

# Rolling Bearing Fault Feature Extraction Method based on SSA-VMD and MOMEDA

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## Abstract

To address the challenge of extracting bearing fault features, this study proposes a new rolling bearing fault feature extraction method based on the Sparrow Search Algorithm (SSA) to optimize Variational Mode Decomposition (VMD) and Multipoint Optimal Minimum Entropy Deconvolution with Convolution Adjustment (MOMEDA). Firstly, SSA is employed to identify optimal parameters in VMD, followed by the utilization of correlation coefficients and kurtosis to filter relevant Intrinsic Mode Function (IMF) components. Subsequently, MOMEDA is applied to denoise the reconstructed signal, mitigating the interference caused by pulse fault signals. Finally, the envelope spectrum analysis is conducted on the denoised signal. Experimental results demonstrate the efficacy of the proposed method in extracting fault features and mitigating noise interference.

## Keywords

Sparrow Search Algorithm; MOMEDA; Variational Mode Decomposition; Steepness; Correlation.

## 1. Introduction

After rotating equipment operates, the weak vibration signals from rolling bearings sensed by sensors are often rich in various components due to environmental factors and internal interferences [1]. They exhibit strong noise and multiple interferences. Moreover, when rolling bearings fail, modulation phenomena between different components during operation are common. Additionally, the signals vary over time, demonstrating non-stationarity and impulsiveness, which poses challenges for fault monitoring and extraction in rolling bearings.

Fault diagnosis technology originated during the Apollo lunar landing program in the United States, prompted by serious equipment failures resulting in astronaut fatalities. To prevent such incidents, fault monitoring and diagnosis technologies are developed following accident investigation conferences. These technologies involve detecting signals generated by mechanical equipment and determining whether faults have occurred by extracting signal features. Classical signal analysis methods such as resonance demodulation, wavelet analysis, Empirical Mode Decomposition (EMD) [2], and Complementary Ensemble Empirical Mode Decomposition with Adaptive Noise (CEEMDAN) [3] have been successfully applied in fault diagnosis across various mechanical equipment. These methods can be categorized into three main types: time-domain, frequency-domain, and time-frequency domain analysis.

For instance, a combination of wavelet denoising and short-time Fourier transform effectively processed the unbalanced non-stationary vibration signals of spindles [4]. Literature [5] applied an improved VMD method in monitoring spindle bearing wear, achieving satisfactory results. Moreover, a composite fault diagnosis method for rolling bearings was proposed by

combining wavelet packet transform with Complete Ensemble Empirical Mode Decomposition with Adaptive Noise (CEEMDAN), achieving favorable results in simulated and measured signals [6].

To accurately extract fault features, mitigate noise effects, and enhance the impulse characteristics of fault signals, this paper proposes a rolling bearing fault feature extraction method based on SSA-VMD and MOMEDA. Firstly, SSA is employed to determine the optimal parameters for penalization factors and decomposition levels in VMD. After setting the optimal parameters, the original signals undergo VMD decomposition, and the criteria of correlation and steepness are used to select IMF components requiring subsequent denoising treatment. Secondly, MOMEDA is utilized to enhance the fault characteristics of denoised signal frequency bands. Lastly, envelope and demodulation analysis are conducted on the fault signals to accurately extract the characteristic frequencies and high-order harmonics of rolling bearing faults.

## 2. VMD Principle

The VMD method is a signal decomposition method proposed on the basis of EMD, but there are significant differences between the two in nature [7]. The VMD method uses a combination of many adaptive Wiener filters to process the signals, and introduces a quadratic penalty term to transform the constrained variational problem into an unconstrained problem. The central frequency of each IMF is iteratively solved by solving the constrained problem.

Generally speaking, the signal to be decomposed is generally composed of the original signal plus noise, and its expression is given as

$$x(t) = x_0(t) + n(t) \quad (1)$$

where  $x(t)$  is the signal to be decomposed;  $x_0(t)$  is the original signal;  $n(t)$  is noise.

