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Non-fragile Fuzzy Control for Fractional-order Unified Chaotic Systems

Xiaofei Yan

School of Electrical & Information Engineering, Northeast Petroleum University, Daqing, China

Abstract

The fuzzy control of fractal-order chaotic systems are studied. Firstly, according to the T-S fuzzy modeling theory, the fractional order unified chaotic system is reconstructed by T-S fuzzy modeling. Secondly, the steady-state performance of Fractional-order unified chaotic systems is discussed by using Lyapunov stability theory. Then, a Fractional-order Non-fragile fuzzy controller is designed according to the linear matrix inequality (LMI) method. Finally, a simulation example is given to verify the effectiveness of the controller.

Keywords

Fractional Order Systems; Chaos; Fuzzy Control; Non-fragile Fuzzy Controller.

1. Introduction

With the development of computer technology, people study of fractional order chaotic systems is increasingly deepening[1]. Fractional order chaotic systems in the field of life science physical economics confidential communications have been widely used. However, although the researchers at home and abroad in the synchronization of chaotic systems and control and its application has achieved certain results, but research is still in its infancy[2-7]. Therefore, both in the theoretical study of fractional chaotic system and in its engineering application, further improvement is needed.

Fuzzy control is an intelligent control method based on fuzzy set theory, fuzzy language variables and fuzzy logic reasoning,Imitate human fuzzy reasoning and decision-making process. Fuzzy control has strong robustness and has been applied to the control and synchronization of fractional differential equations in recent years[8-9].

2. T·S Fuzzy Reconstruction of the System

Unified chaotic system is a continuous autonomous three-dimensional chaotic system, and its mathematical model is shown as follows：

$$\dot{x}(t)=\begin{cases}\dot{x}_1=(25\sigma+10)(x_2-x_1)\\ \dot{x}_2=(28-35\sigma)x_1+(29\sigma-1)x_2-x_1x_3\\ \dot{x}_3=x_1x_2-\dfrac{8+\sigma}{3}x_3\end{cases} \tag{1}$$

If σ continually fluctuates between [0,1], the system is chaotic.The definition of the corresponding Fractional-order unified chaotic system is shown below:

$$D^{\alpha}x(t)=\begin{cases}D^{\alpha}x_1(t)=(25\sigma+10)(x_2-x_1)\\ D^{\alpha}x_2(t)=(28-35\sigma)x_1+(29\sigma-1)x_2-x_1x_3\\ D^{\alpha}x_3(t)=x_1x_2-\dfrac{8+\sigma}{3}x_3\end{cases} \tag{2}$$

Where,α is the order of the derivative and 0<α<1.

According to the fuzzy rules described by T-S model, the global variable state equation of chaotic system (2) can be obtained by using single point fuzzification, product reasoning and center weighted average solution fuzziness:

$$\begin{aligned}&\omega_i(z(t))\geq 0,\sum_{i=1}^{N}\omega_i(z(t))>0, i=1,2,\ldots,N\\ &h_j(z(t))\geq 0,\sum_{j=1}^{N}h_j(z(t))=1, j=1,2,\ldots,N\end{aligned} \tag{3}$$

In a Fractional-order unified chaotic system with two nonlinear terms and a fuzzy T-S model called Equation (3), the nonlinear part of the equation of state must be linearized.Obtained from the above:

$$f_1(x(t))=x_1(t)x_3(t)=\left\{\sum_{i=1}^{2}h_i g_i(x(t))\right\}x_3(t) \tag{4}$$

$$f_2(x(t))=x_1(t)x_2(t)=\left\{\sum_{i=1}^{2}h_i g_i(x(t))\right\}x_2(t) \tag{5}$$

Where, $g_1(x(t))=M_1, g_2(x(t))=M_2, M_1, M_2$ is the fuzzy set.

3. Fuzzy Controller

Lemma 1 [10]: Let's set Y to be Y matrix of some dimension,there are $Y+M\Delta(t)N+N^T\Delta^T(t)M^T<0$ .If and only if there is a constant $\varepsilon>0$, Satisfy the matrix inequality $Y+\varepsilon MM^T+\frac{1}{\varepsilon}NN^T<0$.

$$D^{\alpha}x(t)=\sum_{i=1}^{2}h_i(z(t))[A_ix(t)+B_iu(t)] \tag{6}$$

According to the fuzzy rule, $K_i\in R^{1\times 3}$ is set as a feedback gain matrix, The established controls are as follows:

$$u(t)=\sum_{i=1}^{2}h_i(z(t))(K_i+\Delta K_i)x(t) \tag{7}$$

Therefore, according to lemma 1, the global closed-loop fuzzy control system composed of system (6) and system (7) is:

$$\begin{aligned} D^{\alpha}x(t) &= \sum_{i=1}^{2}\sum_{j=1}^{2} h_i(z(t))h_j(z(t))\{A_1x(t)+B_1u(t)\} \\ &= \sum_{i=1}^{2}\sum_{j=1}^{2} h_i(z(t))h_j(z(t))\{A_1+B_1(K_j+\Delta K_j)\}x(t) \\ &= \sum_{i=1}^{2} h_i^{\,2}(z(t))\{A_1+B_1(K_i+H_iF_i(t)J_i)\}x(t) \\ &\quad + 2h_1(z(t))h_2(z(t))\times \\ &\quad \left\{\frac{A_1+B_1K_2+A_2+B_2K_1+B_1H_2F_2(t)J_2+B_2H_1F_1(t)J_1}{2}\right\}x(t) \end{aligned} \tag{8}$$

