Mathematical Modelling to Analysis the Behaviour of Monkey Pox using Basic Reproduction Number

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KEYWORDS

Monkeypox Virus, Mathematical model, Routh Hurwitz Criteria, Jacobian Matrix.

ABSTRACT

Monkey pox (Mpox or MPX) is a rare viral zoonotic disease, which is related to the family of the smallpox virus. In epidemiology, the fundamental reproduction number R_0 is a key threshold parameter. Here using Jacobian matrix, we analysis the behaviour of mpox through the various mathematical models using eigen values and applying Routh Hurwitz criteria. This mathematical model also, explore various controlling factors and parameters affecting R_0. The finding of this improved epidemic model, we measure a monkey pox free state and optimal control strategy.

1.0 Introduction:

As the 2022 monkey pox outbreak expanded to areas where the illness was not previously prevalent, it becomes a major worldwide health problem. The zoonotic, double-stranded DNA virus known as monkey pox virus (MPXV) belongs to the Ortho poxvirus genus within the Poxviridae family. The cowpox virus (CPXV), vaccinia virus (VACV), and variola virus (VARV) are also members of this family. The WHO's most recent rapid risk assessment of the global mpox situation was carried out in November 2024. The risk is evaluated as follows using the facts at hand:

- **Clade Ib MPXV**: Mostly impacting areas in the Democratic Republic of the Congo and surrounding nations where mpox was not previously prevalent — **High**
- **Clade Ia MPXV**: mostly impacting areas where mpox is endemic in the Democratic Republic of the Congo. — **High**
- **Clade II MPXV**: Observed in Nigeria and other West and Central African endemic nations — **Moderate**
- **Clade IIb MPXV**: Linked to the worldwide mpox outbreak— **Moderate**

It can cause fever, rash, and swollen lymph nodes in humans. Some of the common symptoms are given below:

Figure 1: Symptoms of MPXV

This study develops and thoroughly analyzes a compartmental mathematical model of monkey-pox dynamics in the presence of many compartments. Ordinary differential equations regulate the recently suggested model, which explains how monkey-pox spreads throughout civilization. The stability analysis is examined using the threshold quantity for the monkey pox-free equilibrium. This study examines the sensitivity of the reproduction number. The dynamical behavior of nonlinear incidence rate epidemiological models was examined by Liu et al. (1987). The general incidence rate

$$g(I)S = \frac{kI^{p}S}{1+\alpha I^{q}}$$

was provided by Liu et al. (1987). Where kI represents the infection transmission rate of the disease and $\frac{1}{1+\alpha I^{q}}$ explains the psychological impact of suspects' altered behavior if there are a lot of infectious agents. Elsonbaty, A. et al. (2024) developed a theoretical solution with bounded behavior and talked about the nonlinear incidence rate for the monkey-pox virus. Alqahtani et al. (2023) talked on how various types of people interact with one another. The effects of particular parameters on the dynamics of the given model are quantified by numerical simulations.

2.0 Formation of Mathematical Model:

2.1 Define the Compartments

Identify the key compartments in the population that are relevant to the disease. Common compartments include:

- S: Individuals who are susceptible to the disease.
- I: Infectious individuals who have ability to spread the illness.
- R: Recovered individuals who have been no longer contagious or who have developed immunity.
- V: Vaccination class.

Establish the relationships between the compartments using differential equations. The flow between compartments is governed by factors such as the death rate, incubation time, recovery rate, and transmission rate.

2.2 Base Model: SIR (Susceptible-Infectious-Recovered) Model

The basic SIR model is one of the simplest compartmental models and it is suitable for infectious diseases where recovered individuals gain immunity. According to Kermack and McKendrick (1927), the governing equations are:

$$\frac{dS}{dt} = -\beta SI \qquad (1)$$

$$\frac{dI}{dt} = \beta SI - \gamma I \qquad (2)$$

$$\frac{dR}{dt} = \gamma I \qquad (3)$$

Where β represents rate at which transmission occur and γ signifies the rate of recovery.