In the decomposition and reconstruction, the original signal  $x_0(t)$  belongs to the unknown signal and belongs to the pathological inverse problem, and the reconstruction problem of the original signal is solved by introducing the Tikhonov regularization method, that is, after transformation, according to Euler-Lagrangian and Fourier transform theory, etc., and it is obtained:

$$\hat{x}_0(w) = \frac{\hat{x}(w)}{1 + \alpha w^2} \quad (2)$$

where  $\hat{x}_0(w)$  is the reconstruction signal which represents the Fourier transform of the original signal;  $\alpha$  is the noise variance.

The different modal components are resolved by the Hilbert transform, and then the generated unilateral spectrum is transferred to the respective central baseband by the exponential factor, and the bandwidth is estimated by demodulating the Gaussian smoothing of the signal, and the mathematical model is given as

$$\left\| \left[ \left( \delta(t) + \frac{j}{\pi t} \right) u_k(t) \right] e^{-jw_k t} \right\|_2^2 \quad (3)$$

where  $\left[ \left( \delta(t) + \frac{j}{\pi t} \right) u_k(t) \right]$  represents the unilateral spectrum generated by the analysis of the modal components;  $e^{-jw_k t}$  denotes the exponential factor;  $\|\bullet\|_2^2$  is the square of the L2 norm, which indicates the degree of Gaussian smoothness of the demodulated signal.

Therefore, the decomposition problem of VMD becomes a variational problem, which is converted into a decomposition signal by finding the minimum sum of the bandwidths of each modal component, and its mathematical model is given as

$$\left\{ \begin{array}{l} \left\{ \begin{array}{l} \{u_K\} \\ \{w_K\} \end{array} \right\} \left\{ \sum_K \left\| \left[ \left( \delta(t) + \frac{j}{\pi t} \right) u_K(t) \right] e^{-jw_K t} \right\|_2^2 \right\} \\ s.t. \sum_K u_K = f \end{array} \right\}_{\min} \quad (4)$$

where  $s.t. \sum_K u_K = f$  is the constraint of the mathematical model, which means that the sum of all modal components should be equal to the original signal;  $\{u_K\} = u_1, u_2, \dots, u_K$  and  $\{w_K\} = w_1, w_2, \dots, w_K$  represent the center frequencies of each modal component and each modal component, respectively.

In order to solve the above problems, the VMD method transforms the variational problem of signal decomposition into an unconstrained problem by introducing the quadratic penalty factor  $\alpha$  and the Lagrange operator  $\lambda(t)$ , and solves (4) by the alternating direction multiplier algorithm to transform the problem of finding the sum of the minimum bandwidth into the problem of iteratively finding the local optimal solution. Among them, the magnitude of the secondary penalty factor determines the level of the noise signal, and its magnitude is inversely proportional to the noise, and the Lagrange operator is mainly to make the implementation of the above problem more stringent. The expression is given as

$$\begin{aligned} L(\{u_K\}, \{w_K\}, \lambda) &= \alpha \sum_K \left\| \left[ \left( \delta(t) + \frac{j}{\pi t} \right) u_K(t) \right] e^{-jw_K t} \right\|_2^2 \\ &+ \left\| f(t) - \sum_K u_K(t) \right\|_2^2 + \left\langle \lambda(t), f(t) - \sum_K u_K(t) \right\rangle \end{aligned} \quad (5)$$

Through the Fourier equidistant transformation and the self-iterative update of  $W$ , the local optimal solution is found. Then they are respectively given as

$$u_K^{n+1}(w) = \frac{2f(w) - 2 \sum_{i \neq K} u_i(w) + \lambda(w)}{2 + 4\varepsilon(w - w_K)^2} \quad (6)$$

$$w_K^{n+1} = \frac{\int_0^\infty w |u_K(w)|^2 dw}{\int_0^\infty |u_K(w)|^2 dw} \quad (7)$$

