_2 + \beta_4)}{6} (t_\beta + t_F)^3 + \right. \\ & \left. \left(\frac{\beta_1}{12} + \frac{\beta_2\beta_4}{8} \right) (t_\beta + t_F)^4 - \right. \\ & \left. \left(\frac{\beta_4\beta_1}{20} - \frac{\beta_2\beta_1}{30} \right) (t_\beta + t_F)^5 \right] \end{aligned} \right\} \quad (13)$$

(iv) Cost of deteriorated units per cycle (C_D)

$$= \beta_{dc} \left\{ \begin{aligned} & \int_0^{(t_\beta + t_F)} \beta_1 t \Pi_{rw}^{fifo}(t) e^{-\beta_4 t} dt + \\ & \int_0^{(t_\beta + t_F)} \beta_1 t \Pi_{ow}^{fifo}(t) e^{-\beta_4 t} dt + \\ & \int_{(t_1 + t_F)}^{(t_1 + t_F)} \beta_1 t \Pi_{ow}^{fifo}(t) e^{-\beta_4 \{t + (t_\beta + t_F)\}} dt \end{aligned} \right\}$$

Appendix B (14)

(v) Shortage cost per cycle (C_S)

$$= \beta_{sc} \left[\int_{(t_1 + t_F)}^{(T + t_F)} -\Pi_{ow}^{fifo}(t) e^{-\beta_4 \{(t_1 + t_F) + t\}} dt \right]$$

$$= \frac{-(\beta_0 + \beta) \beta_{sc} e^{-[\beta_4(t_1 + t_F) + \beta_2(t_\beta + t_F)]}}{\beta_3} \left[\int_{(t_1 + t_F)}^{(T + t_F)} e^{-(\beta_4 + \beta_3)t} dt - e^{-\beta_3(t_1 + t_F)} \int_{(t_1 + t_F)}^{(T + t_F)} e^{-\beta_4 t} dt \right]$$

$$= \left[\frac{(\beta_0 + \beta) \beta_{sc} e^{-[\beta_4(t_1 + t_F) + \beta_2(t_\beta + t_F)]}}{\beta_3 \beta_4 (\beta_3 + \beta_4)} \left\{ \begin{aligned} & \beta_3 e^{-(\beta_3 + \beta_4)(t_1 + t_F)} + \\ & e^{-\beta_4(T + t_F)} \left\{ \begin{aligned} & r e^{-\beta_3(T + t_F)} - \\ & (\beta_3 + \beta_4) e^{-\beta_3(t_1 + t_F)} \end{aligned} \right\} \end{aligned} \right\} \right] \quad (15)$$

(vi) Opportunity cost due to lost sales per cycle (C_0)

$$= \beta_{opc} \int_{(t_1 + t_F)}^{(T + t_F)} (\beta_0 + \beta) (1 - e^{-\beta_3 t}) e^{-\beta_2(t_\beta + t_F)} e^{-\beta_4 \{(t_1 + t_F) + t\}} dt$$

$$= \left[\frac{\beta_{opc} (\beta_0 + \beta) e^{-\{\beta_2(t_\beta + t_F) + \beta_4(t_1 + t_F)\}}}{\beta_4 (\beta_3 + \beta_4)} \left\{ \begin{aligned} & e^{-\beta_4(t_1 + t_F)} \left\{ (\beta_3 + \beta_4) - \beta_4 e^{-\beta_3(t_1 + t_F)} \right\} - \\ & e^{-\beta_4(T + t_F)} \left\{ (\beta_3 + \beta_4) - \beta_4 e^{-\beta_3(T + t_F)} \right\} \end{aligned} \right\} \right] \quad (16)$$

Therefore, the total average cost per unit time of our model is obtained as follows

$$TC(t_1 + t_F, T + t_F) = \frac{1}{(T + t_F)} \left[\begin{array}{l} \text{Ordering cost + Holding cost in OW} \\ + \text{Holding cost in RW + Deterioration cost} \\ + \text{Shortage cost + Opportunity cost} \end{array} \right] = \frac{R(t_1, T)}{T} \quad (17)$$

Appendix C (18)

4. Optimization Methodology

To minimize the total cost per unit time, the optimal values of t_1 and T can be obtained by solving the following equations simultaneously

$$\frac{\partial TC}{\partial (t_1 + t_F)} = 0 \quad (19)$$

$$\text{and } \frac{\partial TC}{\partial (T + t_F)} = 0 \quad (20)$$

provided, they satisfy the following conditions

$$\frac{\partial^2 TC}{\partial (t_1 + t_F)^2} > 0, \frac{\partial^2 TC}{\partial (T + t_F)^2} > 0$$

$$\text{and } \left(\frac{\partial^2 TC}{\partial (t_1 + t_F)^2} \right) \left(\frac{\partial^2 TC}{\partial (T + t_F)^2} \right) - \left(\frac{\partial^2 TC}{\partial (t_1 + t_F) \partial (T + t_F)} \right)^2 > 0 \quad (21)$$

