SUPER CONTRA HARMONIC MEAN LABELING OF GRAPHS

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ABSTRACT
The concept of Contra Harmonic mean labeling was introduced by S. Somasundaram and R. Ponraj [6] . Further, the concept of Super mean labeling was introduced by R. Ponrajand D.Ramya [7]. In this paper we investigate the Super Contra harmonic mean labeling for some subdivision of path , sunlet ,bistar related graphs.

1.INTRODUCTION

The graph S has vertex or point set P = P(S) and the edge or line set L= L(S). The set of vertices adjacent to a vertex u of S is denoted by N(u). In this paper, we consider only undirected and finite graph. For notation and terminology, we refer to *J.A Bondy and U.S.R Murthy[1]*. [3,4] Gallian refered for the latest survey of graph labelling. This paper attempt to prove that super contra harmonic mean labeling in the context of brush , star, bistar,sunlet double brush.

Definition 1.1:
A cycle C_n is a closed lane with n points and having n lines.

Definition 1.2:
A bistar graph is the graph accomplishment by joined the middle vertices of twofold piece of star by a line and it is signified by $B_{n,n}$.

Definition 1.3:
A graph $k_{1,n}$is called star graph for n greater than or equal to one.

Definition 1.4:
A subdivision graph S(G) of a graph G is a graph that can be obtained from G by subdividing each edge of G exactly once.

Definition 1.5:
The graph $P_n \odot k_1$ is called brush.

Definition 1.6: The graph $C_n \odot k_1$ is called sunlet.

Definition1.7: The union of two graphs S1=(V1,E1) and S2=(V2,E2) is a graph

S = S1□□G2with vertex set V= V1□□V2and edge set E = E1□□E2

Definition 1.8: The corona of two graphs S1and S2 is the graph S = S1⊙S2 formed by taking one copy of S1and |V(S1)| copies of S2where the i^{th} vertex of S1is adjacent to every vertex in the i^{th} copy of S2.

2.MAIN RESULT

Theorem:2.1

The (Bn) admits super Contra Harmonic mean labeling.

Proof:

Let S = $P_n \odot K_1$ be a brush obtained from a path P_n with vertices $u_1,\ldots,u_n$ and by Joining the vertex u_i to v_i, $1 \le i \le n$.

Define a function R:P(S) tends to{1,2,…,p+q} by

P(S) = { $u_1,u_2,u_3,\ldots u_n,\ v_1,v_2,v_3\ldots v_n$ }

P((B_n) = 2n is the total number of vertex sets, where n is a positive integer.

Define $R*$: P(S) → {1,2,3,...p+q} as follows

R(u_i) = 4i - 3,

R(vi) $= 4i - 1$,

Where i esteems from 1 to n. In S we must verify the total Contra Harmonic mean labeling.Then the resulting edge different.

R (u_i u_{i+1})= 4i , $1 \le i \le n-1$, $R*$ (u_i v_i) = 4i -2 , $1 \le i \le n$,

so proved the total Contra Harmonic mean labeling graph is S.

S is an total Contra Harmonic mean labeling graph as a result.

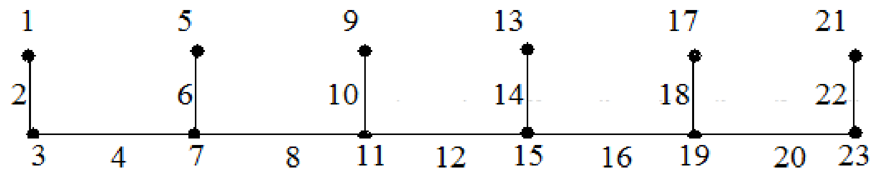


Figure2.1 : The brush graph (B_n) is an super Contra Harmonic mean labeling graph prime graph

Theorem 2.2

The D(Bn) admits super Contra Harmonic mean labeling.

Proof :

Now take S be a union of two identical sets of the brush graph D(Cn), where S is the total number of vertex sets and edges.

P(S) ={ $u_1,u_2,u_3,\ldots u_n,v_1,v_2,v_3\ldots v_n$, $w_1,w_2,w_3\ldots w_n$}

Sum of vertex set is $P(D(Cn) = 3n$, here n is a non negative integer.

Define $R*$: P(S) → {1,2,3,...p+q} this way,

$R*\ (ui) = 6i - 3,$

$R*\ (vi) = 6i - 1$,

$R*\ (wi) = 6i - 5$,

According to this different edge labeling pattern,

$Now\ R(ujwj) = 6j - 4,$

$R(ujvj) = 6j - 2\ ,1 \le j \le n,$

R(u_j u_{j+1})= $6j - 4{,}1 \le j \le n - 1,$

Hence the D(Bn) admits Total Contra Harmonic mean labeling.

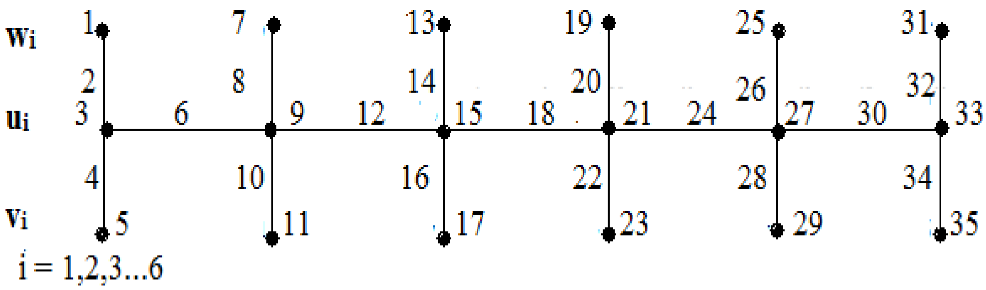


Figure2.2 : The D(B_6) issuper Contra Harmonic mean labeling.

Theorem 2.3

The $k_{1,n}$ admits super Contra Harmonic mean labeling.

Proof:

Now take S be a star graph $k_{1,n}$ .where S is the total number of vertex sets and edges.

P(S) ={ $u_1,u_2,u_3,...u_n,v_1,v_2,v_3...v_n$, $w_1,w_2,w_3...w_n$}

Sum of vertex set is $P(\text{ k1, n }) = n + 1$, here n is a non negative integer.

Let S = $k_{1,n}$ be the star graph .

Here P(T(m,n)) = {u_j; $1 \leq j \leq n$}

$Number\ of\ vertex\ set\ P\ (S) = n + 1$

$Total\ number\ of edge\ set\ L(S) = n$

Define injective $R *$: P(S) ⟶ {1,2,3,...p+q} this way as follows,

$R * (uj) = 2j - 1, 1 \leq j \leq n + 1,$

then the edge labels distinct

$R * (ujuj + 1)) = 2j = 1, 1 \leq j \leq n.$

The $k_{1,n}$ admits super Contra Harmonic mean labeling.

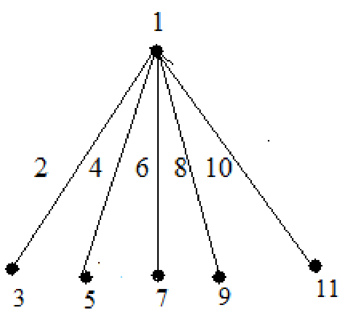


Figure2.3 : Super Contra Harmonic mean labeling of $k_{1,n}$.

Theorem 2.4:

The $B_{n,n}$ admits super Contra Harmonic mean labeling.

Proof:

Now take S be a bistar graph $B_{n,n}$,where S is the total number of vertex sets and edges.

P(S) ={ u,$u_1,u_2,u_3,...u_n$, u' , u'_1,u'_2, $u'_3,.....u'_n$}

Sum of vertex set is $P(\text{ Bn, n }) = 2n + 2$, here n is a non negative integer.

