ISSN: 2688-8645

Study on Noise Reduction of Acoustic Emission Signals based on Improved Wavelet Thresholding

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Abstract

The wavelet transform is extensively utilized in signal denoising due to its benefits of reduced entropy, multiple resolutions, and decorrelation. This paper presents an enhanced wavelet threshold denoising algorithm that combines the existing improved threshold function and threshold selection method, building upon the traditional wavelet threshold denoising algorithm. The enhanced threshold function exhibits improved smoothness and reduced coefficient variation; the novel threshold selection approach integrates the Lipschitz properties of the signal and achieves a higher rate of noise signal elimination. The simulation experiments on denoising demonstrate that the enhanced wavelet threshold denoising algorithm enhances the signal-to-noise ratio (SNR) and mean-square error (MSE) by 14.4% and 58.3% respectively, in comparison to the conventional algorithm. Additionally, it outperforms existing algorithms by 8.4% and 36.5%, showcasing its superior denoising capabilities. These findings validate the performance benefits and practical value of the denoising algorithm proposed in this research paper.

Keywords

Acoustic Emission; Wavelet Threshold Denoising; Threshold Function; Threshold Selection.

1. Introduction

During the detection of acoustic emission, the obtained signals unavoidably include noise produced by the surroundings and the device itself. The existence of this noise impacts the subsequent procedures involving the signals, including feature extraction and localizing the source of acoustic emission[1]. In order to obtain precise characteristics of the signal, it is crucial to remove noise from the acquired acoustic emission signal, making signal denoising a vital process in acoustic emission detection.

Due to its straightforward implementation, low computational requirements, and broad applicability, the wavele threshold denoising technique is considered a prominent approach in the domain of signal denoising. Donoho and Johnstone et al. published their work in 1995.Suggested a conventional technique for reducing noise using wavelet thresholding, however, the described method's soft and hard threshold functions have defects that can lead to distortion after denoising[2-5]. In 2004, Zhang Wei-Qiang et al. Enhanced the threshold function in the traditional denoising function, resulting in improved signal denoising and a more satisfactory denoising outcome. However, the explanation for the selection of the threshold value was incomplete[6]. In the field of literature, enhancements were made to both the threshold function and the method for selecting the threshold. These enhancements were then applied to image denoising, resulting in an improved denoising effect. However, it also resulted in the loss of some of the effective signal[7].

This paper thoroughly examines the drawbacks of the conventional wavelet threshold denoising algorithm and conducts a comprehensive investigation into the existing wavelet threshold function and threshold selection. Based on this analysis, an enhanced wavelet threshold denoising algorithm is proposed[8-9]. The effectiveness of the improved algorithm is then verified through simulation, providing evidence that the denoising effect achieved in this paper is superior.

2. The Concept of Wavelet Threshold Denoising

2.1. Signal Description

Broadly speaking, noise refers to all components of a signal other than the useful components, and a signal containing noise is modelled as follows:

$$s(t) = f(t) + n(t) \tag{1}$$

Where is the noise signal, where is the original signal, and where is the noise signal? The purpose of denoising is to suppress the in and get the desired original signal. The drawback of the conventional noise reduction technique is that it leads to an increase in the entropy of the transformed signal, failing to capture the non-smooth attributes of the signal or achieve signal correlation. Wavelet transform has good time-frequency characteristics, it is advantageous in analysing non-stationary signals and removing signal correlation, and has a broad application prospect in signal denoising[10].

2.2. Wavelet Transform

The continuous wavelet transform of a one-dimensional continuous function $f(t)$ is:

$$WT_f(a,\tau) = \int_{-\infty}^{+\infty} f(t)\psi_{a,\tau}(t)\mathrm{d}t = \frac{1}{\sqrt{a}}\int_{-\infty}^{+\infty} f(t)\psi\left(\frac{t-\tau}{a}\right)\mathrm{d}t \tag{2}$$

The location of the wavelet coefficient can be determined by the wavelet basis function and the wavelet function, while the permissibility condition is satisfied by a. The scale parameter is denoted by, and the time delay parameter is denoted by.

