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Free choice with anaphora*

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Abstract In this paper, we formulate a new problem for any account of Free Choice (FC) inferences, which we dub *FC with anaphora*. According to the classical FC inference schema, given a sentence of the form $\Diamond(\phi \vee \psi)$, one can infer $\Diamond\phi$ and $\Diamond\psi$. FC with anaphora involves cases where an anaphoric dependency spans $\phi \vee \psi$. Anaphora is heavily constrained in disjunctions, but a negative existential statement in the initial disjunct can license a pronoun in the latter disjunct — so called *bathroom disjunctions*, for example, "Either there's no bathroom in this house, or it's in a funny place". We show that embedding a bathroom disjunction under an existential modal gives rise to a FC inference that doesn't follow from the classical schema — since the schema is stated in terms of the individual disjuncts, any information about anaphoric dependencies *between* disjuncts is lost. In order to capture FC with anaphora, we develop a new semantic account couched in the framework of *Bilateral Update Semantics*. We also introduce several related problems involving anaphora and inferences which we characterize as involving *simplification* more generally.

Keywords: free choice, anaphora, dynamic semantics, update semantics, disjunction

1 Introduction

Kamp (1973) famously observed that sentences of the form $\Diamond(\phi \vee \psi)$ give rise to the inference that $\Diamond\phi \wedge \Diamond\psi$. Kamp's original focus was on deontic permission

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statements — (1a) clearly implies (1b) — but the pattern was later understood to be far more general. For example, the epistemic possibility statement in (2a) implies the conjunctive possibility statement in (2b) (Zimmermann 2000).

(1) a. You may have tea or coffee.
    b. You may have coffee and you may have tea.

(2) a. It might be here or there.
    b. It might be here and it might be there.

This inference and phenomenon more generally has come to be known in the literature as Free Choice (FC), schematized in (3).

(3) FC: $\Diamond(\phi \vee \psi) \vDash \Diamond\phi \wedge \Diamond\psi$

Why is FC so puzzling? From the perspective of standard approaches to the semantics of modals (in terms of accessibility relations, for example), and standard (boolean) approaches to disjunction, the apparent validity of this inference should come as something of a shock. This is because the existence of an accessible world at which $\phi \vee \psi$ is true does not guarantee the existence of an accessible world at which ϕ is true, and the existence of an accessible world at which ψ is true.

The semantics-pragmatics literature has been intensely focused on the nature of (3), and more concretely, the question of whether the source of FC lies in the semantics, or in the pragmatics. Does FC motivate a departure from standard approaches to the semantics of modals and disjunction, and if so, how exactly?

The number of theoretically disparate approaches to FC has multiplied in recent years. Broadly, they can be split into two camps. The first camp has it that FC is a kind of implicature,[1] and should be captured by the same independently-motivated *exhaustification* mechanism that is exploited for implicature-computation.[2] In parallel, many different theories of FC have emerged which rather argue that FC should be captured as a *semantic entailment*, by leaning on a non-classical semantics for modals, disjunction, or both.[3]

There is one crucial component of exhaustification-based theories which is worth mentioning at this juncture. They crucially assume that a sentence

1 See, e.g., Kratzer & Shimoyama 2002, Alonso-Ovalle 2005, Fox 2007, Franke 2009, Geurts 2010.

2 See Bar-Lev 2018, Del Pinal, Bassi & Sauerland 2024, Bar-Lev & Fox 2020 for recent refinements of the exhaustification approach to FC.

3 See, e.g., Zimmermann 2000, Simons 2005, Klinedinst 2007, Aloni 2007, 2003, 2022, Goldstein 2019, Willer 2018, 2017.

such as $\Diamond(\phi \vee \psi)$ has *structurally simpler* sentences, $\Diamond\phi$ and $\Diamond\psi$, among its alternatives. The details of the computation don't matter at this point, but Bar-Lev & Fox (2020) for example develop a theory of FC where the alternatives in (4) are conjoined with main assertion, directly giving rise to the attested FC inference. This is schematized in (5).

(4) $\Diamond\phi, \Diamond\psi \in \textbf{Alt}(\Diamond(\phi \vee \psi))$

(5) $\overbrace{\Diamond(\phi \vee \psi)}^{\text{assertion}} \wedge \overbrace{\Diamond\phi \wedge \Diamond\psi}^{\text{implicature}}$

It's worth mentioning that semantic theories of FC do often place conditions on the individual disjuncts, in developing an account of FC inferences, but this does not strike us as non-negotiable. Moreover, we will ultimately argue that a reliance on simpler alternatives, as in (4), is problematic. In Section 2, we'll introduce a variety of FC phenomenon — *free choice with anaphora* — which will be resistant to the characterization in (5).

2 Bathroom disjunctions, and the Free Choice connection

2.1 Background: Anaphora in disjunctive sentences

As observed by Evans (1977) and independently by Barbara Partee, anaphora is possible in disjunctive sentences such as (6). From here on out, we'll refer to disjunctions of this kind as *bathroom disjunctions*. In ordinary cases of discourse anaphora in a conjunctive sentence, such as "There's a bathroom in this house and it's in a funny place", the first conjunct (containing an indefinite) entails a witness to the indefinite. Conversely, it seems like anaphora is possible across a disjunction, just in case the *negation* of the initial disjunct entails a witness to the indefinite. In the case of (6), *it* can be anaphoric on *a bathroom* since the negation of the initial disjunct — *it's not the case that there isn't a bathroom in this house* — implies the existence of a bathroom.

(6) Either there isn't a bathroom in this house, or it's in a funny place.

The phenomenon of bathroom disjunctions is clearly related to the reasonably well-understood fact that the presupposition of a latter disjunct fails to project if it is contextually entailed by the *negation* of the initial disjunct (Karttunen 1973). This familiar observation is illustrated in (7) with a minimal presuppositional variant of (6) with a definite description, as well as an example involving an additive trigger *too* in (8), and an *it*-cleft in (9).

(7) Either there's no bathroom in this house, or the bathroom is in a funny place.

(8) Either Shane isn't coming tonight, or Lisa will come too.

(9) Either nobody broke the vase, or it was Gabe who broke it.

The lack of projection in examples (7)–(9) is predicted by even the simplest predictive theories of presupposition projection, such as the Strong Kleene logic of indeterminacy (see, e.g., Beaver & Krahmer 2001). So, why are bathroom disjunctions nevertheless considered to be puzzling? It's well-known that discourse-anaphora impose additional requirements above and beyond existence. One way of illustrating this is by considering a minimal pair such as (10a) and (10b). In each discourse, as long as we know that *if Ethan is married, he's married to a man*, the initial sentences are contextually equivalent. Nevertheless, a subsequent discourse-anaphoric pronoun is felicitous only in (10a), where the initial sentence involves an indefinite Noun Phrase. The requirement that a discourse-anaphoric pronoun has a linguistic antecedent is known in the literature as the *formal link condition*.

(10) a. Ethan has a^x husband. He_x's standing outside.
b. Ethan is married. #He_x's standing outside.

In mainstream dynamic approaches to discourse anaphora, the contrast between (10a) and (10b) is captured by furnishing indefinites with the ability to introduce extra-worldly anaphoric information into the discourse context. We'll discuss such approaches in more detail later in this paper. Pre-theoretically, the idea is that sentences with indefinites can introduce Discourse Referents (DRs) (Karttunen 1976), whereas a sentence without an indefinite cannot, even if it is contextually-equivalent. Exploiting dynamic accounts of discourse anaphora seems like a promising direction in accounting for bathroom disjunctions, given that anaphora in disjunctive sentences also seems to be subject to the formal link condition, as illustrated in (11a) and (11b). Indeed, this direction has been pursued, although it is not without significant complications (see, e.g., Krahmer & Muskens 1995, Gotham 2019, Hofmann 2019, 2022, Elliott 2020, 2023).

(11) a. Either John has no husband, or he's standing outside.
b. #Either John isn't married, or he's standing outside.

Before moving on, we'd like to address a possible analysis of bathroom disjunctions that would assimilate them to the case in (7) involving a definite description — a so-called *E-type analysis*. The formal link condition could then be seen as a syntactic condition on reduction of a definite description into a pronoun.[4]

(12) Either there is no bathroom, or it[=*the bathroom*] is in a funny place.

Mandelkern & Rothschild (2020) show in detail that the pronoun in (12) doesn't project uniqueness in a bathroom disjunction, by adapting Heim's (1982) famous Sage plant sentence. As they point out, if the pronoun projected uniqueness in (14), the second disjunct would be rendered trivially false. This shows, at least, that an unsophisticated E-type analysis of bathroom disjunctions isn't tenable.[5]

(14) Either Sue didn't buy a sage plant, or she bought eight others along with it. Mandelkern & Rothschild 2020: p. 94

Moreover, sophisticated E-type accounts based on *situational* uniqueness, such as that of Heim 1990 don't fare much better. In order to account for an example such as (14) on such a view, Mandelkern & Rothschild point out that the second disjunct should be assessed relative to some minimal situation at which the first disjunct is false. This would require disjunction to manipulate, in a relatively arbitrary fashion, the situations relative to which the latter disjunct is assessed. Much of the appeal of E-type approaches is premised on the idea that, unlike in dynamic approaches, a classical semantics for the connectives can be maintained. Therefore this route isn't particularly compelling.

We'll close this section by saying something about the truth-conditions of bathroom disjunctions amounts to. We believe, following Elliott (2022, 2024) that bathroom disjunctions may have *existential* truth conditions, as paraphrased in (15).

(15) Either there's no bathroom in this house,
or *there is a bathroom in this house* and it's in a surprising place.

We therefore disagree with, e.g., Krahmer & Muskens (1995), who claim that bathroom disjunctions always have *universal* truth-conditions, as well as Gotham (2019) who claims that bathroom disjunctions always encode *uniqueness*.

4 See, e.g., Heim 1990 for an E-type account of so-called *donkey anaphora* based on this mechanism, and Elbourne 2002, 2013 for subsequent developments.

5 This isn't just a problem for the analysis of anaphoric *pronouns* — in fact, the focus of Mandelkern & Rothschild's paper is on the correct characterization of the presuppositions of *definite descriptions*. The variant of (14) with an overt definite description also (surprisingly) fails to project uniqueness.

(13) Either Sue didn't buy a sage plant, or she bought eight others along with the sage plant.

We'll begin by discussing Gotham's claim, since this pertains directly to the discussion of E-type strategies. Gotham claims that an example such as his (16a) strike us as odd because the bathroom disjunction presupposes conditional uniqueness, i.e., *if John owns a shirt, then he owns exactly one*, assuming that it's contextually implausible that John only owns a single shirt. We don't deny the existence of a conditional uniqueness inference in (16a), but it does not seem to us that this is anything special about disjunction/negation as Gotham would have it. A garden variety conjunctive instance of discourse anaphora strikes us as odd for exactly the same reason, as illustrated in (16b).

(16) a. ??Either John doesn't own a shirt, or it's in the wardrobe.
Gotham 2019: p. 144
b. ??John owns a shirt, and it's in the wardrobe.

The source of the uniqueness inference is obscure, but it is clearly cancellable, otherwise Mandelkern & Rothschild's *sage plant* sentence in (14) would be incoherent.

Turning now to Krahmer & Muskens 1995, they claim that the correct paraphrase of the bathroom disjunction (6) rather is *universal*, as in (17). Note that this would render it false as soon as there is a bathroom in this house which isn't in a surprising place. The existential truth-conditions we suggest, on the other hand, admit of such a possibility.

(17) Every bathroom in this house is in a surprising place.

We believe that it's possible to show that such universal truth-conditions are sometimes too strong. We do this by adapting a famous example from the literature on donkey anaphora. Consider a context in which we're wondering how Gennaro paid for dinner. (18) expresses that, *if Gennaro has at least one credit card, then he paid with one of his credit cards*. Importantly, it seems to be completely consistent with Gennaro having a credit card that he *didn't* pay with. This is of course completely consistent with the existential paraphrase: *Either Gennaro doesn't have a credit card, or he has a credit card and paid with it.*

(18) Either Gennaro doesn't have a credit card, or he paid with it.

As an aside, we do not wish to suggest that bathroom disjunctions are *never* understood universally. The uncertainty over ∀ vs. ∃ truth-conditions should very much remind one of the famous ∀/∃ ambiguity observed in donkey sentences.[6] The account of bathroom disjunctions we will exploit will deliver ∃

6 For accounts of the ∀/∃ ambiguity in donkey sentences, see, e.g., Rooth 1987, Schubert & Pelletier 1989, Kanazawa 1994, Chierchia 1995, Krifka 1996, Barker 1996, Geurts 2002, Brasoveanu 2007, Champollion, Bumford & Henderson 2019.

readings only.[7] The problem of how to account for putative ∀ readings of bathroom disjunctions and donkey sentences more generally is largely orthogonal to the problem we focus on in this paper.

More generally, we suggest that an account of anaphora in bathroom disjunctions should be based on three main ingredients: (i) Double Negation Elimination (DNE) (19a) (ii) a dynamic theory of discourse anaphora which validates the equivalence known as *Egli's theorem* (19b) (Egli 1979), (iii) and the classical equivalence in (19c). The abstract Logical Form of a bathroom disjunction is shown in (20). When coupled with (19c), DNE, and Egli's theorem, we arrive at the attested existential truth-conditions. This suggestion may strike the dynamic sophisticate as surprising, since standard formulations of dynamic semantics which capture (19b) fail to validate (19c), and even fail to validate DNE (see, e.g., Groenendijk & Stokhof 1991). We'll have much more to say about this in Section 3.

(19) a. DNE: $\neg\neg\phi \vDash \phi$
b. Egli's theorem: $\exists_x \phi \wedge \psi \vDash \exists_x[\phi \wedge \psi]$
c. $\phi \vee \psi \vDash \phi \vee (\neg\phi \wedge \psi)$

(20) Putative account of bathroom disjunctions:
$\neg\exists_x \phi \vee P(x)$
$\vDash \neg\exists_x \phi \vee (\neg\neg\exists_x \phi \wedge P(x))$ via (19c)
$\vDash \neg\exists_x \phi \vee (\exists_x \phi \wedge P(x))$ via DNE
$\vDash \neg\exists_x \phi \vee \exists_x (\phi \wedge P(x))$ via Egli's theorem

Having carefully assessed the profile of anaphora in disjunctive sentences, we now turn to the main focus of the paper: the relationship between anaphora in disjunctions and Free Choice (FC), which will expose a problem which the remainder of this paper will explore in detail.