Here $P(\text{ Bn, n }) =$ { u, u'_j , v ,v'_j; $1 \leq j \leq n$}

$Total\ number\ of edge\ set\ L(S)\ = 2\ n$+1
Define injective $R *$: P(S) ⟶ {1,2,3,...p+q} this way as follows,
$R\ (u) = 1, R\ (v) = 4n + 3,$
$R\ (uj) = \ 2j + 1, 1\ \le\ j\ \le\ n,$
$R\ (v\ j) = \ 2j + 2n, 1\ \le\ j\ \le\ n,$
Then the resulting edge be
$R\ (uuj) = \ 2j, 1\ \le\ j\ \le\ n,$
$R\ (uv\ j) = \ 2j + 1 + 2n, 1\ \le\ j\ \le\ n,$
$R\ (uv\ )$ = 4n+2.
Hence the $B_{n,n}$ admits Super Contra Harmonic mean labeling.

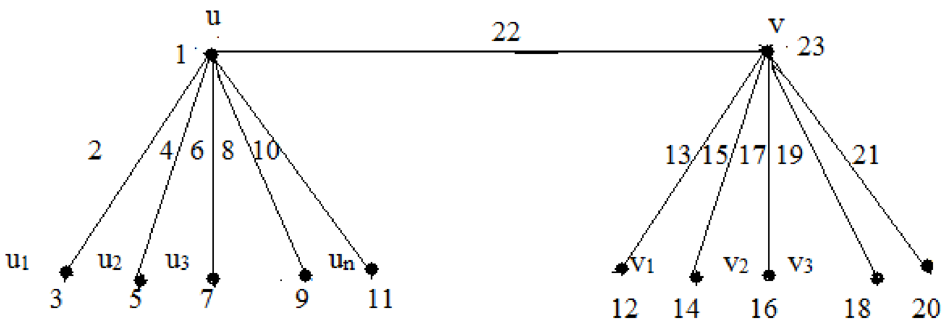


Figure2.4 : Super Contra Harmonic mean labeling of $k_{1,n}$.

Theorem 2.5: The subdividing of star $S(k_{1,n})$ admits super Contra Harmonic mean labeling.

Proof:

Now take S be a star graph $k_{1,n}$ .where S is the total number of vertex sets and edges.
P(S) ={ $u_1,u_2,u_3,...u_n,$ }
Sum of vertex set is $P($ k1, n $) = \ 2n + 1$, here n is a non negative integer.
Let $u_1,u_2,u_3,...u_n,v_1,v_2,v_3...v_n$, *be the* new vertices obtained by subdividing the edges of star graph .
Here P(T(m,n)) = {u_j; 1 ≤j≤ n}
$Total\ number\ of edge\ set\ L(S)\ = \ 2n$
Define injective $R *$: P(S) ⟶ {1,2,3,...p+q} this way as follows,
$R\ (u) = \ 1$
$R\ (uj) = \ 4j, 1\ \le\ j\ \le\ n,$
$R(vj) = \ 4j + 1, 1\ \le\ j\ \le\ n,$
then the edge lbels distinct
$R\ (uuj))\ = 4j - 2 = \ 1, 1\ \le\ j\ \le\ n.$
$R\ (ujvj))\ = 4j - 1 = \ 1, 1\ \le\ j\ \le\ n.$
Hence Subdividing the edges of star graph $k_{1,n}$ admits super Contra Harmonic mean labeling.

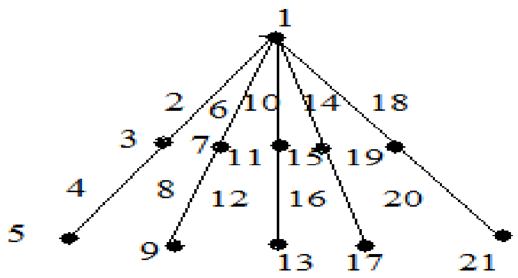


Figure2.5: Subdividing the edges of star graph $k_{1,5}$ super Contra Harmonic mean labeling.

Theorem 2.6:

The subdividing of bistar $S(B_{n,n})$ admits super Contra Harmonic mean labeling.

Proof:

Now take S be a bistar graph $B_{n,n}$ .where S is the total number of vertex sets and edges.

P(S) ={ u,u_1,u_2,u_3,...u_n, v,v_1,v_2,v_3...v_n}

Sum of vertex set is $P(\text{ Bn, n}) = 2n + 2$, here n is a non negative integer.

Let u'_1,u'_2,u'_3,...u'_n,v'_1,v'_2,v'_3...v'_n , *be the* new vertices obtained by subdividing the pendent edges of bistar graph .

Here $P(B_{n,n})$= {u,u_j, u'_j v,v_j, v'_j; $1 \leq j \leq n$}

$Total\ number\ of edge\ set\ L(S) = 4n + 2$

Define injective R: P(S) → {1,2,3,...p+q} this way as follows,

$R\ (u) = 1,$

$R\ (v') = 8n+2$

$R\ (v) = 8n + 10$

$R\ (uj) = 4j - 1, 1 \leq j \leq n,$

$R(u'j) = 4j - 1, 1 \leq j \leq n,$

$R(vj) = 4n + 4j - 2, 1 \leq j \leq n,$

$R\ (v'j) = 4n + 4j, 1 \leq j \leq n,$

then the edge labels distinct

$R(uuj)) = 4j - 2, 1 \leq j \leq n.$

$R\ (uju'j)) = 4j, 1 \leq j \leq n.$

$R(vv'j)) = v'n + 1 + j, 1 \leq j \leq n,$

$R(vjv'j)) = 4n + 4j - 1, 1 \leq j \leq n,$

$R(uv') = 8n + 1, R(v'v) = 8n + 6,$

Hence Subdividing the edges of bistar graph $B_{n,n}$ admits super Contra Harmonic mean labeling.

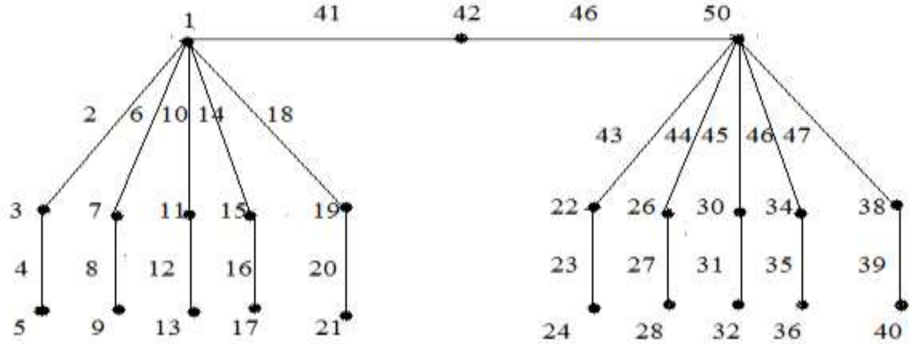


Figure2.6: subdividing of bistar $S(B_{5,5})$ on super Contra Harmonic mean labeling.

Theorem 2.7:

The ($B_{n,n}$ $ OK_{1,2}$) admits super Contra Harmonic mean labeling.

Proof:

Now take S be a bistar graph $B_{n,n}$ .where S is the total number of vertex sets and edges.

P(S) ={ u,u_1,u_2,u_3,...u_n, v,v_1,v_2,v_3...v_n}

Sum of vertex set is $P(\text{ Bn, n}) = 6n + 2$, here n is a non negative integer.

Let u'_1,u'_2,u'_3,...u'_n, x'_1,x'_2,x'_3,...x'_n,v'_1,v'_2,v'_3...v'_n , y'_1, y'_2,y'_3,...y'_n, be the new vertices obtained by corono product the pendent edges of bistar graph .