The reproduction number R_0 is given by

$$R_0 = \frac{\beta}{\gamma} = 0.39 < 1 \qquad (4)$$

The characteristics equation of the given system is given by $|J_1 - \lambda I| = 0$;

$$|J_1 - \lambda I| = \begin{vmatrix} -\beta I - \lambda & -\beta S & 0 \\ \beta I & \beta S - \gamma - \lambda & 0 \\ 0 & \gamma & -\lambda \end{vmatrix}$$

That is $\lambda[\lambda^2 + \lambda(\beta I - \beta S + \gamma) + \beta I\gamma] = 0 \qquad (5)$

Which implies, $\lambda = 0, \frac{-(\beta I - \beta S + \gamma) \pm \sqrt{(\beta I - \beta S + \gamma)^2 - 4\beta I\gamma}}{2}$

Numerical Eigenvalues using MATLAB software:

$\lambda_1 = 0, \lambda_2 = 0.0650 - 0.1494i$ and $\lambda_3 = 0.0650 + 0.1494i$

because all of the eigenvalues are positive and zero. The system will therefore be stable.

Table 1: Depiction of various Parameters

S. No.	Symbols	Description of Parameters	Initial Values (WHO report 24)
1	Λ	Birth rate	12.6
2	β	Transmission rate	0.032
3	α	Cure rate of infected patient	10
4	γ	Recovery rate	0.83
5	μ	Death rate	7.473
6	ξ	Proportion of unsuccessful vaccination	0.1
7	ϑ	Proportion of receiving vaccination	0.1

Table 2: Depiction of all Variables

S. No.	Symbols	Description of Parameters	In India
1	S	Suspected compartment	31
2	I	Infected compartment	1
3	V	Vaccinated compartment	1
4	R	Recovered compartment	31
5	T	Incubation time	5 – 21 days

Graphical Analysis:

Using MATLAB, base model provides the graphically analysis for behaviour of monkey pox virus.

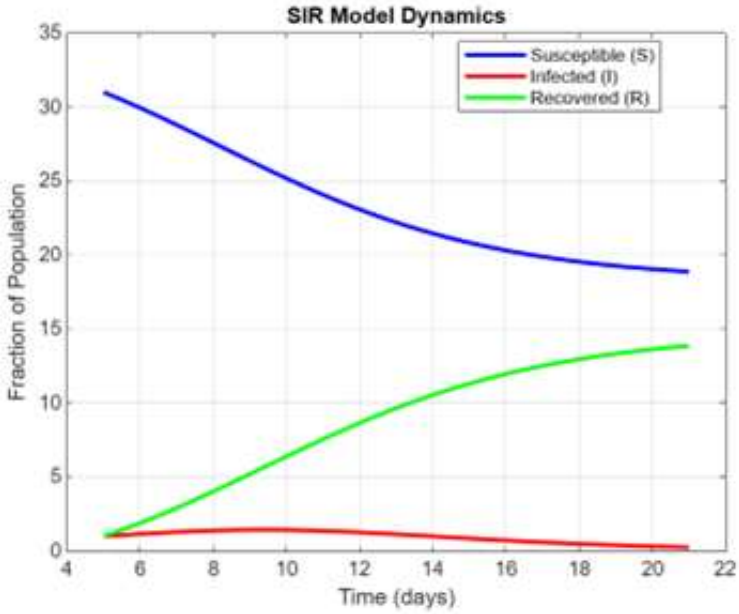

Figure 2: Model SIR

2.3 SIR (Susceptible-Infectious-Recovered) Model with Monod Haldane (M-H) function

The SIR model with Monod Haldane (M-H) function is given by the governing equations like:

$$\frac{dS}{dt} = \Lambda - \frac{\beta S I}{1+\alpha I^2} - \mu S \quad (6)$$

$$\frac{dI}{dt} = \frac{\beta S I}{1+\alpha I^2} - \gamma I - \mu I \quad (7)$$

$$\frac{dR}{dt} = \gamma I - \mu R \qquad (8)$$

The characteristics equation of the given system is given by $|J_2 - \lambda I| = 0$;

$$|J_2 - \lambda I| = \begin{vmatrix} \left(-\frac{\beta I}{1+\alpha I^2} - \mu - \lambda\right) & -\beta S\left(\frac{1-\alpha I^2}{(1+\alpha I^2)^2}\right) & 0 \\ \frac{\beta I}{1+\alpha I^2} & \beta S\left(\frac{1-\alpha I^2}{(1+\alpha I^2)^2}\right) - \gamma - \mu - \lambda & 0 \\ 0 & \gamma & -\mu - \lambda \end{vmatrix}$$

So,

$$(-\mu-\lambda)\left[\lambda^2 + \lambda\left(\frac{\beta I}{1+\alpha I^2} + 2\mu - \beta S\left(\frac{1-\alpha I^2}{(1+\alpha I^2)^2}\right) + \gamma\right) + \left\{\left(\frac{(\gamma+\mu)\beta I}{1+\alpha I^2}\right) + \mu^2 + \mu\gamma + \mu\beta S\left(\frac{1-\alpha I^2}{(1+\alpha I^2)^2}\right)\right\}\right] = 0 \qquad (9)$$

Numerical Eigenvalues using MATLAB software:

$\lambda_1 = -7.473$, $\lambda_2 = -0.2338 + 0.2400i$ and $\lambda_3 = -0.2338 - 0.2400i$

Since, the system will be stable as the value of the eigen values are negative.