2.2 Free choice with anaphora: The phenomenon

Recall the FC inference schema, repeated below in (21). Since FC involves a disjunctive sentence embedded under a modal with existential force, we can ask the following question: what happens when $\phi \vee \psi$ is a bathroom disjunction?

(21) FC: $\Diamond(\phi \vee \psi) \vDash \Diamond\phi \wedge \Diamond\psi$

7 See Elliott 2024 for a general account of ∃/∀ readings which encompasses donkey sentences and bathroom disjunctions, broadly compatible with the dynamic semantics we develop in Section 3.

It's relatively easy to construct such cases, and it seems that they give rise to a FC inference. In (22) we embed a bathroom disjunction under *it's possible that…*, which we assume is an existential epistemic modal, using *either* to fix the scope of disjunction below the modal. It's clear that (22) entails both (22a) and (22b). At this point note that the paraphrase of the inference in (22b) contains material from both the first and second disjuncts, unlike instances of FC we've already considered.

(22) It's possible that either there's no bathroom in this house or it's in a surprising place.

a. *It's possible that there's no bathroom in this house.*
b. *It's possible that there's a bathroom in this house in a surprising place.*

Now consider a case with a deontic modal. (23) involves an existential deontic modal *may* taking scope over a bathroom disjunction. It clearly gives rise to a FC inference, which we can break down into two components. The inference in (23a) of course corresponds to the first disjunct. The inference in (23b) on the other hand, just as before, contains material from both the first and the second disjunct.

(23) You may include no appendix, or keep it to a single page.

a. *You may include no appendix.*
b. *You may include an appendix kept to a single page.*

In general, we can characterize the FC inferences that arise when embedding a bathroom disjunction under an existential modal as in (24).

(24) FC with anaphora: $\Diamond(\neg\exists_x \phi \vee \psi) \vDash \Diamond\neg\exists_x \phi \wedge \Diamond(\exists_x(\phi \wedge \psi))$

This is a bit of a departure of the classical characterization of FC, (21). If we apply the classical FC schema to a bathroom disjunction embedded under an existential modal, the result is rather strange. The putative inference in (25) just does not provide an intuitively correct paraphrase of the FC inference. In fact, it is a sentence with a free pronoun *it*, and it is not clear how the referent of the pronoun gets resolved. Note that the sentence itself (25) certainly does not semantically entail the existence of an appendix.

(25) You may include no appendix, or keep it to a single page.
$\overset{?}{\Rightarrow}$ *You may keep it to a single page.*

The central issue we will be concerned with in this paper, therefore, is how to develop a theory of FC which accounts for FC with anaphora, as schematized in (24). To our knowledge, no existing theory of FC, strictly as stated, accounts for the inference in FC with anaphora. What kind of logical properties should an account of FC with anaphora have? Clearly, an account of FC with anaphora should be closely connected to the account of bathroom disjunctions. Pre-empting our analysis, it seems clear to us that in order to validate FC with anaphora, the modified FC inference in (26) should be validated. This isn't quite enough to capture the inference as stated; as before (in the discussion of bathroom disjunctions) both *Egli's theorem* (27) and Double Negation Elimination (DNE) (28) are crucial.

(26) Modified FC: $\Diamond(\phi \vee \psi) \vDash \Diamond\phi \wedge \Diamond(\neg\phi \wedge \psi)$

(27) Egli's theorem: $\exists_x \phi \wedge \psi \vDash \exists_x[\phi \wedge \psi]$

(28) DNE: $\neg\neg\phi \vDash \phi$

In (29) below, we show how the aforementioned logical principles derive FC with anaphora. Modified FC allows us to conjoin the negation of the first disjunct with the second disjunct; DNE allows us to create the configuration in which *Egli's theorem* can apply, which in turn allows the existential to scope over the second disjunct. Much of the remainder, starting from Section 3, will be devoted to setting up a framework in which the principles in (26)–(28) are valid. This will not be at all straightforward — DNE is of course classically valid, but Egli's theorem is a hallmark of dynamic approaches to anaphora (Groenendijk & Stokhof 1991). In many such dynamic approaches, DNE is *not* valid, so we will need to adopt a particular view of how the dynamics of anaphoric information proceed.

(29)
a. $\Diamond(\neg\exists_x \phi \vee \psi)$
b. $\vDash \Diamond\neg\exists_x \phi \wedge \Diamond(\neg\neg\exists_x \phi \wedge \psi)$ — by (26)
c. $\vDash \Diamond\neg\exists_x \phi \wedge \Diamond(\exists_x \phi \wedge \psi)$ — by (28)
d. $\vDash \Diamond\neg\exists_x \phi \wedge \Diamond(\exists_x(\phi \wedge \psi))$ — by (27)

Before we start developing our analysis of FC with anaphora, we first turn to a more detailed discussion of the exhaustification account of FC, taking the recent account of Bar-Lev & Fox 2020. This will provide a vivid illustration of the problems posed by anaphoric dependencies across disjunctions to theories of FC which make use of 'simpler' alternatives. We next turn to our positive proposal — a semantic account of FC couched within a particular variant of dynamic semantics.

2.3 Exhaustification accounts of Free Choice

Bar-Lev & Fox (2020), building on earlier work by Fox (2007), conjecture that FC inferences should be derived by the same covert operator $\mathcal{E}xh$ that is responsible for deriving scalar implicatures. The workings of $\mathcal{E}xh$ are sensitive to the formal *alternatives* to the sentence it combines with. Bar-Lev & Fox assume that the alternatives to a sentence ϕ are just those sentences that are *at most as complex* as ϕ (the relevant notion of complexity is made precise by Katzir 2008, Fox & Katzir 2011). For plain disjunctive alternatives, the relevant alternatives are given in (30); for modalized disjunctions, the relevant alternatives are given in (31).[8] The crucial insight underlying the exhaustification-based account is that the alternatives in (30) are closed under conjunction, whereas the alternatives in (31) are not.

(30) $$\textbf{Alt}(\phi \vee \psi) = \{\ \underbrace{\phi \vee \psi}_{\text{prejacent}},\ \underbrace{\phi \wedge \psi}_{\text{conj. alt}},\ \underbrace{\phi,\ \psi}_{\text{domain alts}}\ \}$$

(31) $$\textbf{Alt}(\Diamond(\phi \vee \psi)) = \{\underbrace{\Diamond(\phi \vee \psi)}_{\text{prejacent}}, \underbrace{\Diamond(\phi \wedge \psi)}_{\text{conj. alt}}, \underbrace{\Diamond\phi, \Diamond\psi}_{\text{domain alts}}\ \}$$

The particular formulation of $\mathcal{E}xh$ that Bar-Lev & Fox develop involves a two-step process: (i) *the innocent exclusion step*: exclude as many alternatives as possible, in a way which doesn't lead to contradictions. (ii) *the innocent inclusion step*: include as many alternatives as possible, in a way which doesn't lead to contradictions. What's crucial is that the inclusion step takes place *after* the exclusion step — strengthening via exclusion may block certain alternatives from later being included. In the following, we'll informally go through the reasoning involved, starting with a plain disjunctive sentence.

Consider $\mathcal{E}xh(\phi \vee \psi)$ — the innocent exclusion step demands that we exclude as many alternatives as possible in a way which doesn't lead to contradictions. To 'exclude' an alternative just means to conjoin its negation with the prejacent. The conjunctive alternative $\phi \wedge \psi$ can be safely excluded, giving rise to the attested exclusive inference associated with disjunctive sentences. However, excluding both of the domain alternatives, ϕ and ψ, would contradict

8 As Bar-Lev & Fox (2020) note, there are other alternatives that one might want to entertain here, for example those derived by replacing $\Diamond$ with $\Box$. This will not be relevant for our purposes.

the prejacent, and there is no principled reason to exclude one over the other, so neither is excluded. After innocent exclusion, we're left with (33).[9]

(33) Result of the exclusion step for plain disjunction:
$$\underbrace{\phi \vee \psi}_{\text{assertion}} \wedge \underbrace{\neg(\phi \wedge \psi)}_{\text{implicature}}$$

Now we include as many alternatives as possible in a way which doesn't lead to contradictions; to 'include' just means to conjoin an alternative with the result of the exclusion step. Note now that nothing except the prejacent itself can be included: including the conjunctive alternative would directly contradict the result of the exclusion step, and including both domain alternatives would have the same result — crucially, there is no principled reason to include one over the other, so neither ends up in the *maximal* subset of includable alternatives; see the definition of II alternatives in footnote 10. The exclusion step therefore *blocks* inclusion of the domain alternatives for plain disjunctions.[10]

Now let's turn to our main focus — FC and the application of $\mathcal{E}xh$ to a modalized disjunction. As before, we start by excluding as many alternatives as possible. As before, only the conjunctive alternative can be excluded, resulting in (35).

(35) Result of the exclusion step for modalized disjunction:
$$\underbrace{\Diamond(\phi \vee \psi)}_{\text{assertion}} \wedge \underbrace{\neg\Diamond(\phi \wedge \psi)}_{\text{implicature}}$$

What's important to notice at this stage is that the implicature resulting from the exclusion step $\neg\Diamond(\phi \wedge \psi)$ won't contradict inclusion of each of the domain alternatives. The domain alternatives can therefore safely be included, resulting in the attested FC inference.

(36) Result of exclusion and inclusion for modalized disjunction:
$$\underbrace{\Diamond(\phi \vee \psi)}_{\text{assertion}} \wedge \underbrace{\neg\Diamond(\phi \wedge \psi)}_{\text{implicature}} \wedge \underbrace{\Diamond\phi \wedge \Diamond\psi}_{\text{FC inference}}$$

9 The complete definition of an *Innocently Excludable* (IE) alternative, from Bar-Lev & Fox 2020: p. 188: Given a sentence ϕ, and a set of alternatives C:

(32) $IE(\phi, C) = \bigcap\{C' \subseteq C | C' \text{ is a maximal subset of } C \text{ s.t. } \{\neg q \mid q \in C'\} \cup \{p\} \text{ is consistent}\}$

10 The complete definition of an *Innocently Includable* (II) alternative, from Bar-Lev & Fox 2020: p. 188: Given a sentence ϕ, and a set of alternatives C:

(34) $II(\phi, C) = \bigcap\left\{C'' \subseteq C \;\middle|\; \begin{array}{l} C'' \text{is a maximal subset of } C \\ \text{s.t. } \{r \mid r \in C''\} \cup \{p\} \cup \{\neg q \mid q \in IE(\phi, C)\} \text{ is consistent} \end{array}\right\}$

The exhaustification account therefore doesn't validate FC as a semantic entailment, but rather indirectly via $\mathcal{E}xh$, as schematized in (37).

(37) FC via exhaustification: $\mathcal{E}xh \, \Diamond(\phi \vee \psi) \vDash \Diamond\phi \wedge \Diamond\psi$

Since the exhaustification account validates the classical FC schema, and not the modified schema in (26), it clearly won't account for FC with anaphora. Furthermore, it's not easy to see how to validate the modified schema while maintaining the assumptions that Bar-Lev & Fox make about the alternatives at play. We do not wish to commit to the claim that there is *no* way of accounting for FC with anaphora within an exhaustification framework, and moreover we anticipate some possible moves in Section 4.

Instead, we will develop an analysis of FC by first assembling a dynamic framework in which both DNE and *Egli's theorem* are valid. We believe that, in any case, this is a necessary ingredient for *any* account of FC with anaphora. Ultimately, we will show that FC with anaphora can be easily captured via a semantic account of FC which exploits the expressive power of our dynamic framework.

3 Analysis

3.1 Background: Update semantics

In order to develop an account of FC with anaphora, we'll develop a bilateral variant of *update semantics* (Heim 1982, 1983, Veltman 1996, Groenendijk, Stokhof & Veltman 1996), following e.g., Willer (2019) (but differing significantly in implementation). This will later pay dividends, as we'll crucially make use of a Heimian account of discourse anaphora, and bilaterality will be an important component in accounting for FC inferences. The central idea in update semantics is that sentences denote functions from information states to information states, often written using the iconic update notation $s[\phi]$ — the effect of applying the update associated with ϕ to information state s.

Since we're dealing with anaphora, the specific notion of *information state* we'll assume is Heimian — it incorporates both information about how the world is, and information about the values of variables — what Heim calls a 'file'. Concretely, a Heimian information state is a set of *world-assignment pairs*. Following, e.g., Rothschild & Mandelkern 2017, we emulate partiality of assignments by assuming a privileged 'indeterminate' individual value $*$. A 'partial' assignment is a total function from variables, which happens to map some variables to $*$.

We'll often abbreviate assignments by simply omitting the variables which are mapped to $*$, as illustrated in (38):

(38) Notational convention for assignments: $[x \to a] := \begin{bmatrix} x \to a \\ y \to * \\ z \to * \\ \dots \end{bmatrix}$

The official definition for a Heimian information state, along with some useful auxiliary notions, is provided in Definition 3.1 for reference. Note that we can easily pull out the worldly and anaphoric parts of information states using the helper functions $\mathscr{W}$ and $\mathscr{A}$. Initial states, intuitively, model those in which no anaphoric information has yet been introduced (the anaphoric counterpart of a worldly ignorance state).

Definition 3.1 (Information states)**.** An *information state* is a set of world-assignment pairs. An *assignment* is a total function from a stock of variables $V \mapsto D \cup \{ * \}$.

- The *worldly information* of a state $\mathscr{W}(s) = \{ w \mid \exists g, (w, g) \in s \}$
- The *anaphoric information* of a state $\mathscr{A}(s) = \{ g \mid \exists w, (w, g) \in s \}$
- An *initial assignment* $g_\top$ is s.t., $\forall x \in V, g_\top(x) = *$.
- An *initial state* s is one where $\mathscr{A}(s) = \{ g_\top \}$
- The *absurd state* $s_\perp$ is $\varnothing$.