The equation (19) is equivalent to the following equation

$$\frac{\partial TC}{\partial(t_1+t_F)} = \frac{(\beta_0 + \beta)}{(T+t_F)} \left[\beta_{dc}\beta_1 e^{-\beta_2\left(\frac{t_\beta+t_F}{+\beta_4}\right)} \left\{ \left(\frac{(t_1+t_F) - \frac{\beta_4(t_1+t_F)^2}{2} + \frac{\beta_1(t_1+t_F)^3}{3} - \frac{\beta_4\beta_1(t_1+t_F)^4}{4}}{(t_\beta+t_F) - 3\beta_1(t_\beta+t_F)(t_1+t_\beta F)^2 + \frac{\beta_4(t_\beta+t_F)^2}{2} + \frac{\beta_4\beta_1(t_\beta+t_F)^2(t_1+t_F)^2}{2} + \frac{\beta_1(t_\beta+t_F)^3}{6}} \right) \right\} + \left\{ \left(\frac{\frac{(t_1+t_F)^2}{2} - \frac{\beta_4(t_1+t_F)^3}{3} + \frac{\beta_1(t_1+t_F)^4}{8} - \frac{\beta_4\beta_1(t_1+t_F)^5}{6}}{\frac{(t_\beta+t_F)^2}{2} - \frac{\beta_1(t_\beta+t_F)^2(t_1+t_F)^2}{4} - \frac{\beta_4(t_\beta+t_F)^3}{3} - \frac{\beta_4\beta_1(t_\beta+t_F)^3(t_1+t_F)^2}{6} + \frac{\beta_1(t_\beta+t_F)^4}{8}} \right) \right\} + \frac{\beta_{sc} e^{-\beta_2(t_\beta+t_F)}}{\beta_3\beta_4(\beta_3+\beta_4)} \left\{ \left(\frac{-\beta_3(\beta_3+2\beta_4)e^{-(\beta_3+2\beta_4)(t_1+t_F)} - \beta_4^2 e^{-\beta_4(t_1+t_F) - (\beta_4+\beta_3)(T+t_F)} + (\beta_3+\beta_4)^2 e^{-(\beta_4+\beta_3)(t_1+t_F) - \beta_4(T+t_F)}} \right) \right\} + \frac{\beta_{opc} e^{-\beta_2(t_\beta+t_F)}}{\beta_4(\beta_3+\beta_4)} \left\{ \left(\frac{-2\beta_4(\beta_3+\beta_4)e^{-2\beta_4(t_1+t_F)} + \beta_4(\beta_3+2\beta_4)e^{-(\beta_3+2\beta_4)(t_1+t_F)} + \beta_4 e^{-\beta_4(t_1+t_F)} \left(\frac{(\beta_3+\beta_4)e^{-\beta_4(T+t_F)}}{\beta_4 e^{-(\beta_3+\beta_4)(T+t_F)}} \right) \right) \right\} \right] = 0 \tag{22}$$

Also equation (20) is equivalent to

$$\left[R - \frac{(\beta_0 + \beta)}{\beta_3} \left\{ \frac{e^{-\beta_4(T+t_F)} - e^{-(\beta_4+\beta_3)(T+t_F)}}{+\beta_2\left(\frac{t_\beta+t_F}{+\beta_4}\right)} \right\} \left(\beta_{sc} + \beta_{opc}\beta_3 \right) \right] = 0 \tag{23}$$

Equations (22) and (23) are highly nonlinear equations. Therefore, numerical solution of these equations can be obtained by using the software **MATLAB 7.0.1**.

5. Numerical Illustration

To illustrate the model numerically the following parameter values are considered.

$$\beta_1 = 0.0022 \text{ unit}, \beta = 702 \text{ units}, \beta_0 = 502 \text{ units}, \beta_2 = 0.22 \text{ unit}, \beta_3 = 0.12 \text{ unit}, \beta_4 = 0.052 \text{ unit},$$

$$t_\beta = 0.12 \text{ year}, \beta_{OC} = \text{Rs. } 1002 \text{ per order}, \beta_{hcow} = \text{Rs. } 4.02 \text{ per unit per year},$$

$$\beta_{hcrw} = \text{Rs. } 20.0 \text{ per unit}, T = 12 \text{ year}, \beta_{sc} = \text{Rs. } 22.0 \text{ per unit per year}, \beta_{opc} = \text{Rs. } 42.0 \text{ per unit}$$

Then for the minimization of total average cost and with help of software. the optimal policy can be obtained such as: $t_1 = 0.399224$ year, $S = 28.597235$ units and $TC = \text{Rs. } 658.115354$ per year.