The N-point discrete signal is obtained by discretely sampling a one-dimensional continuous signal, and the discrete wavelet transform is applied.

$$WT_f(j,k) = 2^{-j/2}\sum_{0}^{N-1} f(n)\psi\left(2^{-j}n-k\right) \tag{3}$$

The wavelet coefficient can be found by using the discretised scale parameter (j) and the discretised time delay parameter (k).

Accordingly, the wavelet inverse is converted to:

$$f(n) = \frac{1}{A}\sum_{j,k} WT_f(j,k)\psi_{j,k}(n) \tag{4}$$

A is the bound of the wavelet frame consisting of.

2.3. Wavelet Thresholding Denoising

In general, the initial efficient signal is a signal with low frequency, while the noise signal is primarily found in the high-frequency area. In the spatial (or temporal) domain, the efficient signal exhibits spatial (or temporal) coherence, and the predominant value of the wavelet coefficients generally increases upon transformation to the wavelet domain. Conversely, the non-continuous noise signal in the spatial (or temporal) domain retains significant discrete randomness even after undergoing wavelet transformation, resulting in a smaller corresponding predominant value of the coefficients in the wavelet domain. The fundamental concept of the wavelet threshold denoising technique suggested by Donoho and Johnstone is

ISSN: 2688-8645

based on the aforementioned attributes. It involves establishing a threshold for wavelet coefficients, nullifying the wavelet coefficients produced by the noisy signal, preserving the wavelet coefficients generated by the useful signal, and subsequently reconstructing the coefficients through wavelet processing to acquire the denoised signal[11].

The wavelet threshold noise cancellation algorithm relies heavily on the processing of the threshold, encompassing both the estimation of the threshold and the selection of the threshold function. The wavelet coefficient threshold processing directly determines the signal noise cancellation quality. Hence, the main concern and challenge of threshold noise cancellation method lie in estimating the threshold and selecting the appropriate threshold function. The design ideas of threshold processing in hard and soft threshold noise cancellation algorithms are divided as follows:

(1) Wavelet decomposition of the noise-containing signal *f(t)* using the Mallat tower decomposition. The wavelet coefficients $WT_f(j,k)$ of *f(t)* at each scale are obtained from equation (3).

Calculate the threshold value by analyzing the wavelet coefficients, and apply the threshold function to filter the wavelet coefficients at each scale. The function that acts as a strict limit is the hard threshold threshold function.

$$\widehat{W_{j,k}} = \begin{cases} W_{j,k}, & \left|W_{j,k}\right| \geqslant \lambda \\ 0, & \left|W_{j,k}\right| < \lambda \end{cases} \tag{5}$$

(2) the function of the threshold that is gentle.

$$\widehat{W_{j,k}} = \begin{cases} \operatorname{sgn}\left(W_{j,k}\right)\left(\left|W_{j,k}\right| - \lambda\right), & \left|W_{j,k}\right| \geqslant \lambda \\ 0, & \left|W_{j,k}\right| < \lambda \end{cases} \tag{6}$$

The location of the wavelet coefficients in the signal containing noise is represented by; the wavelet coefficients after applying a threshold are represented by; represents the decomposition scale; represents the sequence number of the coefficient; represents the threshold for the wavelet; and the sign function is denoted by sgn().

(3) The empirical formula for threshold threshold estimation is:

$$\lambda = \gamma \cdot \sigma \cdot \sqrt{2\ln(n)} \tag{7}$$

The constant is assumed to be 0.1, while n represents the count of wavelet coefficients. The threshold relies on the estimation of the noise variance, and the noise variance is estimated using the median estimator.

$$\sigma = \operatorname{Median}\left(W_{j,k} / 0.6745\right) \tag{8}$$

The fixed threshold wavelet noise cancellation method is applied to each layer of wavelet decomposition, where the threshold remains constant. Both the hard and soft thresholding techniques possess disadvantages, including the presence of discontinuities in the hard threshold function and the consistent deviations between the estimated wavelet coefficients by the soft threshold function and the signal wavelet coefficients.

3. Improved Wavelet Threshold Function

3.1. Algorithmic Process

(1) The following are the precise procedures of the wavelet threshold denoising algorithm.