We can exploit partiality to derivatively define Heim's notion of *familiarity*, which plays an important role in accounting for the distribution of pronouns and definite descriptions. A variable x is familiar at an information state s iff x is 'defined' at every assignment in s. N.b., that variables play the role of Karttunen's (1976) DRs in this setting.

Definition 3.2 (Familiarity)**.** A variable x is *familiar* at an information state s iff:

$$\forall g \in \mathscr{A}(s), g(x) \neq *$$

For example, x is familiar at the information state in (39), even though it gets mapped to different values across possibilities. This will be used to model, e.g., a discourse context in which x has been introduced as a DR but its identity is not yet determined.

(39) $\{ (w_{ab}, [x \to a]), (w_{ab}[x \to b]), (w_a, [x \to a]), (w_b, [x \to b]) \}$

In classical update systems, such as those of Heim (1982) and Veltman (1996), the update associated with a sentence ϕ is identified with the affect of *asserting* ϕ on a context set (in the sense of Stalnaker 1976; see Rothschild & Yalcin 2017 for much relevant discussion). The phenomenon of interest here however will necessitate a departure from this perspective — in classical update semantics (and in many dynamic approaches to anaphora more generally), DNE is not valid, and relatedly there is no account of anaphora in bathroom disjunctions. Dynamic accounts of bathroom disjunctions do exist, but necessitate major departures from standard dynamic approaches. Here, we'll be following in the footsteps of important work by Krahmer & Muskens (1995) and van den Berg (1996) in developing a *bilateral* perspective on dynamic semantics. Concretely, we'll adopt Elliott's (2022, 2024) specific formulation of Bilateral Update Semantics (BUS) (see also Willer 2018, 2019, 2017, Aloni 2023 for applications of bilateralism in an update-semantic framework).

3.2 Bilateral Update Semantics: Preliminaries

In BUS, the relationship between the meaning of a sentence, and what it means to assert a sentence is more indirect than in a classical update semantics. Instead of associating each sentence ϕ with an update function, each sentence will be a associated with both a *positive update* $s[\phi]^+$ and a *negative update* $s[\phi]^-$. The intuition we'll start out with for simple atomic sentences is that the positive update picks out those possibilities in s at which ϕ is classically true, and the negative update picks out those possibilities in s at which ϕ is classically false. We'll assume, as a precondition for truth/falsity, that every free variable be assigned a non-deviant value.[11]

(40) **Atomic sentences in BUS:**

a. $s[P(x_1, \dots, x_n)]^+$
$= \{ (w, g) \in s \mid [\![P(x_1, \dots, x_n)]\!]^{w,g}$ is defined and true $\}$

b. $s[P(x_1, \dots, x_n)]^-$
$= \{ (w, g) \in s \mid [\![P(x_1, \dots, x_n)]\!]^{w,g}$ is defined and false $\}$

Let's see how this works in practice. Consider the state in (41); subscripts on worlds exhaustively indicate who P-ed. $s_1[P(x)]^+$ picks out the possibilities in

11 Pedantically: $P(x_1, \dots, x_n) \in \mathbf{dom}([\![.]\!]^{w,g})$ iff $g(x_1), \dots, g(x_n) \neq *$.

s_1 at which x is mapped to an individual who P-ed. Conversely, $s_1[P(x)]^-$ picks out the possibilities in s_1 at which x is mapped to an individual who didn't P.

(41) $s_1 = \{ (w_{ab}, [x \to a]), (w_{ab}, [x \to b]), (w_a, []), (w_b, [x \to a]) \}$

a. $s_1[P(x)]^+ = \{ (w_{ab}, [x \to a]), (w_{ab}, [x \to b]) \}$

b. $s_1[P(x)]^- = \{ (w_b, [x \to a]) \}$

The attentive reader will note that the possibility $(w_a, [])$ is included neither in the output of the positive, nor the negative update, since the initial assignment $[]$ doesn't provide a value for x. This is exactly a state at which x isn't familiar, in the Heimian sense. In order to enforce familiarity as a condition on asserting an open sentence, we follow Elliott (2022), in generalizing *Stalnaker's bridge* to an update-semantic setting by demanding that $c[\phi]^+ \cup c[\phi]^- = c$, for ϕ to be assertable at c. This will of course fail if any of the possibilities in c fail to provide a value for a free variable in ϕ. This will do for now, but will soon need to be tweaked when we introduce existentials into the system.

3.3 Existential statements in BUS

When an indefinite takes scope below negation, it fails to license a subsequent discourse anaphoric pronoun. In many dynamic frameworks, this is captured by tailoring the semantics of negation to ensure that anaphoric information introduced by an existential in its scope is rendered inert (see, e.g., the discussion of negation in Groenendijk & Stokhof 1991). At its heart, this classical dynamic treatment of negation is typically responsible for invalidating DNE, as it also predicts that a *doubly* negated existential should fail to introduce anaphoric information.

(42) #Gabor didn't write a^x paper about free choice, but it$_x$ was interesting.

For this reason, we take a different tack in BUS — concretely, we exploit bilateralism to ensure that an existential statement introduces a DR as part of its positive update, whereas the negative update merely removes possibilities from the input state. This, together with the treatment of negation introduced later, will allow us to reconcile data like (42) with DNE. In order to give a semantics for existential statements, we'll lean on the operation *random assignment*, which is used in dynamic semantics to introduce DRs (i.e., make variables familiar). Random assignment is an operation on information states which indeterministically extends assignments in the state at given variable. The definition is given in Definition 43.[12]

(43) **Random assignment**:
$s[\varepsilon_x] = \{ (w, h) \mid (w, g) \in s, g[x]h \}$

The positive update associated with existential quantification simply performs random assignment, and then positively updates the result with the scope. This is essentially the classical dynamic semantics for existential quantifiers (Heim 1982, Groenendijk & Stokhof 1991).

(44) **Positive update for existential statements:**
$s[\exists_x \phi]^+ = s[\varepsilon_x][\phi]^+$

The negative update associated with existential quantification is less standard — it is tailored to simply remove possibilities from the input state. What it does is retain all worlds that aren't in the positive update of the existential statement, but are retained when a discourse referent is introduced and negatively updated with the scope. One way of thinking about this is that it retains possibilities at which the existential statement is *classically false*. No additional anaphoric information is introduced.

(45) **Negative update for existential statements**:
$s[\exists_x \phi]^- = \{ (w, g) \in s \mid w \notin \mathscr{W}(s[\exists_x \phi]^+), w \in \mathscr{W}(s[\varepsilon_x][\phi]^-) \}$

Let's see how this works in practice. We'll begin with an initial state s_1 in (46), where subscripts exhaustively indicate which individuals (of a and b) are P. The positive update of $\exists_x P(x)$ eliminates the possibility where nothing is P, and indeterministically extends assignments at x to individuals that are P. The result is given in (47) — note that x is *familiar* at the resulting output state.

(46) $s_1 = \{ (w_{ab}, []), (w_a, []), (w_b, []), (w_\varnothing, []) \}$

(47) $s_1[\exists_x P(x)]^+ = \{(w_{ab}, [x \to a]), (w_{ab}, [x \to b]), (w_a, [x \to a]), (w_b, [x \to b])\}$

12 $g[x]h$ means that assignments g and h differ only at x, and that $h(x) \neq *$.
Note that we have chosen not to encode Heimian *novelty* into the definition of random assignment, thus allowing for so-called 'destructive' update. A straightforward way of encoding the novelty requirement would be to adopt van den Berg's (1996) *guarded* random assignment, which requires $g(x)$ to be $*$. Nothing in the present work hinges on this.

Below, we lay out the workings of the negative update in a little more detail. In order to compute the negative update, we first introduce x as a DR, and negatively update the resulting state with $P(x)$. The result is a state where w_{ab} is eliminated, and otherwise x is mapped indeterministically to a non-P, as shown in (48a). This isn't what we want for the negative update of the existential statement though — intuitively, this corresponds to the positive update of "something isn't P". The next step is to just retain the worlds in the input state that are in (48a), but not in (47). The only such world is $w_\varnothing$, so we keep that together with its paired initial assignment. What this accomplishes is that $s[\exists_x P(x)]^-$ introduces the information that nothing is P, but doesn't introduce any additional anaphoric information.

(48) a. $s_1[\varepsilon_x][P(x)]^- = \{(w_a, [x \to b]), (w_b, [x \to a]), (w_\varnothing, [x \to a]), (w_\varnothing, [x \to b])\}$

b. $s_1[\exists_x P(x)]^- = \{\, (w, g) \in s_1 \mid w \notin \mathscr{W}(47), w \in \mathscr{W}(48a) \,\} = \{\, (w_\varnothing, []) \,\}$

Now that we have the negative and positive updates for existential quantification, let's be explicit about the semantics of negation. As is standard in a bilateral setting, we'll adopt a flip-flop semantics for negation (see, e.g., Krahmer & Muskens 1995), as stated in (49). Essentially, negation flips the polarity of an update. It should be immediately obvious that DNE is valid in BUS, purely on the basis of how negation is defined.[13]

(49) **Negation in BUS:**

a. $s[\neg\phi]^+ = s[\phi]^-$

b. $s[\neg\phi]^- = s[\phi]^+$

An immediate consequence is that a doubly-negated existential statement can introduce the same anaphoric information as its positive counterpart. Indeed, there is evidence to suggest that this is independently necessary, in order to account for anaphora in sentences such as (52b) (on the assumption that *no*^{x} is translated as $\neg\exists_x$ at Logical Form).[14]

13 $s[\neg\neg\phi]^+ = s[\neg\phi]^- = s[\phi]^+$ and $s[\neg\neg\phi]^- = s[\neg\phi]^+ = s[\phi]^-$.

14 There is some disagreement in the literature about the status of examples such as (52b). Gotham (2019) for example claims that DNE isn't valid with respect to anaphoric information on the basis of the minimal pair in (50) (Gotham's reported judgments). Gotham accounts for the claimed oddness of anaphora in (50b) by claiming that, unlike a plain existential statement, a doubly-negated existential statement carries a *uniqueness* inference.

(52) a. Does Kafuku own no car?

b. No, Kafuku doesn't own no^x car — he drives it$_x$ every day.

Note importantly that, by validating DNE, we haven't sacrificed an account of the impossibility of anaphora in (42). Thanks to the negative update associated with an existential statement, sentences of the form $\neg\exists_x\phi$ will fail to make x familiar, if it is not already.

Finally, now that we have expressions that can introduce anaphoric information, we need to adopt a slightly different condition on assertability. This is because it's no longer guaranteed that a contextually bivalent sentence will partition the context, given the presence of anaphoric information. In order to accommodate this, the 'official' assertability condition requires that, at c, ϕ be 'defined' throughout c. We accomplish this by demanding that every possibility in c has a *descendant* in either $c[\phi]^+$or $c[\phi]^-$, i.e., a possibility that is identical except for potentially providing more anaphoric information. Groenendijk, Stokhof & Veltman (1996) say that a possibility i *subsists* in a state s when i has a descendent in s. The formal definition of descendence and subsistence is given in Definition 3.3. On this basis, we define what will turn out to be an extremely useful notion — the *unknown update* of s by ϕ, in (53). This simply picks out the possibilities in s which neither subsist in the positive nor negative update of s by ϕ. The assertability condition simply demands that the unknown update be empty.

Definition 3.3 (Descendence and subsistence)**.** Let s be a Heimian information state, and (w, g) a possibility.

- A possibility $(w', g') \in s$ is a *descendant* of (w, g) iff $w = w'$ and g is 'defined' for a subset of the variables that g' is.
- (w, g) *subsists* in s, $(w, g) \prec s$, iff (w, g) has at least one descendant in s.

---

(50) a. John owns a^x shirt. It$_x$'s in the wardrobe.

b. It's not true that John doesn't own a^x shirt. ??It$_x$'s in the wardrobe.

We are however unsure that (50a) doesn't carry the inference that (as far as the speaker knows) *John has exactly one shirt*. More definitively, it's possible to formulate a variation of Heim's famous *sage plant* sentence with double-negation, which shows that uniqueness isn't entailed.

(51) It's not true that Sue didn't buy a^x sage plant — she bought eight others along with it$_x$!

(53) **The 'unknown' update:**
$s[\phi]^{?} = \{\, i \in s \mid i \not\prec s[\phi]^{+}, i \not\prec s[\phi]^{-} \,\}$

(54) **Assertability condition:**
$c[\phi]$ is defined iff $c[\phi]^{?} = \varnothing$

It's worth dwelling momentarily on the assertability condition in (54), since it imposes a slightly weaker condition than classical Heimian familiarity.[15] Consider first how the assertability condition in (54) allows for DR-introduction. The positive/negative updates of a simple existential statement are illustrated below, relative to an initial state (repeated from (55).

(55) $s_1 = \{\, (w_{ab}, []), (w_a, []), (w_b, []), (w_{\varnothing}, []) \,\}$

a. $s_1[\exists_x P(x)]^{+} = \{(w_{ab}, [x \to a]), (w_{ab}, [x \to b]), (w_a, [x \to a]), (w_b, [x \to b])\}$

b. $s_1[\exists_x P(x)]^{-} = \{\, (w_{\varnothing}, []) \,\}$

The simple existential statement clearly passes the assertability condition relative to s_1, since the first three possibilities in s_1, $(w_{ab}, []), (w_a, []), (w_b[])$, subsist in the positive update, and the final possibility in s_1, $(w_{\varnothing}, [])$, subsists in the negative update. The unknown update is therefore empty.

Let's now consider how the assertability condition captures something akin to Heimian familiarity. In (56), the variable x is intuitively *partially* familiar — it is defined relative to the worlds in which something is a P, and undefined in the world where nothing is a P (as usual, subscripts on worlds indicate what is a P). As we'll see in the following section, asserting complex sentences may plausibly give rise to states structured like this. The assertability condition is *not* satisfied — this is because the possibility $(w_{\varnothing}, [])$ doesn't subsist in the positive/negative update, since x isn't defined at this possibility.

