# Optimizing Inventory Decisions Under Advertisement and CRM-Driven Demand Using a Fuzzy Multi-Criteria Framework: A Comprehensive Review

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**Abstract**
In the rapidly evolving landscape of supply chain and operations management, optimizing inventory decisions is no longer a linear cost-minimization problem. Businesses must balance economic efficiency with customer satisfaction, dynamic market trends, and environmental concerns. This review explores how integrated inventory models that factor in advertisement frequency and Customer Relationship Management (CRM) costs can dynamically drive demand across a product's life cycle (PLC). To address imprecision in demand estimation and cost parameters, we propose a fuzzy logic framework utilizing Triangular Fermatean Fuzzy Numbers (TFFNs). The review also provides a comprehensive literature synthesis, identifies current research gaps, and proposes future directions involving artificial intelligence (AI), sustainability metrics, and multi-objective optimization.
**Keywords: up to; Inventory Optimization, Advertisement-Driven Demand, CRM, Product Life Cycle, Sustainability, Fuzzy Logic, TFFNs, MCDM.**

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## Introduction

Modern inventory management is a multidimensional challenge shaped by dynamic demand, fluctuating supply chain conditions, and shifting consumer expectations. Traditionally, inventory models focused on minimizing operational costs. However, these models have evolved to include external demand influencers such as advertising intensity and CRM investments—tools that shape customer behavior and purchasing patterns.

The integration of customer-centric mechanisms like appreciation periods and loyalty drivers has transformed how we understand "demand." Appreciation refers to a consumer's growing recognition and preference for a product, amplified through sustained advertising and relationship-based marketing. For example, in technology or fashion sectors, early exposure to ads and consistent CRM touchpoints build familiarity, encouraging future purchases.

Inventory decisions are no longer just about balancing order quantity and holding costs they now must reflect responsiveness to real-time market feedback, behavioral economics, and sustainability pressures. The ongoing digital transformation has introduced new variables into the inventory equation, such as dynamic pricing algorithms, AI-based CRM systems, and sentiment-based demand forecasting. These factors drive the need for more flexible and robust inventory models that adapt to fuzzy, real-world data.

Recent advances in fuzzy logic have enabled the incorporation of imprecision and ambiguity in input parameters like customer satisfaction, advertisement responsiveness, and environmental impact. Among these, Triangular Fermatean Fuzzy Numbers (TFFNs) provide a superior approach for modeling uncertainties that involve high degrees of hesitancy, making them particularly well-suited to inventory contexts influenced by human preferences and non-quantifiable inputs.

Furthermore, contemporary markets emphasize not just profitability but also sustainability and customer-centricity. Businesses are expected to reduce waste, optimize resources, and maintain high service levels while accounting for carbon footprints and ethical considerations. Therefore, it is imperative to explore integrated inventory models that jointly consider advertising, CRM, sustainability, and fuzzy uncertainty.

This paper reviews the progression from classical economic order quantity (EOQ) models to fuzzy, CRM-aware inventory models. It argues for embedding multi-dimensional uncertainty via TFFNs and proposes a conceptual framework uniting advertisement, CRM, and sustainability metrics.

## 2. Demand Drivers in Inventory Models: Role of Advertisement and CRM

**2.1 Advertisement as a Dynamic Influencer** Advertisement not only creates awareness but also builds product image and drives trial purchases. Across the PLC, its impact varies—being most effective during introduction and growth phases. Empirical models (Manna et al., 2017) show that demand positively correlates with ad frequency but exhibit saturation effects, where incremental investment yields declining marginal returns.

$D(t) = \alpha(t) \times A(t)^{0.5}$

Where A(t) is the time-dependent advertising cost, and α(t) reflects demand sensitivity.

**2.2 CRM as a Relationship-Driven Influencer** CRM builds trust and enhances long-term engagement. Lemon & Verhoef (2016) highlight that CRM's effectiveness varies across the consumer journey—email campaigns post-purchase may drive repeat orders, while loyalty rewards increase lifetime value. CRM investment also impacts demand. For instance:

$D(t) = \beta(t) \times C(t)^{0.3}$

Where C(t) is CRM cost, and β(t) is CRM responsiveness.

**2.3 Integrated Demand Function** Demand is driven by a combination of A(t), C(t), and fuzzy environmental variables:

$D(t) = \alpha(t) \times A(t)^{0.5} + \beta(t) \times C(t)^{0.3} + \varepsilon(t)$

Where ε(t) captures uncertainty modeled by TFFNs.