The value of reproduction number R_0 is given as

$$R_0 = \frac{\beta}{\gamma}\frac{1}{(1+\mu/\gamma)}\frac{1}{(1+\alpha I^2)} = 0.0003 < 1 \qquad (10)$$

Where μ is the mortality rate.

In this compartmental model $\frac{\beta SI}{1+\alpha I^2}$ is the Monod Haldane (M-H) function. Also, the important terms as β = transmission rate, and α = cure rate of the infected people.

Graphical Analysis:

Using MATLAB, SIR model with M-H function provides the graphically analysis for behaviour of monkey pox virus.

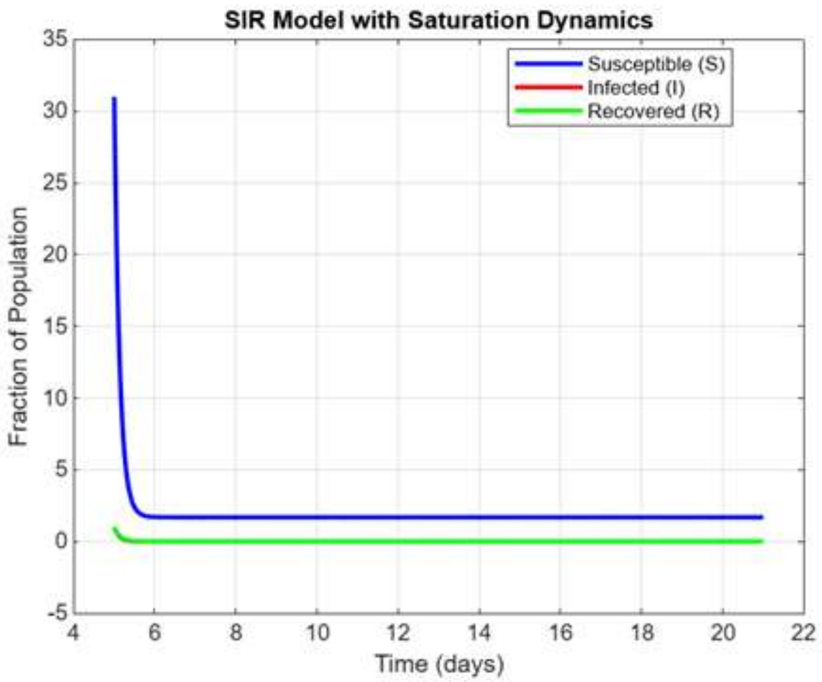

Figure 3: Modified SIR Model

2.4 SIVR (Susceptible-Infectious-Vaccinated-Recovered) Model

The SIVR model is given by the governing equations like:

$$\frac{dS}{dt} = \Lambda + \xi V - \frac{\beta SI}{1+\alpha I^2} - \nu S - \mu S \qquad (11)$$

$$\frac{dI}{dt} = \frac{\beta SI}{1+\alpha I^2} - \gamma I - -\nu I - \mu I \qquad (12)$$

$$\frac{dV}{dt} = \nu S - \xi V - \mu V \quad (13)$$
$$\frac{dR}{dt} = \gamma I + (1-\xi)V - \mu R \quad (14)$$
$$R_0 = \frac{\beta}{\gamma(1+\mu/\gamma+\nu/\gamma)(1+\alpha I^2)} = 0.0003 < 1 \quad (15)$$

Where μ is the mortality rate.