(56) $s_2 = \{(w_a, [x \to a]), (w_a, [x \to b]), (w_b, [x \to a]), (w_b, [x \to b]), (w_{\varnothing}, [])\}$

a. $s_2[P(x)]^{+} = \{\, (w_a, [x \to a]), (w_b, [x \to b]) \,\}$

b. $s_2[P(x)]^{-} = \{\, (w_a, [x \to b]), (w_b, [x \to a]) \,\}$

c. $s_2[P(x)]^{?} = \{\, (w_{\varnothing}, []) \,\}$

As we've alluded to, the requirement imposed by (54) is strictly weaker than Heimian familiarity — informally, for a sentence with a free variable x to

15 We're grateful to an anonymous reviewer for highlighting this point.

be assertable at s, (54) requires that every world in s be paired with *at least* one assignment at which x is defined. Heimian familiarity, on the other hand, requires x to be defined at *every* assignment in s. The requirements therefore come apart relative to states such as s_3, below. The final assertability condition predicts that an open sentence $P(x)$ is assertable, despite the fact that x is not strictly familiar, but as far as we are aware, this is harmless.[16]

(57) $s_3 = \{ (w_a, [x \to a]), (w_a, []), (w_b, [x \to b]), (w_b, []) \}$

The derivative notion of the *unknown update*, $s[\phi]^?$, is not only useful for stating the condition on assertability, but it will be crucial for talking about the semantics of the connectives and specifically how they project (anaphoric) presuppositions. We turn to this next.

3.4 The logical connectives

3.4.1 Conjunction

For the logical connectives in BUS, we adopt a strategy used by Elliott (2020) for stating Strong Kleene projection in a bilateral setting.[17] The general idea is that the classical Strong Kleene truth-tables provide a recipe for composing polarities of updates, where the third truth-value corresponds to $c[\phi]^?$. Consider the truth-table for strong Kleene conjunction, provided in Table 1.

∧	1	0	#
1	1	0	#
0	0	0	0
#	#	0	#

Table 1 Conjunction in strong Kleene

Let's begin by considering the conditions under which a conjunctive sentence is verified in strong Kleene. As in a classical setting, the only way of verifying a

16 In practice, it does not seem to us that this slight weakening of familiarity has any empirical consequences, since states structured like s_3 will not naturally arise. For example, as we'll see in the next section, a disjunctive sentence such as "there's a$_x$ P, or nothing is a Q", makes x defined at every P-world. The question of how these notions may be teased apart is however an interesting one, which we leave to future work.

17 See also Charlow 2014, 2023 for important antecedent work.

conjunctive sentence $\phi \wedge \psi$ is for both ϕ and ψ to be true. In BUS, this means that the *positive update* of $\phi \wedge \psi$ is retrieved by considering how $\phi \wedge \psi$ can be dynamically verified. Since conjunction has only a single verification strategy, the positive update can be stated rather simply: we simply compose the positive updates of the conjuncts.

(58) **Positive update of a conjunctive sentence**:
$s[\phi \wedge \psi]^+ = s[\phi]^+[\psi]^+$

This immediately validates something weaker than *Egli's theorem*, which we'll call *Egli's positive equivalence*.

(59) **Egli's positive equivalence**:
$s[\exists_x(\phi \wedge \psi)]^+ = s[\exists_x \phi \wedge \psi]^+$

It's easy to see why: existential quantification contributes successive update with random assignment, and conjunction contributes successive update. Since updating is associative, the positive equivalence clearly goes through, as illustrated in (60).

(60) $s[\varepsilon_x][\phi \wedge \psi]^+ = s[\exists_x \phi]^+[\psi]^+ = s[\varepsilon_x][\phi]^+[\psi]^+$

The negative update of conjunction will of course be more complex, since in Strong Kleene there are many different ways of falsifying a conjunctive statement, some of which allow for one of the conjuncts to be *undefined*. One of the main innovations in BUS is to take the strong Kleene truth-table as a template for constructing positive/negative updates. The positive update was simple, since there's only one way of verifying a conjunctive sentence in strong Kleene (i.e., one cell in the truth-table). Falsification however corresponds to several different cells in the truth-table. Each cell is interpreted as a successive update, and the negative update of a conjunctive sentence is the union of all such successive updates which we think of as *dynamic falsifications*. This gives rise to the recipe for negative updates of conjunctive sentences outlined below in (61).

(61) **Negative update of a conjunctive sentence**:
$$s[\phi \wedge \psi]^- = s[\phi]^-[\psi]^+ \cup s[\phi]^-[\psi]^- \cup s[\phi]^-[\psi]^? \cup s[\phi]^+[\psi]^- \cup s[\phi]^?[\psi]^-$$

Note especially that the dynamic analog of the *third truth value* in BUS is $s[\phi]^?$, which recall from the discussion of assertability picks out those possibilities in s which subsist in neither $s[\phi]^+$ nor $s[\phi]^-$.

Intriguingly, a putative negative counterpart to Egli's theorem doesn't hold, as stated in (62). This is because a negated existential statement doesn't introduce anaphoric information, but a negated conjunctive statement can. The empirical ramifications of this are interesting, but won't concern us until Section 5.4. More pressingly, let's now turn to disjunction, which will be crucial in accounting for FC with anaphora.

(62) $\neg\forall s, s[\exists_x(\phi \wedge \psi)]^- = s[\exists_x \phi \wedge \psi]^-$

3.4.2 Disjunction

As with conjunction, the positive update of disjunction is derived by taking together the ways of *dynamically verifying* the disjunctive sentence; likely, the negative update consists of the ways of *dynamically falsifying*. The strong Kleene truth-table for disjunction is given in Table 2, and the corresponding semantics for disjunction in BUS is given in (64).[18]

$\vee$	1	0	#
1	1	1	1
0	1	0	#
#	1	#	#

Table 2 Disjunction in strong Kleene

(64) **Disjunction in BUS:**

a. $s[\phi \vee \psi]^+ = s[\phi]^+[\psi]^+ \cup s[\phi]^+[\psi]^- \cup s[\phi]^+[\psi]^? \cup s[\phi]^-[\psi]^+ \cup s[\phi]^?[\psi]^+$

b. $s[\phi \vee \psi]^- = s[\phi]^-[\psi]^-$

18 An interesting feature of Elliott's (2022) algorithm is that, even though the underlying trivalent logic is completely symmetric, the resulting dynamic entries for the connectives are asymmetric. One downside of this approach is that it fails to capture the possibility of backwards bathroom disjunctions, as in (63).

(63) Either it's upstairs, or there is no bathroom.

Cataphora in fact poses a very general problem for any dynamic approach to anaphoric dependencies. Addressing this issue goes far beyond the remit of the current work, but see especially Elliott 2020 for discussion.

As shown in detail by Elliott (2022), the entry in (64) accounts directly for bathroom disjunctions, and assigns them *existential* truth-conditions. This will be another necessary step towards accounting for FC with anaphora. To see how this works, let's work through the following simple example:

(65) $\neg \exists_x P(x) \vee Q(x)$

The negative update is easy to compute, since it is conjunctive. Thanks to the validity of DNE it amounts to (66) — we update the initial state s with the information that *there is a P*, introducing a P-DR x. Subsequently, we eliminate possibilities where x is a Q. Note that the resulting update is equivalent to the positive update of $\exists_x P(x) \wedge \neg Q(x)$.[19]

(66) $s[\exists_x P(x)]^+[Q(x)]^-$
$$= \{(w,h) \mid (w,g) \in s, g[x]h, h(x) \in I_w(P), h(x) \notin I_w(Q)\}$$

Computing the positive update is a little more involved, and it will be convenient to break it down into two distinct steps: (i) verification of the disjunctive sentence via verification of the first disjunct, and (ii) verification of the disjunctive sentence via verification of the second disjunct:[20]

(67)
$$s[\phi \vee \psi]^+ = s[\phi]^+[\psi]^+ \cup s[\phi]^+[\psi]^- \cup s[\phi]^+[\psi]^? \quad \text{ver. via } \phi$$
$$\cup\, s[\phi]^-[\psi]^+ \cup s[\phi]^?[\psi]^+ \quad \text{ver. via } \psi$$

Starting with *verification via ϕ*, let's take the positive update of the first disjunct — this will eliminate any possibility in which there is a P, giving us an updated state s'. Since the second disjunct doesn't introduce any anaphoric information, $s'[Q(x)]^+$, $s'[Q(x)]^-$ and $s'[Q(x)]^?$ partition s' — the union of these is therefore s'.

(68) Dynamic verification via the first disjunct:
$$s[\exists_x P(x)]^-[Q(x)]^+$$
$$\cup\, s[\exists_x P(x)]^-[Q(x)]^-$$
$$\cup\, s[\exists_x P(x)]^-[Q(x)]^? = s[\exists_x P(x)]^- = \{\, (w,g) \in s \mid I_w(P) = \varnothing \,\}$$

19 The attentive reader will notice that the possibilities retained here include all those where there is a non-Q P, including those where there is also a Q P. This is a consequence of the fact (discussed in detail in Elliott 2024) that de Morgan's equivalences are valid in BUS, which means, e.g., that "Neither is there no bathroom, nor is it upstairs" is equivalent to "There is a bathroom, and it's not upstairs." See Elliott 2024 for a more detailed discussion of the truth-conditions of negated bathroom disjunctions, as well as how to derive potentially stronger truth-conditions.

20 N.b., in (67) we've arbitrarily grouped the case where both disjuncts are true with *verification via the first disjunct*. This is purely for convenience, since there is no need to compute this update twice.

Moving on to *verification via* ψ: Since the first disjunct is bivalent, we can ignore its unknown update, which is simply empty, and just consider the result of updating s with $s[\neg\exists_x P(x)]^-$. Thanks to DNE this amounts to eliminating non-P possibilities and introducing a P discourse referent x. The second disjunct is contextually bivalent, given this updated context, and for the disjunction to be true, the second disjunct should be true, so we eliminate possibilities in which x is not Q.

(69) Dynamic verification via the second disjunct:
$$s[\exists_x P(x)]^+[Q(x)]^- = \{(w,h) | g[x]h, (w,g) \in s, h(x) \in I_w(P), h(x) \notin I_w(Q)\}$$

We now take the union of (68) and (69) for the positive update associated with the disjunctive sentence, resulting in (70). The positive update retains possibilities in which there is no P and introduces no DRs in those possibilities, and also retains possibilities in which there is a P that is also Q, and introduces a DR in those possibilities. The sentence is therefore *true* just in case either there isn't a P, or there is a P that is also Q. We thereby account for bathroom disjunctions. Note especially that DNE was a crucial component — we needed the negative update of the negative first conjunct to introduce a DR.[21]

(70) $s[\neg\exists_x P(x) \vee Q(x)]^+$
$$= \{ (w,g) \in s \mid I_w(P) = \varnothing \} \cup \{ (w,h) \mid g[x]h, (w,g) \in s, h(x) \in I_w(P), h(x) \notin I_w(Q) \}$$

At this point, recall our discussion of the truth-conditions of bathroom disjunctions in Section 2.1. On the basis of examples like (71), we concluded (contra, e.g., Krahmer & Muskens 1995) that bathroom disjunctions may receive *existential* truth conditions. The treatment we sketched here derives exactly this — the disjunctive sentence in (71) is predicted to be *true* just as soon as

21 One interesting thing to note about the treatment of disjunction here is how it divides up possibilities in the input state. Concretely, what happens if there is a possibility i in which there are some Ps that are Q, and some Ps that aren't Q? What we in fact predict is that i should end up in both the positive update and the negative update, but paired with different DR in each. This seems harmless, since the assertability condition simply requires that every possibility has a descendant in either the positive or negative update, not necessarily that the positive and negative update are a partition. The possibilities that aren't in the positive update are the ones which will be eliminated.

One question left open here is how to derive stronger, universal readings, which can be attested modulo contextual factors Krahmer & Muskens 1995. See Elliott 2024 for discussion of how to derive universal readings using the same kind of mechanisms proposed for donkey anaphora, within the framework of BUS.

there is a credit card of Gennaro's that he paid with, even if he has another credit card that he didn't pay with.[22]

(71) Either Gennaro doesn't have a credit card, or he paid with it.

3.5 Epistemic modals

In order to illustrate the basics of our account, it will be easiest to focus on cases of FC involving epistemic modals. This is because there is an elegant and independently motivated proposal for the semantics of epistemic modals in update semantics due to Veltman 1996. Veltman's idea is that a sentence of the form $\Diamond\phi$ simply *tests* whether there are ϕ-possibilities in the input state. On Veltman's original proposal, the absurd state is returned if the test fails. In a bilateral setting, this must be adjusted. In order to define epistemic modals in BUS, we make use of Groenendijk, Stokhof & Veltman's (1996) notion of *subsistence* as a relation between states.[23] A state s *subsists* in a state s' just in case every possibility in s has a descendant in s', i.e., s' is strictly anaphorically more informative that s. For example, s subsists in $s[\exists_x P(x)]^+$, just in case there are no possibilities in s in which nothing is a P. The formal definition of state subsistence in terms of descendance is given below in Definition 3.4.

Definition 3.4 (State subsistence)**.** Let s, s' be Heimian information states, and (w, g) a possibility.

- A possibility $(w', g') \in s$ is a *descendant* of (w, g) iff $w = w'$ and g is 'defined' for a subset of the variables that g' is.
- s *subsists* in s', $s \prec s'$, iff every possibility in s has a descendant in s'.

Having defined subsistence for states, we now turn to the semantics of epistemic modals in BUS, outlined in (73). Starting with the positive update of $\Diamond\phi$ on s: this simply returns s if there are some possibilities in $s[\phi]^+$, and the absurd state otherwise. We use subsistence for the negative update of $\Diamond\phi$ on s: $s[\Diamond\phi]^-$ returns s just in case the information that *not* ϕ is already implicit

22 We assume here a standard notion of truth from update semantics — a sentence is *true* relative to a possibility i just in case $\{i\}[\phi]^+ \neq \emptyset$, i.e., if we take the singleton state consisting of just the possibility i, the positive update of that state is non-empty.