Finally, by setting the judgment precision  $e$ , it is decided whether the iteration will be stopped. The expression is given as

$$\sum_K \frac{\|u_K^{n+1} - u_K^n\|_2^2}{\|u_K^n\|_2^2} < e \quad (8)$$

### 3. SSA Principle

The sparrow search algorithm is a swarm intelligence optimization algorithm derived from simulating the foraging process of sparrow flocks[8], and the sparrows in the flock are mainly divided into finders, followers and vigilantes, which are responsible for searching for food, following the discoverer to forage for food, and vigilant and detecting respectively. In the

sparrow search algorithm, the sparrow that obtains food earlier is the finder with the best fitness value, and then indicates the foraging direction for all followers, and compared with the followers, the finder can obtain a larger foraging search range, find potential excellent solutions, and pass them on to the followers. According to the solution provided by the finder and their own search experience, the follower conducts a refined search of the search space, aiming to find a better solution in the search space and pass it on to the vigilant. The Watcher is the control center of the Sparrow search algorithm and is responsible for overseeing and coordinating the activities of finders and followers. Based on the current search status and the information of the global search process, the vigilante decides whether to change the search strategy and adjust the parameters to ensure the convergence of the algorithm and the global search effect. Compared with other search algorithms, the sparrow search algorithm can iteratively adjust the parameters of the model, and the sparrow search algorithm has the characteristics of strong global search ability, fast convergence speed and wide range of application, so it is widely used in the establishment of combination models.

The three important parameters of SSA are the initial size (number of sparrow populations), the maximum number of iterations, and the early warning value, and the three roles are finder, follower, and alert, and their values have an important impact on the search performance and accuracy of SSA. The pop of the sparrow population and the maximum number of overlapping generations Max\_iteration determine the breadth and depth of the search, respectively, and with the increase of the population number and the number of iterations, the search effect of SSA will be better, and it will be easier to find the global optimal solution, but the search time will also be longer. If the population size or the number of iterations is too large, the search time will be too long, which is not conducive to the acquisition of multiple parameters of the VMD algorithm, so the pop and Max\_iteration are obtained within the range of 10~100. The size of the early warning value ST directly affects the search speed and accuracy of the algorithm: when the early warning value is small, the algorithm pays more attention to the local optimal solution, the search speed will be slower but the accuracy will be higher, and vice versa, so the ST is adjusted in the range of 0.5~1. Because the sample data of the surface subsidence coefficient is a high-dimensional small data sample, and the search effect, speed and performance of SSA need to be taken into account, the average values of pop and Max\_iteration of SSA after searching by the grid search method are 30, and the ST value is 0.8. In order to discover the global optimal solution in time and prevent other individuals from falling into the local optimal solution, 20% of the sparrow population is selected as the proportion of discoverers and alerters in the sparrow population, and the remaining 60% sparrow are used as followers to expand the search range and increase the search breadth, and the parameter values of the sparrow search algorithm are shown in Table 1.

**Table 1.** The parameter values of the sparrow search algorithm

Parameter	Meaning	Adjustment range	Valid values
Max_iteration	Maximum number of iterations	10~100	20
pop	Sparrow population	10~100	10
ST	Warning value	0.5~1	0.8

#### 4. MOMEDA Principle

MOMEDA is an improved method of Minimum Entropy Deconvolution (MED) and maximum correlated kurtosis deconvolution (MCKD), which overcomes the defects of many parameters and is susceptible to pulses. The non-iterative method is used to find the optimal finite impulse response filter, and the objective function is given as

$$MOMEDA(y_r, t) = \max_f \frac{t^T y_r}{\|y_r\|} \tag{9}$$

where  $y_r$  is the original impact signal;  $f$  is the filter vector;  $T$  is the failure cycle. The target vector  $t$  is the position and weight of the deconvolution impact component. The derivatives and extrema of the inverse filter coefficient  $f$  are respectively given as

$$\frac{d}{df} \left( \frac{t^T y_r}{\|y_r\|} \right) = \frac{d}{df} \left( \frac{t_1 y_{r1}}{\|y_r\|} \right) + \frac{d}{df} \left( \frac{t_1 y_{r2}}{\|y_r\|} \right) + \dots + \frac{d}{df} \left( \frac{t_{N-L} y_{rN-L}}{\|y_r\|} \right) \tag{10}$$

$$\begin{aligned} \|y_r\|^{-1} X_0 t - \|y_r\|^{-3} t^T y_r X_0 y_r &= 0 \\ \frac{t^T y_r}{\|y_r\|^2} X_0 y_r &= X_0 t \end{aligned} \tag{11}$$