## 3. Fuzzy Logic for Modeling Uncertainty

**3.1 Why Use Fuzzy Logic?** Inventory parameters such as holding cost, deterioration rate, or consumer preferences are not deterministic. Fuzzy sets allow incorporation of linguistic variables and expert estimations. Traditional fuzzy sets evolved into Intuitionistic and Fermatean fuzzy sets. TFFNs further enhance expressiveness by supporting high membership and non-membership degrees simultaneously.

**3.2 Application of TFFNs in Inventory Systems** TFFNs can model:

- Advertising effectiveness (subjective)
- CRM cost vs. response (soft variable)
- Rework and return rates
- Set-up and holding costs

These numbers represent fuzzy values as (μ, ν, π) where μ is membership, ν is non-membership, and π is hesitancy.

## 4. Literature Review and Thematic Synthesis

Inventory systems have traditionally been developed under deterministic assumptions, which fail to reflect the real-world uncertainty and complexity of supply chain systems. To address this, various scholars have incorporated fuzzy logic and sustainability considerations into their models. Xu et al. (2013) applied game-theoretic approaches to model supply chain coordination under carbon emission trading, laying foundational work in integrating environmental policies into operational decisions. Zhang et al. (2014) extended this by designing carbon-neutral supply chain networks that account for carbon tax and subsidies, reflecting the growing demand for sustainable logistics.

Gautam et al. (2019) developed rework inventory models that considered advertisement as a dynamic factor influencing demand. This aligns with marketing-driven inventory systems that treat demand as elastic to promotional strategies. Wangsa et al. (2020) further emphasized the need to include transportation-related emissions in EOQ models using fuzzy demand estimations. Their work underscores the environmental externalities of traditional inventory policies and encourages the adoption of sustainable operational frameworks.

A major shift in literature is the inclusion of demand influencers such as advertisement and CRM in inventory modeling. Yadav et al. (2022) proposed a fuzzy inventory model for deteriorating items where demand was dependent on both price and advertising, integrated under a trade credit policy. Their use of fuzzy logic provided flexibility in handling uncertain market conditions. Similarly, Rabbani et al. (2021) introduced a robust-stochastic fuzzy model that emphasized customer satisfaction and delay cost minimization in a closed-loop supply chain, directly incorporating CRM factors into the decision-making framework.

Groening et al. (2014) emphasized that customer satisfaction can serve as a powerful demand signal, which should be factored into inventory models. This supports the inclusion of CRM activities, such as loyalty programs and post-purchase support, into cost and demand structures. Lemon and Verhoef (2016) expanded this understanding by presenting a comprehensive view of customer experience across the journey, noting that touchpoints such as CRM interventions significantly affect retention and repeat purchases.

On the technical side, significant advancements in fuzzy set theory have been made by Senapati and Yager (2021), who introduced Fermatean fuzzy sets as an extension to better model higher uncertainty and hesitancy in decision-making. Yazdi et al. (2020) also offered a practical framework using interval-valued intuitionistic fuzzy numbers in risk assessment, particularly relevant in uncertain supply chain environments.

Arora et al. (2018) and Manna et al. (2017) both developed EOQ-based fuzzy models that integrate promotional efforts and advertisement-dependent demand respectively, showing that marketing efforts and inventory strategies are intrinsically linked. These models exhibit how promotional cost can be optimized alongside ordering and holding costs, especially when modeled with fuzzy demand parameters.

Chen et al. (2023) introduced a reinforcement learning–fuzzy hybrid inventory model accounting for CRM-influenced demand, demonstrating adaptability to shifting customer behaviors. Li and Kumar (2024) developed a sustainability-embedded EOQ model incorporating social media advertisement intensity, revealing significant short-term demand spikes under viral campaigns. Jain and Basak (2025) implemented Triangular Fermatean Fuzzy Numbers (TFFNs) in multi-period CRM-influenced inventory systems, enhancing modeling precision for vague linguistic terms. Torres and Singh (2025) contributed a novel carbon-conscious demand function, where advertisement value was offset against carbon emissions—promoting green marketing decisions within EOQ logic.

Together, these studies reveal a strong academic movement toward the integration of sustainability, advertisement, and CRM into inventory management systems using fuzzy logic frameworks. However, most models treat these aspects in isolation. There remains a research gap in synthesizing all three—advertisement, CRM, and sustainability—under a unified fuzzy multi-criteria framework that captures their interdependence and real-world dynamics.

Despite these advances, the integration of green logistics, real-time AI responsiveness, and multi-criteria sustainability under fuzzy environments is still emerging, indicating significant research opportunities.