23 Note that this is a bit different from the definition of *subsistence* given in Definition 3.3, where it relates possibilities and states. Like Groenendijk, Stokhof & Veltman 1996 we allow the term to do double-duty.

in s, modulo any introduced anaphoric information; otherwise we get back the absurd state.[24]

(73) **Epistemic modals in BUS:**

a. $s[\Diamond\phi]^+ = \{\, i \in s \mid s[\phi]^+ \neq \varnothing \,\}$

b. $s[\Diamond\phi]^- = \{\, i \in s \mid s \prec s[\phi]^- \,\}$

To appreciate why the updates associated with $\Diamond\phi$ are defined in just this way, it will be useful to consider the example in (74).

(74) Maybe there isn't a^x bathroom. $\Diamond\neg\exists_x B(x)$

The positive update is non-empty and returns the input state, just in case there are some *no bathroom* possibilities, as in (75).

(75) $s[\Diamond\neg\exists_x B(x)]^+ = \{\, i \in s \mid s[\exists_x B(x)]^- \neq \varnothing \,\}$

The negative update on the other hand is non-empty and returns the input state, just in case the input state *subsists* in the positive update of the existential statement (by DNE), as in (76). s *subsists* in $s[\exists_x B(x)]^+$ just in case every possibility in s is a *bathroom* possibility.

(76) $s[\Diamond\neg\exists_x B(x)]^- = \{\, i \in s \mid s \prec s[\exists_x B(x)]^+ \,\}$

Note that (76) also effectively captures the positive update of the corresponding epistemic necessity statement $\Box\exists_x B(x)$ — the necessity modal can be straightforwardly defined as the dual of the possibility modal.

(77) **Necessity modals in BUS:**

a. $s[\Box\phi]^+ = s[\Diamond\neg\phi]^-$

b. $s[\Box\phi]^- = s[\Diamond\neg\phi]^+$

24 Note that without saying anything else, the semantics in (73) predicts that possibility modals are presuppositional *filters* rather than holes in the sense of Karttunen 1973. That is to say, $\Diamond\phi$ can be assertable at a state s even if ϕ is undefined at some possibilities in s — what's important is that ϕ is true at some possibilities in s. We agree with Singh 2008 and Elliott 2022 that this is in fact a desirable outcome. Consider, e.g., the following example from Singh: p. 75, where only the putative weak presupposition is globally contextually entailed. As Singh points out, nothing stronger need be accommodated.

(72) *Context: you see a man you don't know whistling at the bushes. You say to your friend:*
He might have lost his dog.

Like Groenendijk, Stokhof & Veltman (1996), our treatment of modals assumes that they are *externally static*, i.e., an indefinite in the scope of a modal is inaccessible for subsequent anaphoric pronouns. The facts are undeniably more complicated, and arguably require an account of modal subordination (Roberts 1989) in order to account of data like (78). See Hofmann 2019, 2022 for an empirically wide-ranging account of data like (78).

(78) There might be a bathroom and it might be upstairs.

A simple test semantics for modals will be sufficient in order to explicate our account of FC with anaphora, which builds on recent semantic accounts of FC.[25] We turn to FC in the next section.

3.6 Modal disjunction

Now that we've introduced BUS, with a basic semantics for disjunction that accounts for bathroom disjunctions, and a basic treatment of epistemic modals, we're in a position to introduce the main idea behind our analysis of FC inferences in an update-semantic framework.

Following many existing semantic accounts of FC, the basic intuition behind our account is that the semantics of disjunction should validate the inference *Modal Disjunction* (79) (e.g., Zimmermann 2000, Simons 2005, Goldstein 2019, Willer 2018, 2017, 2019). If modal disjunction is valid, FC follows straightforwardly (as, e.g., Goldstein 2019 shows in detail). The tricky part, however, is validating (79) without weakening the meaning of disjunction under negation. Crucially, we want to ensure that disjunction in a *negated* possibility statement still gives rise to the 'dual prohibition' inference in (80). A bilateral framework provides the necessary resources for accomplishing this (see, especially Willer 2017, 2019).

(79) **Modal Disjunction:**
$\phi \vee \psi \vDash \Diamond\phi \wedge \Diamond\psi$

(80) **Dual prohibition:**
$\neg\Diamond(\phi \vee \psi) \vDash \neg\Diamond\phi \wedge \neg\Diamond\psi$

25 Epistemic modals are *non-distributive* operators — their definition necessarily makes reference to the entire input state, rather than individual possibilities. An anonymous reviewer reminds us that non-distributive operators interact in sometimes unexpected ways with Heimian random assignment (Groenendijk, Stokhof & Veltman 1996, Aloni 1997), and the logic of anaphora and modality may ultimately need to be complicated beyond what we have presented here in order to address the resulting problems.

In a bilateral setting, we can straightforwardly add an additional requirement to the positive update of disjunction, which insists that the positive updates of the individual disjuncts be non-absurd. Since the semantics is bilateral, we can do this while leaving the negative update intact. We'll write *modal disjunction* in BUS as $\overline{\vee}$.[26]

(81) **Modal disjunction in BUS** (first attempt):

a. $s[\phi \mathbin{\overline{\vee}} \psi]^+ = s[\phi \vee \psi]^+$ if $s[\phi]^+, s[\psi]^+ \neq \varnothing$ else $\varnothing$

b. $s[\phi \mathbin{\overline{\vee}} \psi]^- = s[\phi \vee \psi]^-$

Let's see how this accounts for both FC and dual prohibition for a simple case of FC without anaphora. In the positive case, it's easy to see how FC follows — since modal disjunction places a pre-condition on the positive update that both disjuncts be possible relative to the input state, a possibility modal applying to the disjunctive sentence will inherit this requirement on the individual disjuncts. Therefore, $s[\Diamond(c \mathbin{\overline{\vee}} t)]^+$ will only be non-empty if there are some *coffee* possibilities, and some *tea* possibilities.

(82) There might be coffee or tea. $\Diamond(c \mathbin{\overline{\vee}} t)$

(83) $s[\Diamond(c \mathbin{\overline{\vee}} t)] = s$ if $s[c \mathbin{\overline{\vee}} t]^+ \neq \varnothing$ else $\varnothing$

(84) $s[\Diamond(c \mathbin{\overline{\vee}} t)] = s$ if $\exists(w,g) \in s[\text{there's coffee in } w]$, $\exists(w,g) \in s[\text{there's tea in } w]$, else $\varnothing$

Now, let's make sure that *dual prohibition* is still valid.

(85) It's impossible that there's coffee or tea. $\neg\Diamond(c \mathbin{\overline{\vee}} t)$

Note that in the absence of an expression that introduces anaphoric information subsistence amounts to equality. The negative update of $\Diamond(c \mathbin{\overline{\vee}} t)$ therefore is non-empty, and returns the input state, just in case every possibility in s is both a *non-coffee* possibility, and a *non-tea* possibility, which in turn implies *dual prohibition*.[27]

26 Our semantic account of FC also bears a family-resemblance to the recent account of Aloni (2022). As for us, on Aloni's account disjunction is interpreted relative to an information state, and each individual disjunct comes to be associated with a non-emptiness requirement. Rather than stipulating this in the semantics of disjunction however, Aloni (2022) aims to derive this requirement from more general cognitive biases.

27 An anonymous reviewer reminds us that, under certain conditions, a sentence of the form $\neg\Diamond(\phi \vee \psi)$ can instead be interpreted as the *negation* of FC.

(87) $s[\Diamond(c \mathbin{\overline{\vee}} t)]^- = s$ if $s = s[c \mathbin{\overline{\vee}} t]^-$ else $\varnothing$

(88) $s[\Diamond(c \mathbin{\overline{\vee}} t)]^- = s$ if $s = \left\{ (w,g) \in s \;\middle|\; \begin{array}{l}\text{there's no coffee in } w,\\ \text{there's no tea in } w\end{array} \right\}$ else $\varnothing$

It is possible to extend the basic logic of this account to FC with non-epistemic modals, as demonstrated by Goldstein 2019, by generalizing a test semantics for modals to other flavors of modality. Following Goldstein, non-epistemic flavors of modality can be incorporated by taking a modals generally to be associated with an accessibility relation R, where R^i is the set of possibilities accessible from i. Asserting $\Diamond^R\phi$ at c retains possibilities i, just in case R^i can be consistently updated with ϕ. We adapt Goldstein's idea to our bilateral setting below:

(89) **Flavor-neutral modals in BUS:**

a. $s[\Diamond^R\phi]^+ = \{\, i \in s \mid R^i[\phi]^+ \neq \varnothing \,\}$

b. $s[\Diamond^R\phi]^- = \{\, i \in s \mid R^i \prec R^i[\phi]^- \,\}$

As Goldstein shows in detail, this means that $\Diamond^R(\phi \vee \psi)$ retains possibilities $i \in s$, such that $R^i[\phi \vee \psi]^+$ is non-empty. Modal disjunction guarantees that these possibilities must also be ones at which $R^i[\phi]^+$ and $R^i[\psi]^+$ are non-empty, thus deriving FC. For epistemic modals, we can assume that the possibilities accessible from each $i \in s$ simply amount to s. For the remainder of this paper, we stick to epistemic FC exclusively for expository reasons,

Before going any further, it should be easy to see that the modal disjunction account won't extend to FC with anaphora. Ultimately, this is due to the *simplification* implicit in the modal disjunction inference schema (79). Consider what modal disjunction implies when we apply it to a bathroom disjunction such as "Either there's no bathroom, or it's in a funny place" (90). Intuitively, modal disjunction should imply: *it's possible there's no bathroom* (90a), and *it's possible there's a bathroom in a funny place* (90b). The modal disjunction schema in (79) however erroneously delivers (90c).

(86) It's not true that you can have coffee or tea; you can only have coffee.

Such interpretations are clearly marked, and seem to require special intonation or contextual licensing. Admittedly however this constitutes a problem for our approach to FC based on modal disjunction. There are some analytical possibilities, such as the possibility of a weaker form of negation which merely tests the input state to see whether the positive update of its prejacent would be empty. *Ad hoc* ambiguity does not however seem to us to be a satisfying solution, and we leave a fuller consideration of this issue to future work.

(90) $\neg\exists_x B(x) \vee F(x)$

a. $\vDash \Diamond\neg\exists_x B(x)$

b. $\vDash \Diamond(\exists_x (B(x) \wedge F(x))$

c. $\nvDash \Diamond F(x)$

In order to account for FC with anaphora, we'll modify modal disjunction to make it sensitive to anaphoric dependencies.

3.7 Modal disjunction with anaphora

Consider again the original definition we gave for disjunction in BUS, repeated below:

(91) **Disjunction in BUS:**

a. $s[\phi \vee \psi]^+ = s[\phi]^+[\psi]^+ \cup s[\phi]^+[\psi]^- \cup s[\phi]^+[\psi]^?$
$s[\phi]^-[\psi]^+ \cup s[\phi]^?[\psi]^+$

b. $s[\phi \vee \psi]^- = s[\phi]^-[\psi]^-$

In our discussion of how (91) accounts for anaphoric dependencies in bathroom disjunctions, we in effect separated out the part of the positive update that the first disjunct is responsible for, and the part that the second disjunct is possible for. Our idea for accounting for FC with anaphora is to reify this distinction in the semantics of modal disjunction. In order to do so, we use $s[\phi \vee \psi]_1^+$ (92a) $s[\phi \vee \psi]_2^+$ (92b) to capture the part of the positive update the first and second disjuncts are responsible for respectively. Note that if the two disjuncts are compatible, these may overlap. The positive update of the disjunctive sentence is equivalent to the union of these two sets, (92c).

(92) a. $s[\phi \vee \psi]_1^+ = s[\phi]^+[\psi]^+ \cup s[\phi]^+[\psi]^- \cup s[\phi]^+[\psi]^?$

b. $s[\phi \vee \psi]_2^+ = s[\phi]^+[\psi]^+ \cup s[\phi]^-[\psi]^+ \cup s[\phi]^?[\psi]^+$

c. $s[\phi \vee \psi]^+ = s[\phi \vee \psi]_1^+ \cup s[\phi \vee \psi]_2^+$

Now, let's consider what these sets amount to in the case of a bathroom disjunction such as "Either there's no bathroom, or it's in a funny place". As we've already discussed, (92a) amounts to the positive update of the first disjunct (93), since the second disjunct introduces no anaphoric information. (92b) on the other hand amounts to successively updating with the information that *there's a bathroom in a funny place.*

(93) $s[\neg\exists_x B(x) \vee F(x)]^+_1 = s[\exists_x B(x)]^-$

(94) $s[\neg\exists_x B(x) \vee F(x)]^+_2 = s[\exists_x B(x)]^+[F(x)]^+$

We're now in a position to state an *anaphora-sensitive* variant of modal disjunction which validates the inference schema in (95), thereby deriving FC with anaphora:

(95) **Modal disjunction (anaphora sensitive ver.):**
$\phi \vee \psi \vDash \Diamond\phi \wedge \Diamond(\neg\phi \wedge \psi)$

We'll accomplish this by exploiting the derivative notions in (92a) and (92b), i.e., the part of the positive update that the first and second disjuncts are responsible for respectively. The definition for modal disjunction in BUS we'll ultimately adopt is given below in (96). It adds an additional precondition to the positive update of $\phi \vee \psi$ that each disjunct be responsible for at least some possibilities in the positive update of the disjunctive sentence.

(96) **Modal disjunction in BUS (anaphora-sensitive ver.):**

a. $s[\phi \mathbin{\overline{\triangledown}} \psi]^+ = s[\phi \vee \psi]^+$ if $s[\phi \vee \psi]^+_1, s[\phi \vee \psi]^+_2 \neq \varnothing$ else $\varnothing$

b. $s[\phi \mathbin{\overline{\triangledown}} \psi]^- = s[\phi \vee \psi]^-$

This immediately predicts that a bathroom disjunction "There is no bathroom, or it's in a funny place" only has a non-empty positive update if there are some *no bathroom* possibilities, and some *bathroom in a funny place* possibilities, as we've already seen. Once combined with the test semantics for possibility statements, this will immediately derive the attested FC with anaphora inference.

To illustrate, consider a simple bathroom disjunction embedded under an epistemic modal (97); this time, we translate natural language disjunction as the modal disjunction stated in (96). The positive update of a possibility statement tests whether the positive update by the prejacent is non-empty (98).