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Apply wavelet transformation to the signal containing noise. To acquire the wavelet coefficients at various scales, ascertain the wavelet basis function and the number of wavelet transform stages based on the signal containing noise.

(2) Thresholding. Choose an appropriate threshold function for thresholding the wavelet coefficients at various transformation scales in order to obtain the estimated wavelet coefficients.

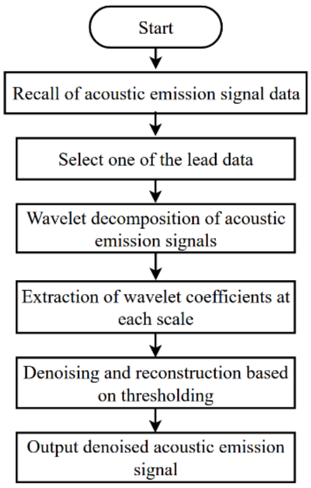


Fig 1. Flowchart of acoustic emission signal based on wavelet threshold denoising

(3) Signal reconstruction. The denoised signal is obtained by performing wavelet inversion on the wavelet coefficients acquired in step (2).

Figure illustrates the procedure of wavelet threshold denoising technique for acoustic emission signal.1.

3.2. Improved Wavelet Threshold Function

Constructing an appropriate threshold function is of utmost importance in wavelet threshold denoising algorithms.

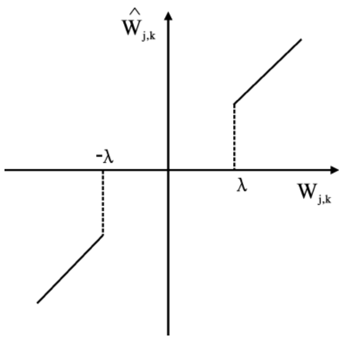


Fig 2. Hard threshold function image

The hard threshold function, depicted in Figure 2, exhibits intermittent and discontinuous behavior at the threshold point. This function effectively preserves the spiking characteristics of the signal, but it also introduces the drawback of inducing oscillations in the denoised signal at the threshold point, leading to the occurrence of Pseudo Gibbs phenomenon[12]. As depicted in the illustration. The soft threshold function, with its excellent continuity, exhibits a consistent deviation λ in its processed wavelet coefficients when compared to the hard threshold function. As a result, the denoised signals processed by the soft threshold function are relatively smoother, albeit with occasional issues of imprecision and distortion.

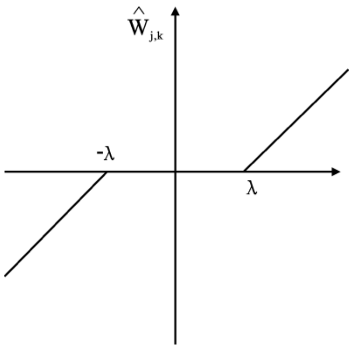


Fig 3. displays an image depicting the soft threshold function.

Wavelet research has focused on enhancing the threshold function due to the unsatisfactory outcomes provided by the conventional soft and hard threshold functions in denoising applications. In the field of literature, a technique was suggested to create a fresh threshold function by merging the approaches of soft and hard thresholding.

$$\widehat{W_{j,k}} = \begin{cases} uW_{j,k} + (1-u)\operatorname{sgn}\left(W_{j,k}\right)\left(\left|W_{j,k}\right| - \lambda\right) & ,\left|W_{j,k}\right| \geqslant \lambda \\ 0, & \left|W_{j,k}\right| < \lambda \end{cases} \tag{9}$$

where $u = 1 - e - \alpha\left(\left|W_{j,k}\right| - \lambda\right)2$, α are adjustment factors with a range of $0 \le \alpha \le 1$.