Here P($B_{n,n}$)= {u,u_j, u'_j, v,v_j, x'_j , v'_j,y'_j; $1 \leq j \leq n$}

Total number ofedge set $\text{L(S)} = 6\text{n} + 1$

Define injective R: P(S) → {1,2,3,...p+q} this way as follows,

$R(u) = 1,$

$R(v) = vn + 2n + 1$

$R(uj) = 6j - 3, 1 \leq j \leq n,$

$R(vj) = (6n + 1) + 5j, 1 \leq j \leq n,$

$R(u'j) = 6j, 1 \leq j \leq n,$

$R\ (x'j) = 1 + 6j, 1 \leq j \leq n,$

$R(v'j) = (6n + 2) + 5j - 5, 1 \leq j \leq n,$

$R(y'j) = (6n + 3) + 5j - 5, 1 \leq j \leq n,$

then the edge labels distinct

$R * \ (uuj)) = 6j - 4, 1 \leq j \leq n.$

$R * \ (uju'j)) = 6j - 2, 1 \leq j \leq n.$

$R * \ (ujx'j)) = 6j - 1, 1 \leq j \leq n.$

$R * \ (vvj)) = v'n + 1 + j, 1 \leq j \leq n,$

$R * \ (vv'j) = 6n + 4j, 1 \leq j \leq n,$

$R * \ (vjx'j)) = 6n + 5j, 1 \leq j \leq n,$

$R(uv') = vn + 2\text{n-1},$

Hence bistar graph ($B_{n,n}$ $ OK_{1,2}$) admits super Contra Harmonic mean labeling.

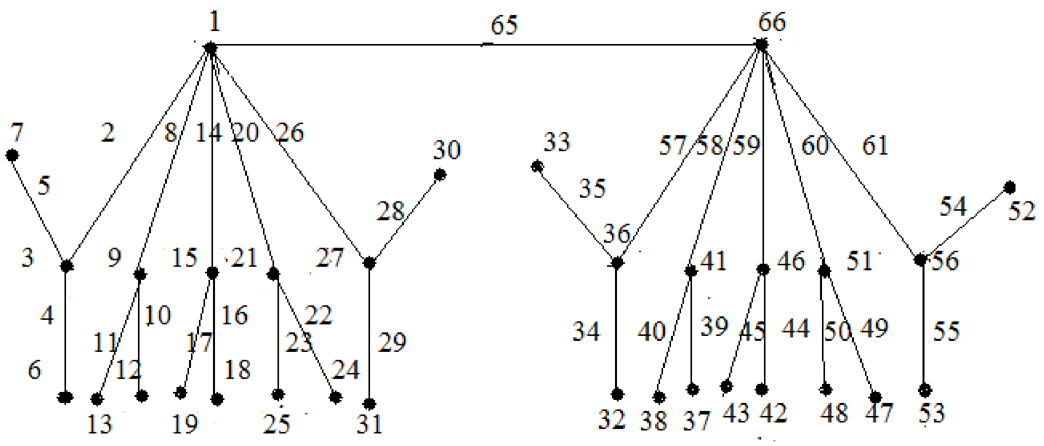


Figure2.7: subdividing of bistar S($B_{5, n5}$) on super Contra Harmonic mean labeling.

Theorem 2.8:

The ($B_{n,n}$ ʘK_3) admits super Contra Harmonic mean labeling.

Proof:

Now take S be a bistar graph $B_{n,n}$ .where S is the total number of vertex sets and edges.

P(S) ={ u,u_1,u_2,u_3,...u_n, v,v_1,v_2,v_3...v_n}

Sum of vertex set is $P(\mathrm{Bn,n} \odot \mathrm{K3}) = 6n + 2$, here n is a non negative integer.

Let u'_1,u'_2,u'_3,...u'_n, x'_1,x'_2,x'_3,...x'_n,v'_1,v'_2,v'_3...v'_n , y'_1, y'_2,y'_3,...y'_n, be the new vertices obtained by corono product K3 tothe pendent edges of bistar graph .

Here P($B_{n,n}$)= {u,u_j, u'_j, v,v_j, x'_j, v'_j,y'_j; 1 ≤j≤ n}

Total number ofedge set L(S) = 8n + 1

Define injective R: P(S) → {1,2,3,...p+q} this way as follows,

$R(u) = 1,$

$R(v) = vv'n + 2n$

$R(uj) = 7j - 4, 1 \le j \le n,$

$R(u'j) = 7j - 2, 1 \le j \le n,$

$R\ (x'j) = 1 + 7j, 1 \le j \le n,$

$R(vj) = (x'n + 1) + 6j - 6, 1 \le j \le n,$

$R(v'j) = (x'n + 4) + 6j - 6, 1 \le j \le n,$

$R(y'j) = (x'n + 5) + 6j - 6, 1 \le j \le n,$

then the edge labels distinct

$R * (uuj) = 7j - 5, 1 \le j \le n.$

$R * (uju'j) = 7j - 3, 1 \le j \le n,$

$R * (ujx'j) = 7j - 1, 1 \le j \le n,$

$R * (u'jx'j) = 7j, 1 \le j \le n,$

$R * (vvj) = y'n + j, 1 \le j \le n,$

$R * (vjv'j) = (x'n + 2) + 6j - 6, 1 \le j \le n,$

$R * (vjy'j)) = (x'n + 3) + 6j - 6, 1 \le j \le n,$

$R * (v'jy'j)) = (x'n + 5) + 6j - 6, 1 \le j \le n$

$R(uv) = vv'n + 2n\text{-}1$

Hence ($B_{n,n}$ $ʘK_3$) admits total Contra Harmonic mean labeling.

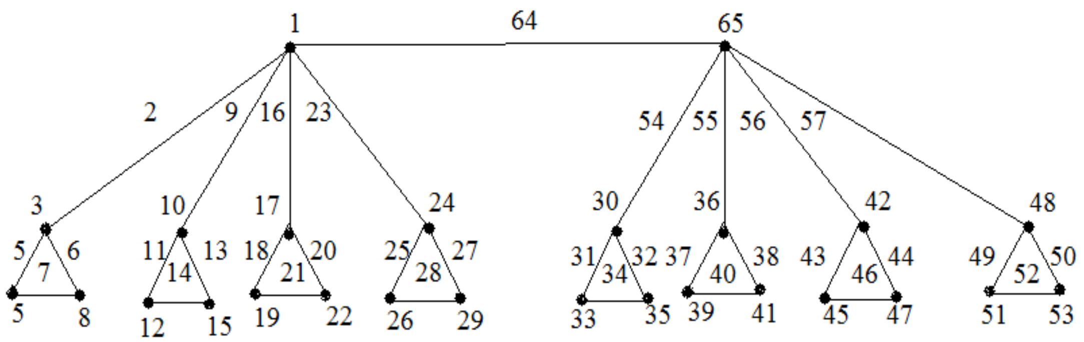


Figure 2.9: ($B_{4,4}$ ʘK_3) on super Contra Harmonic mean labeling.

Theorem 2.9:

The S_n is allows super Contra Harmonic mean labeling

Proof:

Let S be the graph of S_n. In a S_n graph sum of vertices is 2n.
P(S) = { u_1, u_2, u_3...u_n...v,v_2, v_3 ...v_n} and
Here we define P(s_n) = 2n when n is odd and even.
Define a function $R*$:(P(S) → {1,3,5...2n-1} .
Let $R*(u_j) = 4j - 3$ $for\ 1 \le j \le n-1$,
$R*(v_j) = 4j - 1$ $for\ 1 \le j \le n-1$.
$R*(v_n)$ = 4n,

Then different edge labels
$R*(ujuj+1) = 4j, 1 \le j \le n.$
$R*(unu1) = 4n - 3,$
$R*(vjvj+1) = 4j, 1 \le j \le n.$
$R*(ujvj) = 4j - 2, 1 \le j \le n.$
$R*(unvn) = 4n - 1,$
Hence S_n admits super Contra Harmonic mean labeling.

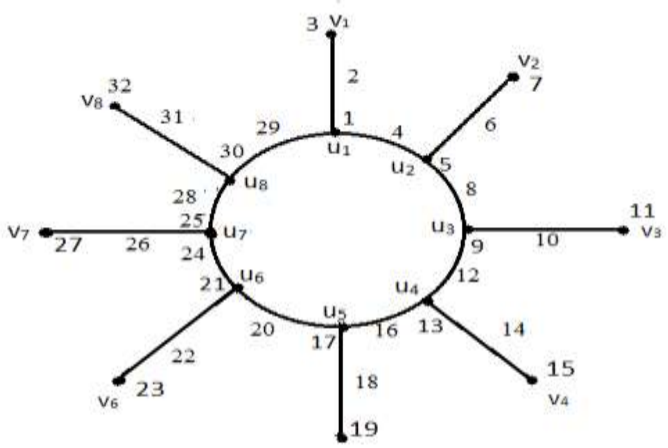


Figure 2.9:super Contra Harmonic mean labeling of S_8.