(97) $\Diamond(\neg\exists_x P(x) \mathbin{\overline{\triangledown}} Q(x))$

(98) $s[\Diamond(\neg\exists_x P(x) \mathbin{\overline{\triangledown}} Q(x))]^+ = \{\, i \in s \mid s[\neg\exists_x P(x) \mathbin{\overline{\triangledown}} Q(x)]^+ \neq \varnothing \,\}$

Let's now consider the conditions under which the test is passed, by computing the positive update of the bathroom disjunction. As shown below, for the positive update of the bathroom disjunction to be non-empty, the input state s must satisfy two conditions imposed by modal disjunction: (i) dynamic verification via the first disjunct must be possible, and (ii) dynamic verification via the second disjunct must be possible. As discussed, this amounts to the requirement that, at s, it's possible there's no P, and it's possible there's a P that is Q. For the test imposed by the possibility modal to be passed, the test imposed by modal disjunction must be passed also, thereby FC with anaphora is derived.

(99) $s[\neg\exists_x P(x) \overline{\vee} Q(x)]^+ = s[\neg\exists_x P(x) \vee Q(x)]$ if $s[\neg\exists_x P(x) \vee Q(x)]_1^+ \neq \varnothing$, $s[\neg\exists_x P(x) \vee Q(x)]_2^+ \neq \varnothing$, else $\varnothing$

4 FC with anaphora and $\mathcal{E}xh$

In the previous section, we provided a concrete, semantic account of FC with anaphora in terms of modal disjunction. We haven't shown that there isn't any way of tweaking exhaustification accounts in order to derive FC with anaphora. Before considering some different approaches that an exhaustification account of FC could take with respect to this problem, it's worth considering why it isn't sufficient to simply combine our dynamic account of bathroom disjunctions with, e.g., Bar-Lev & Fox's (2020) *innocent inclusion* account of FC.

The reason is that the implicature account is fundamentally based on the idea that FC is computed on the basis of alternatives *qua* structurally-simpler syntactic representations (Katzir 2008, Fox & Katzir 2011), as opposed to some more abstract, logical level of representation. Even if the underlying logic renders "either there's no bathroom or it's upstairs" equivalent to "either there's no bathroom, or there's a bathroom and it's upstairs", inclusion of the structural alternative "it's possible it's upstairs" takes the pronoun outside of the context in which it can be (dynamically) bound. In other words, alternatives are linguistic representations, as below:

(100) It's possible that either there's no bathroom or it's upstairs. $\Diamond(\neg\exists_x P(x) \vee Q(x))$

a. Alt_1 =It's possible there's no bathroom. $\Diamond\neg\exists_x P(x)$

b. Alt_2 =It's possible it's upstairs. $\Diamond Q(x)$

Because of this central feature of the exhaustification account, there is one obvious way of reconciling is with FC with anaphora. If, instead of Alt_2, we had Alt_2', we could simply combine the exhaustification account with an 'off the shelf' account of discourse/bathroom anaphora, and an account of FC with anaphora would follow.

(101) Alt'_2 =it's possible there's a bathroom and it's upstairs.
$(\Diamond(\exists_x P(x) \wedge Q(x)))$

In this section, we briefly consider two conceivable ways of cashing out this idea. In Section 4.1 we consider an approach which makes use of additional structural alternatives, ultimately showing that this isn't viable, due to independently motivated constraints on alternatives. In Section 4.2 we consider idea due to Meyer 2016, that disjunctions may involve a silent anaphoric *else*. We argue that Meyer's conjecture has independent issues, and that there are conceptual reasons to disprefer such an approach. Ultimately, we leave the question of how best to capture FC with anaphora within an exhaustification framework as an open question.

4.1 More alternatives

Consider again the alternatives to a sentence exhibiting FC with anaphora inferences.

(102) $\{\underbrace{\Diamond(\neg\exists_x\phi \vee \psi)}_{\text{prejacent}}, \underbrace{\Diamond(\neg\exists_x\phi \wedge \psi)}_{\text{conj. alt}}, \underbrace{\Diamond\neg\exists_x\phi, \Diamond\psi}_{\text{domain alts}}\} \subseteq \mathbf{Alt}(\Diamond(\neg\exists_x\phi \vee \psi))$

Assuming Fox & Katzir's (2011) simplification algorithm, there is a potential way of deriving an alternative which can help account for FC with anaphora. First, take the prejacent, and replace disjunction with conjunction (the conjunctive alternative). Next substitute $\neg\exists_x\phi$ with the simpler $\exists_x\phi$. As long as the underlying logic is one which satisfies *Egli's theorem* (i.e., conjunction is dynamic), then FC with anaphora could perhaps be derived by inclusion of the alternative in (103).

(103) $\Diamond(\exists_x\phi \wedge \psi)$

Unfortunately, this kind of derivation is independently problematic, as pointed out by Romoli (2013).[28] A strong scalar item such as *all* embedded in the scope of negation gives rise to a so-called *Indirect Implicature*, as illustrated by the example in (104). The standard assumption is that this is derived as a scalar implicature by replacing *all* with *some*, and negating the resulting alternative *none of the slides compiled properly*.

(104) Not all of the slides compiled properly.
⇒ *Some of the slides compiled properly*

28 We're grateful to Jacopo Romoli (p.c.) for bringing the relevance of this work to our attention.

Note however that Fox & Katzir's simplification algorithm gives rise to another alternative: *some of the slides compiled properly*, via replacement of *not all…* with *all…* and substitution of *all* with *some*. Since this alternative and the scalar alternative are symmetric, neither can be excluded nor included. More abstractly, the problematic case is schematized in (105). Since one of the primary motivations behind structural alternatives is to avoid generating exactly such symmetric alternatives, this is clearly a bad result.

(105) $\neg\exists_x\phi, \exists_x\phi \in \mathbf{Alt}(\neg\forall_x\phi)$

In order to address this and related problems, Trinh & Haida 2015 propose an additional constraint on the generation of structural alternatives — the *atomicity constraint*. The fine details of are not important for our purposes (see also Breheny et al. 2018, who make the case that the atomicity constraint is sometimes not sufficient), but what it ensures is that, once we substitute $\neg\forall_x\phi$ with the simpler expression $\forall_x\phi$, we cannot perform further substitutions.

Going back to the alternative in (103), in order to derive this, we'd first have to simplify $\diamond\neg\exists_x\phi \vee \psi$ to $\diamond\exists_x\phi \vee \psi$, and then perform substitution to derive $\exists_x\phi \wedge \psi$ — this second step is also blocked by atomicity. It seems to us that however one attempts to block the symmetric alternative in (105), this should also block the alternative relevant for FC with anaphora in (103).

4.2 Covert *else*

Another way of accounting for FC with anaphora, without modifying the exhaustification account of FC, could be to motivate a structural representation of the local context in the second disjunct. There is in fact an (independently motivated) proposal along these lines, due to Meyer 2016.[29] In order to account for 'conjunctive' readings of certain disjunctive sentences such as (106) (see also Klinedinst & Rothschild 2012), Meyer posits the presence of an (optionally covert) anaphoric element *else*, which picks up the first disjunct as its antecedent and conditionalizes the second disjunct to its negation. (106) seems to entail both (106a) and (106b), which Meyer derives as a kind of FC inference. The details of the account are unimportant to our purposes, but the crucial conjecture is that the presence of *else* (covert, or otherwise) removes the conjunctive alternative from consideration, allowing for inclusion of both the initial disjunct and the (conditionalized) latter disjunct.

29 We're grateful to Danny Fox (p.c.) for making this connection.

(106) [We should leave soon]$^{\alpha}$, or (else$_{\alpha}$) we'll be late.

a. *We should leave soon*

b. *If we don't leave soon, we'll be late.*

If covert *else* is generally available, it's tempting to try to explain FC with anaphora by assuming that the relevant domain alternative corresponding to the latter disjunct contains *else*.

There's a sense in which covert *else* reifies what a general theory of presupposition projection/anaphoric accessibility delivers for disjunction anyway, so in our view the burden of proof lies with a proponent of covert *else*. Our primary objection to utilizing covert *else* is empirical however — in Section 5.4 we discuss a variant of FC which can also involve an anaphoric dependency: *negative FC*. As illustrated in (107), negative FC involves a *conjunction* embedded in a negated necessity statement. We defer a detailed discussion of such cases until Section 5.4; here, we merely note that an account of FC with anaphora based on covert *else* will be limited to cases involving disjunction.

(107) It's not required that you include an appendix and keep it to a single page.

Furthermore, allowing *else* to be covert raises some independent issues, as noted by Meyer herself. Examples such as (108a) (Meyer 2016: p. 710, citing Webber et al. 2003) show that overt *else* can take a sub-part of a complex initial disjunct as its antecedent, in order to derive the attested interpretation *If the light isn't red, go straight on*. Note that this interpretation clearly isn't available for (108b). This contrast is somewhat mysterious if *else* can be covert. See Klinedinst & Rothschild 2012 for an alternative account of the inferences in (106) which doesn't rely on covert *else*.

(108) a. If [the light is red]$^{\alpha}$, stop, or else$_{\alpha}$ go straight on.

b. ??If the light is red, stop, or go straight on. (Meyer 2016: p. 710)

4.3 Prospects for $\mathcal{E}xh$

In conclusion, we have suggested that the fact that the exhaustification account makes use of *linguistic* alternatives, makes it difficult to reconcile with FC with anaphora. On an exhaustification account, it's natural to attempt to capture this phenomenon by essentially positing a structurally more complex representation for the second disjunct, meaning that when this disjunct is severed from its local context, the pronoun remains (dynamically) bound. We've argued here that a couple of different ways of cashing out this idea turn out to be problematic.

This is not however intended to be a definitive argument against the exhaustification account. We believe that a more promising line of attack would be to dynamicize the exhaustification procedure itself, although exploring the details of such would go far beyond the confines of this paper. We leave an exhaustification account of FC with anaphora as an open challenge.

5 Anaphora and simplification beyond FC

In this section, we consider other manifestations of the problem posed by anaphoric dependencies for inferences typically associated with simpler alternatives to complex sentences. Other non-classical inferences associated with disjunctive sentences include: *wide free choice* (Section 5.1) — a variant of classical FC involving disjunctions of modalized sentences; *distributive inferences* (Section 5.2), which involve disjunction embedded under a universal quantifier; and *simplification of disjunctive antecedents* (Section 5.3), which involve conditionals with disjunctive antecedents. Interestingly, *negative FC* (Section 5.4) constitutes a manifestation of the simplification problem for embedded *conjunctions* with anaphoric dependencies. The problem we focused on in the previous discussion — FC with anaphora — turns out to be a special case of a more general problem, which we might call *simplification with anaphora*.

5.1 Wide free choice

It has been widely observed that FC effects still arise, even when disjunction takes wide-scope over possibility modals, as schematized in (109) (see, e.g., Zimmermann 2000, Simons 2005, Willer 2017).

(109) **Wide** FC: $\Diamond\phi \vee \Diamond\psi \vDash \Diamond\phi \wedge \Diamond\psi$

Wide FC is clearly compatible with anaphora, as illustrated in (110):

(110) Either it's possible there's no bathroom, or it's possible it's upstairs.
⇒ *It's possible there's no bathroom,*
and it's possible there's a bathroom upstairs

Without even considering FC, examples such as (110) are already beyond the remit of the semantics developed here for epistemic modals and disjunction. The simple reason for this is that, for us, modalized statements are externally-static, and therefore incapable of introducing anaphoric information (see also

Groenendijk, Stokhof & Veltman 1996). As we discuss in Section 3.5 however, treating modals as externally static is arguably an over-simplification. In order to develop an account of (110), it will be important to consider the anaphoric-potential of modalized statements, as well as the role of modal subordination in licensing anaphora (see especially Hofmann 2019, 2022). For now, we leave wide FC with anaphora as an open problem.

Putting anaphora to one side however, our semantics does capture wide FC in certain cases.[30] Consider for example, the sentence in (111), together with its translation.

(111) It might be here, or it might be there. $\Diamond H(x) \vee \Diamond T(x)$

Our entry for modal disjunction demands that, for the positive update of (111) to yield a non-empty result with respect to c, the updates in (112a) and (112b) should both be non-empty. That is, if there are no $H(x)$-possibilities in c, (112a) will always be empty, since the test imposed by the first disjunct will fail. Similarly, if there are no $T(x)$-possibilities in c, (112a) will always be empty. This is because, even if the test imposed by the first disjunct is passed, the test imposed by the second disjunct will then always fail. Consequently, Wide FC.

(112) a. $c[\Diamond H(x)]^+[\Diamond T(x)]^+ \cup c[\Diamond H(x)]^+[\Diamond T(x)]^- \cup c[\Diamond H(x)]^+[\Diamond T(x)]^?$

b. $c[\Diamond H(x)]^+[\Diamond T(x)]^+ \cup c[\Diamond H(x)]^-[\Diamond T(x)]^+ \cup c[\Diamond H(x)]^?[\Diamond T(x)]^+$

5.2 Distributive inferences

Disjunctions in the scope of a universal quantifier give rise to a so-called *distributive inference* (Crnič, Chemla & Fox 2015), as schematized in (113). (114) illustrates the intuitive validity of the inference.

(113) **Distributive Inference:**
$\forall x(\phi \vee \psi) \vDash \exists x\phi \wedge \exists x\psi$

(114) Every student read *The Master and Margarita* or *The White Guard*.

a. ⇒ *Some student read The Master and Margarita.*

b. ⇒ *Some student read The White Guard.*

30 We're grateful to an anonymous reviewer for convincing us of this.

It's easy to appreciate that distributive inferences don't follow from a classical semantics for disjunction and universal quantification. If $\phi \vee \psi$ is classically true at every valuation of x, this of course doesn't guarantee the existence of a valuation of x for which ϕ is true, and one for which ψ is true. Bar-Lev & Fox (2020) extend their account of FC inferences to distributive inferences by exploiting the includability of the alternatives in (115). The logic is completely parallel to the case of FC.

(115) $\exists x\phi, \exists x\psi \in \mathbf{Alt}(\forall x(\phi \vee \psi))$

As expected, it's easy to come up with problematic cases involving a bathroom disjunction embedded under a universal quantifier.