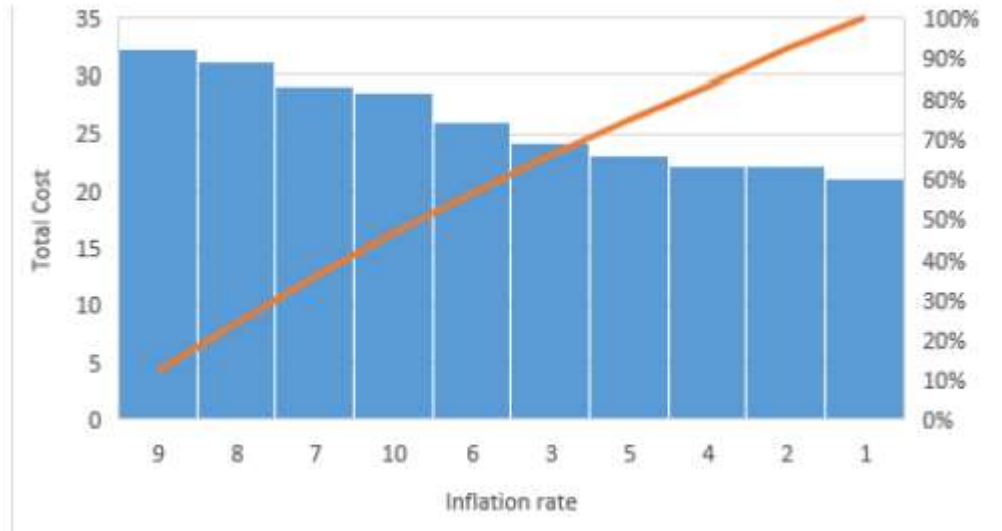


Figure 1: Variation between total cost and inflation rate.

A comprehensive grasp of the dynamics within the two-warehouse degrading inventory model is provided by the graphical representations in this section. The way that changes in inflation impact overall operating and production costs is demonstrated by the difference between the inflation rate and total cost. As inflation increases, higher labor, raw material, and service prices typically translate into higher overall costs (figure 1).

6. Conclusion

In summary, the Medical Device Manufacturing's two-warehouse inventory model for shortage and inflation spoiled items under the FIFO shipping policy provides a comprehensive framework for optimizing inventory management in the Medical Device Manufacturing. Medical Device Manufacturing. By incorporating theories of inventory management, spoilage, shortages and inflation, the model enables dairy companies to make informed decisions regarding inventory replenishment, shipping and cost optimization.

The model recognizes the perishability of dairy products and takes into account their rate of decomposition to minimize spoilage losses. It also addresses potential shortages and aims to balance stock levels with customer demand to avoid stock-outs and lost sales. Additionally, the model includes the FIFO shipping policy, which ensures that products with a shorter remaining shelf life are shipped first to reduce the risk of product wastage.

The model also helps companies adapt to changing economic conditions and make profitable inventory decisions by considering inflation and its impact on various cost elements. By optimizing inventory levels, order quantities, and shipping policies, the model aims to minimize overall inventory costs while maintaining product quality and customer satisfaction.

The Dual Warehouse Inventory Model for the Medical Device Manufacturing provides dairy companies with a valuable tool to improve their inventory management practices, increase

operational efficiency and improve profitability. Through the use of mathematical techniques and optimization algorithms, the model supports data-driven decision making and allows companies to better control their inventory systems.

Overall, the model contributes to the advancement of inventory management in the Medical Device Manufacturing, providing a theoretical basis and practical guidance for effective inventory control, mitigating losses and meeting the demand of customers in a dynamic and competitive business environment.