Suppose  $(X_0 X_0^T)^{-1}$  exists, and  $y_r = X_0^T f$  is brought into Eq. (11), then Eq. (11) becomes:

$$\frac{t y_r}{\|y_r\|^2} f = (X_0 X_0^T)^{-1} X_0 t \tag{12}$$

If the special solution of the filter coefficient  $f$  is optimal, then we have

$$f = (X_0 X_0^T)^{-1} X_0 t \tag{13}$$

Substituting Eq. (13) into  $y_r = X_0^T f$ , we can find the original  $y_r$  of the impact signal.

### 5. Experimental Verification

To verify the effectiveness of the proposed method in the actual rolling bearing fault diagnosis, the bearing fault data of the Bearing Center of Western Reserve University in the United States is used as the analysis object for fault diagnosis, and the experimental platform is composed of a motor with a power of 1.5kw, a torque sensor/decoder, a power tester and an electronic controller. The bearing type is 6205-2RS JEM SKF deep groove ball bearings with 9 rolling elements. The specific bearing data is shown in Table 2.

**Table 2.** 6205-2RS JEM SKF deep groove ball bearings

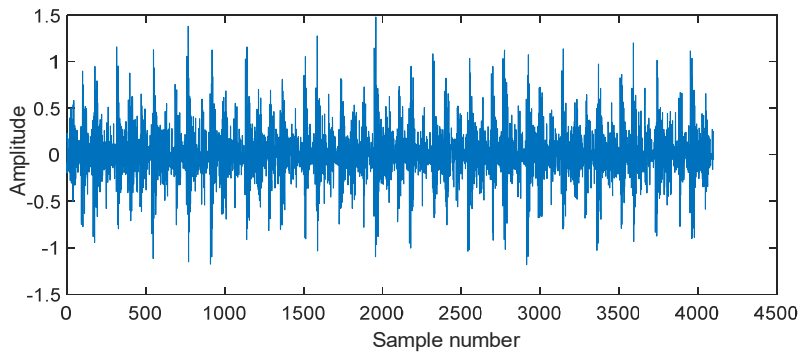
Contact angle /°	Section diameter /mm	Ball diameter /mm	Outside diameter /mm	Inner diameter /mm	Number of rolling bodies
0	39	7.5	52	25	9

The experimental sampling frequency is  $f_s=12kHz$ , the motor speed is  $1797r/min$ , and the corresponding conversion frequency is  $f_r=29.95Hz$ . According to the bearing parameters and frequency, the fault frequency  $f_i=160.69Hz$  and the outer ring failure frequency  $f_o=108.86Hz$  can be calculated.

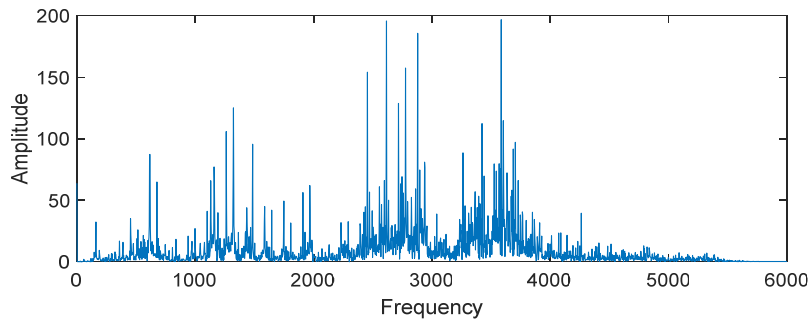
According to Figure 1, it can be seen that the time and frequency domains of the signal contain a large number of noise signals, and it is difficult to judge the failure mode of the bearing, and subsequent noise reduction is required to extract the fault signal.