Definition 1 [11]: The QTH fractional derivative of $V(x(t),t)=x^T(t)Px(t)$ with respect to time t is

$$ {}_0D_t^qV(x,t)=x^T(t)PD_t^qx(t)+L_x \tag{9}$$

Where, P is a positive definite symmetric matrix, and

$$L_x=\sum_{i=1}^{\infty}\frac{\Gamma(1+q)}{\Gamma(1+q)\Gamma(1-k-q)}\,{}_0D_t^kx(t)P\,{}_0D_t^{q-k}x(t) \tag{10}$$

Where, L satisfies the following conditions:

$$\|L_x\|\le\delta\|x^2\| \tag{11}$$

Definition 2 [12]: For system (6), the Lyapunov function is chosen as $V(x(t))=2x^T(t)Px(t)$,P is a positive definite symmetric matrix.If there is a real number β that satisfies $D^{\alpha}V(x(t))\le 2\beta V(x(t))$, Then system (6) is said to be globally asymptotically stable with decay rate P.

$$\begin{aligned} D^{\alpha}V(x(t)) &= 2x^T(t)PD^{\alpha}x(t)+2L_x \\ &= \sum_{i=1}^{2} h_j^2(z(t))2x^T(t)[PA_1+B_iK_i+B_iH_iF_i(t)J_i+\delta I]x(t) \\ &\quad + 2h_1(z(t)h_2(z(t)) \\ &\quad \left\{2x^T(t)[2\delta I+\frac{PB_1H_2F_2(t)J_2}{2}+\frac{PB_2H_1F_1(t)J_1}{2}+\frac{P(A_1+B_1K_2+A_2+B_2K_1)}{2}]\right\}x(t) \end{aligned}$$

4. Numerical Simulation

When σ =1, system (2) is chen system. So let's take a = 0.9, The Other Parameter is $\beta=0.5$, $B_1=[1,0,0]^T$, $H_1=[0.5,0.8,0.5]$, $H_2=[0.6,0.9,0.8]$,

$$J_1=\begin{bmatrix}0.1 & 0.2 & 0.2\\ 0.1 & 0.1 & 0.01\\ 0.3 & 0.4 & 0.01\end{bmatrix},\ J_2=\begin{bmatrix}0.2 & 0.1 & 0.1\\ 0.2 & 0.1 & 0.02\\ 0.3 & 0.4 & 0.01\end{bmatrix}$$

Take the initial value $(x_1(0), x_2(0), x_3(0)) = (5,-10,-10)$,Set the step size to 0.03, plus the amount of control after 100 steps of the original system iteration.Figure 1 and Figure 2 respectively represent the state response and phase trajectory of the fractional Chen chaotic system.

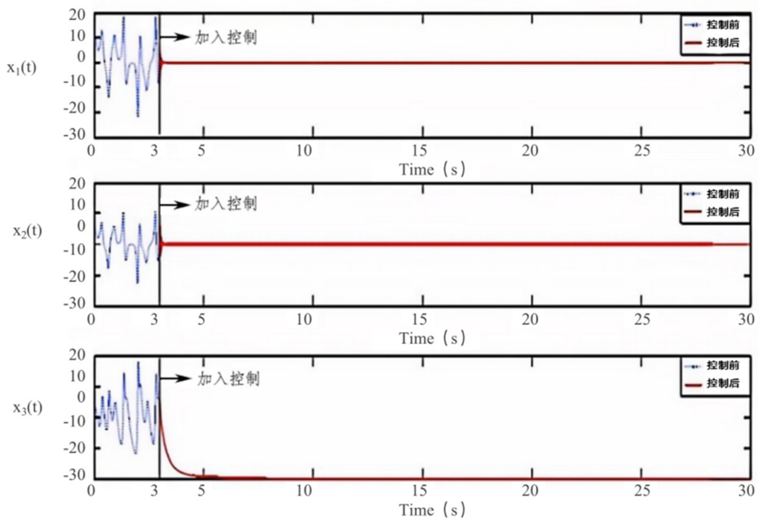

Figure 1. State response of fractional Chen system

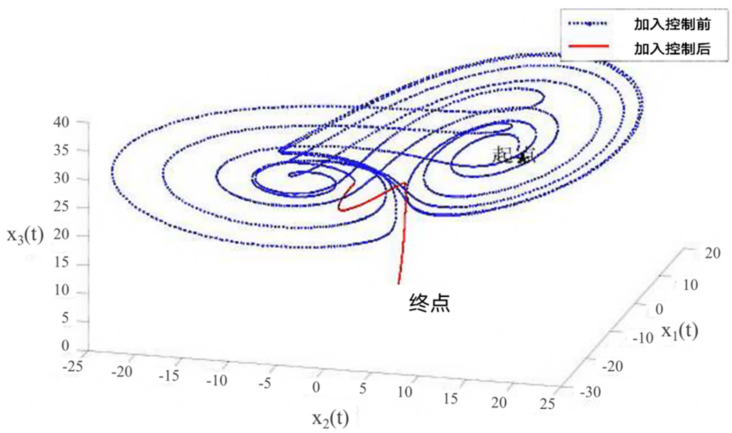

Figure 2. Phase trajectory of the controlled fractional Chen system

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