Although Eq. Although (9) enhances the efficiency of the wavelet threshold denoising algorithm to a certain degree, it remains inadequate in addressing the issue of consistent deviation between the estimated wavelet coefficients and the wavelet coefficients of the noisy signals. Additionally, there is a noticeable distortion in the denoised signals after reconstruction. In order to tackle the aforementioned issues, a threshold estimator with an exponential parameter is introduced in literature [13] to enhance the threshold function by addressing the issue of inconsistent deviation. The function for combining soft and hard thresholds has been enhanced.

$$\widehat{W_{j,k}} = \begin{cases} uW_{j,k} + (1-u)\operatorname{sgn}\left(W_{j,k}\right)\left(\left|W_{j,k}\right| - T\right) & ,\left|W_{j,k}\right| \geqslant \lambda \\ 0, & \left|W_{j,k}\right| < \lambda \end{cases} \tag{10}$$

where $T = \lambda e^{(1-\alpha)(\lambda - |W_{j,k}|)}$, α are adjustment factors with a range of $0 \le \alpha \le 1$.

Eq.By acting as an asymptote, (10) addresses the issue of persistent bias between and, thereby enhancing the signal's denoising effectiveness. According to literature [7], achieving an optimal denoising of the signal may not be possible when the deviation is reduced to zero (in the hard threshold case). To address the limitations of the aforementioned algorithms, this study develops a novel threshold function by leveraging the findings from the literature [9].

$$\widehat{W_{j,k}} = \frac{W_{j,k}\left[1 + 2^{e\alpha\left(-W_{j,k} - T\right)} + e^{-2\alpha W_{j,k}}\right]}{\left(1 + e^{\alpha\left(-W_{j,k} - T\right)}\right)\left(1 + e^{\alpha\left(-W_{j,k} + T\right)}\right)} \tag{11}$$

where $T = \begin{cases} \lambda * \alpha^{\left(1-\frac{1}{\alpha}\right)\left(\lambda - |W_{j,k}|\right)}, \left|W_{j,k}\right| > \lambda \\ \alpha * \alpha^{\left(1-\frac{1}{\alpha}\right)\left(\lambda - |W_{j,k}|\right)}, \left|W_{j,k}\right| \leqslant \lambda \end{cases}$, α are adjustment factors with a range of $\alpha \ge 10$.

To improve the denoising effect, this paper introduces a threshold estimator to correct the threshold function mentioned in literature [9], which does not effectively handle wavelet coefficients below the threshold. The threshold estimator in this paper utilizes an exponential parameter. It considers the threshold as the critical point. For wavelet coefficients exceeding the threshold, the threshold is used as the coefficient of the estimator. For wavelet coefficients below the threshold, the adjustment factor serves as the coefficient. This approach addresses the issue of imperfect denoising at the threshold and further reduces the constant deviation between and.

To simplify the examination of the novel threshold equation, Eq. (11) is transformed to obtain:

$$y(x)=\frac{x\left[1+2e^{\alpha(-x-T)}+e^{-2\alpha x}\right]}{\left(1+e^{\alpha(-x-T)}\right)\left(1+e^{\alpha(-x+T)}\right)} \tag{12}$$

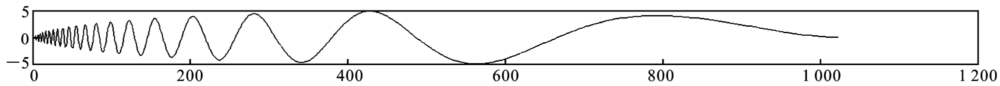
In the formula, $T=\begin{cases}\lambda*\alpha^{\left(1-\frac{1}{\alpha}\right)(\lambda-|x|)}, |x|>\lambda \\ \alpha*\alpha^{\left(1-\frac{1}{\alpha}\right)(\lambda-|x|)}, |x|\leqslant\lambda\end{cases}$.

4. Aimulation Analysis

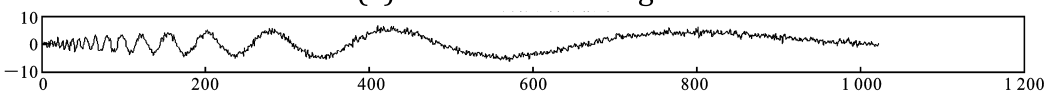
This paper evaluates the effectiveness of the suggested wavelet threshold denoising technique by measuring the signal-to-noise ratio (SNR) and mean square error (MSE). The initial signal in the trial is the Doppler signal, the signal containing noise has a SNR of 12.7906 dB, the chosen wavelet function is the dB4 wavelet, the decomposition stages amount to 4, and the new thresholding function is modified by a factor.