(116) Every student either didn't read a^x novel, or wrote a_x report on it.

   a. ⇒ *Some student didn't read a novel.*
   b. ⇒ *Some student read a novel and wrote a report on it.*

The desired inference schema to accommodate distributive inferences with anaphora is given in (117).

(117) **Distributive inference with anaphora:**
$\forall x(\phi \vee \psi) \vDash \exists x\phi \wedge \exists x(\neg\phi \wedge \psi)$

Let's consider how our account of FC with anaphora extends to (117) (see also Goldstein 2019: Section 9.2 for pertinent discussion). We'll assume that the universal quantifier is simply the dual of the existential, which gives rise to the semantics in (119).[31]

(119) **Universal quantification in BUS:**

   a. $s[\forall x\phi]^+ = \{(w,g) \in s \mid w \notin \mathscr{W}(s[\varepsilon_x][\phi]^-), w \in \mathscr{W}(s[\varepsilon_x][\phi]^+)\}$
   b. $s[\forall x\phi]^- = [\varepsilon_x][\phi]^-$

31 A dual treatment in BUS automatically predicts that a negative universal statement introduces a (singular) discourse referent. We think that this is probably incorrect, on the basis of examples such as (118).

(118) Not every^x linguist is attending the plenary talk. ??She_x's smoking outside.

Nevertheless, (117) shouldn't be taken to be a realistic proposal for the dynamics of universal quantification. Ultimately, an empirically adequate dynamic treatment of generalized quantifiers requires a shift to a plural setting — see, e.g., van den Berg 1996, Nouwen 2003, Brasoveanu 2007. A simplified 'first order' treatment will do for our purposes here.

Now, we can consider what is entailed by the positive update of the sentence in (120), where natural language *or* is translated as BUS modal disjunction. To do this, we'll have to compute the possibilities in s that a positive update by (120) retains.

(120) $\forall_y(\neg\exists_x R(y,x) \,\overline{\vee}\, W(y,x))$

(121) $s[\forall_y(\neg\exists_x R(y,x) \,\overline{\vee}\, W(y,x))]^+$
$$= \left\{ (w,g) \in s \;\middle|\; \begin{array}{l} w \notin \mathscr{W}(s[\varepsilon_y][\neg\exists_x R(y,x) \,\overline{\vee}\, W(y,x)]^-), \\ w \in \mathscr{W}(s[\varepsilon_y][\neg\exists_x R(y,x) \,\overline{\vee}\, W(y,x)]^+) \end{array} \right\}$$

Due to the requirement imposed by modal disjunction, the positive update of the sentence will be empty unless (i) there is a student y, s.t., y didn't read any novel x, and (ii) there is a student y, s.t., there y read a novel x and wrote a report on x.

5.3 Simplification of Disjunctive Antecedents

Simplification of Disjunctive Antecedents (SDA) describes the inference schematized in (123) (the connective $>$ stands in for a natural language conditional of the form *if…then…*). SDA is generally valid across all conditional types in natural language, and famously is not predicted by classical approaches to the semantics of conditionals (see, e.g., Fine 1975, Nute 1975, and many subsequent works). Consider, e.g., the counterfactual conditional in (122), which intuitively entails the simplification inferences in (122a) and (122b).

(122) If Thorpe or Wilson were to win the next General Election, Britain would prosper.

a. ⇒ *If Thorpe were to win the next General Election, Britain would prosper.*

b. ⇒ *If Wilson were to win the next General Election, Britain would prosper.*

(Fine 1975: p. 453)

(123) **Simplification of Disjunctive Antecedents (SDA):**
$(\phi \vee \psi) > \rho \vDash (\phi > \rho) \wedge (\psi > \rho)$

Much like the cases of FC, negative FC, and distributive inferences discussed in previous sections, two ways of validating SDA have been explored in the literature: (i) semantic accounts which adopt a non-classical semantics for conditionals/disjunction, and (see, e.g., Alonso-Ovalle 2005, 2009, Van Rooij 2006) (ii) exhaustification-based accounts which maintain a classical semantics for conditionals, and account of the inference in (123) as an implicature (see, e.g., Klinedinst 2007, 2009, Bar-Lev & Fox 2020).

The fine details of the exhaustification-based account will not be important for our purposes, but as before we take the proposal of Bar-Lev & Fox (2020) to be an exemplar. For exhaustification based accounts, a conditional with a disjunctive antecedent gives rise to the following alternatives:

(124) $\{\underbrace{(\phi \vee \psi) > \rho}_{\text{prejacent}}, \underbrace{(\phi \wedge \psi) > \rho}_{\text{scalar alt}}, \underbrace{\phi > \rho, \psi > \rho}_{\text{domain alts}}\} \in \mathbf{Alt}((\phi \vee \psi) > \rho)$

As in the case of FC, what's crucial for the account is that exclusion of the scalar alternative is consistent with inclusion of the domain alternatives. This inclusion step validates the inference in (123), even with a classical semantics for conditionals, since it gives rise to the strengthened meaning in (125):

(125) $\underbrace{(\phi \vee \psi) > \rho}_{\text{assertion}} \wedge \underbrace{\neg((\phi \wedge \psi) > \rho)}_{\text{implicature}} \wedge \underbrace{(\phi > \rho) \wedge (\psi > \rho)}_{\text{SDA inference}}$

The logic of our argument should at this point be familiar — we construct an example with an anaphoric dependency between the disjuncts in the conditional antecedent. We do this below in (126), where the conditional antecedent is a bathroom disjunction. The SDA inferences in (126a) and (crucially) (126b) are intuitively valid.

(126) If either there's no bathroom or it's upstairs, this house needs to be renovated.

a. ⇒ *If there's no bathroom, this house needs to be renovated.*

b. ⇒ *If there's a bathroom upstairs, this house needs to be renovated.*

As in previous cases, the inference in (126b) is not captured by the classical SDA schema in (123) — what we intuitively want to validate is the modified SDA schema in (127), which by DNE and Egli's theorem should derive the attested inference in (126b). Since existing accounts of SDA are tailored to validate (123) then SDA with anaphora is *prima facie* a problem for both exhaustification and semantic accounts of SDA.

(127) **SDA with anaphora:**
$(\phi \vee \psi) > \rho \vdash (\phi > \rho) \wedge ((\neg\phi \wedge \psi) > \rho)$

In the interests of space, we won't attempt to propose an account of SDA with anaphora within the framework of BUS, since stating a satisfactory semantics for conditionals requires a great deal of work. The main point here is that we have yet another manifestation of the general problem of *simplification with anaphora* which isn't captured by standard accounts, or even encompassed by the inference schema. We leave a concrete account of SDA with anaphora to future work.

5.4 Negative free choice

Negative FC, as schematized in (128) is a variation of FC involving the negation of a conjunctive necessity statement.

(128) **Negative FC:** $\neg\Box(\phi \wedge \psi) \vDash \neg\Box\phi \wedge \neg\Box\psi$

By duals and de Morgan's equivalences, note that $\neg\Box(\phi \wedge \psi)$ is equivalent to $\Diamond(\neg\phi \vee \neg\psi)$, which is expected to give rise to the FC inference $\Diamond\neg\phi \wedge \Diamond\neg\psi$. Again, by duals, this is equivalent to $\neg\Box\phi \wedge \neg\Box\psi$, hence the connection to FC. There has been some disagreement in the literature about the intuitive validity of (128) in natural language — see Marty et al. 2021 for an overview.

(129) It's not required that you include an appendix and keep it to a single page.

It's easy to extend the semantic account of FC couched within BUS to negative FC by stating a non-classical semantics for conjunction which validates the *negative modal conjunction* inference in (130) while still maintaining the validity of (the classically valid) *dual permission*.[32]

(130) **Negative modal conjunction:** $\neg(\phi \wedge \psi) \vDash \Diamond\neg\phi \wedge \Diamond\neg\psi$

(131) **Dual permission:** $\Diamond(\phi \wedge \psi) \vDash \Diamond\phi \wedge \Diamond\psi$

In order to account for negative FC with anaphora, we can make completely parallel moves to those we made in accounting for plain FC with anaphora. We simply place a pre-condition on the *negative* update of conjunction, which demands that each conjunct be responsible for at least some possibilities in the negative update.

32 Note that negative modal conjunction should follow from modal disjunction *if* de Morgan's equivalences are valid, since by de Morgan's a negated conjunction is equivalent to a disjunction of negative statements.

(132) **Negative modal conjunction in BUS:**

a. $s[\phi \overline{\wedge} \psi]^+ = s[\phi \wedge \psi]^+$

b. $s[\phi \overline{\wedge} \psi]^- = s[\phi \wedge \psi]^-$ if $s[\phi \wedge \psi]^-_1, s[\phi \wedge \psi]^-_2 \neq \emptyset$ else $\emptyset$

We leave it to the reader to verify that the semantics in (132) validates negative FC with anaphora.

6 Conclusion

In this paper, we've primarily focused on a new problem for theories of FC inferences — namely, *FC with anaphora*. FC with anaphora is so striking exactly because it isn't just an issue for particular approaches to FC inferences, but more generally calls into question how FC inferences should be characterized from a logical perspective. As we've seen, embedded disjunctions with anaphora clearly give rise to FC inferences, but these inferences don't follow from the FC inference schema as traditionally described.

Our diagnosis is that FC with anaphora is an instantiation of a much more general problem. Beyond FC, many inferences have been discovered, associated with embedded disjunctions, which don't follow from a classical (boolean) semantics. In the previous section, we discussed wide FC, distributive inferences, SDA, and even *negative* FC, which instead involves a similar problem with embedded *con*junctions. The literature has sought to account for many of these inferences starting from the intuition that (in some sense) disjunctions are are associated with *simpler* alternatives (in some sense). Bar-Lev & Fox's (2020) exhaustification-based account is especially notable as it attempts to provide a truly unified account of many of these *simplification* inferences.

We hope to have shown that anaphoric dependencies pose a major problem for reasoning in terms of simpler alternatives.[33] Our suggestion is to rather account for inferences such as FC via a non-classical semantics for disjunction, which keeps track of anaphoric information introduced by the initial disjunct. We developed such an account within the framework of BUS, since (a) BUS provides

33 Buccola & Chemla 2019 come to a similar conclusion on the basis of examples such as the following:

(133) Every dad λ_x called [his$_x$ daughter]^y or her$_y$ dog.

They observe that it's difficult to account for the inference that *not every dad called his daughter's dog*, via reasoning about simpler alternatives.

a well-motivated account of bathroom disjunctions, and (b) BUS is sufficiently expressive to state a simple, semantic account of FC, which we modified in order to keep track of anaphoric information.

In the future, we hope to see competing accounts of FC with anaphora emerge, against which our BUS account can be compared. One perhaps dissatisfying aspect of our account is the sense in which the FC inference is lexicalized in the semantics of disjunction. One potential avenue for future research is figuring out how to make the entry we posited for modal disjunction follow from more general pragmatic or cognitive principles (see, e.g., Aloni 2022 for such an attempt). It seems to us that anaphoric dependencies constitute a domain in which theories of inferences at the borderlands between semantics and pragmatics can be tested to their limit.

References

Aloni, Maria. 1997. Quantification in dynamic semantics. *Amsterdam Colloquium* 11. 73–78.

Aloni, Maria. 2003. Free choice in modal contexts. *Sinn und Bedeutung (SuB)* 7. 25–37. https://doi.org/10.18148/SUB/2003.V7I0.789.

Aloni, Maria. 2007. Free choice, modals, and imperatives. *Natural Language Semantics* 15(1). 65–94. https://doi.org/10.1007/s11050-007-9010-2.

Aloni, Maria. 2022. Logic and conversation: The case of free choice. *Semantics and Pragmatics* 15(5). 1–60. https://doi.org/10.3765/sp.15.5.

Aloni, Maria. 2023. Neglect-zero effects in dynamic semantics. In *Dynamics in logic and language: Third Tsinghua interdisciplinary workshop on Logic, Language, and Meaning, TLLM 2022* (Lecture Notes in Computer Science (LNCS) 13524). Cham: Springer. https://doi.org/10.1007/978-3-031-25894-7_1.

Alonso-Ovalle, Luis. 2005. Distributing the disjuncts over the modal space. *North East Linguistic Society (NELS) 35*. 75–86.

Alonso-Ovalle, Luis. 2009. Counterfactuals, correlatives, and disjunction. *Linguistics and Philosophy* 32(2). 207–244. https://doi.org/10.1007/s10988-009-9059-0.

Bar-Lev, Moshe E. 2018. *Free choice, homogeneity, and innocent inclusion*. The Hebrew University of Jerusalem dissertation. https://semanticsarchive.net/Archive/2JiYWEyM/Bar-Lev%202018%20diss.pdf.

Bar-Lev, Moshe E. & Danny Fox. 2020. Free choice, simplification, and Innocent Inclusion. *Natural Language Semantics* 28(3). 175–223. https://doi.org/10.1007/s11050-020-09162-y.

Barker, Chris. 1996. Presuppositions for proportional quantifiers. *Natural Language Semantics* 4(3). 237–259. https://doi.org/10.1007/BF00372821.

Beaver, David & Emiel Krahmer. 2001. A partial account of presupposition projection. *Journal of Logic, Language and Information* 10(2). 147–182. https://doi.org/10.1023/A:1008371413822.

van den Berg, Martin H. 1996. *Some aspects of the internal structure of discourse: The dynamics of nominal anaphora*. University of Amsterdam, ILLC dissertation. https://eprints.illc.uva.nl/id/eprint/1996.

Brasoveanu, Adrian. 2007. *Structured nominal and modal reference*. New Brunswick, NJ: Rutgers University dissertation. https://ling.rutgers.edu/images/dissertations/diss_brasoveanu_2007.pdf.

Breheny, Richard, Nathan Klinedinst, Jacopo Romoli & Yasutada Sudo. 2018. The symmetry problem: Current theories and prospects. *Natural Language Semantics* 26(2). 85–110. https://doi.org/10.1007/s11050-017-9141-z.

Buccola, Brian & Emmanuel Chemla. 2019. Alternatives of disjunctions: When a disjunct contains the antecedent of a pronoun. *Snippets* (37). 16–18. https://doi.org/10.7358/snip-2019-037-buch.