In order to remove random noise and accurately extract the fault feature frequency, we use VMD to decompose the original signal. According to Table 1, the parameters of the initialized sparrow search algorithm, Figure 2 is the iterative diagram of the SSA algorithm, and it can be seen that it tends to stabilize after 14 steps, and the obtained optimal decomposition mode  $K$  is 8, and the penalty factor  $a$  is 199. VMD decomposition of the signal is shown in Figure 3.

The autocorrelation coefficient and steepness values for each MRA component are calculated as shown in Table 3.



(a) Time domain waveform



(b) Frequency domain waveform

Figure 1. The characteristics of outer ring signal

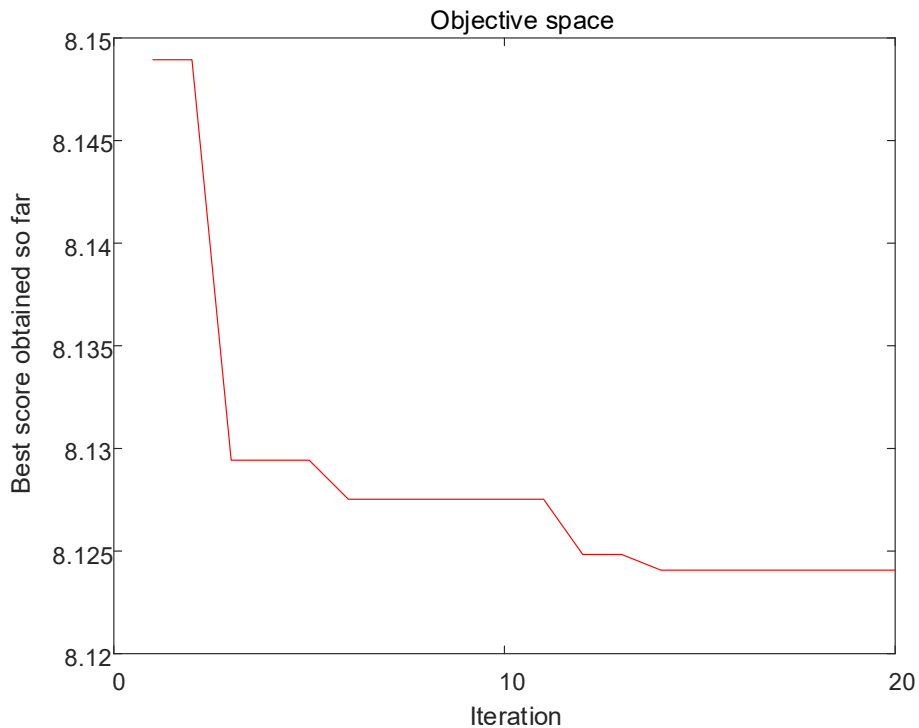
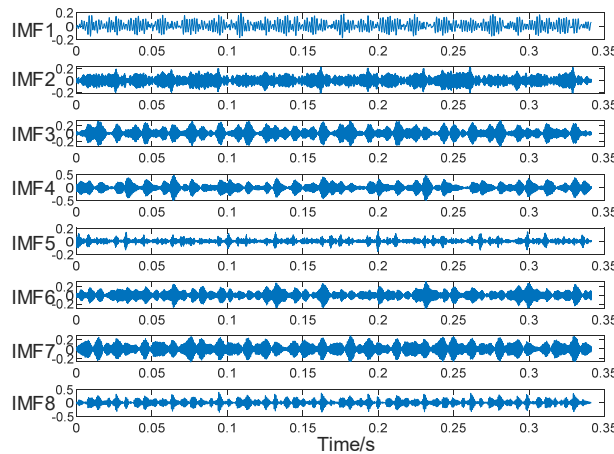


Figure 2. Iterative diagram of the SSA algorithm

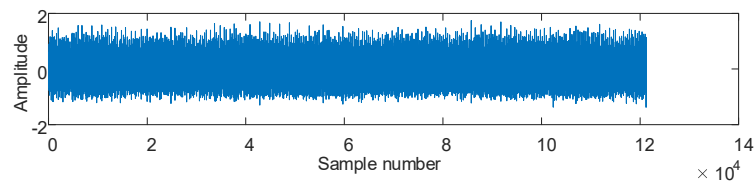
Table 3. The specific parameters of IMFs