Theorem2.10: The sunlet graph S_n**($C_n \odot K_1$)** admits Total Contra Harmonic mean labeling.
Proof:
Let u_1,u_2,…,u_n be the cycle C_n and and vi,wi be the vertices which are joined to the vertex u_i , 1≤i≤n-1of the cycle .
Let S = $S_n(C_n \odot K_1)$. Define R:P(S)→{1,2,…,p+q}by
R(u_1)=1,
R(u_i)=6i-3, 2≤i≤n-1 ,
R(u_n)=6n
R(v_1)=3
R(v_i)=6i-6, 2≤i≤n-1 ,
R(w_i)=6i-1, 1≤i≤n-1 ,
R(w_i)=6n-5,
With edge labels
R(u_1u_2)=8,
R(u_iu_{i+1})=6i+1, 2≤i≤n-1 ,
R($u_{n-1}u_n$)=6(n-1)+2,
R(u_nu_1)=6n-1
R(u_1v_1)=2,
R(u_2v_2)=7,

$R(u_iv_i)=6i-4,\ 3\leq i\leq n-1$
$R(u_nv_n)=6n-3$,
$R(u_iw_i)=6i-2,\ 1\leq i\leq n$,
Here the vertices and the edges together get distinctl abels{1,2,…,p+q}.
Hence $Sn(C_n\odot K_1)$ admits Total Contra Harmonic mean labeling.

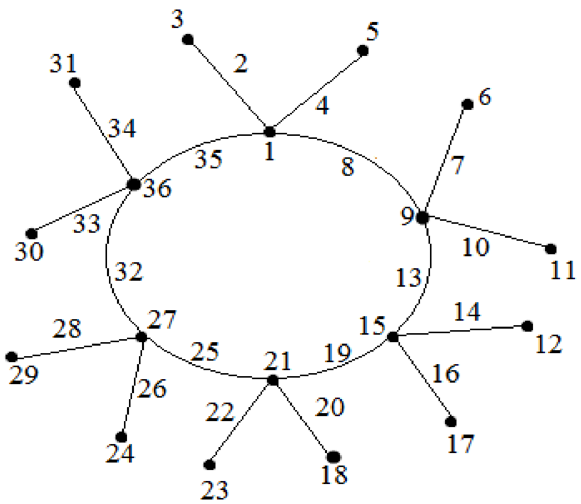


Figure2.10: A Super Contra Harmonic Mean Labeling of $S_n(C_n\odot K_1)$

Theorem: 2.11;

$S_n\odot K_3$ is a super Contra Harmonic mean graph.

Proof:

Let $u_1,u_2,...,u_n$ be a cycle C_n and let v_i, be the vertex adjacent to u_i, $1\leq i\leq n$.The resultant graph is $C_n\odot K_1$.Let x_i, y_i be the vertices of K_3which are attached to each of the vertex v_i.
The resultant graph is $S_n\odot K_3$
Definea function $R:P(S)\rightarrow\{1,2,\ldots,p+q\}$by

$R(u_1)=1$,
$f(u_2)=10$,
$R(u_i)=9i-5,\ 3\leq i\leq n-1$
$R(v_1)=3$,
$R(v_2)=12$,
$R(v_i)=9i-1,\ 3\leq i\leq n-1$,
$R(v_n) = 9i-8$,
$R(x_1)=5$, $R(x_2)=14$,
$R(x_i)=9i-8\ 3\leq i\leq n-1$,
$R(x_n) = 9n -2$, $R(y_1)=8$,
$R(y_2)=17$,
$R(y_i)=9i-6,\ 3\leq i\leq n-1$ and
$R(y_n) = 9n$

With the edges labels
$R(u_iu_{i+1})=9i,\ 1\leq i\leq n-1$,
$R(u_nu_1)=9n-6$,$R(u_1v_1)=2$,$R(u_2v_2)=11$,
$R(u_iv_i)=9i-2,\ 3\leq i\leq n-1$,
$R(u_nv_n)=9n-7$,

$R(v_1x_1)=4$, $R(v_2x_2)=13$,
$R(v_ix_i)=9i-4$, $3\le i\le n$,
$R(v_1y_1)=7$, $R(v_iy_i)=9i-3$, $2\le i\le n$
$R(x_iy_i)=10i-4$, $1\le i\le 2$,
$R(x_iy_i)=9i-7$, $3\le i\le n-1$,
$R(x_ny_n)=9n-1$, Clearly, R is a super Contra Harmonic mean labeling of S .

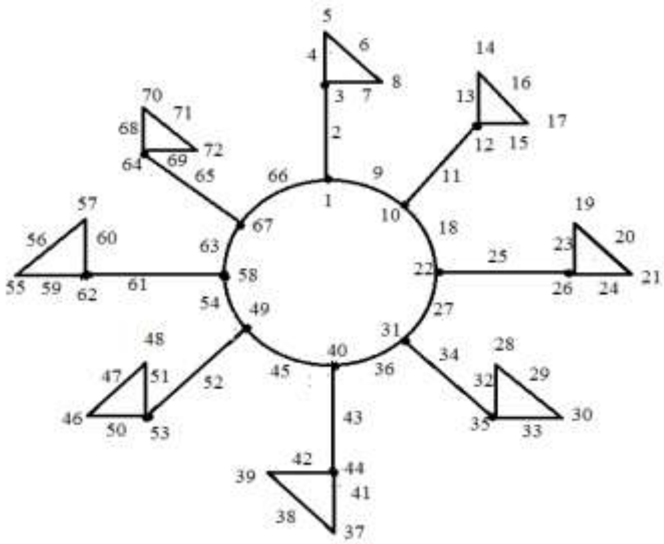


Figure2.11: Super Contra Harmonic mean labeling of $S_n\odot K_3$ **is**

Theorem2.12;

The $S_n\odot K_{1,2}$ is a super Contra Harmonic mean graph.

Proof:

Let C_n be the cycle $u_1,u_2,...,u_n$. Let v_i, x_i, y_i, z_i be the vertices of i^{th} copy of $K_{1,2}$ in which v_i is the central vertex. Identify z_i with u_i, $1\le i\le n$. Let the resultant graph be G .

Let $S = S_n\odot K_{1,2}$

Define a function $R:P(S)\to\{1,2,\ldots,p+q\}$ by

$R(u_1)=1$, $R(u_2)=9$,

$R(u_i)=8i-1$, $3\le i\le n$
$R(v_1)=3$, $R(v_2)=11$,
$R(v_i)=8i-6$, $3\le i\le n$
$R(w_1)=5$, $R(w_2)=13$,
$R(w_i)=8i-8$, $3\le i\le n$
$R(x_1)=7$, $R(x_2)=15$,
$R(x_i)=8i-2$, $3\le i\le n-1$, $R(x_n) = 8n$
with edge labels
$R(u_1u_2)=8$, $R(u_iu_{i+1})=8i+3$, $2\le i\le n-1$,
$R(u_nu_1)=8n-2$, f$R(u_iv_i)=8i-6$, $1\le i\le 2$,
$R(u_iv_i)=8i-4$, $3\le i\le n$
$R(w_iv_i)=8i-4$, $1\le i\le 2$,
$R(w_iv_i) = 8i-7$, $3\le i\le n$
$R(v_ix_i)=8i-2$, $1\le i\le 2$,
$R(v_ix_i)=8i-3$, $3\le i\le n$

Clearly, R is a super Contra Harmonic mean labeling of S.