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Appendix A

$$C_{HO} = \beta_{hcow} \left\{ \left(\beta_w \left\{ \left(t_\beta + t_F \right) - \frac{\beta_4 (t_\beta + t_F)^2}{2} - \frac{\beta_1 (t_\beta + t_F)^3}{6} \right\} + \right. \right. \\
 \left. \left. (\beta_0 + \beta) e^{-\beta_2((t_\beta + t_F) + \beta_4)} \left\{ \begin{aligned} & \frac{(t_1 + t_F)^2}{2} - \frac{\beta_4 (t_1 + t_F)^3}{6} + \\ & \frac{\beta_1 (t_1 + t_F)^4}{12} - \frac{\beta_4 \beta_1 (t_1 + t_F)^5}{20} - \\ & \frac{(t_\beta + t_F)}{2} (2(t_1 + t_F) - (t_\beta + t_F)) - \\ & \frac{\beta_1 (t_\beta + t_F)}{24} (4(t_1 + t_F)^3 - (t_\beta + t_F)^3) + \\ & \frac{\beta_4 (t_\beta + t_F)^2}{6} (3(t_1 + t_F) - 2(t_\beta + t_F)) + \\ & \frac{\beta_4 \beta_1 (t_\beta + t_F)^2}{30} (5(t_1 + t_F)^3 - 3(t_\beta + t_F)^3) + \\ & \frac{\beta_1 (t_\beta + t_F)^3}{24} (4(t_1 + t_F) - 3(t_\beta + t_F)) \end{aligned} \right\} \right) \right\} \quad (12)$$

Appendix B

$$\begin{aligned}
 &= \beta_{dc}\beta_1 \left\{ (\beta_0 + \beta) \left[\begin{aligned} &\left(\frac{1}{6}(t_\beta + t_F)^3 - \right. \\ &\left. \left(\frac{\beta_2}{4} + \frac{\beta_4}{12} \right) (t_\beta + t_F)^4 + \right. \\ &\left. \left(\frac{\beta_1}{40} + \frac{\beta_4\beta_2}{15} \right) (t_\beta + t_F)^5 - \right. \\ &\left. \left. \left(\frac{\beta_4\beta_1}{36} - \frac{\beta_2\beta_1}{24} \right) (t_\beta + t_F)^6 \right) \right] + \right. \\ &\left. \beta_W \left(\frac{(t_\beta + t_F)^2}{2} - \frac{\beta_4(t_\beta + t_F)^3}{3} - \frac{\beta_1(t_\beta + t_F)^4}{8} \right) \right\} + \\ &\left. \left\{ (\beta_0 + \beta) e^{-(t_\beta + t_F)(\beta_2 + \beta_4)} \left[\begin{aligned} &\frac{(t_1 + t_F)^3}{6} - \frac{\beta_4(t_1 + t_F)^4}{12} + \\ &\frac{\beta_1(t_1 + t_F)^5}{40} - \frac{\beta_4\beta_1(t_1 + t_F)^6}{36} - \\ &\frac{(t_\beta + t_F)^2}{6} (3(t_1 + t_F) - 2(t_\beta + t_F)) - \\ &\frac{\beta_1(t_\beta + t_F)^2}{60} (5(t_1 + t_F)^3 - 2(t_\beta + t_F)^3) - \\ &\frac{\beta_4(t_\beta + t_F)^3}{12} (4(t_1 + t_F) - 3(t_\beta + t_F)) - \\ &\frac{\beta_4\beta_1(t_\beta + t_F)^3}{36} (2(t_1 + t_F)^3 - (t_\beta + t_F)^3) - \\ &\frac{\beta_1(t_\beta + t_F)^4}{40} (5(t_1 + t_F) - 4(t_\beta + t_F)) \end{aligned} \right] \right\} \quad (14)
 \end{aligned}$$