	IMF1	IMF2	IMF3	IMF4	IMF5	IMF6	IMF7	IMF8
Correlation Coefficient	0.254	0.348	0.478	0.528	0.195	0.398	0.461	0.391
Kurtosis	2.599	2.484	3.123	3.396	5.416	3.252	2.603	3.633

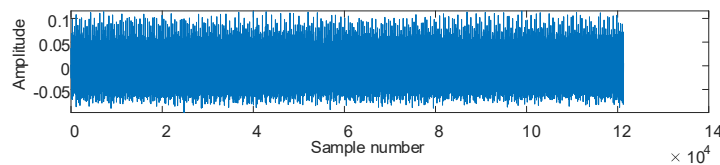
As can be seen from Table 2, the autocorrelation coefficient value of IMF5 is the smallest, and the kurtosis value is much greater than 3, which is judged to be a noisy signal. IMF1~4 and IMF6~8 is reconstructed. The reconstructed signal is shown in Figure 4, the overlay signal is used for MOMEDA noise reduction, and the time domain plot of the noise reduction signal is shown in Figure 5.



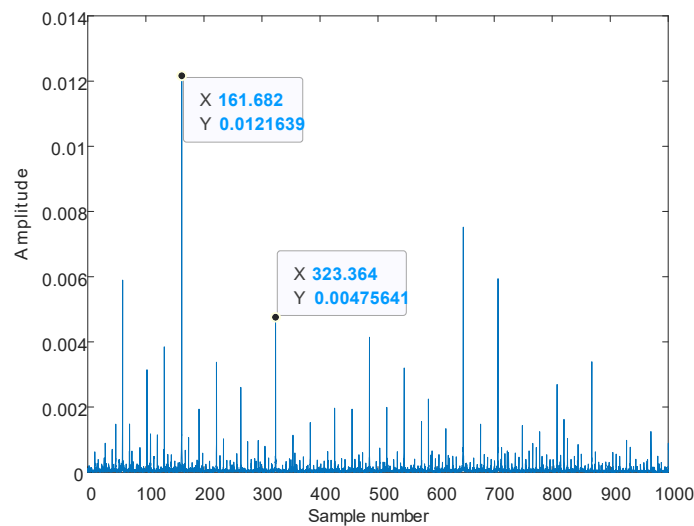
**Figure 3.** The schematic diagram of each component after the VMD decomposes the outer ring signal



**Figure 4.** The reconstruction signal of the outer ring signal



**Figure 5.** The inner ring signal after screening



**Figure 6.** The envelope spectrum of the denoised signal

The envelope spectrum of the denoised signal is analyzed, and the envelope spectrum of the reconstructed signal is shown in Figure 6, and the conversion frequency and fault frequency are clearly visible, and the relative error of the fault frequency is 0.62%. As a result, fault features can be extracted accurately.

## 6. Conclusion

In this paper, a new bearing fault diagnosis method is proposed, which first uses SSA to find the decomposition layer  $K$  and penalty factor  $a$  in the VMD, and then uses the adaptive VMD to decompose the original signal to obtain  $K$  IMFs, and then combines the steepness value and the autocorrelation coefficient to screen the components, remove the noise component, and then we perform MOMEDA noise reduction on the reconstructed signal. Finally, the envelope spectrum analysis of the noise reduction signal is carried out, and the fault characteristics are effectively extracted and the noise interference is reduced through the analysis of the measured fault signal.

In order to accurately extract the fault features of the bearing under the background of noise, the results show that a rolling bearing fault feature extraction method of SSA-VMD-MOMEDA is verified by the experimental measured signals. The specific conclusions of this paper are given as follows:

- (1) The VMD after SSA optimization has strong adaptability, and can well retain the structure and details of the signal.
- (2) The comprehensive autocorrelation coefficient and kurtosis value can effectively denoise and avoid the loss of fault information.
- (3) The MOMEDA noise reduction method has a stable effect, and the appropriate number of filter layers can be artificially set to better improve the fault characteristics.

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