$$SNR=10\lg\frac{\sum_{1}^{N}f(n)^2}{\sum_{1}^{N}(f(n)-\hat{f}(n))^2} \tag{13}$$

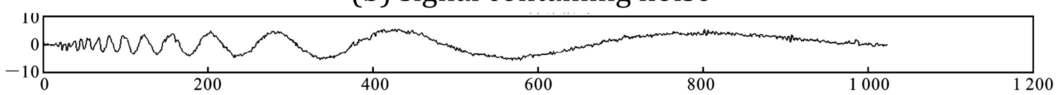
$$MSE=\frac{\sum_{1}^{N}(f(n)-\hat{f}(n))^2}{N} \tag{14}$$



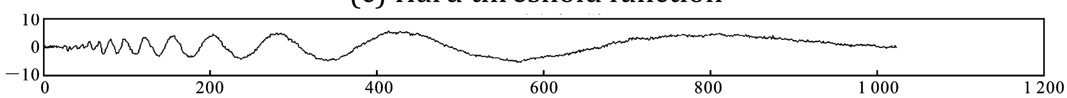
(a) Raw noise-free signal



(b) signal containing noise



(c) Hard threshold function



(d) Soft thresholding function

Fig 4. illustrates the comparison between various threshold functions in term of denoising outcomes

ISSN: 2688-8645

The original signal is denoted as , while the denoised signal after undergoing the wavelet threshold denoising algorithm is represented as ; N refers to the signal's length. In the denoising simulation experiments, a total of two sets of controlled experiments were carried out. In the case of universal thresholding, experiment one focuses on denoising various threshold functions, including unified thresholding which involves the selection of a threshold using equation (13).Fig.4 and Fig.5 show the simulation of denoising effect of experiment I. The comparison table of the signal-to-noise ratio and mean square error is displayed in Table 1.

Based on the aforementioned findings, it is evident that the new thresholding function exhibits the highest signal-to-noise ratio and the lowest mean-square error among the alternative thresholding functions in the context of universal thresholding. This suggests that its denoising capability is the most efficient.

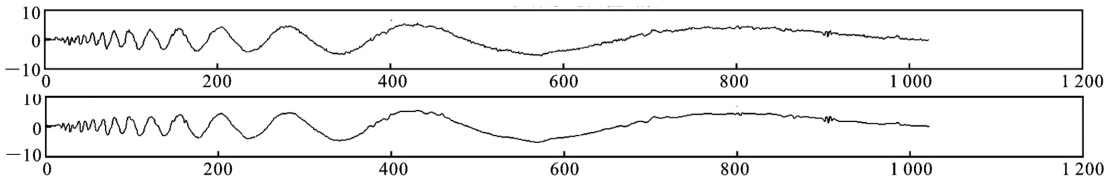
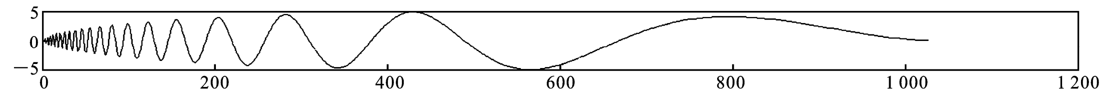


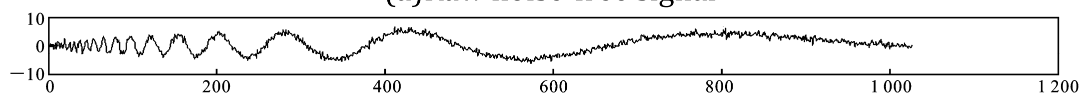
Fig 5. illustrates the comparison of denoising outcomes between the threshold function and the novel threshold function mentioned in the literature [9]

Table 1. Comparison of signal-to-noise ratio and mean square error for different threshold functions for generic thresholding