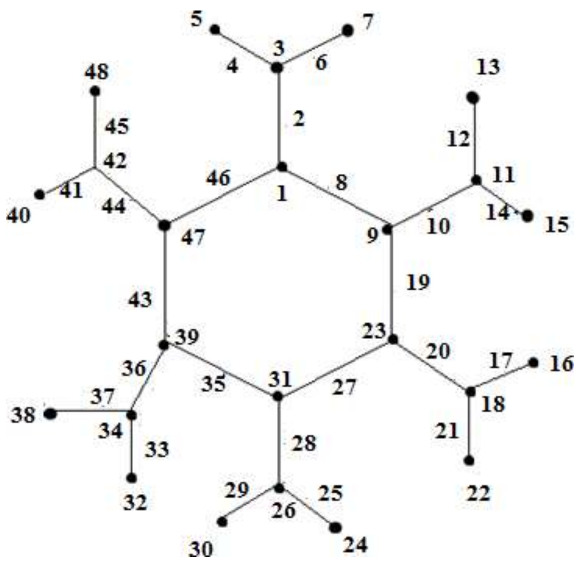


Figure2.12: super Contra Harmonic mean labeling of $S_6 \odot K_{1,2}$

Theorem2.13:

Let S be the graph obtained from the graph $P_n \Theta K_1$by subdividing the edges of path P_n. Then S is a super contra harmonic mean graph for all values of n.

Proof:

Let us takeP_n=u_1, u_2,…,u_n and let v_i be the vertex which is joined to $\leq$ i≤n of the path P_n then the resultant graph is $P_n \Theta K_1$
Let w_1, w_2,…,w_{n-1}be the new vertices obtained by subdividing the edges e_1,e_2,…,$e_{n-1}$of the path P_n respectively, where e_i= $u_i u_{i+1}$for 1≤ i ≤ n-1.

Define a function R :V(G)→{1,2,...,p+q}
by R(u) = 1 ;R(u_i) = 7for 1≤ i ≤ n-1
R(v_i)=7i–2 for1≤i≤n-1

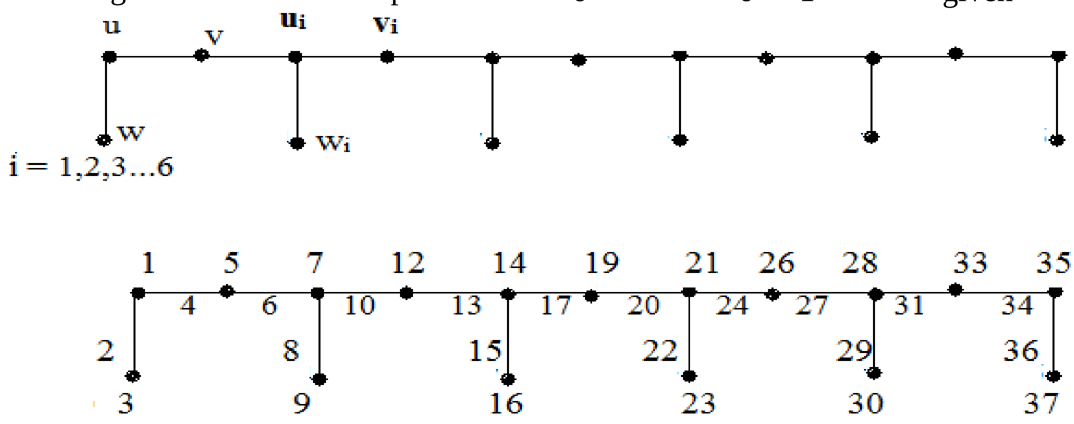
R(w_i) = 3 ;R (w_i) = 7i+2 for1≤i≤n-1 Then
the resulting edge labels are distinct.
R(vu_1)=6;R($u_{i+1}v_i$)=7i+6 for1≤ i≤ n-2

R(uw)=2;R($u_i w_i$)=7i+1 for1≤i≤n-1

R(uv) = 4;R($u_i v_i$) = 7i+3 for 1≤ i ≤ n-2

Thus R provides a super contra harmonic mean labeling of S. Hence S is a total contra harmonic mean labeling graph.

Figure 2.13: A super contra harmonic mean labeling of graph S obtained by subdividing the edges of path $P6$ in $P6ʘK1$ is given in



Theorem2.14:

Let S be the graph obtained from the graph $PnʘK1$ by subdividing the pendent edges of path Pn. Then S is a super contra harmonic mean graph for all values of n.

Proof:

Let us take Pn=u1, u2,…,un and let vi be the vertex which is joined to ≤ i≤n of the path Pn then the resultant graph is Pn ʘ$K1$

Let $w1$, $w2$,…,$wn-1$ be the new vertices obtained by subdividing the pendent edges $e1$,$e2$,…,en of the path Pn respectively, where ei= uivi for 1≤ i ≤ n.

Define a function R :P(S)→{1,2,...,q+P}by

R(u) = 1 ;R(ui) = 6i+1 for1≤i≤n-1

R(vi)= 6i-3 for1≤i≤n

R(vi)= 6i-1 for1≤i≤n

Then the resulting edge labels are distinct.

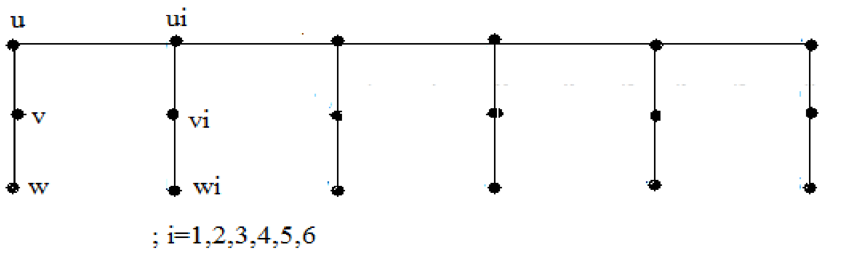
R(uu$_1$)=6;R(ui+1ui)=6i+4 for1≤ i≤ n-2

R(vw)=4;R(viwi)=6i+5 for1≤i≤n-2

R(v$_{n-1}$w$_{n-1}$)= v$_{n-2}$w$_{n-2}$+5

R(uv) = 2;R(uivi) = 6i+2 for 1≤ i ≤ n-1

Thus R provides a super contra harmonic mean labeling of S. Hence S is a super contra harmonic mean labeling graph.



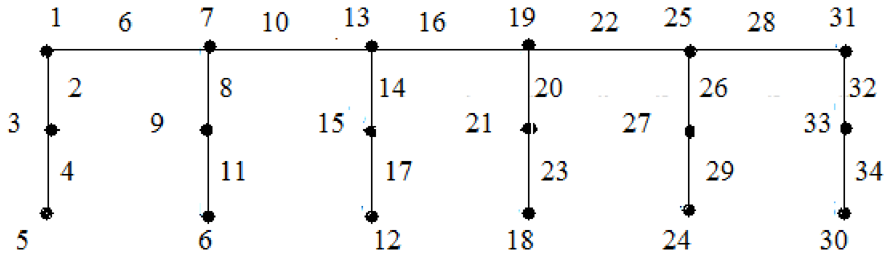


figure 2.14 : A super contra harmonic mean labeling of graph S obtained by subdividing the pendent edges of path $P6$ in$P6ʘK1$

Theorem2.15:

The subdivision graph of $PnʘK1$ is a super contra harmonic mean graph for all values of n.

Proof:

Let us takePn = u1, u2,…,un and let vi be the vertex which is joined to 1 ≤ i≤n of the path Pn then the resultant graph is Pn ʘ$K1$.
Let $w1, w2,\dots,wn-1$ and $x_1,x_2,x_3\dots x_n$ be the new vertices obtained by subdividing edges ei= uiui+1 and uivi respectively.
Define a function R :P(S)→{1,2,...,q +P}by
R(ui) = 8i-7 for 1≤ i ≤ n
R(vi)= 8i-5 for1≤i≤n
R(wi)= 8i-1 for1≤i≤n-1
R(xi)= 8i-3 for1≤i≤n-1
R(xn)= x_{n-1}+7 for1≤i≤n-1
Then the resulting edge labels are distinct.
R(uivi)=8i-6 for1≤ i≤ n
R(xivi)=8i-4 for1≤ i≤ n
R(uiwi)=8i-6 for1≤ i≤ n
R(u_1w_1)=6 for1≤ i≤ n
R(uiwi)=8i-3 for2≤ i≤ n-2
R(u_{i+1}wi)=8i for1≤ i≤ n-1

Thus R provides a super contra harmonic mean labeling of S. Hence S is a super contra harmonic mean labeling graph.