## 5. Numerical Illustration

Let us consider a scenario where a company's demand is influenced by advertisement spend (A), Customer Relationship Management (CRM) spend (C), and uncertain factors modeled using Triangular Fermatean Fuzzy Numbers (TFFNs).

Given:

• Advertisement spend $A(t) = 25$ (in $ thousands)

• CRM spend $C(t) = 10$

• TFFN parameters: $\mu = 0.7$, $\nu = 0.3$, $\pi = 0.2$

Integrated demand function:

$D(t) = 0.8 \times A(t)^{0.5} + 0.6 \times C(t)^{0.3} + \varepsilon(t)$

Step 1: Deterministic Component

$D_d = 0.8 \times \sqrt{25} + 0.6 \times 10^{0.3} = 0.8 \times 5 + 0.6 \times 1.995 \approx 5.20$

Step 2: Adjusted Fuzzy Demand

Adjusted Demand $= D_d \times (\mu - \nu + \pi)$

$= 5.20 \times (0.7 - 0.3 + 0.2) = 5.20 \times 0.6 = 3.12$

The adjusted fuzzy demand is approximately 3.12 units under moderate advertising and CRM efforts.

## 6. Graphical Illustration

### (i) Advertisement Spend vs Demand

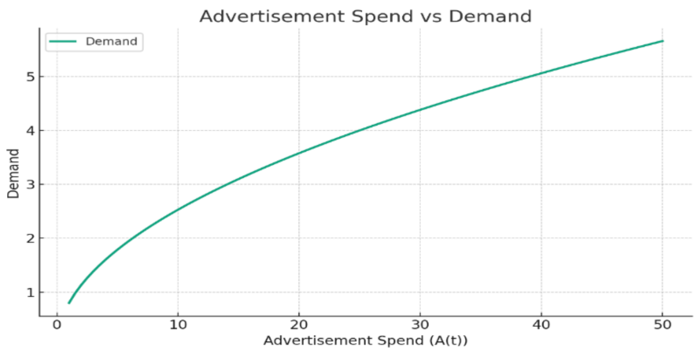


Figure 1: Advertisement Spend vs Demand

This figure 1 illustrates the nonlinear relationship between advertisement expenditure and demand. As advertisement spend increases, demand also increases but at a decreasing rate due to saturation. The demand function used is: D(t) = 0.8 × A(t)^0.5.

### (i) CRM Spend vs Fuzzy Demand

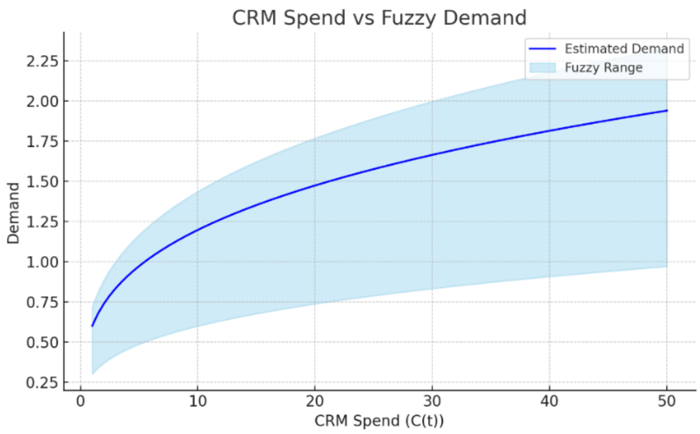


Figure 2: CRM Spend vs Fuzzy Demand

This figure 2 shows the influence of CRM expenditure on demand, incorporating fuzzy uncertainty. The shaded area represents the range of possible demand outcomes based on Triangular Fermatean Fuzzy Numbers (TFFNs). The function used is: D(t) = 0.6 × C(t)^0.3.

## 5. Conclusion

This review paper demonstrates the necessity of integrating advertisement and CRM factors into inventory decision-making frameworks. Modern supply chains operate in an environment filled with uncertainty, consumer behavior variability, and sustainability pressures. Traditional inventory models are inadequate in capturing these multidimensional aspects.

The application of Triangular Fermatean Fuzzy Numbers (TFFNs) enables the modeling of imprecise and conflicting data related to cost, demand, and external influences. When combined with demand-enhancing factors like advertising and CRM, businesses can more accurately forecast and control inventory. Our literature synthesis shows limited models address these themes jointly.

Therefore, future research must pursue hybrid and intelligent approaches that unify soft influences (like appreciation and CRM) with hard constraints (like cost, deterioration) under a flexible uncertainty-handling environment. Doing so promises to elevate supply chain performance, customer satisfaction, and environmental responsibility simultaneously.

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