The characteristics equation of the given system is given by $|J_3 - \lambda I| = 0$;

$$|J_3 - \lambda I| = \begin{vmatrix} \left(-\frac{\beta I}{1+\alpha I^2} - \mu - \nu - \lambda\right) & -\beta S\left(\frac{1-\alpha I^2}{(1+\alpha I^2)^2}\right) & \xi & 0 \\ \frac{\beta I}{1+\alpha I^2} & \beta S\left(\frac{1-\alpha I^2}{(1+\alpha I^2)^2}\right) - \gamma - \mu - \nu - \lambda & 0 & 0 \\ \nu & 0 & -\xi - \mu - \lambda & 0 \\ 0 & \gamma & 1-\xi & -\mu - \lambda \end{vmatrix}$$

$$(\mu+\lambda)\left[\left(-\frac{\beta I}{1+\alpha I^2} - \mu - \nu - \lambda\right)\left(\beta S\left(\frac{1-\alpha I^2}{(1+\alpha I^2)^2}\right) - \gamma - \mu - \nu - \lambda\right)(\xi+\mu+\lambda) - \left(\frac{\beta^2 SI}{(1+\alpha I^2)^3}\right)(\xi+\mu+\lambda)(1-\alpha I^2) + \nu\xi\left(\beta S\left(\frac{1-\alpha I^2}{(1+\alpha I^2)^2}\right) - \gamma - \mu - \nu - \lambda\right)\right] = 0 \quad (16)$$

Numerical Eigenvalues using MATLAB software:

$\lambda_1 = -7.4756, \lambda_2 = -7.6250, \lambda_3 = -7.6951$ and $\lambda_4 = -7.473$

Since, the system will be stable as values of all the eigen values are negative.

Graphical Analysis:

Using MATLAB, improved SIR model with M-H function provides the graphically analysis for behaviour of monkey pox virus.

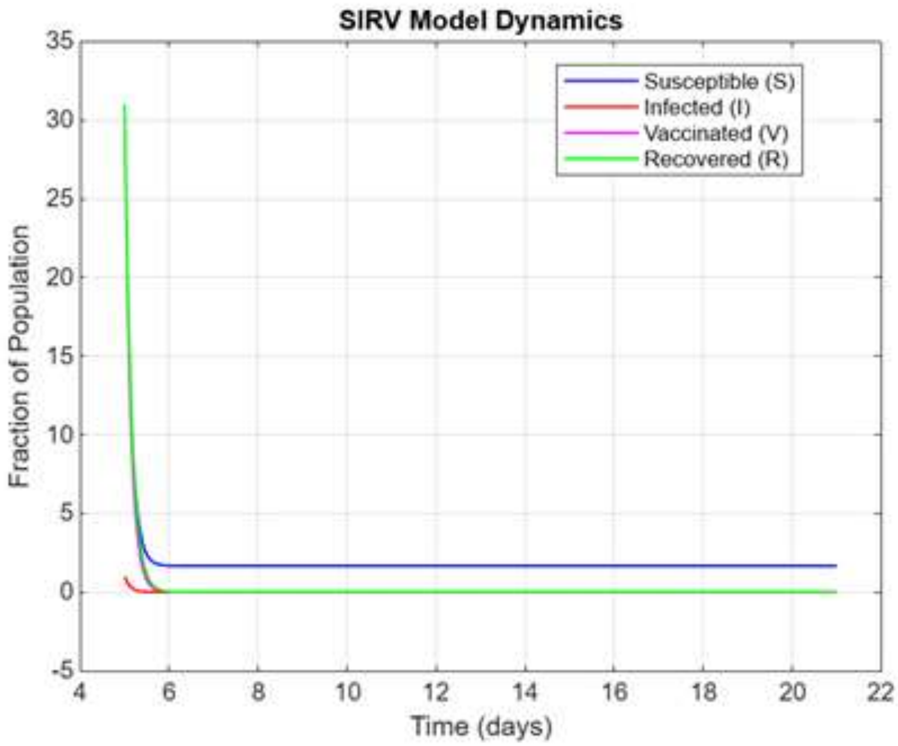

Figure 4: Improved SIVR Model

2.5 **Result and Discussion:**

All three compartmental models like SIR Model, SIR Model with Monod Haldane (M-H) function and SIVR Model satisfy the Routh-Hurwitz stability condition, indicating that the system remains stable with no immediate threat to society. In each scenario, the basic reproduction number R_0 is less than 1, confirming that the disease will not spread under current situations. This stability suggests that while vigilance is necessary, widespread outbreaks are unlikely. These finding offer crucial insights for public health interventions, emphasizing the need for continued monitoring and preventive measures.

3.0 Conclusion:

The spread of epidemiological diseases poses significant challenges to public health. Compartmental models, have been instrumental in understanding the disease progression and guiding interventions. However, the accuracy and predictive strength of these models heavily depends on the parameters used, like transmission rate, recovery rate. Here, using Routh Hurwitz condition all three models have stability that means there is no threat in the society but need to care. In all the provided scenario the basic reproduction number R_0 is below 1. So, these models will stable that means the disease will not spread at this time. The findings provide valuable guidance for public health interventions and health policy makers.

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