Appendix C

$$\begin{aligned}
 TC\left(\frac{t_1+t_F}{T+t_F}\right) &= \frac{1}{(T+t_F)} \left[\beta_{oc} + \beta_{hcow} \left\{ \beta_w \left\{ \left((t_\beta+t_F) - \frac{\beta_4(t_\beta+t_F)^2}{2} - \frac{\beta_1(t_\beta+t_F)^3}{6} \right) \right\} + \right. \right. \\
 &\quad \left. \left. \left(\frac{(t_1+t_F)^2}{2} - \frac{\beta_4(t_1+t_F)^3}{6} + \frac{\beta_1(t_1+t_F)^4}{12} - \frac{\beta_4\beta_1}{20}(t_1+t_F)^5 - \frac{(t_\beta+t_F)}{2} (2(t_1+t_F) - (t_\beta+t_F)) - \right. \right. \right. \\
 &\quad \left. \left. \left. \beta_0 e^{-\beta_2(t_\beta+t_F)+\beta_4} \left\{ \frac{\beta_1(t_\beta+t_F)}{24} (4(t_1+t_F)^3 - (t_\beta+t_F)^3) + \frac{\beta_4(t_\beta+t_F)^2}{6} (3(t_1+t_F) - 2(t_\beta+t_F)) + \right. \right. \right. \\
 &\quad \left. \left. \left. \frac{\beta_4\beta_1(t_\beta+t_F)^2}{30} (5(t_1+t_F)^3 - 3(t_\beta+t_F)^3) + \frac{\beta_1(t_\beta+t_F)^3}{24} (4(t_1+t_F) - 3(t_\beta+t_F)) \right\} \right. \right. \\
 &\quad \left. \left. + \beta_{hcnw} (\beta_0 + \beta) \left\{ \frac{(t_\beta+t_F)^2}{2} - \frac{(3\beta_2+\beta_4)(t_\beta+t_F)^3}{6} + \left(\frac{\beta_1}{12} + \frac{\beta_2\beta_4}{8} \right) (t_\beta+t_F)^4 - \right. \right. \right. \\
 &\quad \left. \left. \left. \left(\frac{\beta_4\beta_1}{20} - \frac{\beta_2\beta_1}{30} \right) (t_\beta+t_F)^5 \right\} \right. \right. \\
 &\quad \left. \left. \beta_{dc}\beta_1 \left\{ (\beta_0 + \beta) \left(\frac{1}{6}(t_\beta+t_F)^3 - \frac{\left(\frac{\beta_2}{4} + \frac{\beta_4}{12} \right) (t_\beta+t_F)^4}{4} + \right. \right. \right. \right. \\
 &\quad \left. \left. \left. \left(\frac{\beta_1}{40} + \frac{\beta_4\beta_2}{15} \right) (t_\beta+t_F)^5 - \left(\frac{\beta_4\beta_1}{36} - \frac{\beta_2\beta_1}{24} \right) (t_\beta+t_F)^6 \right) \right\} + \right. \\
 &\quad \left. \left. \beta_w \left(\frac{(t_\beta+t_F)^2}{2} - \frac{\beta_4(t_\beta+t_F)^3}{3} - \frac{\beta_1(t_\beta+t_F)^4}{8} \right) \right\} + \right. \\
 &\quad \left. \left. \left(\beta_0 + \beta \right) e^{-(t_\beta+t_F)(\beta_2+\beta_4)} \left(\frac{(t_1+t_F)^3}{6} - \frac{\beta_4(t_1+t_F)^4}{12} + \frac{\beta_1(t_1+t_F)^5}{40} - \frac{\beta_4\beta_1(t_1+t_F)^6}{36} - \right. \right. \right. \\
 &\quad \left. \left. \frac{(t_\beta+t_F)^2}{6} (3(t_1+t_F) - 2(t_\beta+t_F)) - \frac{\beta_1(t_\beta+t_F)^2}{60} (5(t_1+t_F)^3 - 2(t_\beta+t_F)^3) - \right. \right. \\
 &\quad \left. \left. \frac{\beta_4(t_\beta+t_F)^3}{12} (4(t_1+t_F) - 3(t_\beta+t_F)) - \frac{\beta_4\beta_1(t_\beta+t_F)^3}{36} (2(t_1+t_F)^3 - (t_\beta+t_F)^3) - \right. \right. \\
 &\quad \left. \left. \frac{\beta_1(t_\beta+t_F)^4}{40} (5(t_1+t_F) - 4(t_\beta+t_F)) \right) \right\} \right. \\
 &\quad \left. + \frac{(\beta_0 + \beta) \beta_{sc} e^{-\beta_4(t_1+t_F)+\beta_2(t_\beta+t_F)}}{\beta_3\beta_4(\beta_3 + \beta_4)} \left\{ \beta_3 e^{-\beta_3+\beta_4(t_1+t_F)+} \right. \right. \\
 &\quad \left. \left. e^{-\beta_4(T+t_F)} \left\{ r e^{-\beta_3(T+t_F)} - (\beta_3 + \beta_4) e^{-\beta_3(t_1+t_F)} \right\} \right\} \right. \\
 &\quad \left. + \frac{\beta_{opc} (\beta_0 + \beta) e^{-\beta_2(t_\beta+t_F)+\beta_4(t_1+t_F)}}{\beta_4(\beta_3 + \beta_4)} \left\{ e^{-\beta_4(t_1+t_F)} \left\{ (\beta_3 + \beta_4) - \beta_4 e^{-\beta_3(t_1+t_F)} \right\} - \right. \right. \\
 &\quad \left. \left. e^{-\beta_4(T+t_F)} \left\{ (\beta_3 + \beta_4) - \beta_4 e^{-\beta_3(T+t_F)} \right\} \right\} \right] = \frac{R(t_1, T)}{T} \tag{18}
 \end{aligned}$$