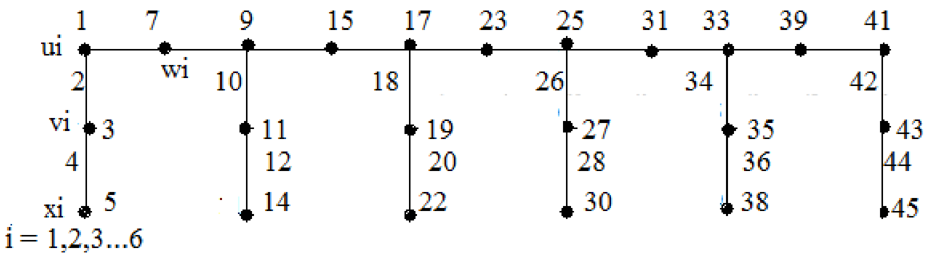


Figure 2.15: A super contra harmonic mean labeling of graph S obtained by subdividing $P6ΘK1$is given in

Theorem2.16:

Let S be the graph obtained from the graph $PnΘ\bar{k2}$ by subdividing the edges of path Pn.Then S is a total contra harmonic mean graph for all values of n.

Proof:

Let us takePn = $u,u_2,\ldots,u_n$ and let $vixi$ be the vertices of $\bar{k2}$ which are joined to the vertex ui of path Pn.1≤i≤n.The resultant graph is$PnΘ$ $\bar{k2}$

Let $w1,2,\ldots,wn-1$be the new vertices obtained by subdividing the edges $e1,e2,\ldots,en-1$ of the path Pn respectively where ei = uiui+1for 1≤ i ≤ n-1

Define a function R:P(S)→{1,2,...,q+p}by

R(u_1) = 1,

R(v_1)=3, R(x_1) = 5,

R(ui)=10i–11 for2≤i≤n

R(vi)=10i-8 for1≤i≤n

R(xi)= 10i-7 for1≤i≤n

R(wi) = 10i-3 for 1≤ i ≤ n-1

Then the resulting edge labels are

R(u_1v_1) = 2; R(u_1x_1) = 4; R(u_1w_1) = 6;

R(uivi) = 10i-10 for 2≤ i ≤ n

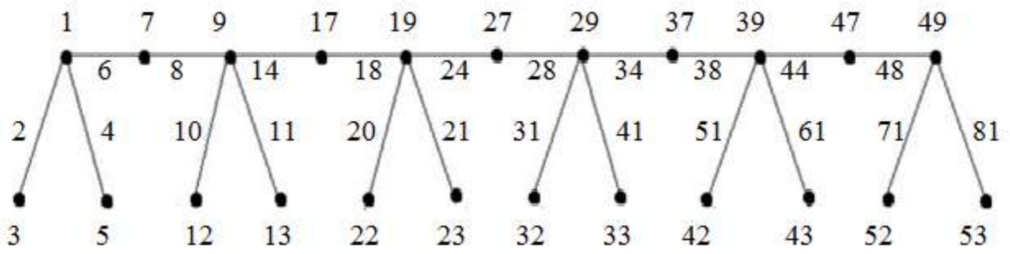
R(uixi) = 10i-9 for2≤i≤n

R(uiwi)= 10i-6 for2≤i≤n-1

R(wiui+1)=8i for1≤i≤ n-1

Thus R provides a super contra harmonic mean labeling of S. Hence S is a super harmonic mean

Figure 2.16:A super harmonic mean labeling of graph S obtained by subdividing the edges of path $P6$ in $P6Θ\bar{K2}$



Theorem Theorem2.17:

Let S be the graph obtained from the graph $Pn \odot \overline{k_2}$ by subdividing the pendant edges.

Then S is a super contra harmonic mean graph for all values of n.

Proof:

Let u1,u2,…,un be the path Pn and let $viwi$1 ≤i≤n be the vertices of $\overline{k}2$ which are joined to

The vertex ui of the path Pn for1≤ i≤ n.

Then the resultant graph is $Pn \odot_2 \overline{k}$ ·Let si, ti be the vertices obtained by subdividing the pendant edges vi ui, wi ui respectively for 1 ≤i ≤ n

Define a injective function R:P(S)⟶{1,2,...,q+p}as follows

$R(u_1) = 5$; $R(v_1) = 3$; $R(x_1) = 1$; $R(y_1) = 7$;

R(ui) = 10i- 9; for 2≤i≤ n

R(vi)=10i–1 for2≤i≤n

R(ui)=10i–11 for2≤i≤n

R(wi)=10i for1≤i≤n-1

R(wn)= wn-1+9 for1≤i≤n-1

R(xi)= 10i-6 for2≤i≤n

R(yi) = 10i-5 for 1≤ i ≤ n

Then the resulting edge labels are

$R(u_1x_1) = 4$; $R(u_1y_1) = 6$; $R(u_1u) = 9$;

$R(x_1v_1) = 2$;

R(uixi) = 10i-8 for 2≤ i n

R(uiyi) = 10i-9 for2≤i≤n

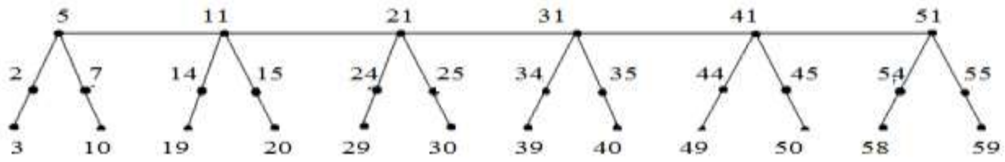
R(xivi)= 10i-4 for2≤i≤n-1

R(wiyi) = 10i-2 for1≤i≤n-1

R(wnyn) = wn-1y_{n-1}+9.

Hence s is a super contra harmonic mean labelin

Figure 2,18 : A super contra harmonic mean labeling of graph obtained by subdividing the pendant edges of the graph $P6\Theta\ \overline{\overline{K}}_2$.



Theorem2.19:

The subdivision graph of $P_n\Theta\overline{\overline{K}}_2$ is a super contra harmonic mean labeling graph for all values of n.

Proof:

Let $u_1,u_2,\ldots, u_n$ be the path P_n, and let $v_i r_i\ 1\le i\le n$ be the vertices of which are joined to the vertex u_i of the path P_n for $1\le i \le n$

Let $w_1, w_2,\ldots,w_{n-1}$ be the vertices obtained by subdividing the edges of the path $e_1,e_2,\ldots,e_{n-1}$ respectively.

Let s_i,t_i be the vertices obtained by subdividing the pendant edges u_iv_i,u_ir_i respectively for $1\le i \le n$.

Define a function $R:P(S)\rightarrow\{1,2,...,q+P\}$ by

$R(u_i) = 10i-9+(2j-2$ for $1\le i,j \le n$.

$R(w_i)= 12i-1$ for $1\le i\le n$.

$R(s_i)=12i-8$ for $1\le i\le n$.

$R(t_i)= 12i-7$ for $1\le i\le n$.

$R(v_i)= 12i-4$ for $1\le i\le n$.

$R(r_i)= 12i-3$ for $1\le i\le n$.

Edge labels with,

$R(u_iw_i)= 12i-2$ for $1\le i\le n$.

$R(w_iu_{i+1})=12i$ for $1\le i\le n-1$.

$R(u_is_i)= 12i-10$ for $2\le i\le n$

$R(u_it_i)= 12i-9$ for $1\le i\le n$,

$R(s_iv_i)= 12i-6$ for $2\le i\le n$

$R(t_ir_i)=12i-5$ for $1\le i\le n$

Thus R provides a super contra harmonic mean labeling for subdivision graph of $P_n\Theta\ \overline{\overline{k}}_2$.

Hence subdivision graph of $P_n\Theta\overline{\overline{k}}_2$ is a super contra harmonic mean graph.