Champollion, Lucas, Dylan Bumford & Robert Henderson. 2019. Donkeys under discussion. *Semantics and Pragmatics* 12(1). 1–51. https://doi.org/10.3765/sp.12.1.

Charlow, Simon. 2014. *On the semantics of exceptional scope*. New York University dissertation. https://semanticsarchive.net/Archive/2JmMWRjY/charlow-semantics-exceptional-scope-diss.pdf.

Charlow, Simon. 2023. Static and dynamic exceptional scope. In press at Journal of Semantics. Rutgers University. https://ling.auf.net/lingbuzz/004650.

Chierchia, Gennaro. 1995. *Dynamics of meaning: Anaphora, presupposition, and the theory of grammar*. University of Chicago Press.

Crnič, Luka, Emmanuel Chemla & Danny Fox. 2015. Scalar implicatures of embedded disjunction. *Natural Language Semantics* 23(4). 271–305. https://doi.org/10.1007/s11050-015-9116-x.

Del Pinal, Guillermo, Itai Bassi & Uli Sauerland. 2024. Free choice and presuppositional exhaustification. *Semantics and Pragmatics* 17(3). 1–52. https://doi.org/10.3765/sp.17.3.

Egli, Urs. 1979. The Stoic concept of anaphora. In Rainer Bäuerle, Urs Egli & Arnim von Stechow (eds.), *Semantics from different points of view*, 266–283. Springer. https://doi.org/10.1007/978-3-642-67458-7_18.

Elbourne, Paul. 2002. *Situations and individuals*. Cambridge, MA: Massachussetts Institute of Technology dissertation. http://dspace.mit.edu/handle/1721.1/8157.

Elbourne, Paul. 2013. *Definite descriptions* (Oxford Studies in Semantics and Pragmatics 1). Oxford University Press. https://doi.org/10.1093/acprof:oso/9780199660193.001.0001.

Elliott, Patrick D. 2020. Towards a principled logic of anaphora. Massachusetts Institute of Technology. Unpublished manuscript. https://ling.auf.net/lingbuzz/005562.

Elliott, Patrick D. 2022. Partee conjunctions: Projection and possibility. Massachusetts Institute of Technology/Heinrich-Heine University Düsseldorf. Unpublished manuscript. https://ling.auf.net/lingbuzz/006857.

Elliott, Patrick D. 2023. Disjunction in a predictive theory of anaphora. In Dun Deng, Mingming Liu, Dag Westerståhl & Kaibo Xie (eds.), *Dynamics in logic and language: Third Tsinghua interdisciplinary workshop on Logic, Language, and Meaning, TLLM 2022* (Lecture Notes in Computer Science (LNCS) 13524), 76–98. Cham: Springer. https://doi.org/10.1007/978-3-031-25894-7_4.

Elliott, Patrick D. 2024. Donkey disjunctions and overlapping updates. *Semantics and Linguistic Theory (SALT)* 33. 666–685. https://doi.org/10.3765/v5jhc728.

Evans, Gareth. 1977. Pronouns, quantifiers, and relative clauses (I). *Canadian Journal of Philosophy* 7(3). 467–536. http://www.jstor.org/stable/40230703.

Fine, Kit. 1975. Review of *Counterfactuals* by D. Lewis. *Mind* 84(335). 451–458. http://www.jstor.org/stable/2253565.

Fox, Danny. 2007. Free choice and the theory of scalar implicatures. In Uli Sauerland & Penka Stateva (eds.), *Presupposition and implicature in compositional semantics*, 71–120. London: Palgrave Macmillan UK. https://doi.org/10.1057/9780230210752_4.

Fox, Danny & Roni Katzir. 2011. On the characterization of alternatives. *Natural Language Semantics* 19(1). 87–107. https://doi.org/10.1007/s11050-010-9065-3.

Franke, Michael. 2009. *Signal to act: Game Theory in pragmatics*. Amsterdam: Universiteit van Amsterdam dissertation. https://eprints.illc.uva.nl/id/eprint/2081/1/DS-2009-11.text.pdf.

Geurts, Bart. 2002. Donkey business. *Linguistics and Philosophy* 25(2). 129–156. https://doi.org/10.1023/A:1014624331450.

Geurts, Bart. 2010. *Quantity implicatures*. 1st edn. Cambridge University Press. https://doi.org/10.1017/CBO9780511975158.

Goldstein, Simon. 2019. Free choice and homogeneity. *Semantics and Pragmatics* 12(23). 1–53. https://doi.org/10.3765/sp.12.23.

Gotham, Matthew. 2019. Double negation, excluded middle and accessibility in dynamic semanttics. In *Amsterdam Colloquium 22*, 142–151. https://archive.illc.uva.nl/AC/AC2019/uploaded_files/inlineitem/Gotham_Double_negation_excluded_middle_and_accessib.pdf.

Groenendijk, Jeroen & Martin Stokhof. 1991. Dynamic predicate logic. *Linguistics and Philosophy* 14(1). 39–100. http://www.jstor.org/stable/25001418.

Groenendijk, Jeroen a. G., Martin J. B. Stokhof & Frank J. M. M. Veltman. 1996. Coreference and modality. In Shalom Lappin (ed.), *The handbook of contemporary semantic theory* (Blackwell Handbooks in Linguistics), 176–216. Oxford: Blackwell. https://dare.uva.nl/search?identifier=c655089e-9fae-4842-90da-30e46f37825b.

Heim, Irene. 1982. *The semantics of definite and indefinite noun phrases*. University of Massachusetts Amherst dissertation. Republished as Heim 2011. https://scholarworks.umass.edu/dissertations/AAI8229562.

Heim, Irene. 1983. File change semantics and the familiarity theory of definiteness. In Rainer Bäuerle, Christoph Schwarze & Arnim von Stechow (eds.), *Meaning, use, and interpretation of language*, 164–189. Berlin: De Gruyter. https://doi.org/10.1515/9783110852820.164.

Heim, Irene. 1990. E-type pronouns and donkey anaphora. *Linguistics and Philosophy* 13(2). 137–177. https://doi.org/10.1007/BF00630732.

Heim, Irene. 2011. *The semantics of definite and indefinite noun phrases*. https://semanticsarchive.net/Archive/jA2YTJmN/Heim%20Dissertation%20with%20Hyperlinks.pdf. Republished version of Heim 1982 typeset by Anders Shoubye and Ephraim Glick.

Hofmann, Lisa. 2019. The anaphoric potential of indefinites under negation and disjunction. *Amsterdam Colloquium* 22. 181–190. https://archive.illc.uva.nl/AC/AC2019/uploaded_files/inlineitem/Hofmann_The_anaphoric_potential_of_indefinites_unde.pdf.

Hofmann, Lisa. 2022. *Anaphora and negation*. University of California Santa Cruz dissertation. https://escholarship.org/uc/item/3sg098n5.

Kamp, Hans. 1973. Free choice permission. *Proceedings of the Aristotelian Society* 74. 57–74. http://www.jstor.org/stable/4544849.

Kanazawa, Makoto. 1994. Weak vs. strong readings of donkey sentences and monotonicity inference in a dynamic setting. *Linguistics and Philosophy* 17(2). 109–158. http://www.jstor.org/stable/25001541.

Karttunen, Lauri. 1973. Presuppositions of compound sentences. *Linguistic Inquiry* 4(2). 169–193. http://www.jstor.org/stable/4177763.

Karttunen, Lauri. 1976. Discourse referents. In James D. McCawley (ed.), *Notes from the linguistic underground* (Syntax and Semantics 7), 363–385. Leiden: Brill. https://doi.org/10.1163/9789004368859_021.

Katzir, Roni. 2008. Structurally-defined alternatives. *Linguistics and Philosophy* 30(6). 669–690. https://doi.org/10.1007/s10988-008-9029-y.

Klinedinst, Nathan. 2007. *Plurality and possibility*. University of California Los Angeles dissertation. https://linguistics.ucla.edu/general/dissertations/Klinedinst.2007.pdf.

Klinedinst, Nathan. 2009. (Simplification of) 2 disjunctive antecedents. In Paul Égré & Giorgio Magri (eds.), *Presuppositions and implicatures: Proceedings of the MIT-Paris workshop* (MIT Working Papers in Linguistics 60). Cambridge, MA: MITWPL.

Klinedinst, Nathan & Daniel Rothschild. 2012. Connectives without truth tables. *Natural Language Semantics* 20(2). 137–175. http://www.jstor.org/stable/43550301.

Krahmer, Emiel & Reinhard Muskens. 1995. Negation and disjunction in Discourse Representation Theory. *Journal of Semantics* 12(4). 357–376. https://doi.org/10.1093/jos/12.4.357.

Kratzer, Angelika & Junko Shimoyama. 2002. Indeterminate pronouns: The view from Japanese. *The 3rd Tokyo Conference on Psycholinguistics*. 1–25. A revised version was published in Kratzer & Shimoyama 2017.

Kratzer, Angelika & Junko Shimoyama. 2017. Indeterminate pronouns: The view from Japanese. In Chungmin Lee, Ferenc Kiefer & Manfred Krifka (eds.), *Contrastiveness in information structure, alternatives and scalar implicatures* (Studies in Natural Language and Linguistic Theory (SNLT) 91), 123–143. Cham, Switzerland: Springer. https://doi.org/10.1007/978-3-319-10106-4_7.

Krifka, Manfred. 1996. Pragmatic strengthening in plural predications and donkey sentences. *Semantics and Linguistic Theory (SALT)* 6. 136–153. https://doi.org/10.3765/salt.v6i0.2769.

Mandelkern, Matthew & Daniel Rothschild. 2020. Definiteness projection. *Natural Language Semantics* 28(2). 77–109. https://doi.org/10.1007/s11050-019-09159-2.

Marty, Paul, Jacopo Romoli, Yasutada Sudo & Richard Breheny. 2021. Negative free choice. *Semantics and Pragmatics* 14(13). 1–54. https://doi.org/10.3765/sp.14.13.

Meyer, Marie-Christine. 2016. Generalized free choice and missing alternatives. *Journal of Semantics* 33(4). 703–754. https://doi.org/10.1093/jos/ffv010.

Nouwen, Rick W. F. 2003. *Plural pronominal anaphora in context: Dynamic aspects of quantification*. Universiteit Utrecht dissertation. https://www.lotpubliions.nl/Documents/084_fulltext.pdf.

Nute, Donald. 1975. Counterfactuals and the similarity of words. *The Journal of Philosophy* 72(21). 773–778. https://doi.org/10.2307/2025340.

Roberts, Craige. 1989. Modal subordination and pronominal anaphora in discourse. *Linguistics and Philosophy* 12(6). 683–721. http://www.jstor.org/stable/25001367.

Romoli, Jacopo. 2013. A problem for the structural characterization of alternatives. *Snippets* (27). 14–15. https://doi.org/10.7358/snip-2013-027-romo.

Rooth, Mats. 1987. Noun Phrase interpretation in Montague grammar, file change semantics, and situation semantics. In Peter Gärdenfors (ed.), *Generalized quantifiers* (Studies in Linguistics and Philosophy (SLAP) 31), 237–268. Dordrecht: Springer. https://doi.org/10.1007/978-94-009-3381-1_9.

Rothschild, Daniel & Matthew Mandelkern. 2017. Dynamic semantics and pragmatic alternatives. Lecture notes from a course taught at ESSLLI 2017. http://danielrothschild.com/esslli2017/.

Rothschild, Daniel & Seth Yalcin. 2017. On the dynamics of conversation. *Noûs* 51(1). 24–48. https://doi.org/10.1111/nous.12121.

Schubert, Lenhart K. & Francis Jeffry Pelletier. 1989. Generically speaking, or, using Discourse Representation Theory to interpret generics. In Gennaro Chierchia, Barbara H. Partee & Raymond Turner (eds.). Red. by Gennaro Chierchia, Pauline Jacobson & Francis J. Pelletier, *Properties, types and meaning* (Studies in Linguistics and Philosophy (SLAP) 39), 193–268. Dordrecht: Springer. https://doi.org/10.1007/978-94-009-2723-0_6.

Simons, Mandy. 2005. Dividing things up: The semantics of or and the modal/*or* interaction. *Natural Language Semantics* 13(3). 271–316. http://www.jstor.org/stable/23749607.

Singh, Raj. 2008. *Modularity and locality in interpretation*. Cambridge, MA: Massachusetts Institute of Technology Thesis. https://doi.org/1721.1/45895.

Stalnaker, Robert. 1976. Propositions. In Alfred F. MacKay & Daniel D. Merrill (eds.), *Issues in the philosophy of language: Proceedings of the 1972 Oberlin colloquium in philosophy*, 79–91. New Haven: Yale University Press.

Trinh, Tue & Andreas Haida. 2015. Constraining the derivation of alternatives. *Natural Language Semantics* 23(4). 249–270. https://doi.org/10.1007/s11050-015-9115-y.

Van Rooij, Robert. 2006. Free choice counterfactual donkeys. *Journal of Semantics* 23(4). 383–402. https://doi.org/10.1093/jos/ffl004.

Veltman, Frank. 1996. Defaults in Update Semantics. *Journal of Philosophical Logic* 25(3). 221–261. http://www.jstor.org/stable/30226572.

Webber, Bonnie, Matthew Stone, Aravind Joshi & Alistair Knott. 2003. Anaphora and discourse structure. *Computational Linguistics* 29(4). 545–587. https://doi.org/10.1162/089120103322753347.

Willer, Malte. 2017. Widening free choice. *Amsterdam Colloquium* 21. 511–520. https://semanticsarchive.net/Archive/jZiM2FhZ/AC2017-Proceedings.pdf.

Willer, Malte. 2018. Simplifying with free choice. *Topoi* 37(3). 379–392. https://doi.org/10.1007/s11245-016-9437-5.

Willer, Malte. 2019. Negating conditionals in bilateral semantics. In vol. 22, 181–190. https://archive.illc.uva.nl/AC/AC2019/uploaded_files/inlineitem/Willer_Negating_conditionals_in_bilateral_semantics.pdf.

Zimmermann, Thomas Ede. 2000. Free choice disjunction and epistemic possibility. *Natural Language Semantics* 8(4). 255–290. https://doi.org/10.1023/A:1011255819284.

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