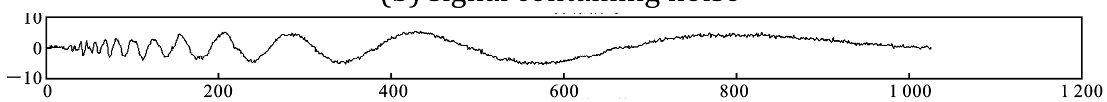
Denoising method	SNR/dB	MSE
signal containing noise	12.7906	2.2046
hard threshold function	19.1482	0.1369
soft threshold function	19.7266	0.1277
Literature [9] Functions	20.6903	0.0879
new threshold function	22.4854	0.0586



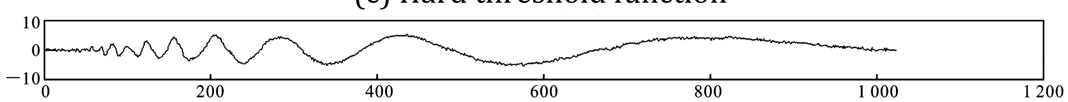
(a)Raw noise-free signal



(b) signal containing noise



(c) Hard threshold function



(d)Soft thresholding function

Fig 6. Evaluating the denoising outcomes of various threshold functions through comparison

In this paper, we propose an improved threshold selection method for denoising the signal in Experiment II, which is combined with various threshold functions.Fig.6 and Figure.7

Experiment II result plots are displayed in Figure 7, while Table 2 presents the comparison table for its signal-to-noise ratio and mean square error.

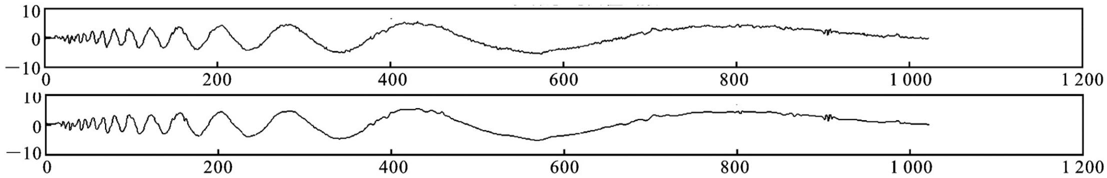


Fig 7. illustrates the comparison of denoising outcomes between the threshold function and the novel threshold function mentioned in the literature [9]

Table 2. Comparison of signal-to-noise ratio and mean square error for different threshold functions with improved thresholds

Denoising method	SNR/dB	MSE
signal containing noise	12.7906	2.2046
hard threshold function	19.7935	0.1321
soft threshold function	21.9663	0.1222
Literature [9] Functions	22.0483	0.0834
new threshold function	23.8987	0.0529

The results of Experiment 2 demonstrate that the enhanced threshold selection reveals that the new threshold function outperforms the other threshold functions in terms of signal-to-noise ratio, mean-square error, and denoising effect.

By combining the findings from Experiment I and Experiment II, it becomes evident that the wavelet threshold denoising algorithm, as suggested in this study, exhibits the highest signal-to-noise ratio and the lowest mean-square error in the denoised signals. This outcome is achieved by implementing the enhanced threshold selection method and the novel threshold function, thus establishing the superiority of the improved wavelet threshold denoising algorithm over alternative denoising algorithms.

5. Conclusion

To improve the effectiveness of reducing noise in acoustic emission signals, this research paper suggests an enhanced denoising algorithm based on wavelet threshold. The algorithm is developed by analyzing the features of the conventional wavelet threshold denoising algorithm and other enhanced algorithms, and making improvements to the threshold function and selection method. The analysis and comparison of denoising results under different algorithms were conducted through signal denoising simulation experiments. It was confirmed that the enhanced algorithm enhances the signal-to-noise ratio (SNR) and mean square error (MSE) by 14.4% and 58.3% respectively, in comparison to the traditional algorithm. Additionally, it improves the algorithm proposed in literature by 8.4% and 36.5%. These findings demonstrate the effective performance of the wavelet threshold denoising algorithm proposed in this paper in terms of signal denoising and performance. The improved wavelet threshold denoising algorithm proposed in this paper demonstrates its effectiveness and superiority in enhancing signal denoising effect and performance.

Acknowledgments

Fund projects:Electric Youth Top Talent Fund of NEPU(DQKY202303).

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