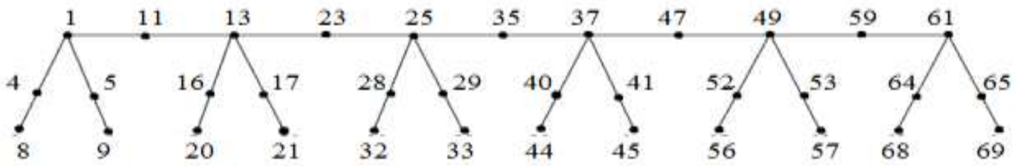
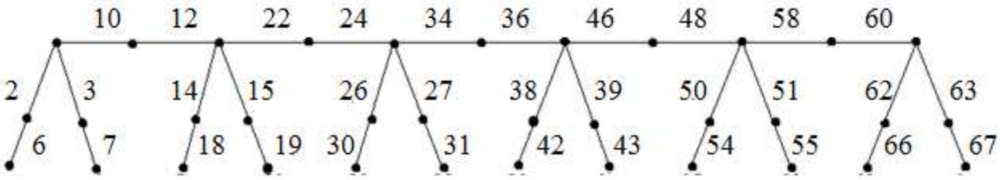


Figure2.19: A super contra harmonic mean labeling of subdivision graph of P6ʘK̄2.



Theorem2.20: Cm□Pn is a super Contra Harmonic mean graph, form □□3 and n □□1

Proof: Let Cm be the cycle u1,u2, u3…um and Pn be the path v1, v2,v3…vn.

Let S = Cm□Pn.

Define R: P(S) → {1,2,3,...p+q} as follows

$R(u_j) = 2i - 1$, for$1\leq j\leq n$

$R(v_i) = 2m+2j- 1$, for$1\leq j\leq n$

With edge labels

$R(u_iu_{i+1})=2i+ 2$, $1\leq j\leq n -1$;

$R(v_iv_{i+1})=2m+ 2j$, $1\leq j\leq n -1$

Clearly,Cm□Pn is super Contra Harmonic mean graph.

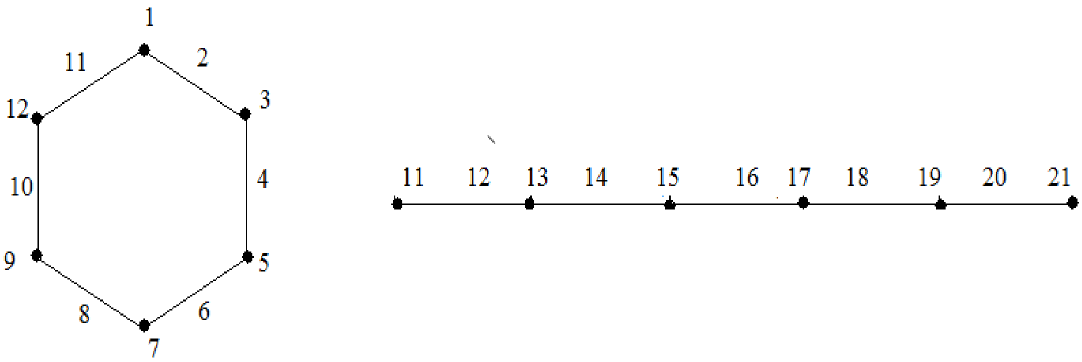


Figure2.20: The super Contra Harmonic mean labeling of C6□P6

Theorem 2.21: Cm□□bn is a super Contra Harmonic mean graph.

Proof:

Let Cm be a cycle with vertices u1,u2,u3,…,um and Let v1, v2,v3,..vn be the path Pn.and let wi be the vertices which is joined to the vertex vi,11≤ j≤ n of the path Pn.

The resultant graph is b_n. LetS = Cm□ b_n

Define R: P(S) → {1,2,3,...p+q} as follows

$R(u_j) = 2i - 1$, for$1\leq j\leq n-1$;
$R(u_n) = 2m - 1$,
$R(v_i) = 2m+4j-3$, for$1\leq j\leq n$
$R(w_i) = 2m+4j$, for$1\leq j\leq n-1$
$R(w_n) = w_{n-1} +3$
With edge labels
$R(u_iu_{i+1})=2i$, $1\leq j\leq n -1$;
$R(u_nu_1)=2n-1$, $1\leq j\leq n -2$;
$R(v_iv_{i+1})=2m+ 4j-1$,$1\leq j\leq n -1$;
$R(v_iw_i)=2m+ 4j-2$, $1\leq j\leq n$;
Clearly, C_m□b_n is super Contra Harmonic mean graph.

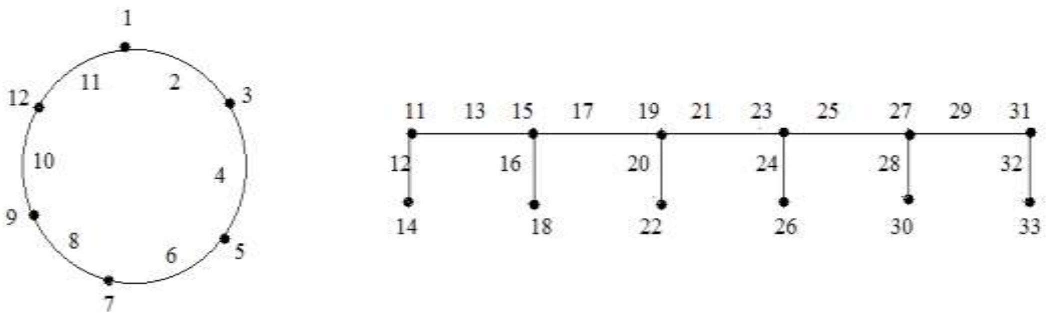


Figure2.21: The super Contra Harmonic mean labeling of C_6 □□bn

Theorem2.22: S_n□P_n is a super Contra Harmonic mean graph.

Proof:

Let $u_1,u_2\ldots,u_m$ be a cycle C_m and v_i be the vertex which is joined to the Vertex $u_i$of the cycle C_m, $1\leq j\leq n$.

The resultant graph is S_n. Let $w_1,w_2,\ldots.w_n$be the path P_n .

Let S = **S_n□□P_n**.

Define R: P(S) → {1,2,3,...p+q} as follows
$R(u_j) = 4i - 3$, for$1\leq j\leq n-1$;
$R(u_n) = 4n$
$R(v_j) = 4j - 1$, for$1\leq j\leq n-1$; $R(v_n) = 4n-3$
$R(w_i) =4n+2j-1$, for$1\leq j\leq n$
With edge labels
$R(u_ju_{j+1}) =4j$, $1\leq j\leq n -1$;
$R(u_jv_j) =4j-2$, $1\leq j\leq n$;
$R(u_{n-1}u_1) = 4n-1$,
$R(w_jw_{j+1})=4n+2j-1$, $1\leq j\leq n$;
Hence **S_n**□P_n is a super Contra Harmonic mean graph.

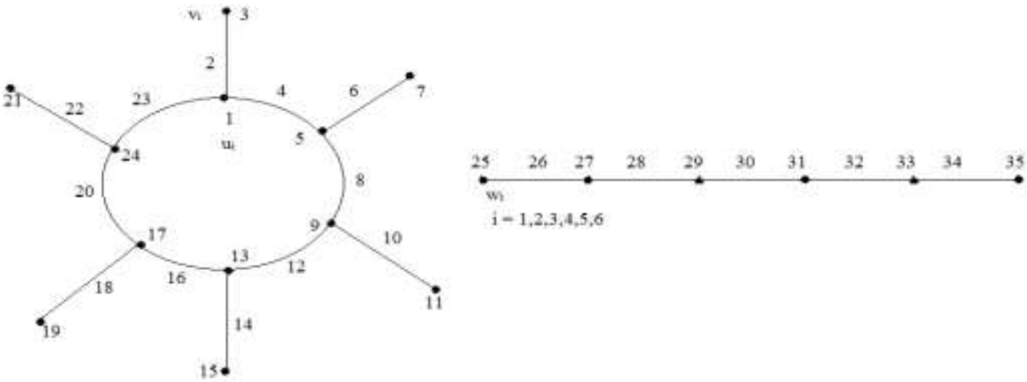


Figure2.22: S_n□Pn is a super Contra Harmonic mean graph.

Theorem2 :23: S_m□ bn is a super Contra Harmonic mean graph.

Proof: Let $u_1u_2.....u_m$be the cycle $C_m$and let $v_i$be the pendent vertex joined to the vertex $u_i$of C_m1≤ j≤ m. The resultant graph is S_n. Let $w_1......w_n$ be the path P_n and x_i be the vertex which is joined to the vertex w_i, 1≤ j≤ n of the path P_n. The resultant graph is b_n.

Let S = S_m□□bn

Define R: P(S) → {1,2,3,...p+q} as follows

$R(u_j)$ = 4i - 3, for1≤ j≤ n-1;$R(u_n)$ = 4n

$R(v_j)$ = 4j - 1, for1≤ j≤ n-1; $R(v_n)$ = 4n-3

$R(w_i)$ =(4n+1)+4j-4, for1≤ j≤ n;

$R(x_i)$ =(4n+4)+4j-4, for1≤ j≤ n

With edge labels

$R(u_ju_{j+1})$ =4*j*, 1≤ j≤ n -1;

$R(u_jv_j)$ =4*j-2*, 1≤ j≤ n;

$R(u_{n-1}u_1)$ = 4*n-1*,

$R(w_jw_{j+1})$=4*n+3+4j-4*, 1≤ j≤ n ;

$R(w_jx_j)$=4*n++4j*, 1≤ j≤ n-1 ; $R(w_nx_n)$=w_{n-1}+3

Clearly, **S_n□bn** is a super Contra Harmonic mean graph.

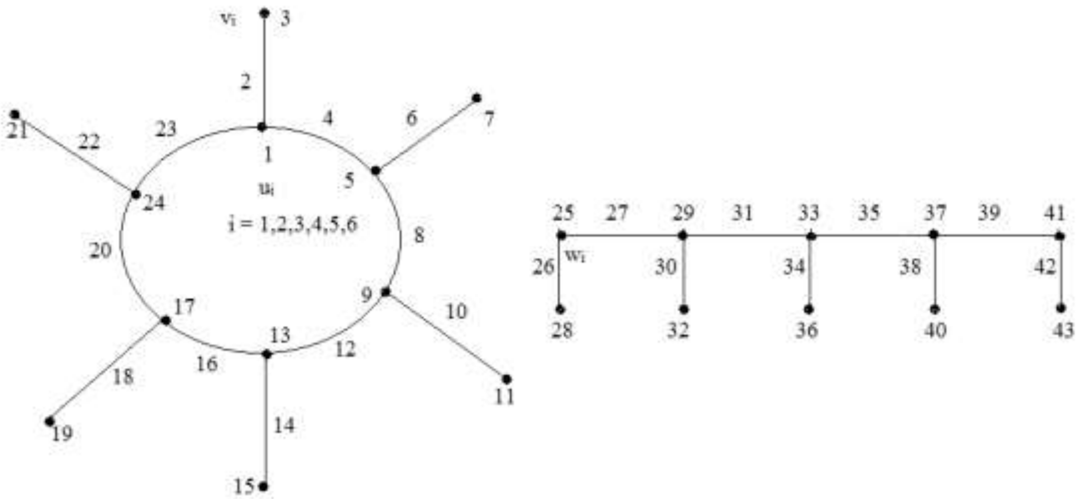


Figure2.22: Super Contra Harmonic mean graph **S_n□□bn** .

Theorem2:24 **S_n**U D(**bn)** is a super Contra Harmonic mean graph.

Proof: Let $u_1, u_2 \ldots u_m$ be the cycle C_m and let v_i be the vertex joined to the vertex u_i of C_m, $1 \le j \le m$. The resultant graph is Sn. Let $w_1, w_2 \ldots w_n$ be the path P_n and let x_i and y_i be the vertices which are joined to the vertex w_i of path P_n, $1 \le j \le n$.

The resultant graph in $P_n K_2$. Let S = **Sn**U D(bn)

Define R: P(S) → {1,2,3,...p+q} as follows

$R(u_j) = 4i - 3$, for $1 \le j \le n-1$;

$R(u_n) = 4n$

$R(v_j) = 4j - 1$, for $1 \le j \le n-1$;

$R(v_n) = 4n-3$

$R(w_i) = (4n+1)+6j-6$, for $1 \le j \le n$;

$R(x_i) = (4n+3)+6j-6$, for $1 \le j \le n$;

$R(y_i) = (4n+5)+6j-6$, for $1 \le j \le n$

With edge labels

$R(u_j u_{j+1}) = 4j$, $1 \le j \le n-1$;

$R(u_j v_j) = 4j-2$, $1 \le j \le n$;

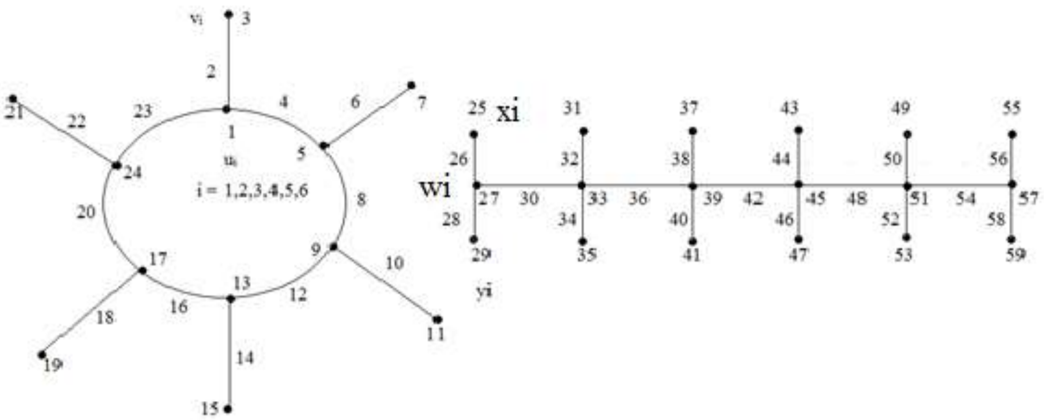
$R(u_{n-1} u_1) = 4n-1$,

$R(w_j w_{j+1}) = (4n+6)+6j-6$, $1 \le j \le n-1$;

$R(w_j x_j) = (4n+2)+6j-6$, $1 \le j \le n$;

$R(w_j y_j) = (4n+3)+6j-6$, $1 \le j \le n$;

Clearly, **Sn**U D(bn) is a super Contra Harmonic mean graph.



CONCLUSION

The study of labeled graph is important due to its diversified applications of graphs like the brush, sunlet, Subdivision of some graph. All graphs are not super contra harmonic mean graphs. Similar findings for additional graph families can be investigated is an open area of research.

References:

[1] J.A. Gallian 2016, "A Dynamic Survey of Graph Labeling". The electronic journalof Combinatorial .

[2] F.Harary,1988,Graphtheory,NarosaPublicationHousereading,NewDelhi.

[3] R.PonrajandD.Ramya“Supermeanlabelinggraphs”,ArsCombin.112(2013),65-72.

[4] S.SomasundramandS.S.Sandhya 'Harmonic Mean Labeling of Graphs 'Journalof Combinatorial Mathematics and Combinatorial computing.

[5] R.Ponrajandd.Ramya“Super Meanlabelinggraphs”,Arscombin.112(2013),65-72.

[6] S.S.Sandhya,S.Somasundaram,J.Rajeshnigolda,“Contra Harmonic Mean Labeling Of Disconnected Graphs International Research Publication House .Issn0974-3200volume9,Number3(2017),Pp.399-411.

[7] S.S.Sandhya,S.Somasundram,andJ.RajeshniGolda" super Contra Harmonic Mean Labeling of Graphs”, International Journal of Engineering , Science and Mathematics, Vol.7Issue5,May2018,
ISSN:2320-0294ImpactFactor:6